Drexel University

Office of the Dean of the College of Engineering

ENGR 232 – Dynamic Engineering Systems

Week 1 – Pre Lab

Study and review the week 1 lecture notes – example with equilibrium points, d-field, stability, and solutions in MATLAB

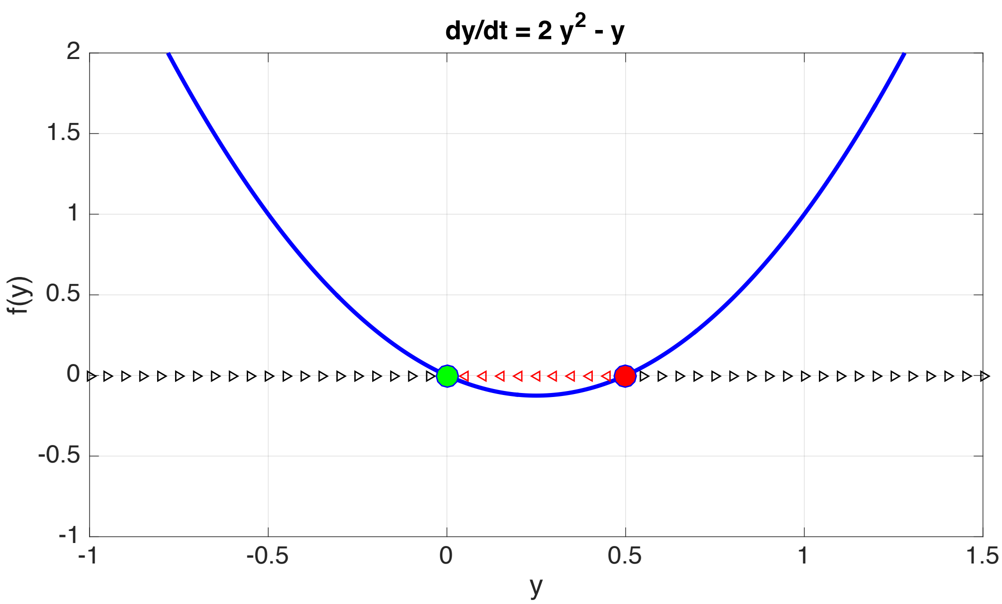
Stability of equilibrium points in first-order equations:

For first-order **autonomous** systems, the derivative does not change with time, so we can plot dy/dt vs. y and use this plot to determine the stability of the equilibrium points.

Review: Stable, unstable and semi-stable equilibrium points from ENGR 231.

Example: Consider the first-order, non-linear, autonomous differential equation given below.

$$\frac{dy}{dt} = 2y^2 - y$$



This is a graph of $\frac{dy}{dt}$ vs y. Here, we can clearly see the equilibrium points at 0 and $1/2$ which can be verified by setting $dy/dt = 0$ and solving for y. The equilibrium point 0 is stable (shown in green) and the equilibrium point ½ is unstable (shown in red).

Region of attraction of the stable point is: $(-\infty, 0.5)$ and the ROA for the unstable point is [0.5].

Direction Fields:

Given a first-order ordinary differential equation,

$$\frac{dy}{dt} = f(t, y)$$

we have an expression for the derivative of the variable for each value t and y. Recall that the derivative is the slope of the line tangent to the curve at that point. One can construct a figure on which the value of the derivative is used to draw an arrow of corresponding slope over a grid of points t, y.

Example: Consider the specific differential equation:

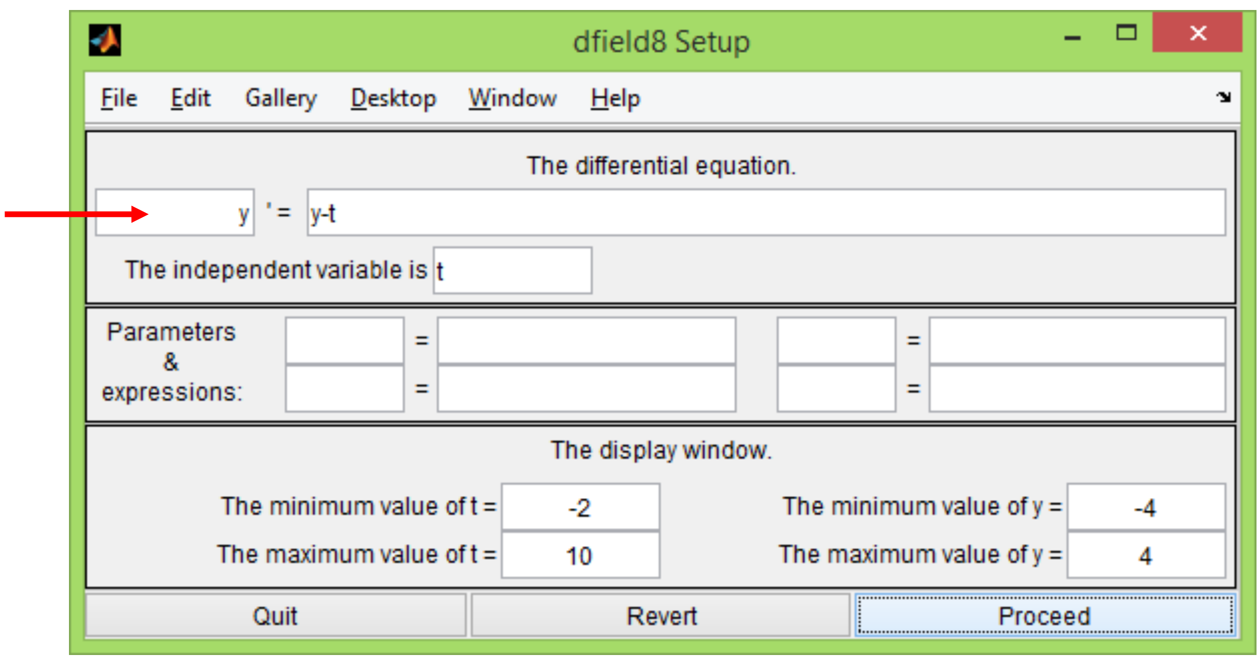
$$\frac{dy}{dt} = y - t$$

On a set of axes with t on the x-axis and y on the y-axis, the value of the derivative can be determined at each point by calculating dy/dt at that point. Representing this derivative as a directional arrow of slope = dy/dt yields a direction field plot.

We will not do this by hand, but rather using the Rice University's d-field tool.

You can download the m-file from the course website under the Software Folder.

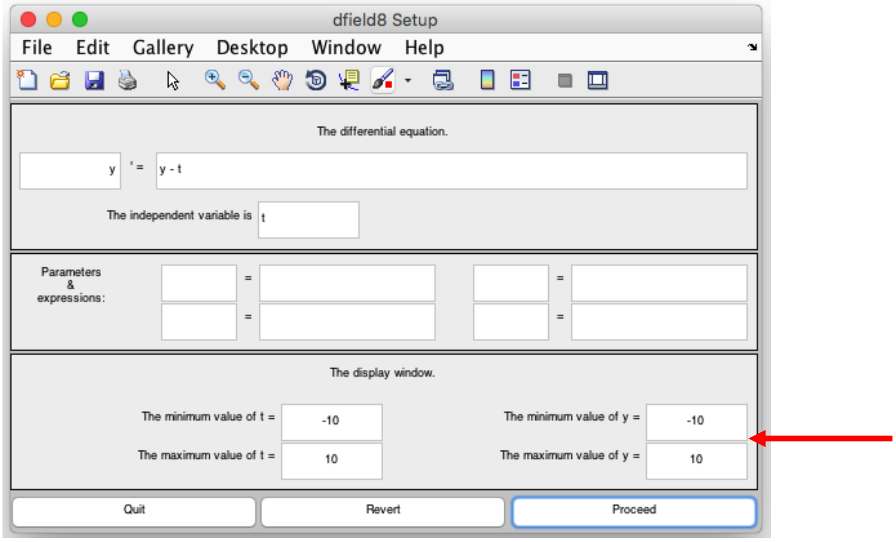
Look for the script file dfield8.m inside **Software** > **MATLAB versions of pplane8 and dfield8**



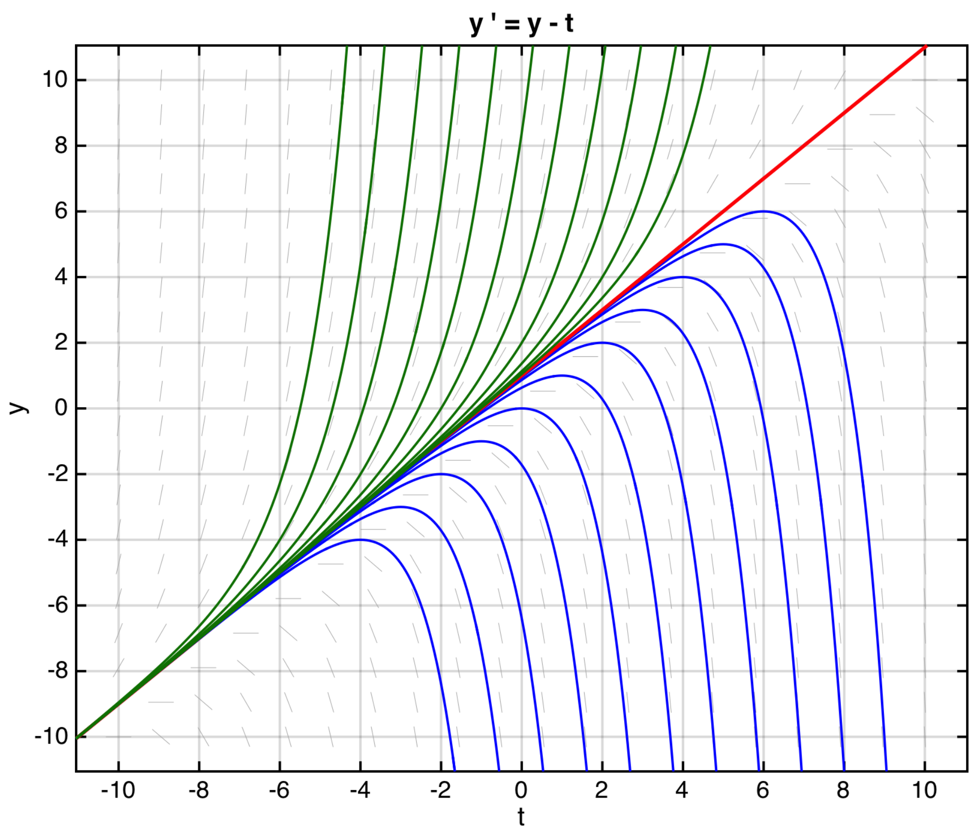
Note: You can type in the differential equation (MATLAB syntax needed) and set the time-scale and the limits for the y-axis.

The online applet uses arrows, the MATLAB version does not have arrows – we assume time is moving forward so all arrows point accordingly.

Now adjust both axes to go from -10 to +10.



Create a plot of selected solutions similar to that shown below.



The colors were created by selecting the **Property Editor** in MATLAB. The same editor allows you to specify line widths and the font size. You can see one of the solutions is a straight line (see red line above.) Be sure to include it amongst your **dfield8** solutions.

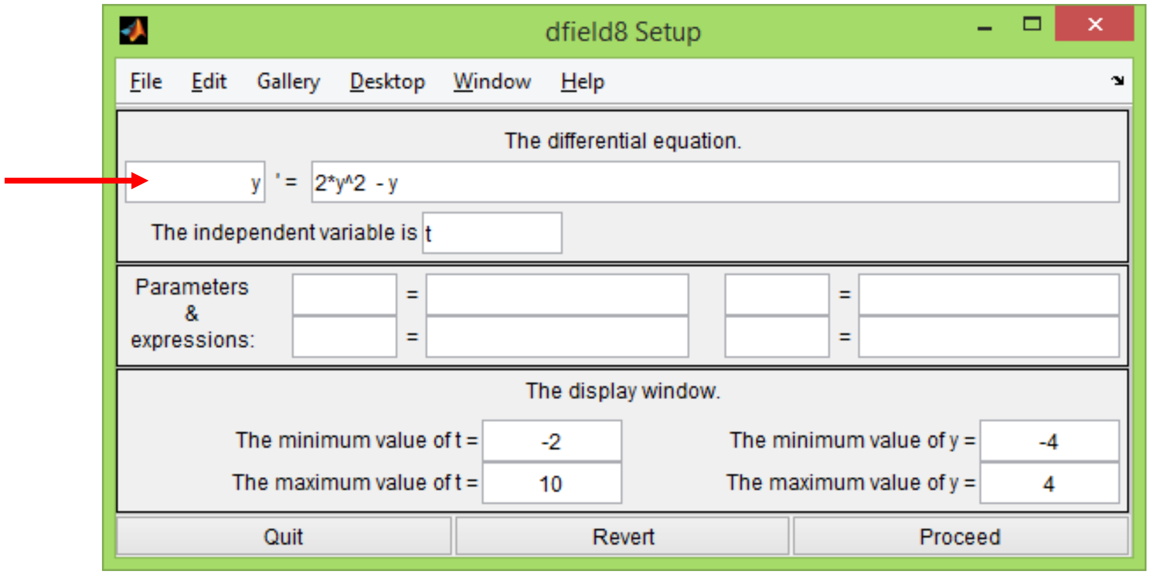
What is the equation for the red line?

a. $y = t$ **b.** $y = -t$ **c.** $y = t + 1$ **d.** $y = t + 1/2$

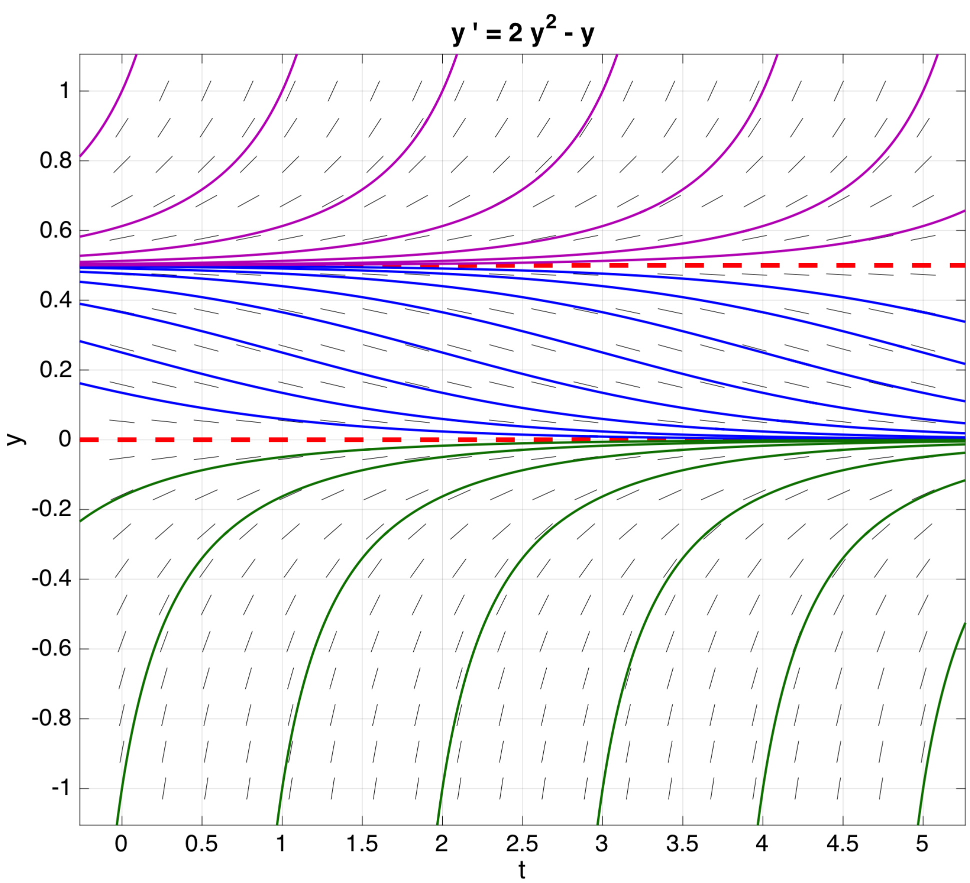
Previous Example: Consider again the non-linear, autonomous differential equation:

$$\frac{dy}{dt} = 2y^2 - y$$

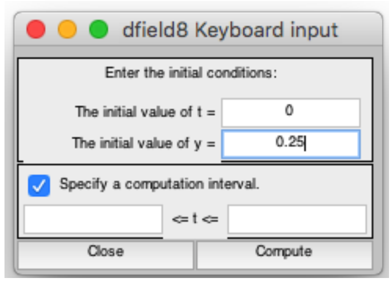
Enter the equation using MATLAB notation as seen below.



Produce a graph of the solutions similar to that shown below. Adjust the time axis to go from 0 to 5 and the *y* axis to go from –1 to +1. Color code each solution as shown using the Property Editor.



To obtain the perfectly symmetrical curves shown above, the **keyboard input** option was used.



Using keyboard input, or just by clicking on the graph, you are asking MATLAB to generate a solution trajectory which will correspond to a unique set of initial conditions. Notice how the lines all move away from the unstable critical point $y = 0.5$. The ROA of equilibrium point 0 is $(-\infty, 0.5)$ and so any lines that pass through this region will be attracted to the equilibrium point 0 – as seen.

Recognize that we have not solved the differential equation. We can see what the solution will look like simply based on knowing the function (derivative). This is a numerical solution and gives us quantitative values for a solution. While we cannot write down the equation for the solution based solely on this approach, we can certainly study the behavior of the system by analyzing the solution trajectories.