Total No. of Questions—**8**] [Total No. of Printed Pages—**4**

Seat No.	

[5559]-195

S.E. (Comp/IT) (II Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Three Hours **Maximum Marks : 60**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* differential equations : [8]

(*i*) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^{2x}\sin 4x + 2^{3x} + 6$

(*ii*) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = x^4 + 3x + 1$

(*iii*) $\dfrac{d^2y}{dx^2} + 9y = \tan 3x$, by using the method of variation of parameters.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x)\cos\lambda x\, dx = \begin{cases} 2-\lambda, & 0 \le \lambda \le 2 \\ 0, & \lambda > 2 \end{cases}$$

P.T.O.

Or

2. (*a*) An inductor of 0.25 henries, with negligible resistance, a capacitor of 0.04 farads are connected in series and having an alternating voltage [12 sin 6*t*]. Find the current and charge at any time *t*. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Obtain $z[4^k e^{-6k}],\ k \geq 0.$

(*ii*) Obtain $z^{-1}\left[\dfrac{13z}{(5z+1)(4z+1)}\right].$

(*c*) Solve the difference equation : [4]

$$f(k+2) - 7f(k+1) + 12f(k) = 0$$

where $f(0) = 0,\ f(1) = 3,\ k \geq 0.$

3. (*a*) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the first four central moments, β_1 and β_2. [4]

(*b*) Fit a straight line of the form $Y = aX + b$ to the following data by the least square method : [4]

X	1	3	4	5	6	8
Y	–3	1	3	5	7	11

(*c*) A riddle is given to three students whose probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that the riddle is solved. [4]

Or

4. (*a*) In a sample of 1,000 cases, the mean of a certain examination is 14 and standard deviation is 2.5. Assuming the distribution to be normal. Find the number of students scoring between 12 and 15. [4]

[Given : $Z_1 = 0.4$, $A_1 = 0.1554$, $Z_2 = 0.8$, $A_2 = 0.2881$]

(*b*) During working hours, on an average 3 phone calls are coming into a company within an hour. Using Poisson distribution, find the probability that during a particular working hour, there will be at the most one phone call. [4]

(*c*) For a bivariate data, the regression equation of Y on X is $8x - 10y = -66$ and the regression equation of X on Y is $40x - 18y = 214$. Find the mean values of X and Y. Also, find the correlation coefficient between X and Y. [4]

5. (*a*) Find the directional derivative of $\phi = xy^2 + yz^2 + zx^2$ at (1, 1, 1) along the line $2(x - 2) = y + 1 = z - 1$. [4]

(*b*) Find constants *a*, *b*, *c* so that

$$\overline{F} = (x + 2y + az)\overline{i} + (bx - 3y - z)\overline{j} + (4x + cy + 2z)\overline{k}$$

is irrotational. [4]

(*c*) Find the workdone by the force

$$\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$$

in taking a particle from (0, 0, 0) to (1, 2, 1). [5]

Or

6. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \cdot \left(\dfrac{\overline{a} \times \overline{r}}{r^n} \right) = 0$ (*ii*) $\nabla^2 \left[\nabla \cdot \left(\dfrac{\overline{r}}{r^2} \right) \right] = \dfrac{2}{r^4}$.

(*b*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2$ at $(2, -1, 2)$ along the tangent to the curve

$x = e^t \cos t,\ y = e^t \sin t,\ z = e^t$ at $t = 0$. [4]

(*c*) Find the workdone by, $\overline{\mathrm{F}} = 2xy^2\overline{i} + (2x^2y + y)\overline{j}$ in taking a particle from (0, 0, 0) to (2, 4, 0) along the parabola $y = x^2$, $z = 0$. [5]

7. (*a*) Determine the analytic function $f(z) = u + iv$ if $u = 2x - x^3 + 3xy^2$. [4]

(*b*) Find the bilinear transformation that maps to points $z = -i, 0, i$ into the points $\mathrm{W} = 1, 0, \infty$. [4]

(*c*) Evaluate $\displaystyle\int_c \frac{z^3}{z^2 - 4}\, dz$, where c is the circle $|z| = 3$. [5]

Or

8. (*a*) Determine the analytic function $f(z) = u + iv$ if $u = 3x^2y + 2x^2 - y^3 - 2y^2$. [4]

(*b*) Find image of the circle $|z - 2i| = 2$, under the mapping $w = \dfrac{1}{2}$. [4]

(*c*) Evaluate $\displaystyle\int_c \frac{2z^2 + z}{z^2 - 1}\, dz$, where c is the circle $|z| = \dfrac{3}{2}$. [5]