$$
\begin{array}{lcll}
\hat{S} & ::= & [\hat{s}, \ldots] & \textit{abstract programs} \\
\hat{s} & ::= & \hat{\ell} : \hat{\ell} : \hat{d} & \textit{abstract statements} \\
\hat{d} & ::= & \hat{x} = \hat{v} \mid \hat{x} = \hat{x} & \textit{abstract directives} \\
\hat{v} & ::= & \texttt{int}^{+} \mid \texttt{int}^{-} \mid \texttt{int}^{0} & \textit{abstract values} \\
\hat{x} & & & \textit{abstract variables} \\
\hat{\ell} & & & \textit{labels}
\end{array}
$$

Figure 1: Normalized Python Language Grammar

$$
\begin{array}{lcll}
\hat{G} & ::= & \{\hat{g}, \ldots\} & \textit{control flow graphs} \\
\hat{g} & ::= & \hat{o} \blacktriangleleft \hat{o} \mid \hat{o} \ll \hat{o} & \textit{control flow graph edge} \\
\hat{o} & ::= & \textsc{Start} \mid \textsc{End} \mid \hat{s} & \textit{control flow graph nodes}
\end{array}
$$

Figure 2: Control Flow Graph Grammar

1 CoPylot

1.1 Grammar

The grammar of the language to be analyzed appears in Figure 1.

We assume throughout the rest of this document that a fixed program $\hat{S}$ is under analysis. *(TODO: describe here the idea of a bijection between labels and statements in this fixed program. – ZP)*

1.2 Control Flow

The grammar of control flow graphs appears in Figure 2. *(Discuss construction of initial graph. – ZP)*

We write $\hat{o} \overset{?}{\blacktriangleleft} \hat{o}'$ to denote $(\hat{o} \blacktriangleleft \hat{o}' \in \hat{G}$ when $\hat{G}$ is understood from context. Likewise, we write $\hat{o} \overset{?}{\ll} \hat{o}'$ to denote $(\hat{o} \ll \hat{o}' \in \hat{G}$ when $\hat{G}$ is understood from context.

We define a relation $\longrightarrow^1$ to perform control flow graph closure.

Definition 1.1. *Let $\hat{G} \longrightarrow^1 \hat{G}'$ be the least relation satisfying the rules appearing in Figure 3. Throughout these rules, the predicates $\overset{?}{\ll}$ and $\overset{?}{\blacktriangleleft}$ refer to graph $\hat{G}$.*

1.3 Value Lookup

The value lookup function uses the additional grammar in Figure 4.

Definition 1.2. *Given a control-flow graph $\hat{G}$, let $\hat{G}(\hat{o}_0, \hat{K})$ be the function returning the least set $\hat{V}$ which satisfies the following conditions:*

1. ***Value Manipulation***

$$\frac{\textsc{Start} \overset{?}{\blacktriangleleft} \hat{o}}{\hat{G} \longrightarrow^1 \hat{G} \cup \{\textsc{Start} \ll \hat{o}\}} \quad \textsc{Lexical Start}$$

$$\frac{\hat{o}_1 = (\hat{x} = \hat{v}) \qquad \hat{o}_1 \overset{?}{\blacktriangleleft} \hat{o}_2}{\hat{G} \longrightarrow^1 \hat{G} \cup \{\hat{o}_1 \ll \hat{o}_2\}} \quad \textsc{Literal Assignment}$$

$$\frac{\hat{o}_1 = (\hat{x} = \hat{x}') \qquad \hat{o}_1 \overset{?}{\blacktriangleleft} \hat{o}_2 \qquad \hat{v} \in \hat{G}(\hat{o}_1, [\hat{x}']) \qquad \hat{v} \neq \textsc{Undefined}}{\hat{G} \longrightarrow^1 \hat{G} \cup \{\hat{o}_1 \ll \hat{o}_2\}} \quad \textsc{Variable Accessible}$$

Figure 3: Control Flow Graph Closure

$\hat{K}$	$::=$	$[\hat{k}, \ldots]$	*lookup stacks*
$\hat{k}$	$::=$	$\hat{x} \mid \hat{v} \mid \textsc{Capture}(\mathbb{N}) \mid \textsc{Jump}(\hat{o})$	*lookup stack elements*

Figure 4: Value Lookup Grammar

(a) $\boxed{\textsc{Result}}$
If $\hat{K} = [\hat{v}]$, *then* $\hat{v} \in \hat{V}$.

2. ***Variable Lookup***

(a) $\boxed{\textsc{Value Discovery}}$
If $\hat{o}_1 \overset{?}{\ll} \hat{o}_0$, $\hat{o}_1 = \hat{\ell}_1 : \hat{\ell}_2 : \hat{x} = \hat{v}$, *and* $\hat{K} = [\hat{x}] \,||\, \hat{K}'$, *then* $\hat{G}(\hat{o}_1, [\hat{v}] \,||\, \hat{K}') \subseteq \hat{V}$.

(b) $\boxed{\textsc{Value Skip}}$
If $\hat{o}_1 \overset{?}{\ll} \hat{o}_0$, $\hat{o}_1 = \hat{\ell}_1 : \hat{\ell}_2 : \hat{x}' = \hat{v}$, $\hat{K} = [\hat{x}] \,||\, \hat{K}'$, *and* $\hat{x} \neq \hat{x}'$, *then* $\hat{G}(\hat{o}_1, \hat{K}) \subseteq \hat{V}$.

(c) $\boxed{\textsc{Value Aliasing}}$
If $\hat{o}_1 \overset{?}{\ll} \hat{o}_0$, $\hat{o}_1 = \hat{\ell}_1 : \hat{\ell}_2 : \hat{x} = \hat{x}'$, *and* $\hat{K} = [\hat{x}] \,||\, \hat{K}'$, *then* $\hat{G}(\hat{o}_1, [\hat{x}'] \,||\, \hat{K}) \subseteq \hat{V}$.