Part (d)
For AL data
The AR(1) model summary is:

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 30 |
| **Model:** | AutoReg(1) | **Log Likelihood** | -311.642 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 11241.209 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 18.862 |
| **Time:** | 15:54:39 | **BIC** | 19.003 |
| **Sample:** | 1 | **HQIC** | 18.906 |
| | 30 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 1.188e+04 | 3181.643 | 3.733 | 0.000 | 5640.480 | 1.81e+04 |
| **y.L1** | 0.0183 | 0.185 | 0.099 | 0.921 | -0.344 | 0.380 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | 54.6006 | +0.0000j | 54.6006 | 0.0000 |

```
The predictions results are: MAPE:  15.282073921372355
```

For AR(3) model

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 30 |
| **Model:** | AutoReg(3) | **Log Likelihood** | -290.252 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 11284.045 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 19.033 |
| **Time:** | 16:04:39 | **BIC** | 19.273 |
| **Sample:** | 3 | **HQIC** | 19.104 |
| | 30 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 1.324e+04 | 4940.127 | 2.681 | 0.007 | 3560.635 | 2.29e+04 |
| **y.L1** | 0.0338 | 0.192 | 0.176 | 0.860 | -0.342 | 0.409 |
| **y.L2** | 0.0864 | 0.194 | 0.445 | 0.656 | -0.294 | 0.467 |
| **y.L3** | -0.1560 | 0.192 | -0.811 | 0.417 | -0.533 | 0.221 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | -1.7253 | -0.0000j | 1.7253 | -0.5000 |
| **AR.2** | 1.1398 | -1.5547j | 1.9277 | -0.1493 |
| **AR.3** | 1.1398 | +1.5547j | 1.9277 | 0.1493 |

```
The prediction result is: MAPE:  15.282073921372355
```

The AR(5) model summary is:

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 23 |
| **Model:** | AutoReg(5) | **Log Likelihood** | -191.373 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 10025.838 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 19.204 |
| **Time:** | 15:55:47 | **BIC** | 19.550 |
| **Sample:** | 5 | **HQIC** | 19.251 |
| | 23 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 3.68e+04 | 1.09e+04 | 3.376 | 0.001 | 1.54e+04 | 5.82e+04 |
| **y.L1** | -0.3824 | 0.229 | -1.673 | 0.094 | -0.830 | 0.066 |
| **y.L2** | -0.0956 | 0.224 | -0.427 | 0.669 | -0.534 | 0.343 |
| **y.L3** | -0.3502 | 0.193 | -1.816 | 0.069 | -0.728 | 0.028 |
| **y.L4** | -0.5581 | 0.210 | -2.656 | 0.008 | -0.970 | -0.146 |
| **y.L5** | -0.0937 | 0.236 | -0.397 | 0.692 | -0.557 | 0.369 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | 0.6748 | -0.8961j | 1.1218 | -0.1473 |
| **AR.2** | 0.6748 | +0.8961j | 1.1218 | 0.1473 |
| **AR.3** | -1.0181 | -0.7569j | 1.2686 | -0.3983 |
| **AR.4** | -1.0181 | +0.7569j | 1.2686 | 0.3983 |
| **AR.5** | -5.2706 | -0.0000j | 5.2706 | -0.5000 |

```
The prediction result is: MAPE:  7.009200685746833
The MAPE for EWMA(0.5) is MAPE:  5.0218770273748845
The MAPE for EWMA(0.8) is MAPE:  7.411584609108143
```

For AL data:
The AR(1) model summary is:

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 30 |
| **Model:** | AutoReg(1) | **Log Likelihood** | -303.843 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 8590.540 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 18.324 |
| **Time:** | 15:48:26 | **BIC** | 18.465 |
| **Sample:** | 1 | **HQIC** | 18.368 |
| | 30 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 1.626e+04 | 4761.136 | 3.415 | 0.001 | 6925.299 | 2.56e+04 |
| **y.L1** | 0.3941 | 0.155 | 2.547 | 0.011 | 0.091 | 0.697 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | 2.5375 | +0.0000j | 2.5375 | 0.0000 |

```
The predictions results are: MAPE = 0.3429774876856654
```

The AR(3) model summary is:

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 30 |
| **Model:** | AutoReg(3) | **Log Likelihood** | -279.649 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 7619.138 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 18.247 |
| **Time:** | 15:51:03 | **BIC** | 18.487 |
| **Sample:** | 3 | **HQIC** | 18.319 |
| | 30 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 2.389e+04 | 7091.236 | 3.369 | 0.001 | 9990.784 | 3.78e+04 |
| **y.L1** | 0.5166 | 0.193 | 2.684 | 0.007 | 0.139 | 0.894 |
| **y.L2** | -0.2457 | 0.200 | -1.229 | 0.219 | -0.638 | 0.146 |
| **y.L3** | -0.1185 | 0.174 | -0.683 | 0.495 | -0.459 | 0.222 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | 0.8649 | -1.2128j | 1.4896 | -0.1514 |
| **AR.2** | 0.8649 | +1.2128j | 1.4896 | 0.1514 |
| **AR.3** | -3.8028 | -0.0000j | 3.8028 | -0.5000 |

```
The predictions results are: MAPE = 0.3429774876856654
```

The AR(5) model summary is:

AutoReg Model Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | y | **No. Observations:** | 23 |
| **Model:** | AutoReg(5) | **Log Likelihood** | -185.128 |
| **Method:** | Conditional MLE | **S.D. of innovations** | 7086.682 |
| **Date:** | Tue, 17 May 2022 | **AIC** | 18.510 |
| **Time:** | 15:50:51 | **BIC** | 18.856 |
| **Sample:** | 5 | **HQIC** | 18.557 |
| | 23 | | |

| | coef | std err | z | P>\|z\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **intercept** | 5.047e+04 | 1.57e+04 | 3.211 | 0.001 | 1.97e+04 | 8.13e+04 |
| **y.L1** | 0.2061 | 0.230 | 0.895 | 0.371 | -0.245 | 0.657 |
| **y.L2** | -0.4323 | 0.227 | -1.904 | 0.057 | -0.877 | 0.013 |
| **y.L3** | -0.1043 | 0.248 | -0.420 | 0.674 | -0.591 | 0.382 |
| **y.L4** | -0.1821 | 0.218 | -0.836 | 0.403 | -0.609 | 0.245 |
| **y.L5** | -0.1250 | 0.205 | -0.608 | 0.543 | -0.528 | 0.278 |

Roots

| | Real | Imaginary | Modulus | Frequency |
|---|---|---|---|---|
| **AR.1** | 0.7895 | -0.9747j | 1.2544 | -0.1416 |
| **AR.2** | 0.7895 | +0.9747j | 1.2544 | 0.1416 |
| **AR.3** | -0.3984 | -1.4534j | 1.5070 | -0.2926 |
| **AR.4** | -0.3984 | +1.4534j | 1.5070 | 0.2926 |
| **AR.5** | -2.2392 | -0.0000j | 2.2392 | -0.5000 |

```
The predictions results are: MAPE 0.37663267836330533
The MAPE for EWMA(0.5) is MAPE:  0.11738066271063642
The MAPE for EWMA(0.8) is MAPE:  0.15902183285036464
```

Part(e)
To perform the pair-T test: We need to first calculate the sample mean of AZ and AL respectively. The sample mean of the difference of two places is D_AZ – D_AL. Then we need to calculate the sample mean and variance of the difference D. With the statistics obtain, we can calculate the test statistics: D_mean/(D_se/np.sqrt(len(D))).

The test statistics is: T = - 3.777. Since |T| > critical value(2.002), we reject the hypothesis that they have the same mean value.