Local assortativeness in scale-free networks

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Addendum

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Network assortativity is typically defined as

$$r=\frac{1}{\sigma_q^2}\left[\left(\sum_{jk} jke_{j,k}\right)-\mu_q^2\right], \tag{1}$$

where $e_{j,k}$ is the link distribution of the network, μ_q is the expectation of the excess degree distribution $q(k)$ and σ_q is the standard deviation of this distribution.

In the original paper the local assortativity of a given node v was introduced using the following formula:

$$\rho_v=\frac{\alpha_v-\beta_v}{\sigma_q^2}=\frac{(j+1)\left(j\overline{k}-\mu_q^2\right)}{2M\sigma_q^2}, \tag{2}$$

where j is the node's excess degree, $\overline{k}$ is the average excess degree of its neighbours, $\sigma_q \neq 0$. The contribution α_v of the node v to the first term in (1), *i.e.*, $\sum_{jk} jke_{j,k}$, and contribution β_v to the second term in (1), *i.e.*, μ_q^2, are

$$\alpha_v=(j+1)\,\frac{j\overline{k}}{2M}, \qquad \beta_v=(j+1)\,\frac{\mu_q^2}{2M}. \tag{3}$$

We subsequently found that this formulation has a bias which favours peripheral nodes over hubs, and provide here a better measure (5) which should replace expression (2) in the original paper. The new derivation is summarised below. While the component α_v captures the precise contribution of each node to the term $\sum_{jk} jke_{j,k}$, the component β_v represents the contribution of each node to the term μ_q^2 with an imprecise scaling. Specifically, the scaling factor $(j+1)/2M$ in (3) is the correct scaling factor for μ_q, rather than μ_q^2, and hence, β_v has a bias towards peripheral nodes. The unbiased contribution instead is given by:

$$\hat{\beta}_v=(j+1)\frac{j\mu_q}{2M}. \tag{4}$$

Hence, local assortativity is

$$\hat{\rho}_v=\frac{\alpha_v-\hat{\beta}_v}{\sigma_q^2}=\frac{j\,(j+1)\left(\overline{k}-\mu_q\right)}{2M\sigma_q^2}. \tag{5}$$

If the neighbours' average $\overline{k}$ is higher than the global average μ_q, then the node is assortative. Otherwise, the node is disassortative. Hence, the local assortativity is a scaled difference between the average excess degree of the node's neighbours and the global average excess degree.

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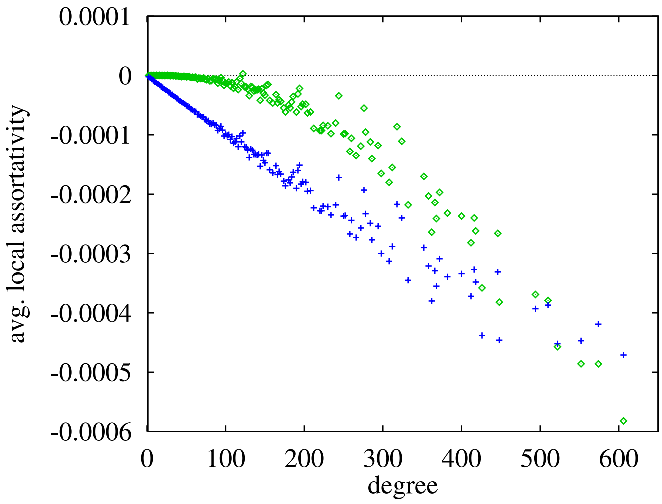

Fig. 1: (Colour on-line) Internet Autonomous System level, August, 2008. Local assortativity profiles ρ (+) and $\hat{\rho}$ ($\Diamond$).

The difference between $\rho(d)$, defined by (2), and $\hat{\rho}(d)$, defined by (5), is illustrated for Internet Autonomous Systems level (August 2008) in fig. 1. Clearly, $\hat{\rho}(d) > \rho(d)$ for nodes with smaller d, and $\hat{\rho}(d) < \rho(d)$ for the hubs.

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