```
::= ALPHANUMERIC | *ALPHANUMERIC
                                                                                                                            variables
                                                                                                                      general labels
                                                                                                                                labels
    T
         ::= [t, \ldots]
                                                                                                                                stack
        ::= \langle \eta, \overset{\star}{\ell}, S \rangle
                                                                                                                       stack\ frames
                                                                                                                           programs
         := \ell : \hat{\ell} : d
                                                                                                                              clauses
         := x = e \mid \text{return } x \mid \text{goto } \ell \mid \text{goto } \ell \text{ if not } x
                                                                                                                           directives
                  | raise x | catch x | pass
    B
         ::=
                 \{x\mapsto m,\ldots\}
                                                                                                                            bindings
    H
         ::= \{m \mapsto v, \ldots\}
                                                                                                                                 heap
         ::= \mathbb{Z} \mid \mathbb{S} \mid [m,\ldots] \mid (m,\ldots) \mid B \mid F \mid M \mid *
                                                                                                                               values
         \mathbb{Z} \mid \mathbb{S} \mid \mathbb{N} None \mid x \mid \operatorname{def} x(x, \ldots) = \{S\} \mid x(x, \ldots) \mid x.x \mid [x, \ldots] \mid (x, \ldots)
                                                                                                                         expressions
    Y
         ::=
                 [y,\ldots]
                                                                                                                   microcode\ stack
    Z
         ::=
                 [z,\ldots]
                                                                                                           microcode\ literal\ stack
                 STORE | WRAP | BIND | LOOKUP | LIST n | TUPLE n
         ::=
                                                                                                           microcode\ instructions
                  | Advance | Pop | Push S | Raise | Goto \ell | Gotoifn \ell
                  Call n | GetCall n | Convert n | Retrieve
                  ALLOCNAMEERROR | ALLOCTYPEERROR | ALLOCATTRERROR
        := x \mid m \mid v
     2.
                                                                                                                 microcode\ literals
   \stackrel{\star}{m} ::= m \mid \eta \mid *
                                                                                                       general memory locations
\eta, m
                <address>
                                                                                                                 memory locations
         ::= \langle \eta, \operatorname{def}(x, \ldots) \to S \rangle \mid \mathfrak{F} \mid \mathfrak{M}
    F
                                                                                                                 general functions
   M
                 \langle m, F \rangle
                                                                                                                   general methods
\mathfrak{F},\mathfrak{M}
         ::= CallFunc | Type
                                                                                                                   magic\ functions
    n
                                                                                                                             integers
```

Figure 1: Expression Grammar

## **Definition 0.1.** *Initialization*

```
\begin{split} H_{\text{init}} &= \{\eta_{\text{init}} \mapsto B_{\text{init}}, m_{\textit{None}} \mapsto *, m_{\textit{AttrError}} \mapsto \{\ \}, m_{\textit{FunType}} \mapsto \{\ \}\} \\ B_{\text{init}} &= \{\text{AttributeError} \mapsto m_{\textit{AttrError}}, \text{FunctionType} \mapsto m_{\textit{FunType}}\} \\ t_{\text{init}} &= \langle \ell_{\text{init}}, S_{\text{init}} \rangle \\ T_{\text{init}} &= [t_{\text{init}}] \end{split} \textbf{GetAttribute} &= \langle \eta, \ \textit{def} \ (o, a) \rightarrow \$1 = o.a; \rangle \\ \textbf{GetCall} &= \langle \eta, \ \textit{def} \ (f) \rightarrow \rangle \end{split}
```

("if \$1 == None then return getattribute(o...class...,a) else return \$1" - TC) (todo: add builtin mappings  $m \mapsto F - TC$ )

$$\begin{split} & m \notin H \quad H' = H[m \mapsto v] \\ & Z \mid |[v, \operatorname{Store}] \mid |Y, T, H \longrightarrow^1 Z \mid |[m] \mid |Y, T, H'] \\ & \\ & W \text{RAP } m \\ & v = \text{GetObj}(H, m) \\ & Z \mid |[m, \operatorname{Wrap}] \mid |Y, T, H \longrightarrow^1 Z \mid |[v] \mid |Y, T, H \\ & \\ & \frac{B \text{BIND } m \text{ to } x}{Z \mid |[m, x, \operatorname{Bind}] \mid |Y, T, H \longrightarrow^1 Z \mid |Y, T, H']} \\ & \frac{B \text{DIM } m \text{ to } x}{Z \mid |[m, x, \operatorname{Bind}] \mid |Y, T, H \longrightarrow^1 Z \mid |Y, T, H']} \\ & \frac{S(\ell) = \ell : \ell' : d \qquad \ell \stackrel{\$}{\blacktriangleleft} \ell''}{Z \mid |[\operatorname{Advance}] \mid |Y, [\langle \eta, \ell, S \rangle] \mid |T, H \longrightarrow^1 Z \mid |Y, [\langle \eta, \ell', S \rangle] \mid |T, H} \\ & \frac{P \text{OP}}{Z \mid |[\operatorname{Pop}] \mid |Y, t \mid |T, H \longrightarrow^1 Z \mid |Y, T, H} \\ & \frac{P \text{Ush } S}{Z \mid |[n, \operatorname{Push } S] \mid |Y, T, H \longrightarrow^1 P', Z \mid |Y, [\langle \eta', \ell, S \rangle] \mid |T, H']} \\ & \text{Look up } x \left( \text{Bound} \right) \\ & \frac{L \text{Ookup}(H, \eta, x) = m}{Z \mid |[x, \operatorname{Lookup}] \mid |Y, [\langle \eta, \ell, S \rangle] \mid |T, H \longrightarrow^1 Z \mid |[m] \mid |Y, [\langle \eta, \ell, S \rangle] \mid |T, H} \\ & \text{Look up } x \left( \text{NameError} \right) \\ & \frac{L \text{Ookup}(H, \eta, x) = *}{Z \mid |[x, \operatorname{Lookup}] \mid |Y, [\langle \eta, \ell, S \rangle] \mid |T, H \longrightarrow^1 [\text{AllocNameError}, \operatorname{Raise}], [\langle \eta, \ell, S \rangle] \mid |T, H} \\ & \frac{M \text{Ake List}}{Z \mid |[m_1, \dots, m_n, \operatorname{List } n] \mid |Y, T, H \longrightarrow^1 Z \mid |[v] \mid |Y, T, H} \\ & \frac{M \text{Ake Tuple}}{Z \mid |[m_1, \dots, m_n, \operatorname{Tuple} n] \mid |Y, T, H \longrightarrow^1 Z \mid |[v] \mid |Y, T, H} \\ & \frac{V = (m_1, \dots, m_n)}{Z \mid |[m_1, \dots, m_n, \operatorname{Tuple} n] \mid |Y, T, H \longrightarrow^1 Z \mid |[v] \mid |Y, T, H} \\ \end{aligned}$$

Figure 2: Microcommands

Raise (no exception label) 
$$S(\ell) = \ell : * : d$$

$$\overline{Z \parallel [\text{Raise}] \parallel Y, [\langle \eta, \ell, S \rangle] \parallel T, H \longrightarrow^1 Z \parallel [\text{Pop, Raise}] \parallel Y, [\langle \eta, \ell, S \rangle] \parallel T, H}$$
Raise (caught) 
$$S(\ell) = \ell : \ell_0 : d \qquad S(\ell_0) = \ell_0 : \ell_1 : \text{ catch } x \qquad Y' = [x, \text{Bind, Advance}]$$

$$\overline{Z \parallel [\text{Raise}] \parallel Y, [\langle \eta, \ell, S \rangle] \parallel T, H \longrightarrow^1 Z \parallel Y' \parallel Y, [\langle \eta, \ell_0, S \rangle] \parallel T, H}$$
Goto 
$$\ell$$

$$S(\ell) = \ell : \overset{\star}{\ell} : d$$

$$\overline{Z \parallel [\text{Goto } \ell] \parallel Y, [\langle \eta, \ell', S \rangle] \parallel T, H \longrightarrow^1 Z \parallel Y, [\langle \eta, \ell, S \rangle] \parallel T, H}$$
Gotoifn 
$$\ell (\text{success})$$

$$H[m] = \text{False} \qquad S(\ell) = \ell : \overset{\star}{\ell} : d$$

$$\overline{Z \parallel [m, \text{Gotoifn } \ell] \parallel Y, T, H \longrightarrow^1 Z \parallel [\text{Goto}] \parallel Y, T, H}$$
Gotoifn 
$$\ell (\text{failure})$$

$$H[m] = \text{True}$$

$$\overline{Z \parallel [m, \text{Gotoifn } \ell] \parallel Y, T, H \longrightarrow^1 Z \parallel [\text{Advance}] \parallel Y, T, H}$$
Call function 
$$v = \langle \eta, \text{ def } (x_1, \dots, x_n) \to S \rangle$$

$$Y' = [\eta, \text{Push } S, m_1, x_1, \text{Bind}, \dots, m_n, x_n, \text{Bind}]$$

$$\overline{Z \parallel [v, m_1, \dots, m_n, \text{Call } n] \parallel Y, T, H \longrightarrow^1 Z \parallel Y' \parallel Y, T, H}$$
Call function (wrong args)
$$v = \langle \eta, \text{ def } (x_1, \dots, x_q) \to S \rangle q \neq n$$

$$\overline{Z \parallel [v, m_1, \dots, m_n, \text{Call } n] \parallel Y, T, H \longrightarrow^1 [\text{AllocTypeError, Raise}], T, H}$$
Get Call 
$$m$$

$$v = H[m_0] [\star x_{\text{value}}], v \text{ is of form } F \text{ or } M$$

$$Z \parallel [m_0, \dots, m_n, \text{GetCall } n] \parallel Y, T, H \longrightarrow^1 [\text{AllocTypeError, Raise}], T, H$$

Figure 3: Microcommands (cont.)

Convert Function 
$$v$$

$$\frac{v = F}{Z ||[v, m_1, \dots, m_n, \text{Convert } n]|| Y, T, H \longrightarrow^1 Z ||[v, m_1, \dots, m_n, \text{Call } n]|| Y, T, H}$$

Convert Method v

CONVERT METHOD 
$$v$$
 
$$v = \langle m_0, F \rangle \qquad v' = F$$
 
$$\overline{Z ||[v, m_1, \dots, m_n, \text{Convert } n] || Y, T, H \longrightarrow^1 Z ||[v', m_0, m_1, \dots, m_n, \text{Call } n+1] || Y, T, H}$$

Retrieve 
$$x$$

$$v = H[m][x] \qquad Y' = [v, \text{Store}]$$

$$Z \mid\mid [m, x, \text{Retrieve}] \mid\mid Y, T, H \longrightarrow^{1} Z \mid\mid Y' \mid\mid Y, T, H$$

Retrieve x (AttributeError)

$$\frac{* = H[m][x]}{Z \, || [m, x, \texttt{Retrieve}] \, || \, Y, T, H \longrightarrow^1 [\texttt{AllocAttrError}, \texttt{Raise}], T, H}$$

Figure 4: Microcommands (cont.)

Figure 5: Magic Functions

$$\frac{S(\ell) = \ell : \overset{\star}{\ell}' : x = \mathtt{None} \qquad Y = [m_{\mathtt{None}}, x, \mathtt{Bind}, \mathtt{Advance}]}{[\;], [\langle \eta, \ell, S \rangle] \mid\mid T, H \longrightarrow^{1} Y, [\langle \eta, \ell, S \rangle] \mid\mid T, H}$$

LITERAL ASSIGNMENT 
$$S(\ell) = \ell : \stackrel{\star'}{\ell} : x = e, \ e \ \text{is of form } \mathbb{Z}, \mathbb{S}$$
 
$$\underline{Y} = [v, \text{Store}, \text{Wrap}, \text{Store}, x, \text{Bind}, \text{Advance}]}$$
 
$$\underline{[\ ], [\langle \eta, \ell, S \rangle] \, ||\ T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] \, ||\ T, H}$$

(TODO: make literal category (ints, str, bool, None) - TC)

Name Assignment

$$\frac{S(\ell) = \ell : \stackrel{\star}{\ell}' : x_1 = x_2 \qquad Y = [x_2, \text{Lookup}, x_1, \text{Bind}, \text{Advance}]}{[\ ], [\langle \eta, \ell, S \rangle] \mid\mid T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] \mid\mid T, H}$$

LIST ASSIGNMENT

$$S(\ell) = \ell : \stackrel{\star'}{\ell} : x = [x_1, \dots, x_n]$$

$$Y = [(x_1, \text{Lookup}), \dots, (x_n, \text{Lookup}), \text{List } n, \text{Store, Wrap, Store, } x, \text{Bind, Advance}]$$

$$[], [\langle \eta, \ell, S \rangle] || T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] || T, H$$

(Parentheses in Y group instructions together for convenience of reading. – TC)

Tuple Assignment

$$S(\ell) = \ell : \stackrel{\star'}{\ell} : x = [x_1, \dots, x_n]$$

$$\underline{Y = [(x_1, \text{LookUp}), \dots, (x_n, \text{LookUp}), \text{Tuple } n, \text{Store, Wrap, Store, } x, \text{Bind, Advance}]}$$

$$[], [\langle \eta, \ell, S \rangle] || T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] || T, H$$

FUNCTIONDEF ASSIGNMENT

$$S(\ell) = \ell : \ell' : x = \operatorname{def}(x_1, \dots, x_n) = \{S'\} \qquad v = \langle \eta, \operatorname{def}(x_1, \dots, x_n) \to S' \rangle$$

$$\frac{Y = [v, \operatorname{Store}, \operatorname{Wrap}, \operatorname{Store}, x, \operatorname{Bind}, \operatorname{Advance}]}{[], [\langle \eta, \ell, S \rangle] || T, H}$$

Attribute Assignment

$$\frac{S(\ell) = \ell : \ell' : x = x_1.x_2}{Y = [x_1, \text{LookUp}, x_2, \text{Retrieve}, \text{Wrap}, \text{Store}, x, \text{Bind}, \text{Advance}]}{[\ ], [\langle \eta, \ell, S \rangle] \ ||\ T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] \ ||\ T, H}$$

Call Assignment

$$S(\ell) = \ell : \ell' : x = x_0(x_1, \dots, x_n)$$

$$\underline{Y = [x_0, \text{LookUp}, \dots, x_n, \text{LookUp}, \text{GetCall } n, \text{Convert } n]}$$

$$[], [\langle \eta, \ell, S \rangle] || T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] || T, H$$

Figure 6: Operational Semantics: Assignment

$$\begin{split} &\operatorname{Pass} \\ & \underbrace{S(\ell) = \ell : \stackrel{\star'}{\ell} : \operatorname{pass}} & Y = [\operatorname{Advance}] \\ & \overline{[\;], [\langle \eta, \ell, S \rangle] \, || \, T, H} \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] \, || \, T, H} \end{split}$$

Return

RETURN 
$$S(\ell) = \ell : \stackrel{\star'}{\ell} : \text{ return } x \qquad T = [\langle \eta', \ell'', S' \rangle] || T'$$
 
$$S(\ell'') = \ell'' : \stackrel{\star''}{\ell} : x' = e \qquad Y = [x, \text{LookUp}, \text{Pop}, x', \text{Bind}, \text{Advance}]$$
 
$$[], [\langle \eta, \ell, S \rangle] || T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] || T, H$$
 Goto 
$$S(\ell) = \ell : \stackrel{\star'}{\ell} : \text{goto } \ell'' \qquad Y = [\text{Goto } \ell'']$$
 
$$[], [\langle \eta, \ell, S \rangle] || T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] || T, H$$

$$\frac{S(\ell) = \ell : \ell' : \text{ goto } \ell'' \text{ if not } x \qquad Y = [x, \text{Lookup}, \text{Gotoifn } \ell'']}{[\ ], [\langle \eta, \ell, S \rangle] \ ||\ T, H \longrightarrow^1 Y, [\langle \eta, \ell, S \rangle] \ ||\ T, H}$$

END OF FUNCTION

$$\frac{t = \langle \eta', \ell, S' \rangle \qquad S(\ell) = \ell : \overset{\star'}{\ell} : x = e \qquad Y = [\text{Pop}, m_{\text{None}}, x, \text{Bind}, \text{Advance}]}{[\;], [\langle \eta, *, S \rangle, t] \, || \, T, H \longrightarrow^1 Y, [\langle \eta, *, S \rangle, t] \, || \, T, H}$$

 $(m_{None} is a memory location reserved for None. - TC)$ 

$$\frac{T = [\langle \eta, *, S \rangle]}{T = [T, T, H]} \frac{Y = [T, T, H]}{T = [T, T, H]}$$

Figure 7: Operational Semantics: Flow

## Definition 0.2.

$$Lookup(H, \eta, x) = (todo - TC)$$

## Definition 0.3.

$$H[m] = v, B_{obj} = \{ \star x_{value} \mapsto v, \_getattribute\_ \mapsto \mathfrak{Getattribute} \}$$

$$GetObj(H, m) = \begin{cases} B, & if \ v = B \\ B = B_{obj}[\_class\_ \mapsto \star_{\mathsf{FUNTYPE}}], & if \ v = F \end{cases} \tag{1}$$

(getob) takes (H, m, memory location of getattribute) - TC

Figure 8: Helper Functions