ENGR 232 – Dynamic Engineering Systems

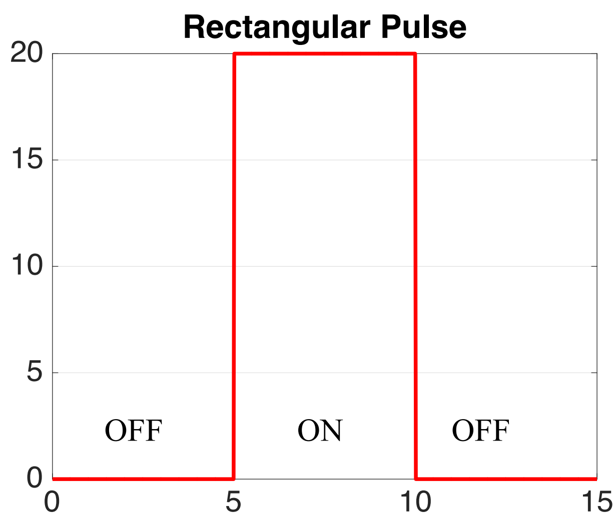
Lab 9: Fall 2016-2017

Here we study the matrix equation:

$$\textbf{DE}: \frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{B}\, f(t) \qquad \textbf{IC}: \mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Above, $\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ is the **state vector**, $A = \begin{bmatrix} 0 & 1 \\ -26 & -2 \end{bmatrix}$ is the system matrix, $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the time-dependent forcing term $f(t)$ is: $f(t) = 20 \cdot [u(t-5) - u(t-10)]$, where $u(t)$ denotes the Heaviside unit step function. This forcing term is a single rectangular pulse which jumps to 20 at time 5, and after 5 seconds is turned off.

Here's a quick plot of this driving function:



While the rectangular pulse is in the **off state**, there is no forcing and the system is "temporarily" homogeneous:

$\textbf{DE1}$: $\mathbf{x}' = A\mathbf{x}$ (Homogeneous: Valid only when rectangular pulse is 0).

or

$$\textbf{DE1}: \mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -26 & -2 \end{bmatrix}\mathbf{x} \qquad \textbf{IC}: \mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

DE1 has the equilibrium point: $\mathbf{x}_{eq1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

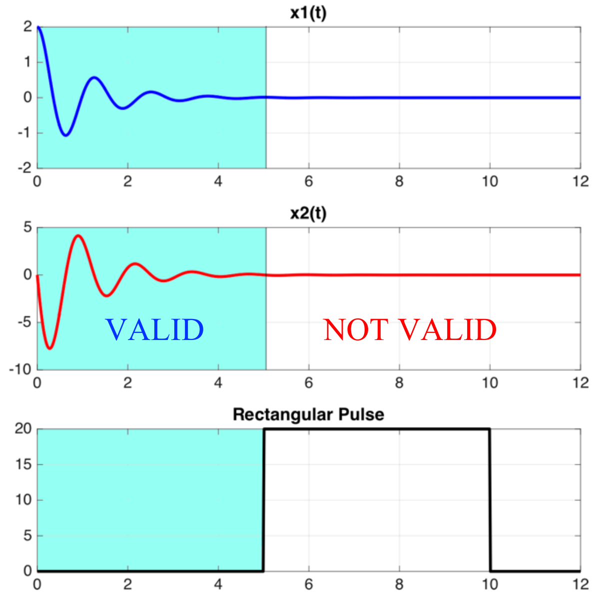
Questions 1 and 2: Using `dsolve`, solve this homogeneous equation exactly, and write out the unique solutions for $x_1(t)$ and $x_2(t)$ which match the initial conditions $x_1(0) = 2$ and $x_2(0) = 0$. Of course, these solutions are only valid for the first five seconds.

Questions 1&2: The first five seconds.

$x_1(t) =$

$x_2(t) =$

Here's a component plot of these solutions, for 12 seconds, but they are <u>only valid for the first five seconds</u>, after which the 20-volt pulse is switched on and the system becomes non-homogeneous.



Turn on the pulse!
Assume now that the rectangular pulse is in the **on state**, so the system becomes non-homogeneous:

DE2: $\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ -26 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 20 \end{bmatrix}$ (Valid only when rectangular pulse is **on** for $5 \le t \le 10$.)

Question 3: Find the new equilibrium point $\mathbf{x}_{eq2}$ for this non-homogeneous equation, using Matlab's `inv()` command.

Question 3:

$\mathbf{x}_{eq2} =$

When the pulse is on, we expect the solution to move away from $\mathbf{x}_{eq1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and towards the new equilibrium point $\mathbf{x}_{eq2}$. Let's verify this now, by finding a solution which is valid for all time, whether the pulse is on or off.

Question 4: For all times t, the forcing term is given by: $f(t) = 20 \cdot [u(t-5) - u(t-10)]$
Define this function in Matlab using `heaviside()` and `matlabFunction()`. Then find its Laplace transform $F(s)$.

Question 4: The Laplace transform *F*(*s*) of the rectangular pulse *f*(*t*) is:

$F(s) =$

Question 5: Next, we will solve the full DE for all times, using the Laplace transform following these steps. Recall the transform of the complete solution is given by the formula:

$$\mathbf{X}(s) = \underbrace{(sI - A)^{-1}\mathbf{x}(0)}_{\text{Zero Input Solution (homog.)}} + \underbrace{(sI - A)^{-1}\mathbf{B}F(s)}_{\text{Zero State Soln. (forcing function)}}$$

First, we'll find the homogeneous term, or **zero-input solution** using just this part.

$$\mathbf{X}_{hom}(s) = (sI - A)^{-1}\mathbf{x}(0)$$

Record both components of $\mathbf{X}_{hom}(s)$ below as functions of s.

Question 5: The Laplace transform $\mathbf{X}_{hom}(s)$ for just the zero-input solution is:

$\mathbf{X}_{hom}(s) =$

Hint: If you take the inverse Laplace transform of the above results, you should recover the exact same functions you found earlier for the homogeneous equation.

Question 6: Now, we'll find the forced response term, or **zero-state solution** using just this part.

$$\mathbf{X}_{forced}(s) = (sI - A)^{-1}\,\mathbf{B}F(s)$$

Record both components of $\mathbf{X}_{forced}(s)$ below as functions of s.

Question 6: The Laplace transform $\mathbf{X}_{forced}(s)$ for just the zero-state solution is:

$\mathbf{X}_{forced}(s) =$

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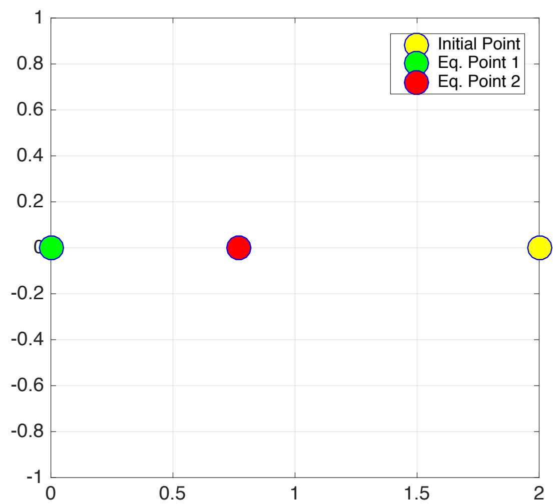
Question 7&8: Combine the zero-state and zero-input solutions to obtain the transform of the complete answer. Then apply the inverse transform to get the solutions(s) in the time domain. Make $x_1(t)$ and $x_2(t)$ into functions. In a **separate** plot, create a **phase plot** going from time 0 to 15 seconds.
Show the initial point (2, 0) with a large **yellow** circle with blue edge as shown below.

figure
```
plot(2, 0,  'o', 'MarkerSize',20, 'MarkerEdgeColor', 'b', 'MarkerFaceColor', 'y')
hold on, grid on
```

Display the two "equilibrium" solutions with a similar large circle, but make the face color for (0,0) **green** and the face color for the second equilibrium point found in question #3, **red**. Add a legend for the three points.

At this point, you should see something similar to:

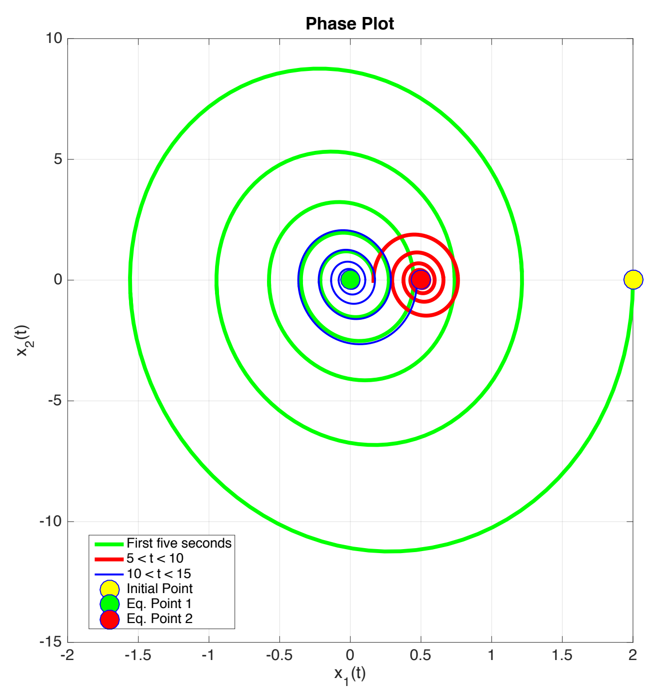


Now add the **phase plot** using the complete solutions for $x_1(t)$ and $x_2(t)$. But add this twist. The solution transitions every five seconds. To make these transitions clear:
i. Draw the solution for the first five seconds using a **green** line of thickness 2.
ii. Draw the solution for the next five seconds using a **blue** line of thickness 2.
ii. Draw the solution for the last five seconds using a **red** line of thickness 2.

Adjust your **legend** to include all six items, these three lines and the three points.

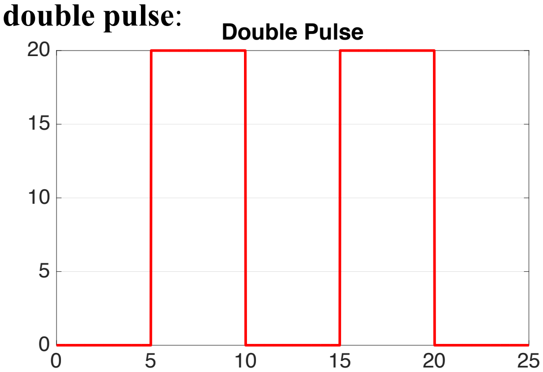
Paste your completed phase plot here.
Just **replace** this sample plot which corresponds to the matrix $A = \begin{bmatrix} 0 & 1 \\ -40 & -1 \end{bmatrix}$ instead.

Sample plot! Your plot will be different!



Questions 9 & 10: Change the forcing function to be the following **double pulse**:

$$f(t) = 20 \cdot [u(t-5) - u(t-10) + u(t-15) - u(t-20)]$$



Clone your code, and produce a new **phase plot** for when the system is driven by this double pulse instead of the single pulse. You should extend the time range to go from 0 to 25 seconds. Add the three points as before. They will not change. Also change the plot color or style every five seconds, breaking the plot into a total of five segments. Because the plot segment for $15 < t < 20$ is almost the same as for, $5 < t < 10$, you might also show this in **red** using a dotted line. Similarly, the plot segment for $20 < t < 25$ is almost the same as for, $10 < t < 15$, you might show this as a **blue** dotted line. Adjust the legend accordingly. Of course, the solution will be exactly the same up until time 15, when the second pulse kicks in. **Paste your completed phase plot here.**

Just **replace** this sample plot which corresponds to the matrix $A = \begin{bmatrix} 0 & 1 \\ -40 & -1 \end{bmatrix}$ instead.

Sample plot! Your plot will be different!