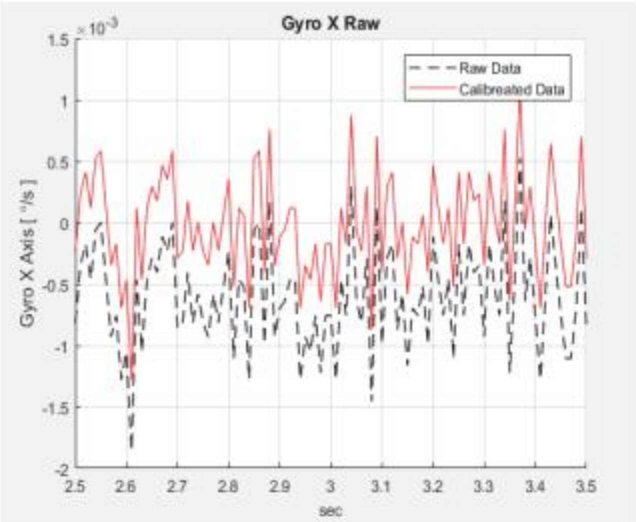
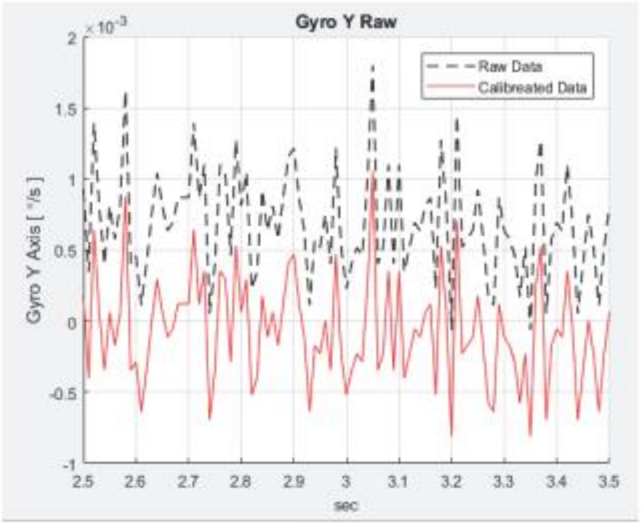
NURA 전국항공우주과학경진대회

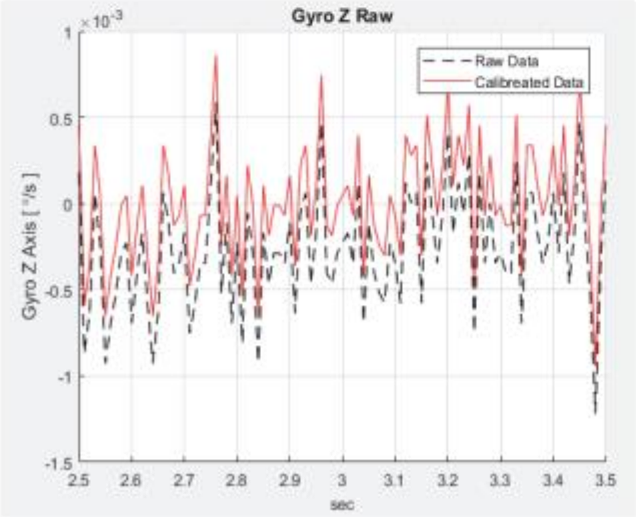
NURA National Aerospace Science Competition

Gyroscope

$$\begin{bmatrix} g_{m_x} \\ g_{m_y} \\ g_{m_z} \end{bmatrix} = \begin{bmatrix} g_{r_x} \\ g_{r_y} \\ g_{r_z} \end{bmatrix} + \begin{bmatrix} b_{g_x} \\ b_{g_y} \\ b_{g_z} \end{bmatrix}$$







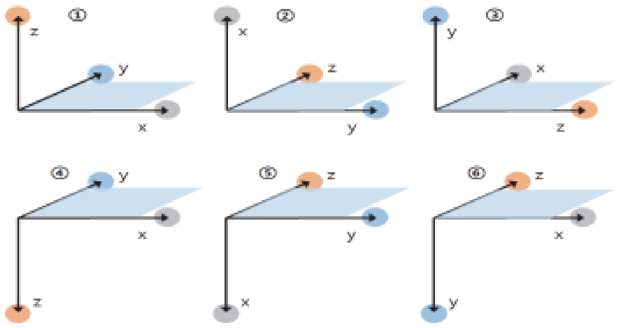
1) Hanlin sheng, Tianhong Zhang, "MEMS-based low-cost strap-down AHRS research", ELSEVIER Journal, 2015, pp63-72
2) WanSeok Jo, "A Study of a Calibration method and Flight Test for MEMS Sensor", Dissertation of the degree of Master of Philosophy, Korea Aerospace University, 2017
3) A Study of a Calibration for Low-grade IMU

NURA
The National Universities' Rocket Association

SEOUL TECH
The National Universities of Science & Technology

Acceleration

$$\begin{bmatrix} a_{m_x} \\ a_{m_y} \\ a_{m_z} \end{bmatrix} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \begin{bmatrix} a_{r_x} \\ a_{r_y} \\ a_{r_z} \end{bmatrix} + \begin{bmatrix} b_{a_x} \\ b_{a_y} \\ b_{a_z} \end{bmatrix}$$



$$a^{m_{x5}} = G_{xx} + b_{a_x},\ a^{m_{y5}} = G_{xy} + b_{a_y},\ a^{m_{z5}} = G_{xz} + b_{a_z}$$

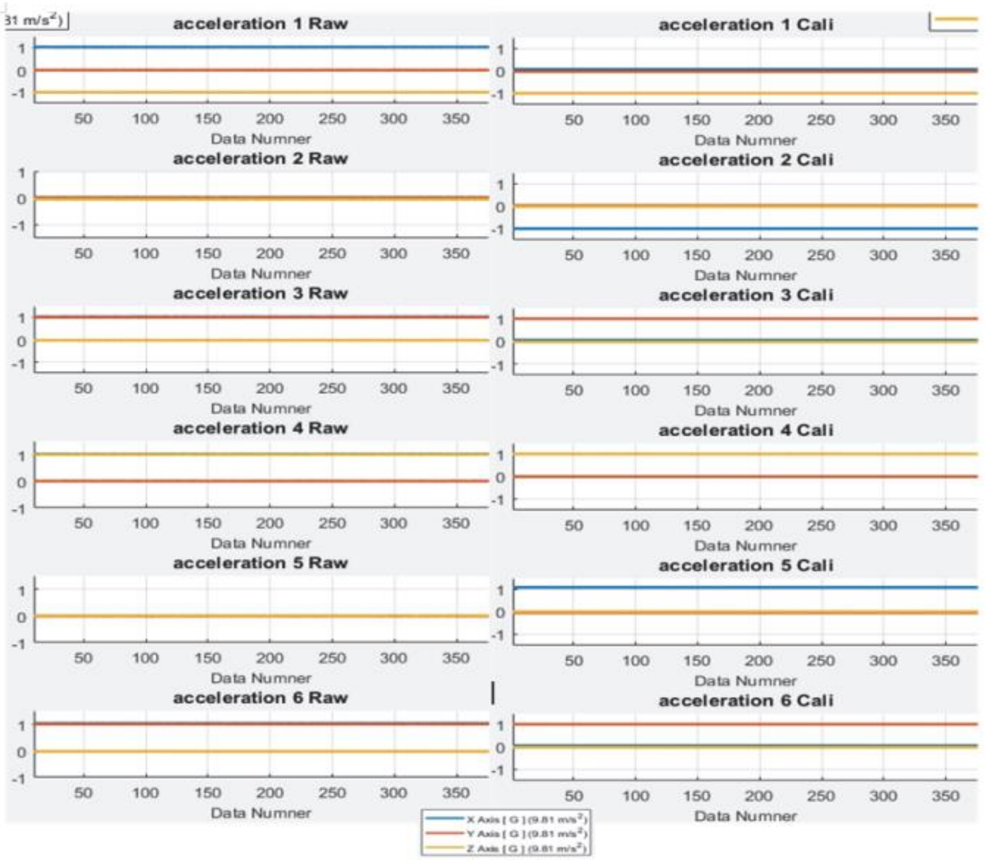
$$a^{m_{x6}} = G_{yx} + b_{a_x},\ a^{m_{y6}} = G_{yy} + b_{a_y},\ a^{m_{z6}} = G_{yz} + b_{a_z}$$

$$a^{m_{x4}} = G_{zx} + b_{a_x},\ a^{m_{y4}} = G_{zy} + b_{a_y},\ a^{m_{z4}} = G_{zz} + b_{a_z}$$

$$a^{m_{x2}} = -G_{xx} + b_{a_x},\ a^{m_{y2}} = -G_{xy} + b_{a_y},\ a^{m_{z2}} = -G_{xz} + b_{a_z}$$

$$a^{m_{x3}} = -G_{yx} + b_{a_x},\ a^{m_{y3}} = -G_{yy} + b_{a_y},\ a^{m_{z3}} = -G_{yz} + b_{a_z}$$

$$a^{m_{x1}} = -G_{zx} + b_{a_x},\ a^{m_{y1}} = -G_{zy} + b_{a_y},\ a^{m_{z1}} = -G_{zz} + b_{a_z}$$



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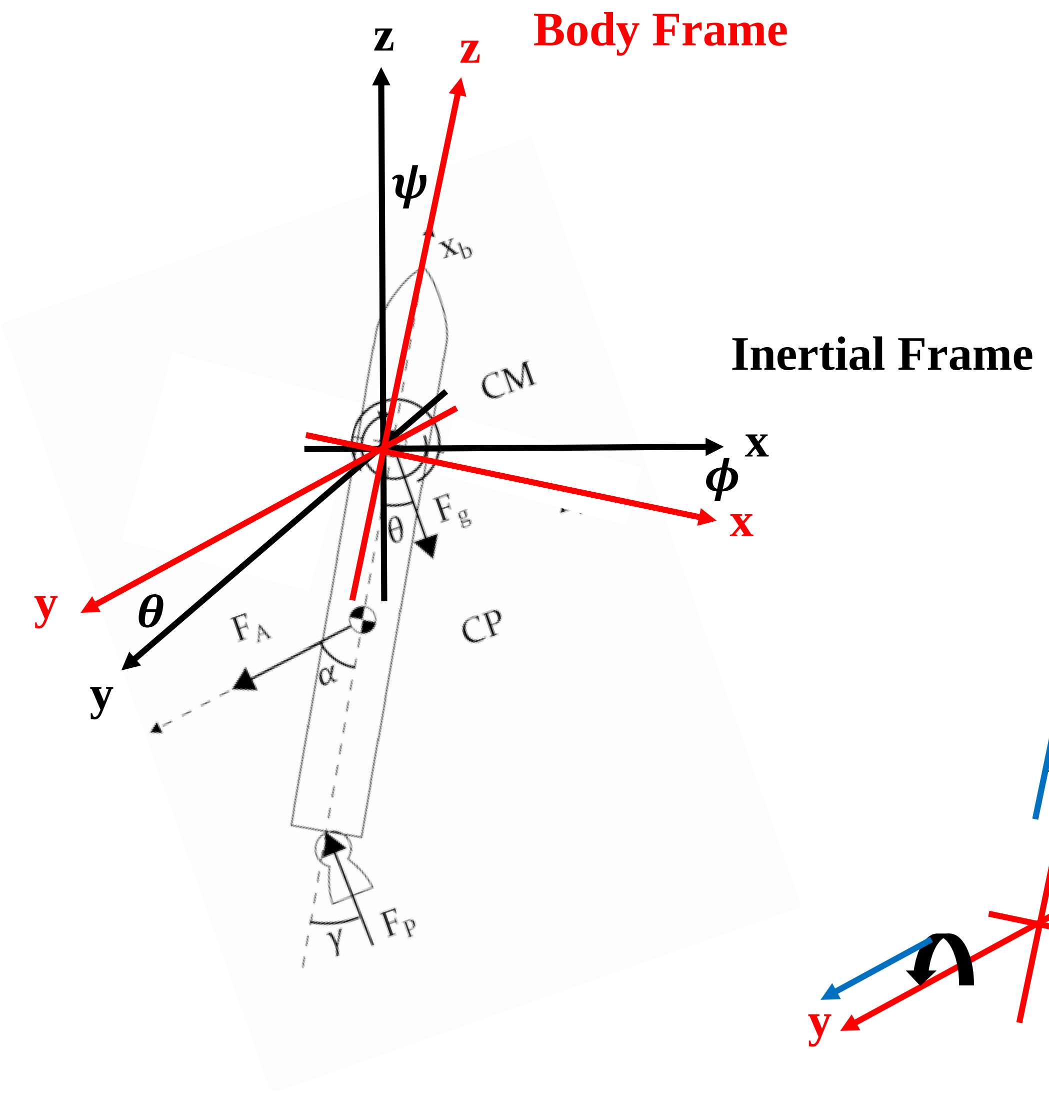
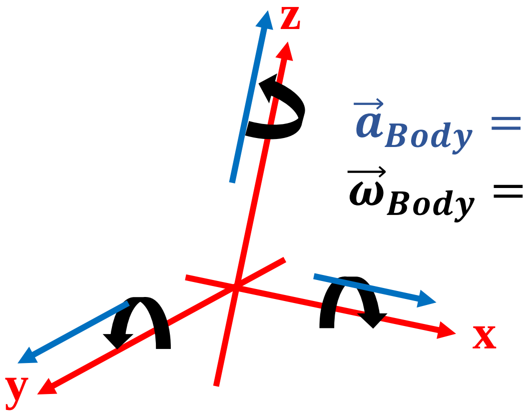
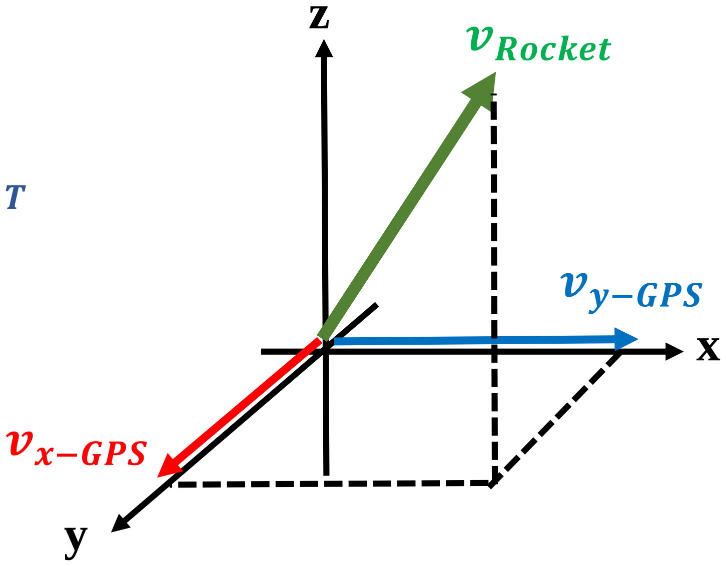
Background



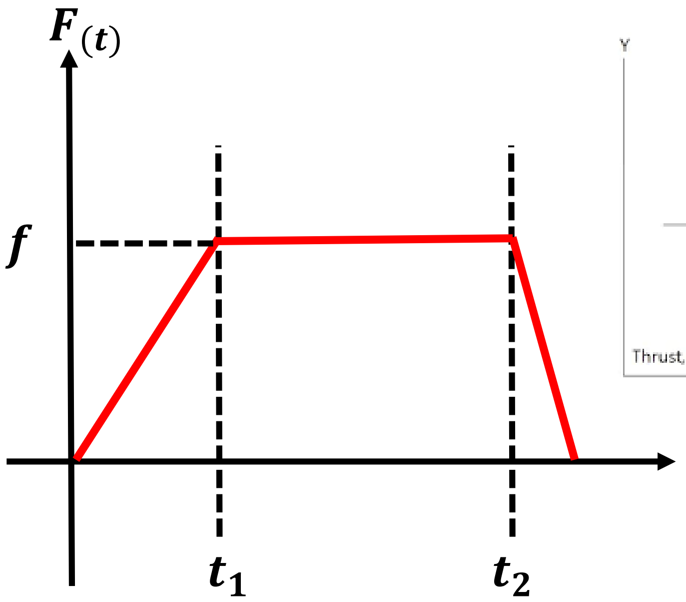
We know that

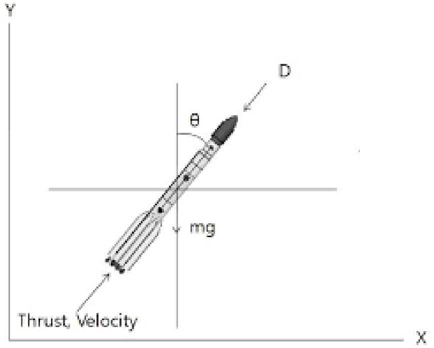
$$\vec{a}_{Body} = [a_x, a_y, a_z]^T$$
$$\vec{\omega}_{Body} = [p, q, r]^T$$
$$v_{x-GPS}$$
$$v_{y-GPS}$$
$$v_{Rocket}$$

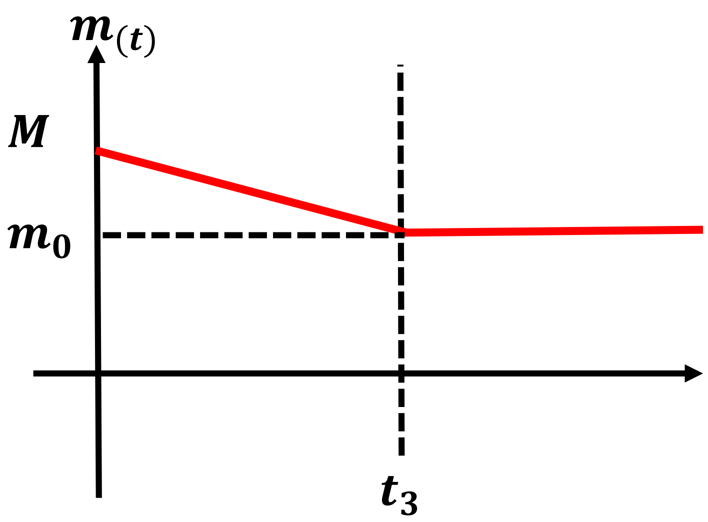




Background







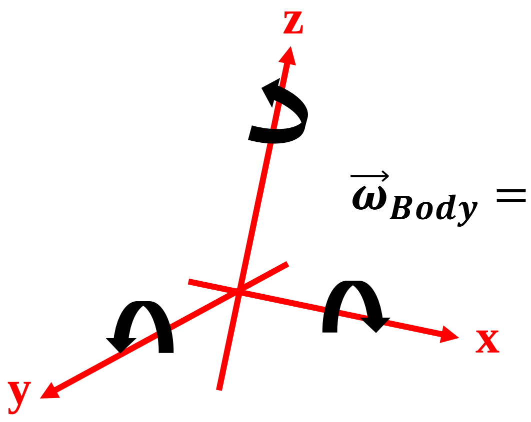
$$F_{(t)} = \begin{cases} \dfrac{f}{t_1} t & (t < t_1) \\ f & (t_1 < t < t_2) \\ f - mt & (t_2 < t) \end{cases}$$

$$m_{(t)} = \begin{cases} \dfrac{m_0 - M}{t_3} t + m \\ m_0 \end{cases}$$

Gyroscope

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = R_x^{-1}(\phi)R_y^{-1}(\theta)R_z^{-1}(\psi)\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + R_x^{-1}(\phi)R_y^{-1}(\theta)\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R_x^{-1}(\phi)\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

so $$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)\sec(\theta) & \cos(\phi)\sec(\theta) \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \boldsymbol{L\,\vec{\omega}_{Body}}$$



$$\boldsymbol{\vec{\omega}_{Body} = [p, q, r]^T}$$

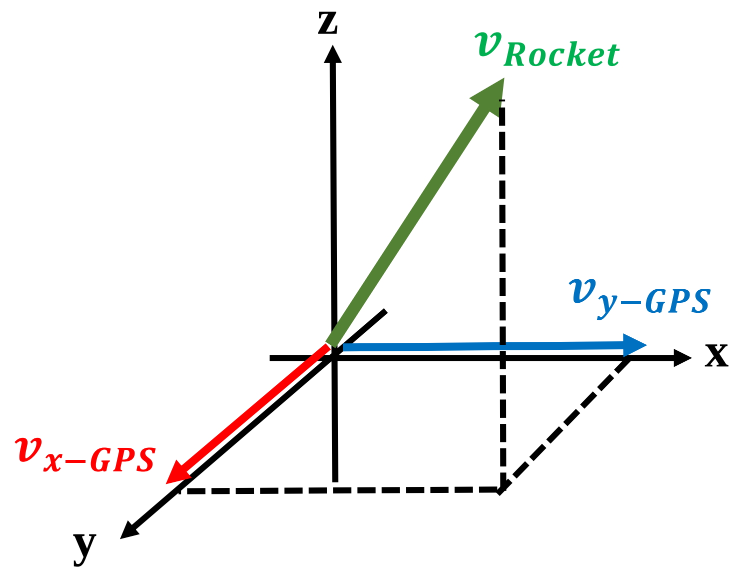
Note*

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix}$$

$$R_y(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix}$$

$$R_z(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Acceleration



$$\sum F \;= m_{(t)}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = m_{(t)}\vec{a}_{Body} = Thrst + Coriolis\ force + Gravity$$

$$Thrust \;= F_{(t)}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = m_{(t)}a_{(t)}\begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix}$$

$$Coriolis\ force = \;2\,m_{(t)}\begin{bmatrix} 0 & R_{xyz(\phi,\theta,\psi)}\sqrt{v^2{}_{Rocket} - (v^2{}_{x-GPS} + v^2{}_{y-GPS})} & -R_{xyz(\phi,\theta,\psi)}v_{y-GPS} \\ -R_{xyz(\phi,\theta,\psi)}\sqrt{v^2{}_{Rocket} - (v^2{}_{x-GPS} + v^2{}_{y-GPS})} & 0 & R_{xyz(\phi,\theta,\psi)}v_{x-GPS} \\ R_{xyz(\phi,\theta,\psi)}v_{y-GPS} & -R_{xyz(\phi,\theta,\psi)}v_{x-GPS} & 0 \end{bmatrix}\vec{\omega}_{Body}$$

$$Gravity \;= m_{(t)}g\begin{bmatrix} sin(\theta) \\ -cos(\theta)sin(\phi) \\ -cos(\theta)cos(\phi) \end{bmatrix}$$

$$\vec{a}_{Body} = \frac{Thrst + Coriolis\ force + Gravity}{m_{(t)}}$$

Model

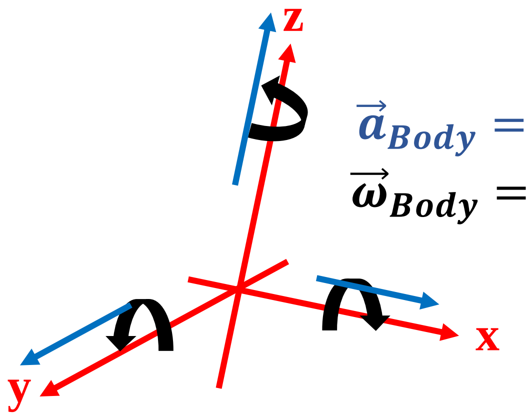
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Change Discrete system

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_{k+1} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_k + \frac{\Delta t}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_k = \left(I + \frac{\Delta t}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\right)\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}_k$$

Let, $\mathbf{X} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$

$$X_{k+1} = \left(I + \frac{\Delta t}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\right)X_k$$

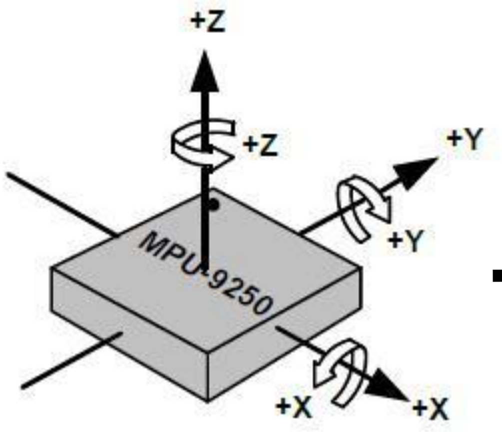


$$\vec{\boldsymbol{a}}_{\boldsymbol{Body}} = [\boldsymbol{a_x}, \boldsymbol{a_y}, \boldsymbol{a_z}]^{\boldsymbol{T}}$$

$$\vec{\boldsymbol{\omega}}_{\boldsymbol{Body}} = [\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}]^{\boldsymbol{T}}$$

$$\boldsymbol{A(model) = I + \frac{\Delta t}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}}$$

Transfer



v_{x-GPS}

v_{y-GPS}

v_{Rocket}

$\vec{a}_{Body} = [a_x, a_y, a_z]^T$

$\vec{\omega}_{Body} = [p, q, r]^T$

Euler → Quaternion

Model Design

Kalman Filter

Quaternion → Euler

Controller

Note*

$$\mathbf{q}_{\mathrm{IB}} = \begin{bmatrix} \cos(\psi/2) \\ 0 \\ 0 \\ \sin(\psi/2) \end{bmatrix} \begin{bmatrix} \cos(\theta/2) \\ 0 \\ \sin(\theta/2) \\ 0 \end{bmatrix} \begin{bmatrix} \cos(\phi/2) \\ \sin(\phi/2) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\phi/2)\cos(\theta/2)\cos(\psi/2) + \sin(\phi/2)\sin(\theta/2)\sin(\psi/2) \\ \sin(\phi/2)\cos(\theta/2)\cos(\psi/2) - \cos(\phi/2)\sin(\theta/2)\sin(\psi/2) \\ \cos(\phi/2)\sin(\theta/2)\cos(\psi/2) + \sin(\phi/2)\cos(\theta/2)\sin(\psi/2) \\ \cos(\phi/2)\cos(\theta/2)\sin(\psi/2) - \sin(\phi/2)\sin(\theta/2)\cos(\psi/2) \end{bmatrix}$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \arctan\left(\dfrac{2(q_0 q_1 + q_2 q_3)}{1 - 2(q_1^2 + q_2^2)}\right) \\ -\pi/2 + 2\arctan\sqrt{\dfrac{1 + 2(q_0 q_2 - q_1 q_3)}{1 - 2(q_0 q_2 - q_1 q_3)}} \\ \arctan\left(\dfrac{2(q_0 q_3 + q_1 q_2)}{1 - 2(q_2^2 + q_3^2)}\right) \end{bmatrix}$$