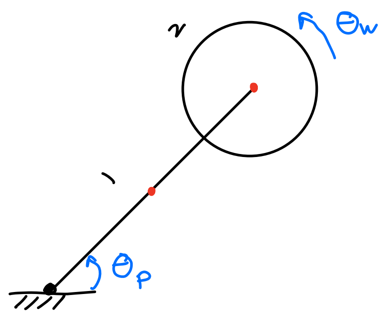


$$\tau = K_m u$$

$$\frac{1}{2}I\omega^2 + \frac{1}{2}m\overbrace{v^2}^{(L\theta_p)^2} = \frac{1}{2}mL^2\theta_p^2 + \frac{1}{2}I_2\theta_p^2$$

EOM

$$V = mg\ell\sin\theta_p + MgL\sin\theta_p$$

$$= (M+m)(L+\ell)g\sin\theta_p$$

shaft

$$T_1 = \frac{1}{2}I_1\theta_p^2 = \frac{1}{2}\left(\frac{mL^2}{12} + \frac{mL^2}{4}\right) = \frac{1}{6}mL^2\theta_p^2$$

wheel

$$T_2 = \frac{1}{2}I_2\dot{\theta}_w^2 + \frac{1}{2}ML^2\theta_p^2 = \frac{1}{2}\frac{Mr^2}{2}\theta_w^2 + \frac{1}{2}ML^2\theta_p^2$$

$$T = \frac{1}{6}m\ell^2\dot{\theta}_p^2 + \frac{1}{2}ML^2\dot{\theta}_p^2 + \frac{1}{2}\frac{Mr^2}{2}\dot{\theta}_w^2$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right): \quad \frac{1}{3}mL^2\ddot{\theta}_p + ML^2\ddot{\theta}_p, \quad \frac{1}{2}Mr^2\ddot{\theta}_w$$

$$\frac{\partial \mathcal{L}}{\partial q} = (M+m)(L+\ell)g\cos\theta_p$$

1) $\left(M + \frac{m}{3}\right)L^2\ddot{\theta}_p - (M+m)(L+\ell)g\cos\theta_p = \cancel{0}$

2) $\frac{1}{2}Mr^2\ddot{\theta}_w = \tau$

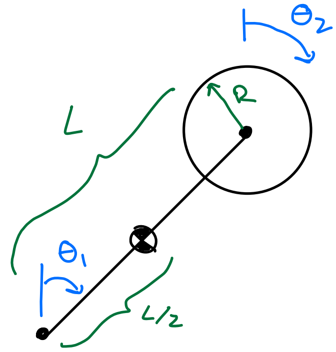
$$M = \Big[$$

$$T = \underbrace{\frac{1}{2}\dot{\theta}^T M \dot{\theta}}_{I_2\dot{\theta}_p\dot{\theta}_w} \qquad \dot{\theta} = \begin{bmatrix} \theta_p \\ \theta_w \end{bmatrix}$$

$$M = \begin{bmatrix} I_1 + I_2 + ml + ML^2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$V = mgl\sin\theta_p + MgL\sin\theta_p$$

$$= (\underbrace{ml + ML}_{m_0})g\sin\theta_p = m_0 g\sin\theta_p$$



EOM

$$V = m_1 \frac{L}{2} g\cos\theta_1 + m_2 L g \cos\theta_1$$

$$= \left(m_1\frac{L}{2} + m_2 L\right) g\cos\theta_1 \qquad v_1 = \frac{L}{2}\dot\theta_1$$

$$T_1 = \frac{1}{2} I_1 \dot\theta_1^2 + \frac{1}{2} m_1 v_1^2 = \frac{1}{2} I_1 \dot\theta_1^2 + \frac{1}{2} m_1 \frac{L^2}{4}\dot\theta_1^2$$

$$= \frac{1}{2}\left(\frac{m_1 L^2}{12}\right)\dot\theta_1^2 + \frac{1}{2}\left(\frac{m_1 L^2}{4}\right)\dot\theta_1^2$$

$$= \frac{1}{6} m_1 L^2 \dot\theta_1^2$$

$$T_2 = \frac{1}{2} I_2 \dot\theta_2^2 + \frac{1}{2} I_2 \dot\theta_1^2 + \frac{1}{2} m_2 v_2^2, \quad v_2 = L\dot\theta_1$$

$$T_2 = \frac{1}{2} I_2 \left(\dot\theta_1 + \dot\theta_2\right)^2 + \frac{1}{2} m_2 L^2 \dot\theta_1^2$$

$$\mathcal{L} = \frac{1}{2} I_1 \dot\theta_1^2 + \frac{1}{2} I_2 \dot\theta_1^2 + \frac{1}{2} I_2 \dot\theta_2^2 + \frac{1}{8} m_1 L^2 \dot\theta_1^2 + \frac{1}{2} m_2 L^2 \dot\theta_1^2 - \frac{1}{2} m_1 L g\cos\theta_1$$
$$+ I_2 \dot\theta_1 \dot\theta_2 \qquad\qquad - m_2 L g \cos\theta_1$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} \left| \begin{array}{l} 1)\ I_1\ddot{\theta}_1 + I_2\ddot{\theta}_1 + \frac{1}{4}m_1L^2\ddot{\theta}_1 + m_2L^2\ddot{\theta}_1 + I_2\ddot{\theta}_2 \\ 2)\ I_2\ddot{\theta}_2 + I_2\ddot{\theta}_1 \end{array} \right.$$

$$\frac{\partial \mathcal{L}}{\partial q} \left| \quad \frac{1}{2}m_1Lg\sin\theta_1 + m_2Lg\sin\theta_1 \right.$$

$$1)\ \overbrace{\left(I_1 + I_2 + \frac{1}{4}m_1l^2 + m_2L^2\right)}^{I_0}\ddot{\theta}_1 \overset{+ I_2\ddot{\theta}_2}{} - \underbrace{\left(\frac{1}{2}m_1 + m_2\right)}_{m_0}Lg\sin\theta_1 = 0$$

$$2)\ I_2\ddot{\theta}_2 + I_2\ddot{\theta}_1 = \tau \Rightarrow \ddot{\theta}_2 = \frac{\tau - I_2\ddot{\theta}_1}{I_2} = \frac{\tau}{I_2} - \ddot{\theta}_1$$

$$I_0\ddot{\theta}_1 + I_2\left(\frac{\tau}{I_2} - \ddot{\theta}_1\right) = I_0\ddot{\theta}_1 + \tau - I_2\ddot{\theta}_1$$

$$(I_0 - I_2)\ddot{\theta}_1 + \tau - m_0Lg\sin\theta_1 = 0$$

$$\ddot{\theta}_1 = \frac{m_0Lg\sin\theta_1 - \tau}{I_0 - I_2}$$

$$I_2\ddot{\theta}_2 + I_2\left(\frac{m_0Lg\sin\theta_1 - \tau}{I_0 - I_2}\right) = \tau$$

$$\ddot{\theta}_2 = \frac{\tau}{I_2} - \frac{m_0Lg\sin\theta_1 - \tau}{I_0 - I_2}$$

$$\ddot{\theta}_2 = \frac{(I_0 - I_2)\tau - I_2 m_0 L g \sin\theta_1 + I_2 \tau}{I_2(I_0 - I_2)}$$

$$\ddot{\theta}_2 = \frac{I_2 m_0 L g \sin\theta_1 - I_0 \tau}{I_2(I_2 - I_0)}$$

$$\mathring{\theta}_1 =$$

$$\theta = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\begin{cases} \ddot{\theta}_1 = \dfrac{m_0 L g \sin\theta_1 - \tau}{I_0 - I_2} \\[2ex] \ddot{\theta}_2 = \dfrac{I_2 m_0 L g \sin\theta_1 - I_0 \tau}{I_2(I_2 - I_0)} \end{cases}$$

$$\left. \left[\begin{matrix} \dfrac{m_0 L g \cos\theta_1}{I_0 - I_2} & 0 \\[2ex] \dfrac{m_0 L g \cos\theta_1}{I_2 - I_0} & 0 \end{matrix} \right] \right|_{\theta_1 = 0}$$

$$\begin{bmatrix} \frac{m_0 L g}{I_0 - I_2} & 0 \\ \frac{m_0 L g}{I_2 - I_0} & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{I_0 - I_2} \\ -\frac{I_0}{I_2 (I_2 - I_0)} \end{bmatrix}$$

$$\ddot{\theta}_1 = \frac{m_0 L g \sin\theta_1 - \tau}{I_0 - I_2}$$

$$\ddot{\theta}_2 = \frac{\tau}{I_2} - \frac{m_0 L g \sin\theta_1 - \tau}{I_0 - I_2}$$

$$\dot{X} = \begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{m_0 L g}{I_0 - I_2} & 0 & 0 \\ \frac{-m_0 L g}{I_0 - I_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{I_0 - I_2} \\ \frac{1}{I_2} + \frac{1}{I_0 - I_2} \end{bmatrix} \tau$$