Autonomous Steering Mechanism
Main Report

Contents

Introduction

This project involves designing a mechatronic system which enables the steering mechanism on the 2022-2023 UTS Motorsports FSAE car to be controlled autonomously. The steering system consists of a steering wheel, universal joint linkages and a steering column into a worm drive steering rack.

The engineer must consider ergonomic constraints as the vehicle must be both manually and autonomously operated. Further, competition regulations must be kept in mind throughout the design process so the final solution can form part of a fully rules-compliant competition car.

Successful completion of this project allows this year’s team to migrate to the newer 2022/23 chassis and progress towards our goal of having a rules compliant autonomous Formula SAE car. The stakeholders involved include UTS Motorsports, UTS and team sponsors.

The proposed design composes of a BLDC mounted on a bracket onto the floor of the car, which is accompanied by a wedge which would help orient the motor to be parallel to the steering column. The motor is then connected via a series of gears and pulleys.

Project Documentation

The project scope includes the following design documentation:

- Timeline
- Ideation
- Morphological Table
- Scoring Matrix

Timeline

Below is a flow chart of the process we followed to achieve the final design.

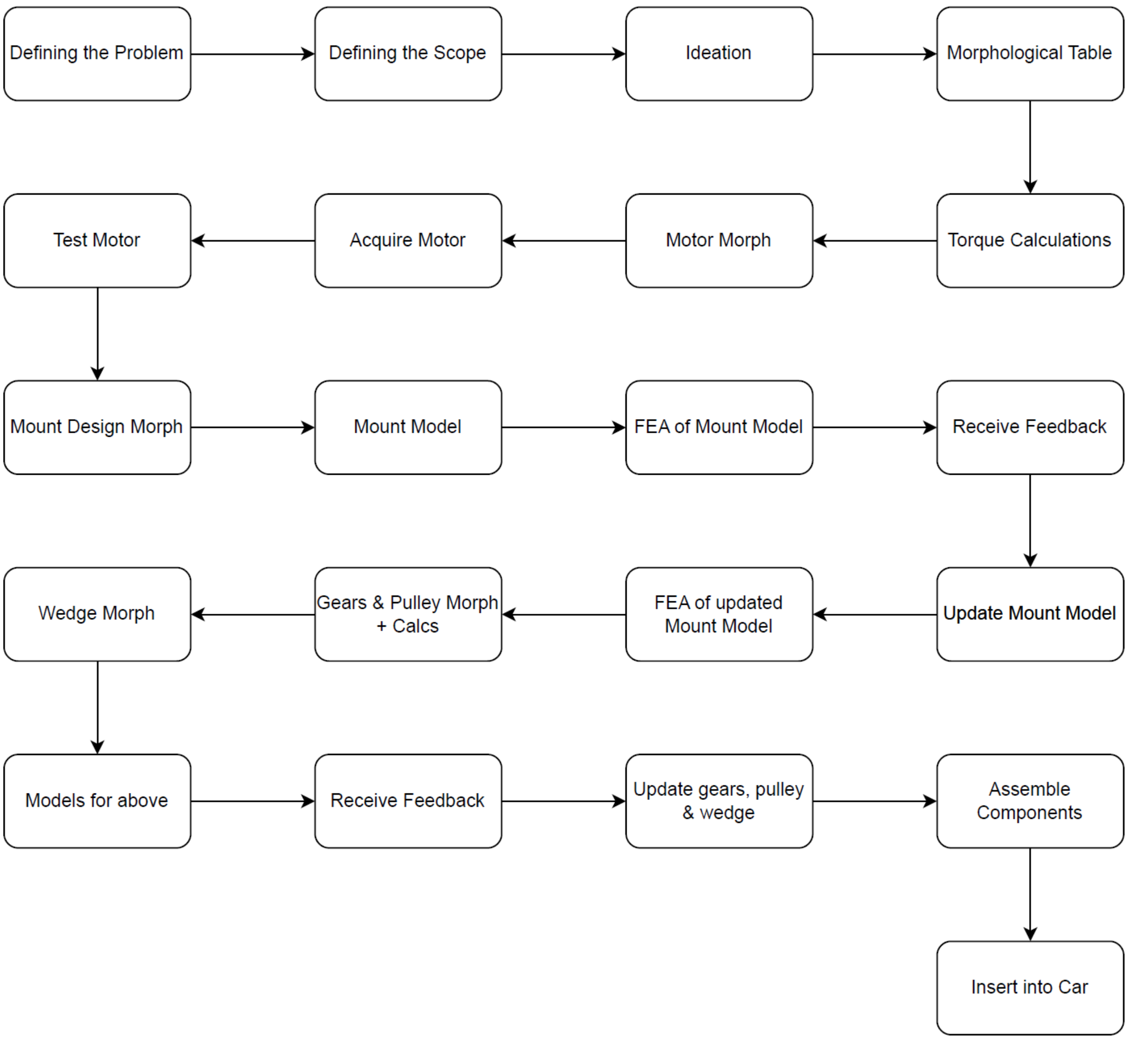


Figure 1: Timeline Flowchart

Problem Statement

Design an electromechanical control system capable of rotating the steering column of the UTSMA autonomous car, ensuring it fits safely and ergonomically within the footwell while complying with the FSAE competition regulations.

Ideation & Morphological Table

Ideating followed the basic principles of discussing how the product could be broken down into sub-assemblies which can be solved individually. These segments include:

1. Motor type

This was mainly condensed to two options: A BLDC or a Stepper. Finally, a BLDC Motor was selected: AK80-9.

2. Coupling Position

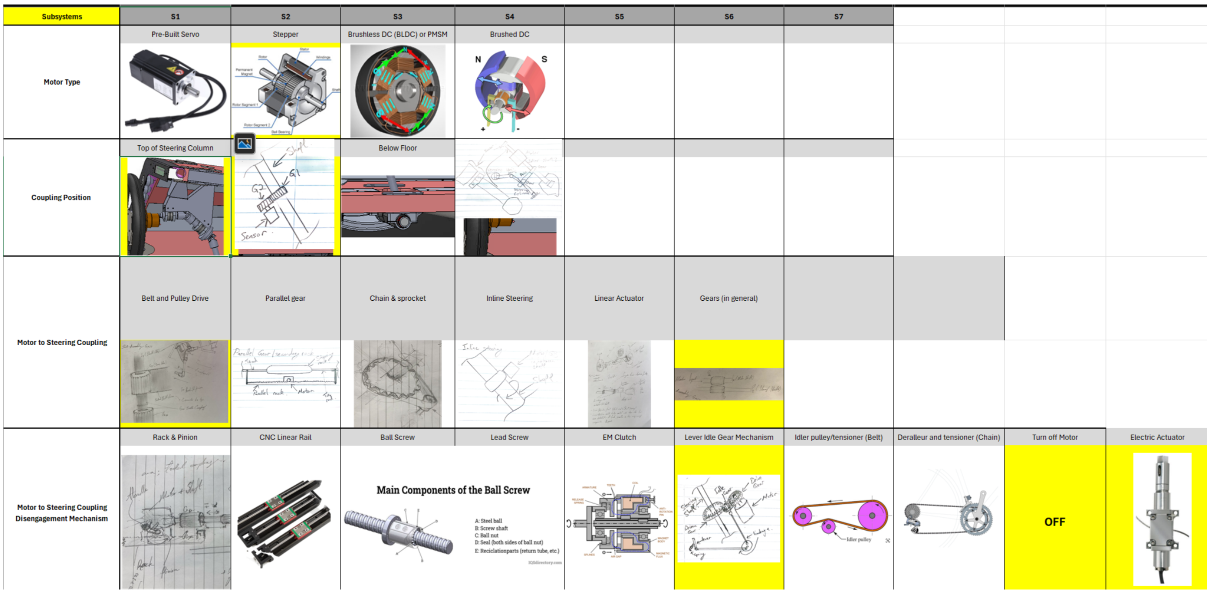
Positioned at the bottom of the steering column to make full use of space constraints with Cockpit Template (Refer to FSAE rules).

3. Motor to Steering Coupling

Belt and Pulley drive

4. Disengagement

Since the selected motor is backdrivable, the system can be electrically disengaged. Therefore, the considered idea of a mechanical disengage was discarded.



Subsystems	S1	S2	S3	S4	S5	S6	S7			
Motor Type	Pre-Built Servo	Stepper	Brushless DC (BLDC) or PMSM	Brushed DC						
Coupling Position	Top of Steering Column		Below Floor							
Motor to Steering Coupling	Belt and Pulley Drive	Parallel gear	Chain & sprocket	Inline Steering	Linear Actuator	Gears (in general)				
Motor to Steering Coupling Disengagement Mechanism	Rack & Pinion	CNC Linear Rail	Ball Screw	Lead Screw	EM Clutch	Lever Idle Gear Mechanism	Idler pulley/tensioner (Belt)	Deralleur and tensioner (Chain)	Turn off Motor	Electric Actuator

Figure 2: Morphological Table

Scoring Matrix

Suitable and relevant components were selected from each of the morphological table categories and made into defined “designs”. This would be rated by the following criteria:

1. Performance

Delivering the calculated amount of torque without any compromise to the motor.

2. Precision & Accuracy

The ability of the steering system to precisely follow the desired path and make accurate adjustments.

3. Reliability & Durability

The system’s ability to consistently perform under various operating conditions and resist wear and tear.

4. Complexity & Integration

How easy the system is to design, implement, and integrate with the rest of the vehicle, including sensors, controllers, and other hardware.

5. Cost-Effectiveness

The overall cost of the steering system, including the cost of components, manufacturing, and maintenance.

6. Maintainability

The ease with which the system can be repaired, upgraded, or serviced.

7. Safety

Built-in safety features, fail-safes, and redundancy to prevent system failures from compromising the vehicle's performance.

8. Modularity

Ease of create a modular variant, that is compliant with FSAE rules.

9. Scalability & Flexibility

Ease of using current design to be used in future iterations.

Using these criteria, the following design was proposed:

BLDC controlled using a magnetic rotary encoder (being positioned directly at the end of the motor shaft). It would be situated in the floor, driven by a set of gears and pulleys. The system can be disengaged by electrically disconnecting the motor, as it is backdrivable. This is shown in the figure below. However, with a slight change in motors led to the sensor being discarded.

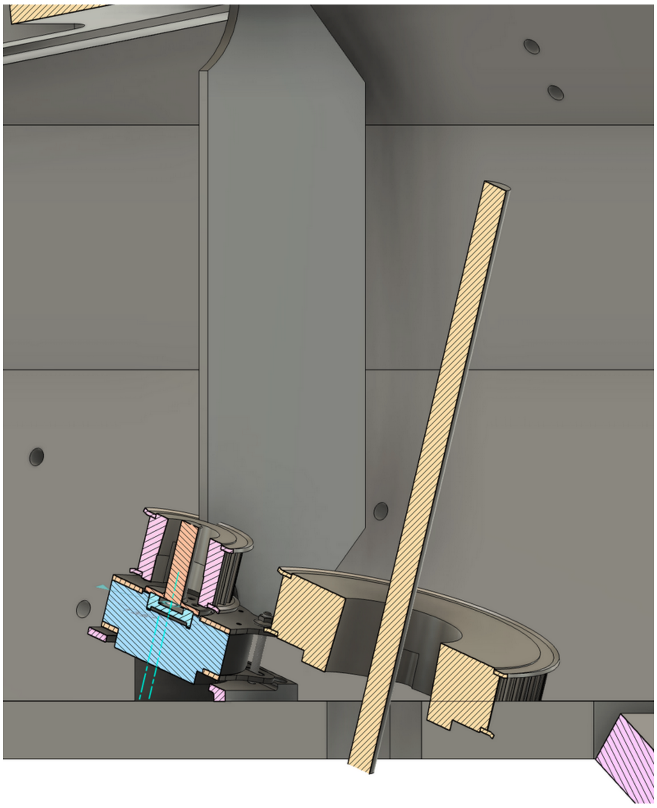


Figure 3: Mechanism fitted in Car Cockpit

Issues & Feedback

- Motor torque calculations took a long time to confirm and validate, resulting in the team being able to purchase component (which has to be verified with tutor using calculations and confirmation from UTSMS) a couple of weeks in.
- Communications were not consistent in the first few weeks, which unfortunately was not completely resolved even by the end of the project.
- Issues with team members not showing up to meetings and not living up to agreements on time.
- Issues with final design not being in accordance with space constraints (refer to CAD and FSAE rules).
- Difficulty to come to decisions about final design.

Design

Pulley/Belt Design

This section covers the complete (for the most part) design for the belt and pulley system for the autonomous steering system.

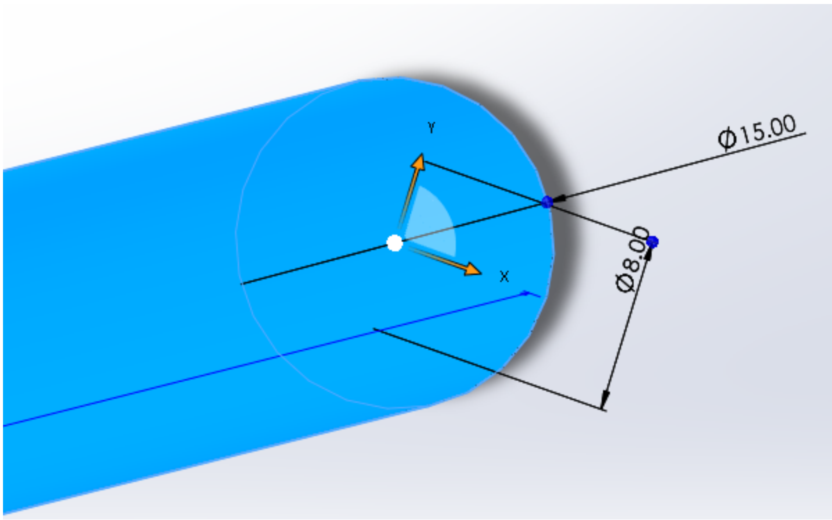
Determine Shaft diameters

From CAD model:

- Steering shaft column outer diameter: 15mm
- Steering shaft column inner diameter: 10mm

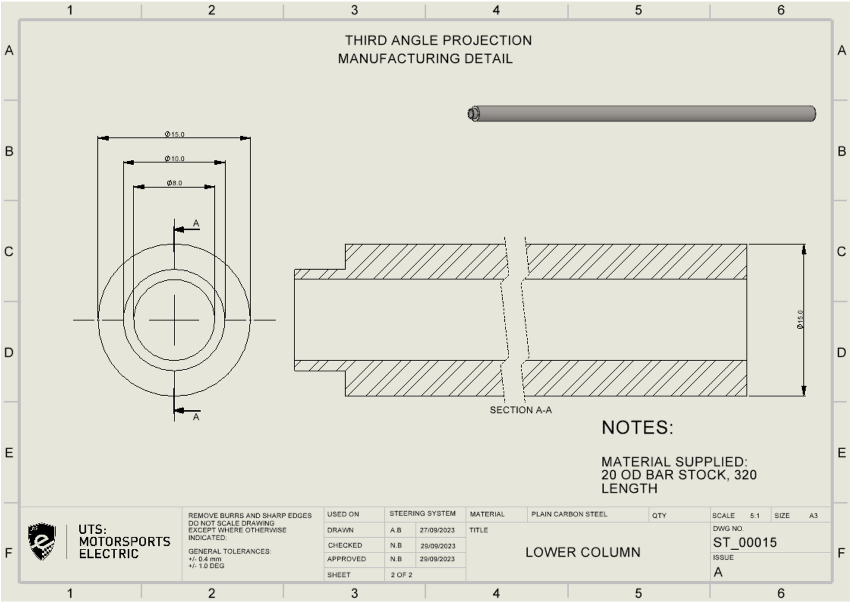
However, on the existing 2022 UTS Motorsports Electric car physical shaft, it is a solid 15mm steering column shaft.

Figure
CAD model close-up of shaft



(Source: UTS Electric Motorsports, 2023)

Figure
Drawing of ’22 steering column



(Source: UTS Electric Motorsports, 2023)

From manual calculations and computer simulations:

Due to the lack of documentation, particularly with the design or the current steering system, step-by-step hand calculations were performed to analyse whether or not the current thickness of 15mm shaft diameter can support the motor’s torque. Please refer to “Analysis of Column Shaft” documentation.
Computer simulations in terms of finite element analysis (FEA) was performed to verify our hand calculations.

Motor shaft diameter:

The motor shaft of the AK1-9 motor will be customised to suit the mounting on the motor hub. The diameter of the shaft will be determined by both torsion, radial forces, along with axial forces.

Figure
Picture of AK10-9 V2.0

(Source: Tmotor, n.d)

Manual calculations and computer simulations:

Both hand calculations and FEA were performed on the design of the motor shaft. Please refer to "Analysis of Column Shaft" documentation.

Belt Selection

There are several types of pulley belts available such as flat, V, Wedge, Synchronous belts along with other types.

Table
Comparison of Belt performance

Table 12.1: Comparison of belt performance.

Parameter	Flat belt drive	V belt drive	Wedge belt drive	Synchronous belt drive
Optimum efficiency	98%	80%	86%	98%
Maximum speed (m/s)	70	30	40	50
Minimum pulley diameter (mm)	40	67	60	16
Maximum speed ratio	20	7	8	9
Optimum tension ratio	2.5	5	5	–

(Source: Childs, Peter R.N.. (2014). Mechanical Design Engineering Handbook. Elsevier)

Figure
Various Belt Cross Sections

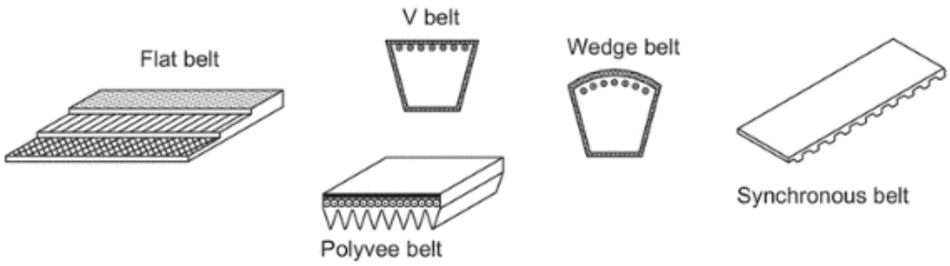


Figure 12.2
Various belt cross sections.

(Various Belts - Source: Childs, Peter R.N.. (2014). Mechanical Design Engineering Handbook. Elsevier)

Referring to the Table 12.1, synchronous belts can output an optimum efficiency of up to 98%, due to their teeth engagement between the belt teeth and the grooves of the pulley. Due to the required precise motion control of an autonomous steering system, a slippage between the belt and the pulley is highly not preferred, which the synchronous belt as the name suggests provides exact shaft synchronization (with the exception of belt creep).
Synchronous belts need significantly lower installation tension compared to V-belts, which results in reduced stress on drive components like shafts and bearings. Hence, along with the Figure 12.4, synchronous belt has been selected.

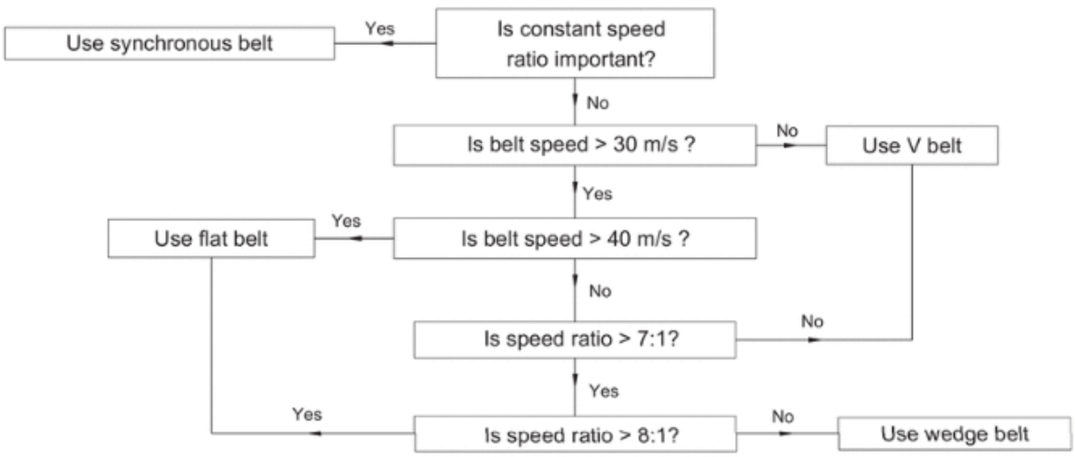


Figure 12.4
Procedure for the selection of belt type. *After Hamilton (1994).*

General Selection Procedure – Synchronous belt pulley system

Define the rotational speeds of the motor shafts

Rated motor (AK10-9 V2) speed:

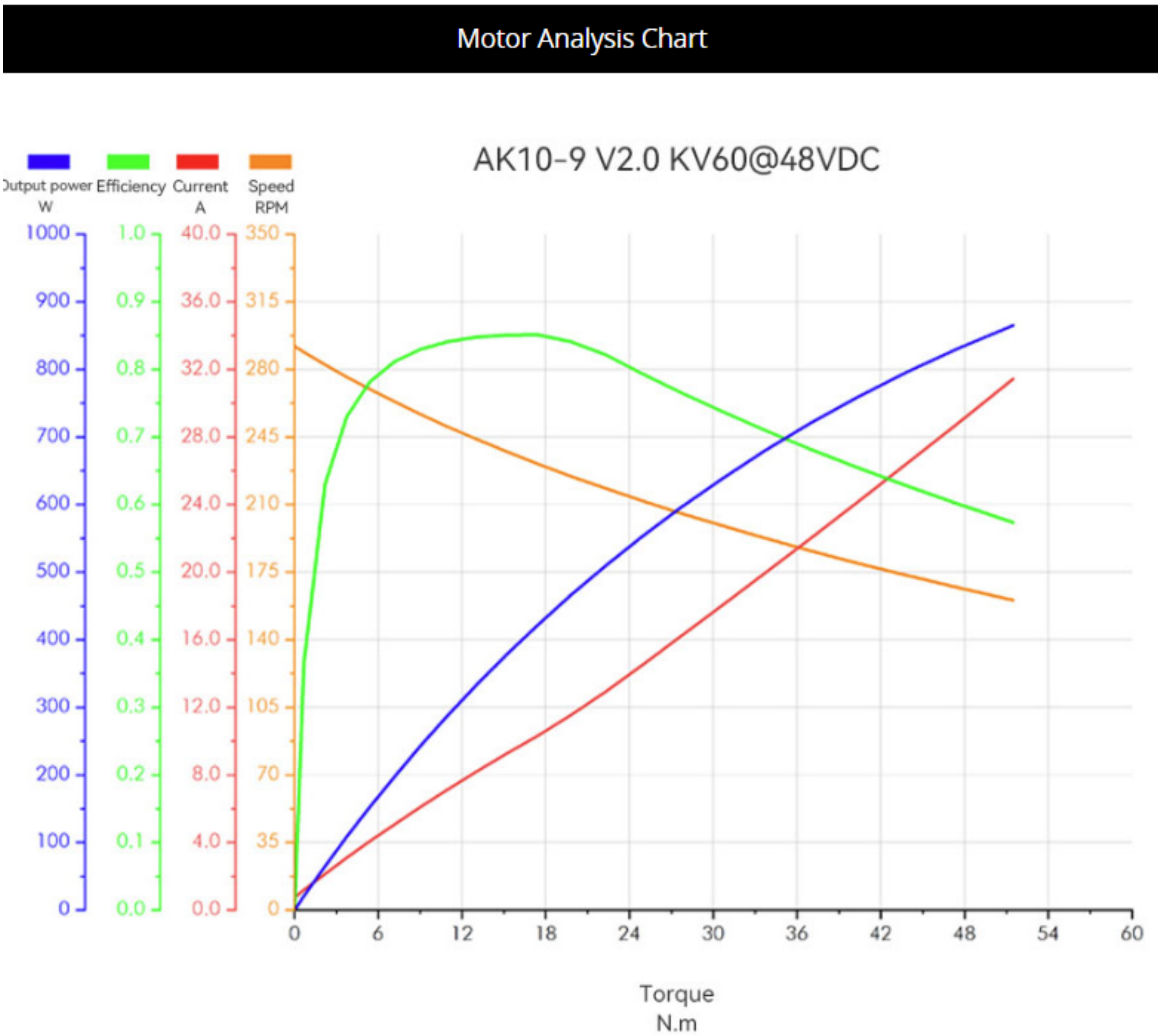
- 228 rpm at 18nm (rated torque)

24.9N Steering force (With Factor of Safety of 1.5 from 16.6N steering force):
- ~200 rpm at 25nm

Motor Analysis Chart:

1) Pulley selection

Table
Motor Analysis Chart



(Source: Tmotor, n.d)

N.m to hp:

$$P(\text{HP}) = \frac{T(\text{Nm}) \times N(\text{RPM})}{7127}$$

For 18nm:

$$P(\text{HP}) = \frac{18(\text{Nm})\times 228(\text{RPM})}{7127} = 0.576\text{HP}$$

For 25nm:

$$P(\text{HP}) = \frac{25(\text{Nm})\times 200(\text{RPM})}{7127} = 0.702\ \text{HP}$$

2) Determine the service factor.

Using Table 7-8 below, using good engineering judgement, a service factor of 1.2 has been selected.

Table

Synchronous Belt Service Factor Table

TABLE 7–8 Service Factor

DriveN machine	DriveR					
The driveN machines listed below are representative samples only. Select a driveN machine whose load characteristics most closely approximate those of the machine being considered.	AC Motors: Normal Torque, Squirrel Cage, Synchronous, Split Phase, Inverter Controlled DC Motors: Shunt Wound Stepper Motors Engines: Multiple Cylinder Internal Combustion			AC Motors: High Torque, High Slip, Repulsion-Induction, Single Phase, Series Wound, Slip Ring DC Motors: Series Wound, Compound Wound Servo Motors Engines: Single Cylinder Internal Combustion, Line Shafts, Clutches		
	Intermittent Service (Up to 8 Hours Daily or Seasonal)	Normal Service (8–16 Hours Daily)	Continuous Service (16–24 Hours Daily)	Intermittent Service (Up to 8 Hours Daily or Seasonal)	Normal Service (8–16 Hours Daily)	Continuous Service (16–24 Hours Daily)
Display, Dispending Equipment Instrumentation Measuring Equipment Medical Equipment Office, Projection Equipment	1.0	1.2	1.4	1.2	1.4	1.6
Appliances, Sweepers, Sewing Machines Screens, Oven Screens, Drum, Conical Woodworking Equipment (Light): Band Saws, Drills Lathes	1.1	1.3	1.5	1.3	1.5	1.7
Agitators for Liquids Conveyors: Belt, Light Package Drill Press, Lathes, Saws Laundry Machinery Wood Working Equipment (Heavy): Circular Saws, Jointers, Planers	1.2	1.4	1.6	1.6	1.8	2.0
Agitators for Semi-Liquids Compressor: Centrifugal Conveyor Belt: Ore, Coal, Sand Dough Mixers Line Shafts Machine Tools: Grinder, Shaper, Boring Mill, Milling Machines Paper Machinery (except Pulpers): Presses, Punches, Shears Printing Machinery Pumps: Centrifugal, Gear Screens: Revolving, Vibratory	1.3	1.5	1.7	1.6	1.8	2.0
Brick Machinery (except Pug Mills) Conveyor: Apron, Pan, Bucket, Elevator Extractors, Washers Fans, Centrifugal Blowers Generators & Exciters Hoists Rubber Calender, Mills, Extruders	1.4	1.6	1.8	1.8	2.0	2.2
Centrifuges Screw Conveyors Hammer Mills Paper Pulpers Textile Machinery	1.5	1.7	1.9	1.9	2.1	2.3
Blowers: Positive Displacement, Mine Fans Pulvertzers	1.6	1.8	2.0	2.0	2.2	2.4
Compressors: Reciprocating Crushers: Gyratory, Jaw, Rol Mils: Ball, Rod, Pebble, etc. Pumps: Reciprocating Saw Mill Equipment	1.7	1.9	2.1	2.1	2.3	2.5

These service factors are adequate for most belt drive applications. Note that service factors cannot be substituted for good engineering judgment. Service factors may be adjusted based upon an understanding of the severity of actual drive operating conditions.

(Source: Mott, Robert L, 2018)

3) Calculate the design power.

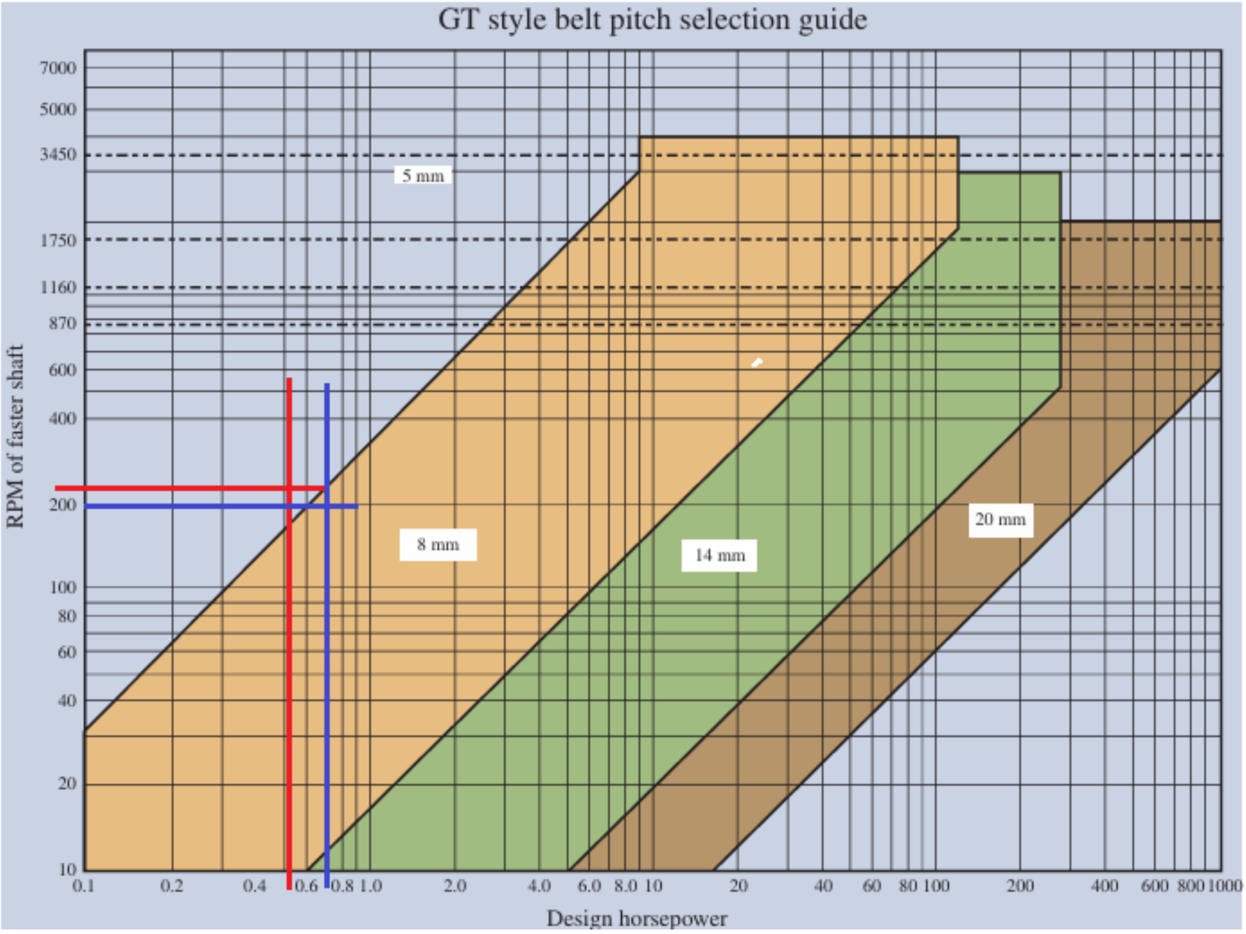
$$Design\ power = P_{des} = P_{rated} \cdot SF$$

Using the formula above;

- the design rated power is = 0.576HP x 1.2 = 0.691 HP (0.691 x 0.7457 = 0.52 kW)
- the design peak power is = 0.702 HP x 1.2 = 0.8424 HP (0.8424 x 0.7457 = 0.63kW)

Determine required pitch of the belt.

Graph
GT Belt Selection Guide



(Source: Mott, Robert L, 2018)

Using the table above, the red lines depict the selection for the rated torque, whilst the blue lines indicate the use of peak torque. Since the majority of the imaginary curve from the red line intersection to the blue line intersection, a 5mm belt pitch would be considered a reasonable design choice. However, 8mm had to be chosen due to belt selection requirements.

Determine the velocity ratio VR belt between the driver and driven sprockets.

$$VR = \frac{\omega_{driving}}{\omega_{driven}} = \frac{PD_{driven}}{PD_{driving}} = \frac{N_{driven}}{N_{driving}}$$

To determine the sizes/ratio of the driven and driver pulleys, giving the following:

Motor operational torque: 18nm
Required torque: 24.9nm

An adequate pulley size ratio is needed to meet the steering force requirement of 24.9nm.

As the torque ratio is directly related to the velocity ratio:

$$\text{Torque ratio} = \frac{Torque\ Required\ \ (Driven)}{Motor\ Torque\ (Driver)}$$

Using the values of 24.9nm of steering torque (with FOS 1.5), with 24.9nm of required torque,

$$\textbf{Torque ratio} = \frac{\mathbf{24.9\ nm}}{\mathbf{24.9 nm}} \approx \mathbf{1}$$

Therefore the driven pulley (on steering column shaft) must be **1** or more greater than the driver pulley (on the motor).

When the car is in motion, using a dynamic torque of 27nm (from 13.5N x FOS 2), At moderate dynamic or during sudden spikes:

Motor rated torque: 18nm
Required torque: 27nm

$$\textbf{Torque ratio} = \frac{\mathbf{27\ nm}}{\mathbf{18 nm}} \approx \mathbf{1.5}$$

Thus, the driven pulley (attached to the steering column shaft) needs to be at least 1.38 times larger than the driver pulley (connected to the motor).

Pulley selection

5mm Pitch Belt:

SYNCHRONOUS BELT DRIVES

HTD 5M Drives

CENTRE DISTANCE IN MILLIMETRES

Speed Ratio	Number of grooves on Driving Pulley	Number of grooves on Driven Pulley	Belt pitch length in millimetres: 305 (61 teeth)	325 (65 teeth)	350 (70 teeth)	375 (75 teeth)	400 (80 teeth)	425 (85 teeth)	450 (90 teeth)	475 (95 teeth)	500 (100 teeth)	575 (115 teeth)	600 (120 teeth)	640 (128 teeth)	700 (140 teeth)	800 (160 teeth)	890 (178 teeth)	980 (196 teeth)	1100 (220 teeth)	1200 (240 teeth)	1420 (284 teeth)	Speed Ratio
1.71	28	48	–	–	78	91	104	116	129	142	154	192	204	224	255	305	350	395	455	505	615	1.71
1.75	32	56	–	–	–	75	88	101	113	126	139	176	189	209	239	289	334	380	440	490	600	1.75
1.75	64	112	–	–	–	–	–	–	–	–	–	–	–	–	–	176	222	267	328	378	489	1.75
1.78	36	64	–	–	–	–	–	65	97	110	123	161	174	194	224	274	319	364	424	474	585	1.78
1.80	40	72	–	–	–	–	–	–	–	94	107	145	158	178	208	259	304	349	409	459	569	1.80
1.82	44	80	–	–	–	–	–	–	–	–	–	129	142	162	193	243	289	334	394	444	554	1.82
1.88	48	90	–	–	–	–	–	–	–	–	–	–	123	144	174	225	270	316	376	426	536	1.88
1.88	34	64	–	–	–	–	–	–	100	113	125	163	176	196	226	276	322	367	427	477	587	1.88
1.89	72	136	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	224	285	336	447	1.89
1.89	38	72	–	–	–	–	–	–	–	–	109	148	160	180	211	261	306	351	412	462	572	1.89
2.00	28	56	–	–	–	79	92	105	118	131	143	181	194	214	244	294	339	384	444	494	605	2.00
2.00	32	64	–	–	–	–	–	89	102	115	127	166	178	198	229	279	324	369	429	479	589	2.00
2.00	36	72	–	–	–	–	–	–	–	98	111	150	163	183	213	263	309	354	414	464	574	2.00
2.00	40	80	–	–	–	–	–	–	–	–	–	134	147	167	197	248	293	339	399	449	559	2.00
2.00	56	112	–	–	–	–	–	–	–	–	–	–	–	–	–	185	231	276	337	387	498	2.00
2.05	44	90	–	–	–	–	–	–	–	–	–	–	127	148	179	230	275	320	381	431	541	2.05
2.11	38	80	–	–	–	–	–	–	–	–	–	136	149	169	200	250	296	341	401	451	562	2.11
2.12	34	72	–	–	–	–	–	–	–	–	113	152	165	185	215	266	311	356	416	467	577	2.12
2.13	64	136	–	–	–	–	–	–	–	–	–	–	–	–	–	–	186	233	294	345	456	2.13
2.22	36	80	–	–	–	–	–	–	–	–	–	138	151	171	202	253	298	343	403	454	564	2.22
2.25	32	72	–	–	–	–	–	–	–	103	116	154	167	187	218	268	313	359	419	469	579	2.25
2.25	40	90	–	–	–	–	–	–	–	–	–	–	132	152	183	234	280	325	385	436	546	2.25
2.29	28	64	–	–	–	–	–	93	106	119	132	170	183	203	233	284	329	374	434	484	594	2.29
2.33	48	112	–	–	–	–	–	–	–	–	–	–	–	–	–	193	240	285	346	397	507	2.33
2.35	34	80	–	–	–	–	–	–	–	–	–	140	153	174	204	255	300	346	406	456	566	2.35
2.37	38	90	–	–	–	–	–	–	–	–	–	120	134	154	185	236	282	327	388	438	548	2.37
2.43	56	136	–	–	–	–	–	–	–	–	–	–	–	–	–	–	195	242	303	354	466	2.43
2.50	32	80	–	–	–	–	–	–	–	–	–	142	155	176	206	257	303	348	408	458	569	2.50
2.50	36	90	–	–	–	–	–	–	–	–	–	122	136	157	188	239	284	330	390	440	551	2.50
2.55	44	112	–	–	–	–	–	–	–	–	–	–	–	–	145	198	244	290	351	401	512	2.55
2.57	28	72	–	–	–	–	–	–	–	107	120	159	171	192	222	273	318	363	424	474	584	2.57
2.65	34	90	–	–	–	–	–	–	–	–	–	125	137	159	190	241	287	332	392	443	553	2.65
2.80	40	112	–	–	–	–	–	–	–	–	–	–	–	–	149	202	248	294	355	406	517	2.80
2.81	32	90	–	–	–	–	–	–	–	–	–	127	140	161	192	243	289	334	395	445	556	2.81
2.83	48	136	–	–	–	–	–	–	–	–	–	–	–	–	–	–	203	250	312	363	475	2.83
2.86	28	80	–	–	–	–	–	–	–	93	107	147	160	180	211	262	307	353	413	463	574	2.86
2.95	38	112	–	–	–	–	–	–	–	–	–	–	–	–	151	204	251	297	358	408	519	2.95
3.09	44	136	–	–	–	–	–	–	–	–	–	–	–	–	–	–	207	254	317	368	479	3.09
3.11	36	112	–	–	–	–	–	–	–	–	–	–	–	–	153	206	253	299	360	411	521	3.11
3.21	28	90	–	–	–	–	–	–	–	–	–	131	144	165	196	248	293	339	399	450	560	3.21
3.29	34	112	–	–	–	–	–	–	–	–	–	–	–	–	155	208	255	301	362	413	524	3.29
3.40	40	136	–	–	–	–	–	–	–	–	–	–	–	–	–	162	211	259	321	372	484	3.40
3.50	32	112	–	–	–	–	–	–	–	–	–	–	–	–	157	210	257	303	364	415	526	3.50
3.58	38	136	–	–	–	–	–	–	–	–	–	–	–	–	–	164	213	261	323	374	486	3.58
3.78	36	136	–	–	–	–	–	–	–	–	–	–	–	–	–	166	215	263	325	377	489	3.78
4.00	28	112	–	–	–	–	–	–	–	–	–	–	–	127	161	215	261	308	369	420	531	4.00
4.00	34	136	–	–	–	–	–	–	–	–	–	–	–	–	–	168	217	265	327	379	491	4.00
4.25	32	136	–	–	–	–	–	–	–	–	–	–	–	–	–	170	219	267	330	381	493	4.25
4.86	28	136	–	–	–	–	–	–	–	–	–	–	–	–	–	174	223	271	334	385	498	4.86

All centre distances are rounded values – Consult your local Authorised Distributor if centre distance is fixed.

POWER RATINGS (KW) FOR 15mm WIDE 5M BELT

Rev/min of small pulley	Number of grooves											
	28	**32**	**34**	**36**	**38**	**40**	**44**	**48**	**56**	**64**	**72**	**80**
20	0.02	0.02	0.02	0.04	0.04	0.04	0.04	0.04	0.06	0.06	0.06	0.08
40	0.04	0.06	0.06	0.06	0.06	0.08	0.08	0.09	0.09	0.11	0.13	0.15
60	0.06	0.08	0.08	0.09	0.09	0.09	0.11	0.13	0.15	0.17	0.19	0.23
100	0.09	0.11	0.13	0.13	0.15	0.17	0.19	0.21	0.25	0.28	0.32	0.36
200	0.21	0.25	0.26	0.28	0.30	0.32	0.38	0.43	0.51	0.59	0.64	0.72
300	0.26	0.32	0.36	0.38	0.42	0.43	0.51	0.57	0.68	0.77	0.87	0.96
400	0.34	0.40	0.43	0.47	0.51	0.55	0.62	0.70	0.83	0.95	1.06	1.17
500	0.40	0.47	0.51	0.55	0.59	0.64	0.72	0.81	0.96	1.10	1.25	1.38
600	0.45	0.55	0.59	0.62	0.68	0.72	0.81	0.93	1.10	1.25	1.42	1.57
720	**0.51**	**0.62**	**0.66**	**0.72**	**0.77**	**0.81**	**0.95**	**1.06**	**1.25**	**1.42**	**1.61**	**1.78**
800	0.57	0.66	0.72	0.77	0.83	0.89	1.00	1.13	1.34	1.53	1.72	1.91
960	**0.64**	**0.76**	**0.79**	**0.83**	**0.89**	**0.95**	**1.15**	**1.29**	**1.53**	**1.74**	**1.95**	**2.15**
1000	0.66	0.79	0.87	0.91	0.98	1.04	1.19	1.32	1.57	1.80	2.00	2.23
1200	0.76	0.89	0.96	1.04	1.12	1.19	1.34	1.49	1.78	2.02	2.27	2.46
1440	**0.87**	**1.02**	**1.12**	**1.19**	**1.27**	**1.36**	**1.53**	**1.70**	**2.00**	**2.29**	**2.57**	**2.84**
1600	0.95	1.12	1.19	1.29	1.36	1.46	1.64	1.83	2.11	2.46	2.76	3.06
2000	1.12	1.30	1.40	1.49	1.61	1.70	1.91	2.14	2.51	2.85	3.19	3.52
2400	1.27	1.47	1.59	1.70	1.83	1.97	2.17	2.40	2.84	3.21	3.57	3.93
2880	**1.44**	**1.70**	**1.83**	**1.97**	**2.08**	**2.19**	**2.44**	**2.70**	**3.16**	**3.57**	**4.01**	**4.33**
4000	1.81	2.12	2.27	2.40	2.55	2.70	3.01	3.31	3.80	4.23	4.61	4.91
5000	2.10	2.44	2.59	2.76	2.91	3.08	3.38	3.69	4.18	4.54	4.80	4.95
6000	2.36	2.70	2.87	3.04	3.19	3.36	3.67	3.93	4.35	4.55	4.59	4.46
8000	2.76	3.10	3.25	3.38	3.52	3.65	3.84	3.97	3.97	3.55		
10000	2.99	3.23	3.31	3.40	3.44	3.48	3.44	3.27				

BELT LENGTH CORRECTION FACTORS (multiplier)

Belt Length (mm)	305 - 400	450 - 500	575 - 800	890 -1200	1270 - 2250
Correction Factor	0.8	0.9	1.0	1.1	1.2

BELT WIDTH FACTORS

Belt width mm	9	15
Width factor	0.53	1.00

8mm Pitch Belt:

Table

Screenshot of 8mm Synchronous Belt Drive Table Part 1

SYNCHRONOUS BELT DRIVES

4 Fenner Torque Drive Plus® *3* 8MXP & HTD 8M Drives

CENTRE DISTANCE IN MILLIMETRES

Speed Ratio	Number of grooves on: Driving Pulley	Number of grooves on: Driven Pulley	Belt pitch length in millimetres: 480 (60 teeth)	560 (70 teeth)	600 (75 teeth)	640 (80 teeth)	720 (90 teeth)	800 (100 teeth)	880 (110 teeth)	960 (120 teeth)	1040 (130 teeth)	1120 (140 teeth)	1200 (150 teeth)	1280 (160 teeth)	1440 (180 teeth)	1600 (200 teeth)	1760 (220 teeth)	1800 (225 teeth)	2000 (250 teeth)	2400 (300 teeth)	2600 (325 teeth)	2800 (350 teeth)	Speed Ratio
1.00	24	24	144	184	204	224	264	304	344	384	424	464	504	544	624	704	784	804	904	1104	1204	1304	1.00
1.00	26	26	136	176	196	216	256	296	336	376	416	456	496	536	616	696	776	796	896	1096	1196	1296	1.00
1.00	28	28	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.00
1.00	30	30	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.00
1.00	32	32	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.00
1.00	34	34	104	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.00
1.00	36	36	–	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.00
1.00	38	38	–	128	148	168	208	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.00
1.00	40	40	–	120	140	160	200	240	280	320	360	400	440	480	560	640	720	740	840	1040	1140	1240	1.00
1.00	44	44	–	–	–	144	184	224	264	304	344	384	424	464	544	624	704	724	824	1024	1124	1224	1.00
1.00	48	48	–	–	–	–	168	208	248	288	328	368	408	448	528	608	688	708	808	1008	1108	1208	1.00
1.00	56	56	–	–	–	–	–	176	216	256	296	336	376	416	496	576	656	676	776	976	1076	1176	1.00
1.00	64	64	–	–	–	–	–	–	184	224	264	304	344	384	464	544	624	644	744	944	1044	1144	1.00
1.00	72	72	–	–	–	–	–	–	–	–	232	272	312	352	432	512	592	612	712	912	1012	1112	1.00
1.00	80	80	–	–	–	–	–	–	–	–	–	240	280	320	400	480	560	580	680	880	980	1080	1.00
1.05	38	40	–	124	144	164	204	244	284	324	364	404	444	484	564	644	724	744	844	1044	1144	1244	1.05
1.06	36	38	–	132	152	172	212	252	292	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.06
1.06	34	36	–	140	160	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.06
1.06	32	34	108	148	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.06
1.07	30	32	116	156	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.07
1.07	28	30	124	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.07
1.08	26	28	132	172	192	212	252	292	332	372	412	452	492	532	612	692	772	792	892	1092	1192	1292	1.08
1.08	24	26	140	180	200	220	260	300	340	380	420	460	500	540	620	700	780	800	900	1100	1200	1300	1.08
1.09	44	48	–	–	–	136	176	216	256	296	336	376	416	456	536	616	696	716	816	1016	1116	1216	1.09
1.10	40	44	–	–	132	152	192	232	272	312	352	392	432	472	552	632	712	732	832	1032	1132	1232	1.10
1.11	36	40	–	128	148	168	208	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.11
1.11	72	80	–	–	–	–	–	–	–	–	216	256	296	336	416	496	576	596	696	896	996	1096	1.11
1.12	34	38	–	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.12
1.13	32	36	104	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.13
1.13	64	72	–	–	–	–	–	–	–	208	248	288	328	368	448	528	608	628	728	928	1028	1128	1.13
1.13	80	90	–	–	–	–	–	–	–	–	–	–	260	300	380	460	540	560	660	860	960	1060	1.13
1.13	30	34	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.13
1.14	28	32	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.14
1.14	56	64	–	–	–	–	–	–	200	240	280	320	360	400	480	560	640	660	760	960	1060	1160	1.14
1.15	26	30	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.15
1.16	38	44	–	–	136	156	196	236	276	316	356	396	436	476	556	636	716	736	836	1036	1136	1236	1.16
1.17	24	28	136	176	196	216	256	296	336	376	416	456	496	536	616	696	776	796	896	1096	1196	1296	1.17
1.17	48	56	–	–	–	–	152	192	232	272	312	352	392	432	512	592	672	692	792	992	1092	1192	1.17
1.18	34	40	–	132	152	172	212	252	292	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.18
1.19	32	38	–	140	160	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.19
1.20	30	36	108	148	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.20
1.20	40	48	–	–	–	144	184	224	264	304	344	384	424	464	544	624	704	724	824	1024	1124	1224	1.20
1.21	28	34	116	156	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.21
1.22	36	44	–	120	140	160	200	240	280	320	360	400	440	480	560	640	720	740	840	1040	1140	1240	1.22
1.23	26	32	124	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.23
1.25	24	30	132	172	192	212	252	292	332	372	412	452	492	532	612	692	772	792	892	1092	1192	1292	1.25
1.25	32	40	–	136	156	176	216	256	296	336	376	416	456	496	576	656	736	756	856	1056	1156	1256	1.25
1.25	64	80	–	–	–	–	–	–	–	–	231	271	311	351	432	512	592	612	712	912	1012	1112	1.25
1.25	72	90	–	–	–	–	–	–	–	–	–	235	275	315	395	475	556	576	676	876	976	1076	1.25
1.26	38	48	–	–	127	147	188	228	268	308	348	388	428	468	548	628	708	728	828	1028	1128	1228	1.26
1.27	30	38	103	144	164	184	224	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.27
1.27	44	56	–	–	–	–	159	199	240	280	320	360	400	440	520	600	680	700	800	1000	1100	1200	1.27
1.29	28	36	112	152	172	192	232	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.29
1.29	56	72	–	–	–	–	–	–	183	223	263	303	343	383	464	544	624	644	744	944	1044	1144	1.29
1.29	34	44	–	123	143	164	204	244	284	324	364	404	444	484	564	644	724	744	844	1044	1144	1244	1.29
1.31	26	34	120	160	180	200	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.31
1.33	24	32	128	168	188	208	248	288	328	368	408	448	488	528	608	688	768	788	888	1088	1188	1288	1.33
1.33	30	40	–	139	159	180	220	260	300	340	380	420	460	500	580	660	740	760	860	1060	1160	1260	1.33
1.33	36	48	–	–	131	151	191	231	272	312	352	392	432	472	552	632	712	732	832	1032	1132	1232	1.33
1.33	48	64	–	–	–	–	–	175	215	255	295	335	375	416	496	576	656	676	776	976	1076	1176	1.33
1.36	28	38	107	147	168	188	228	268	308	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.36
1.38	32	44	–	127	147	167	207	248	288	328	368	408	448	488	568	648	728	748	848	1048	1148	1248	1.38
1.38	26	36	115	155	176	196	236	276	316	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.38
1.40	40	56	–	–	–	–	167	207	247	287	327	367	407	448	528	608	688	708	808	1008	1108	1208	1.40
1.40	80	112	–	–	–	–	–	–	–	–	–	–	–	–	334	414	494	514	615	815	915	1015	1.40

All centre distances are rounded values – Consult your local Authorised Distributor if centre distance is fixed.

(Source: Fenner, 2006)

Table

Screenshot of 8mm Synchronous Belt Drive Table Part 2

Fenner Torque Drive Plus® *3* MXP & HTD 8M Drives

CENTRE DISTANCE IN MILLIMETRES

Speed Ratio	Number of grooves on: Driving Pulley	Number of grooves on: Driven Pulley	Belt pitch length in millimetres: 480 (60 teeth)	560 (70 teeth)	600 (75 teeth)	640 (80 teeth)	720 (90 teeth)	800 (100 teeth)	880 (110 teeth)	960 (120 teeth)	1040 (130 teeth)	1120 (140 teeth)	1200 (150 teeth)	1280 (160 teeth)	1440 (180 teeth)	1600 (200 teeth)	1760 (220 teeth)	1800 (225 teeth)	2000 (250 teeth)	2400 (300 teeth)	2600 (325 teeth)	2800 (350 teeth)	Speed Ratio
1.41	64	90	–	–	–	–	–	–	–	–	–	250	290	330	411	491	571	591	691	891	991	1091	1.41
1.41	34	48	–	–	135	155	195	235	275	315	356	396	436	476	556	636	716	736	836	1036	1136	1236	1.41
1.42	24	34	123	164	184	204	244	284	324	364	404	444	484	524	604	684	764	784	884	1084	1184	1284	1.42
1.43	28	40	103	143	163	183	223	264	304	344	384	424	464	504	584	664	744	764	864	1064	1164	1264	1.43
1.43	56	80	–	–	–	–	–	–	–	206	246	286	327	367	447	527	607	627	727	927	1028	1128	1.43
1.45	44	64	–	–	–	–	–	182	223	263	303	343	383	423	503	583	664	684	784	984	1084	1184	1.45
1.46	26	38	111	151	171	191	231	272	312	352	392	432	472	512	592	672	752	772	872	1072	1172	1272	1.46
1.47	30	44	–	131	151	171	211	251	291	332	372	412	452	492	572	652	732	752	852	1052	1152	1252	1.47
1.47	38	56	–	–	–	–	170	211	251	291	331	371	411	451	532	612	692	712	812	1012	1112	1212	1.47
1.50	24	36	119	159	179	199	240	280	320	360	400	440	480	520	600	680	760	780	880	1080	1180	1280	1.50
1.50	32	48	–	118	139	159	199	239	279	319	359	399	440	480	560	640	720	740	840	1040	1140	1240	1.50
1.50	48	72	–	–	–	–	–	–	198	238	278	319	359	399	479	559	639	659	759	960	1060	1160	1.50
1.54	26	40	107	147	167	187	227	267	307	348	388	428	468	508	588	668	748	768	868	1068	1168	1268	1.54
1.56	36	56	–	–	–	134	174	214	255	295	335	375	415	455	535	615	696	716	816	1016	1116	1216	1.56
1.56	72	112	–	–	–	–	–	–	–	–	–	–	–	267	348	429	509	530	630	830	931	1031	1.56
1.57	28	44	–	134	155	175	215	255	295	335	375	416	456	496	576	656	736	756	856	1956	1156	1256	1.57
1.58	24	38	115	155	175	195	235	275	315	356	396	436	476	516	596	676	756	776	876	1076	1176	1276	1.58
1.60	30	48	–	122	142	162	203	243	283	323	363	403	443	483	564	644	724	744	844	1044	1144	1244	1.60
1.60	40	64	–	–	–	–	149	190	230	270	310	351	391	431	511	591	671	691	791	992	1092	1192	1.60
1.61	56	90	–	–	–	–	–	–	–	–	224	264	305	345	426	506	586	606	707	907	1007	1107	1.61
1.64	44	72	–	–	–	–	–	164	205	245	286	326	366	406	487	567	647	667	767	967	1067	1167	1.64
1.65	34	56	–	–	–	137	178	218	258	299	339	379	419	459	539	619	699	719	820	1020	1120	1220	1.65
1.67	24	40	110	151	171	191	231	271	311	351	391	432	472	512	592	672	752	772	872	1072	1172	1272	1.67
1.67	48	80	–	–	–	–	–	–	179	220	261	301	342	382	462	542	623	643	743	943	1043	1143	1.67
1.68	38	64	–	–	–	–	152	193	234	274	314	354	395	435	515	595	675	695	795	995	1095	1196	1.68
1.69	26	44	–	138	158	179	219	259	299	339	379	419	459	499	580	660	740	760	860	1060	1160	1260	1.69
1.71	28	48	–	125	146	166	206	247	287	327	367	407	447	487	567	647	728	748	848	1048	1148	1248	1.71
1.75	32	56	–	–	–	141	181	222	262	302	343	383	423	463	543	623	703	723	823	1024	1124	1224	1.75
1.75	64	112	–	–	–	–	–	–	–	–	–	–	240	281	363	444	524	545	645	846	946	1046	1.75
1.78	36	64	–	–	–	–	156	197	237	278	318	358	398	439	519	599	679	699	799	999	1099	1199	1.78
1.80	40	72	–	–	–	–	–	171	212	253	293	334	374	414	494	575	655	675	775	975	1075	1175	1.80
1.80	80	144	–	–	–	–	–	–	–	–	–	–	–	–	–	342	424	445	546	748	848	948	1.80
1.82	44	80	–	–	–	–	–	–	186	227	268	309	349	389	470	550	630	650	751	951	1051	1151	1.82
1.83	24	44	–	142	162	182	223	263	303	343	383	423	463	503	583	664	744	764	864	1064	1164	1264	1.83
1.85	26	48	–	129	149	170	210	250	291	331	371	411	451	491	571	651	731	751	852	1052	1152	1252	1.85
1.87	30	56	–	–	–	144	185	226	266	306	346	387	427	467	547	627	707	727	827	1027	1128	1228	1.87
1.88	48	90	–	–	–	–	–	–	–	197	238	279	320	360	441	521	602	622	722	922	1023	1123	1.88
1.88	34	64	–	–	–	–	159	200	241	281	322	362	402	442	523	603	683	703	803	1003	1103	1203	1.88
1.89	38	72	–	–	–	–	–	175	216	256	297	337	378	418	498	578	659	679	779	979	1079	1179	1.89
2.00	24	48	–	132	153	173	214	254	294	335	375	415	455	495	575	655	735	755	855	1056	1156	1256	2.00

(Source: Fenner. 2006)

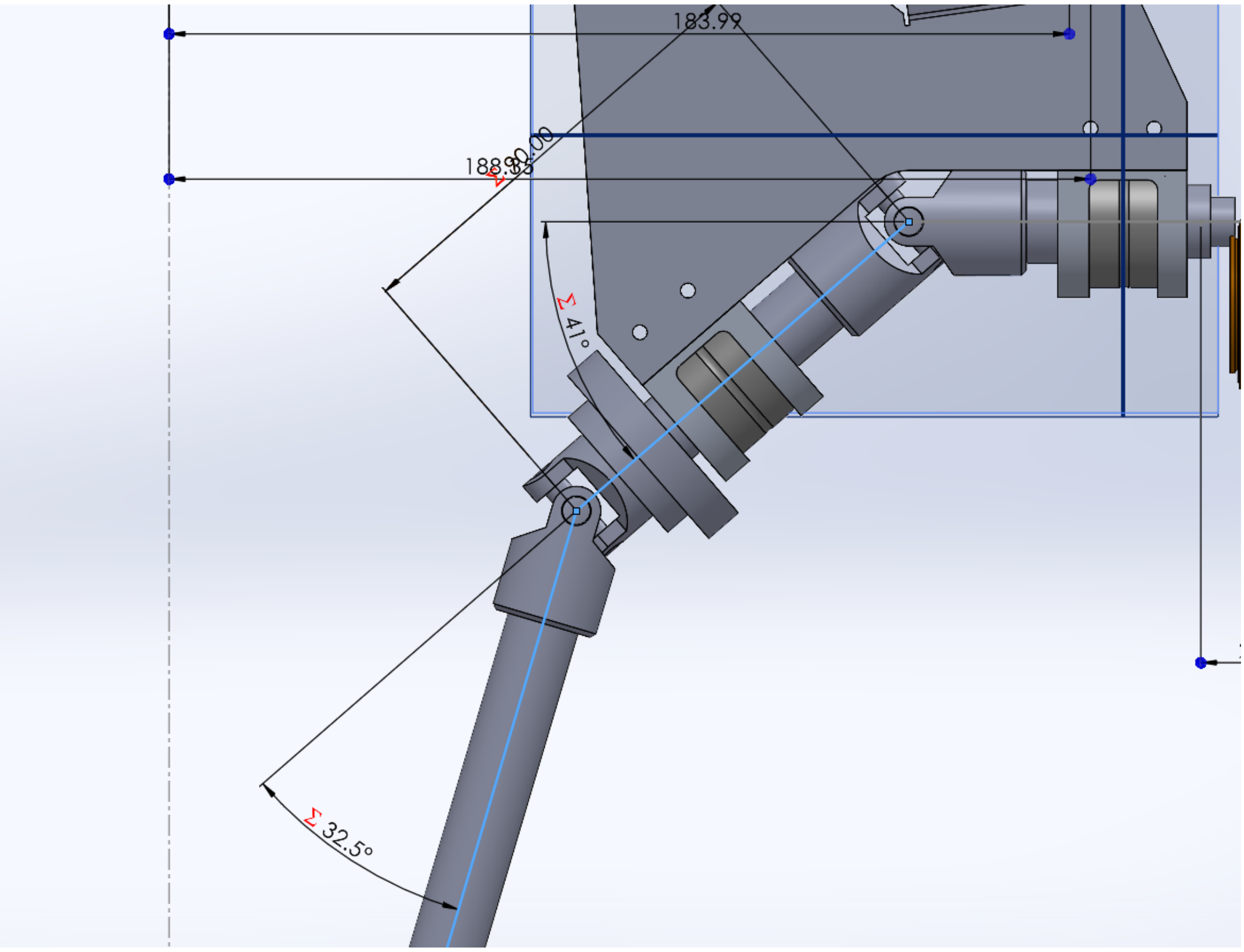
Analysis of Motor and Column Shaft

Obtain dimensions from Solidworks ’22 Full Car Model

Double U-joint Analysis

As there was no documentation on the design of the current steering shaft geometry model, an investigation was conducted.

Figure



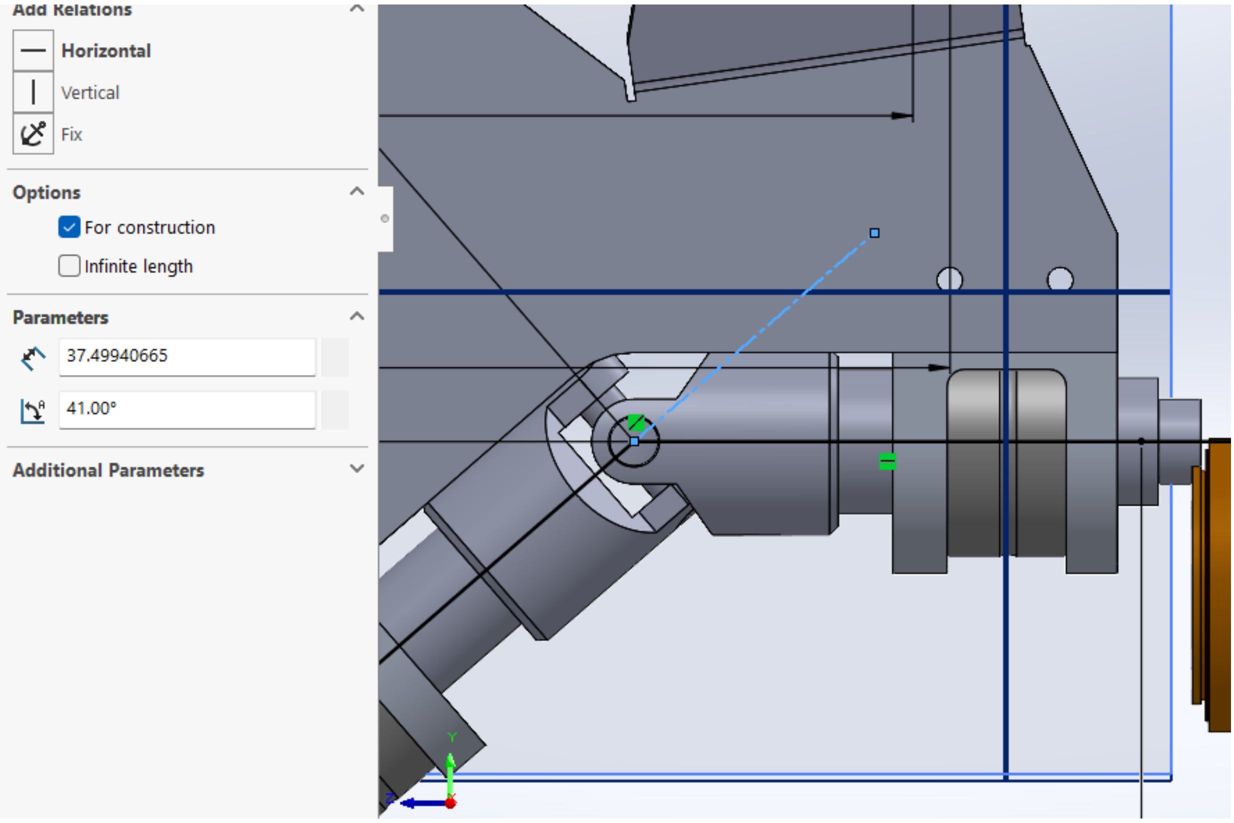
(Screenshot 1 of the lower joint angle and the entire double U-joint)

The current U-joint is double U-joint configuration, however through investigating the joint angles in Solidworks shown in the pictures above and below, the lower U-joint is angled at 32.5 degrees from the centre component axis, whilst the highest U-joint angle is approximately 37.5 degrees.`

As both of the angles are not the same, a constant velocity relation is not present, meaning that the angular velocity on both outer shaft ends are not equal.

This can cause torsional vibrations, which can accelerate wear and tear of the shafts' bearings and joints. (RegalRexnord, nd.), and the overall efficiency of the steering joint mechanism (C.W Spicer, 1915).

Figure



(Screenshot 1 of the upper joint angle - closeup)

Due to time constraints and the lack of viability of Solution 1, Torsion was the only force calculated for the motor shaft and steering column for Solution 1. However, for Solution 2, both torsion and bending stress of the shafts has been hand-calculated and put through finite element analysis (FEA).

Calculation – Motor Shaft Torsion – Solution 1

Polar moment of inertia of a solid bar, J:

$$J = \frac{\pi}{2} r^4$$

$$= \frac{\pi}{32} D^4$$

Using reasonable engineering judgement, 15mm diameter bar has been selected.

$$J = \frac{\pi}{32}(0.015^4) = 4.97 \times 10^{-9} \text{ m}^4$$

Shear Stress, τ:

$$\mathrm{T} = \frac{Tr}{J}$$

Using rated 9nm torque for AK80-9 Motor:

$$\tau = \frac{9 \cdot \left(\frac{0.015}{2}\right)}{4.97 \times 10^{-9} \text{ m}^4} \approx 13.58 \times 10^6 \text{ Pa} = 13.58 \text{ MPa}$$

Using peak 18nm torque Calculation (Solution 1), with AK80-9 Motor, or rated torque for AK10-9 (Solution 2) :

$$\tau = \frac{18 \cdot \left(\frac{0.015}{2}\right)}{4.97 \times 10^{-9} \text{ m}^4} \approx 27.16 \times 10^6 \text{ Pa} = 27.16 \text{ MPa}$$

Allowable shear stress estimate for steel can be given as:

τallow ≈ 0.5*ultimate yield stress

From a resource for general properties of steel, the yield strength of steel is 350 MPa.

τallow ≈ 0.5*350 = 175 MPa

Since the allowable shear stress is 175 MPa, which is 12.89 times higher than the theoretical shear stress of 13.58 MPa in the shaft, the shaft can safely support a torque of 9nm. With 18nm, since 175 MPa is 6.44 times greater than 27.16 MPa, the shaft can also safely support the peak torque of 18nm.

Calculation – Hollow Steering Shaft Torsion – Solution 1

The shaft's outer diameter is 15mm of the '22 CAD model, however for the selected larger pulley for the steering shaft, part-named P72-8MGT-30, the minimum required shaft diameter/bore size for the 2517 taper locking bushing is be used as a pulley-to-shaft hub connection available to purchase in the Australian market (as of writing), is 16mm.
Analysing the current 15mm outer diameter, 10mm inner diameter of the main section for of the steering shaft:

Since the main shaft is hollow, the polar moment J for a hollow circular shaft must be used:

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$$

$$J = \frac{\pi}{32}(0.015^4 - 0.01^4) = 3.99 \times 10^{-9}\ \mathrm{m}^4$$

Shear Stress, τ:

$$\mathrm{T} = \frac{Tr}{J}$$

where T is the applied torque, r is the distance from the centre to the stressed surface.

Autonomous condition: Torque applied to shaft via motor – 9nm rated torque:

Using gear ratio of 2.86, the rated torque at the motor is 9nm, the torque at the driven is:

9nm x 2.86 = 25.74Nm.

Shear Stress, τ:

$$\tau = \frac{25.74 \cdot \left(\frac{0.015}{2}\right)}{3.99 \times 10^{-9}} \approx 48.40\ \mathrm{MPa}$$

Autonomous condition: Torque applied to shaft via motor – 18nm peak AK80-9 torque:

Using gear ratio of 2.86, the rated torque at the motor is 18nm, the torque at the driven is:

18nm x 2.86 = 51.48Nm

Shear Stress, τ:

$$\tau = \frac{51.48 \cdot \left(\frac{0.01}{2}\right)}{3.99 \times 10^{-9}} \approx 96.77 \text{ MPa}$$

Manual drive condition: Torque applied to shaft via steering wheel:

Using 25nm from FOS of 16.6nm steering torque. Due to vector quantity, the torques are not additive. When equal and opposite torques of 25 Nm are applied at both ends of the shaft, it results in a constant torque of 25 Nm on the shaft.

Shear Stress, τ:

$$\tau = \frac{25 \cdot \left(\frac{0.015}{2}\right)}{3.99 \times 10^{-9}} \approx 46.99 \text{ MPa}$$

Allowable shear stress estimate for steel can be given as:

τallow ≈ 0.5*ultimate yield stress

From a resource for general properties of steel, the yield strength of steel is 350 MPa.

τallow ≈ 0.5*350 = 175 MPa

Autonomous condition: Torque applied to shaft via motor – 9nm rated torque:

Since the yield strength is 350 MPa, which is over 7 times higher than the theoretical shear stress of 48.40 MPa in the shaft, the shaft can safely support a torque of 9 Nm.

Autonomous condition: Torque applied to shaft via motor – 18nm rated torque:

Given that the yield strength is 350 MPa, which is more than 3.5 times the theoretical shear stress of 96.77 MPa in the shaft, it can safely withstand a torque of 18 Nm.

Manual drive condition: Torque applied to shaft via steering wheel:

Since the yield strength is 350 MPa, which is over seven times higher than the theoretical shear stress of 46.99 MPa in the shaft, the shaft can safely support a torque of 25 Nm with a reasonable factor of safety.

Calculation – Steering Column Shaft Torsion:

Polar moment of inertia of a solid bar, J:

$$J = \frac{\pi}{2} r^4$$

$$= \frac{\pi}{32} D^4$$

Using 15mm Shaft

$$J = \frac{\pi}{32}(0.015^4) = 4.97 \times 10^{-9} \text{ m}^4$$

Shear Stress τ:

$$\text{T} = \frac{Tr}{J}$$

Using gear ratio of 2.86, the torque at the motor is 25nm (with FOS), the torque at the driven is:

25nm x 2.86 = 71.5Nm

Shear Stress:

$$\tau = \frac{71.5 \cdot \left(\frac{0.02}{2}\right)}{4.97 \times 10^{-9}} \approx 143.86 \times 10^6 \text{ Pa} = 143.86 \text{ MPa}$$

Allowable shear stress estimate for steel can be given as:

τallow ≈ 0.5*ultimate yield stress

From a resource for general properties of steel, the yield strength of steel is 350 MPa.

τallow ≈ 0.5*350 = 175 MPa

Since the allowable shear stress is 175 MPa, which is 1.21 times higher than the theoretical shear stress of 143.86 MPa in the shaft, the shaft can safely support a torque of 25 Nm. This includes a factor of safety (FOS) of 1.5, based on a steering torque of 16.6 Nm.

Calculation – Motor Shaft Torsion – Solution 2

Polar moment of inertia of a solid bar, J:

$$J = \frac{\pi}{2} r^4$$

$$= \frac{\pi}{32} D^4$$

20mm diameter bar has also been selected.

$$J = \frac{\pi}{32}(0.020^4) = 1.57 \times 10^{-8}\ \mathrm{m}^4$$

Shear Stress, τ:

$$\mathrm{T} = \frac{Tr}{J}$$

Using rated 18nm torque for AK10-9 Motor:

$$\tau = \frac{18 \cdot \left(\frac{0.020}{2}\right)}{1.57 \times 10^{-8}\ \mathrm{m}^4} \approx 11.46 \times 10^6\ \mathrm{Pa} = 11.46\ \mathrm{MPa}$$

The minimum diameter shaft of 9mm:

$$\tau = \frac{18 \cdot \left(\frac{0.009}{2}\right)}{J} \approx 125.75\ \mathrm{MPa}$$

The minimum diameter shaft of 12mm for taper lock bushing:

$$\tau = \frac{18 \cdot \left(\frac{0.012}{2}\right)}{J} \approx 53.05\ \mathrm{MPa}$$

Allowable shear stress estimate for steel can be given as:

τallow ≈ 0.5*ultimate yield stress

From a resource for general properties of steel, the yield strength of steel is 350 MPa.

τallow ≈ 0.5*350 = 175 MPa

Since the allowable shear stress is 175 MPa, which is 15.27 times higher than the theoretical shear stress of 11.46 MPa in the 20mm shaft, shaft can safely support a torque of 18nm.
The minimum shaft to resist a torsion of 18nm is shaft size of 9mm, which is under the allowable shear stress of 175MPa. However, the smallest available taper lock bushing for the driver pulley is 12mm. As 53.05 MPa is smaller than the 350MPa yield strength of steel by a factor of 6.6, a 12mm shaft diameter is sound.

Calculation – Steering Column Shaft Torsion:

Using 15mm Shaft:

Polar moment of inertia of a solid bar, J:

$$J = \frac{\pi}{2} r^4$$

$$= \frac{\pi}{32} D^4$$

$$J = \frac{\pi}{32}(0.015^4) = 4.97 \times 10^{-9}\ \text{m}^4$$

Shear Stress, τ:

$$\text{T} = \frac{Tr}{J}$$

Using gear ratio of 1.5, the torque at the motor is 25nm (with FOS), the torque at the driven is:

25nm x 1.5 = 37.5Nm

Shear Stress:

$$\tau = \frac{37.5 \cdot \left(\frac{0.015}{2}\right)}{4.97 \times 10^{-9}} \approx 56.59\ \text{MPa}$$

Using 20mm Shaft:

Polar moment of inertia of a solid bar, J:

$$J = \frac{\pi}{2} r^4$$

$$= \frac{\pi}{32} D^4$$

$$J = \frac{\pi}{32}(0.020^4) = 1.57 \times 10^{-8} \text{ m}^4$$

Shear Stress, τ:

$$\text{T} = \frac{Tr}{J}$$

Using gear ratio of 1.5, the torque at the motor is 25nm (with FOS), the torque at the driven is:

25nm x 1.5 = 37.5Nm

Shear Stress:

$$\tau = \frac{37.5 \cdot \left(\frac{0.02}{2}\right)}{1.57 \times 10^{-8}} \approx 23.89 \times 10^6 \text{ Pa} = 23.89 \text{ MPa}$$

Allowable shear stress estimate for steel can be given as:

τallow ≈ 0.5*ultimate yield stress

Using 22mm shaft:

$$J = \frac{\pi}{32}(0.022^4) = 2.31 \times 10^{-8} \text{ m}^4$$

Shear Stress:

$$\tau = \frac{37.5 \cdot \left(\frac{0.022}{2}\right)}{J} \approx 23.89 \times 10^6 \text{ Pa} = 17.86 \text{ MPa}$$

From a resource for general properties of steel from MatWeb, the yield strength of steel is 350 MPa.

τallow ≈ 0.5*350 = 175 MPa

For 15mm diameter shaft, Since the allowable shear stress is 175 MPa, which is much greater than the theoretical shear stress of 56.69MPa in the shaft, the shaft can safely support a torque of 25Nm, with a FOS of approximately 3.09.

For 20mm diameter shaft, since the allowable shear stress is 175 MPa, which is 7.3 times higher than the theoretical shear stress of 23.89 MPa in the shaft, the 20mm shaft can safely support a torque of 25 Nm. This includes a factor of safety (FOS) of 1.5, based on a steering torque of 16.6 Nm.

Likewise for the 22mm shaft, since it has a factor of safety of 15.59 from the value of 350 MPa divided by 17.86 MPa, the 22mm diameter is extremely over-engineered, on terms of undertaking torsion only force.

Radial Force Calculation – Solution 2

Combined Radial, bending and Torsion Calculation – Solution 2

For Motor Shaft:

Using Von Mises stress criterion for combing loading, we can calculate the combined effects of bending and shear stresses, where the alternating $\boldsymbol{\sigma_a}$ or mean stress $\boldsymbol{\sigma_m}$ can be calculated by:

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2}$$

Assuming bending moment applied to shaft is a mean value, of 12mm shaft, where σb is the bending stress of 81.46 MPa, and τ is the torsional shear stress of 54.05MPa:

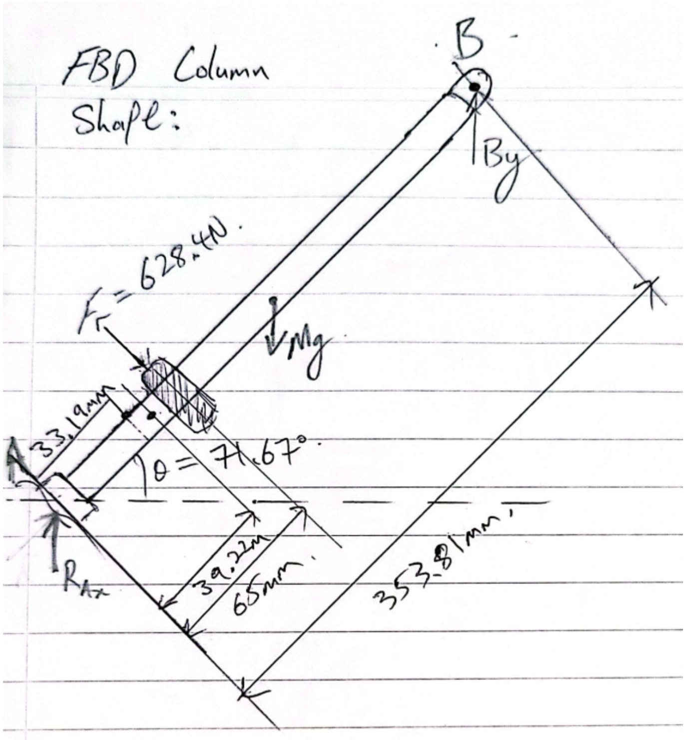
$$\boldsymbol{\sigma = (81.46^2 \mathrm{MPa} + 3 \times 53.05^2)^{1/2}}$$

= 122.80 MPa

As the allowable yield stress of 175MPa, the shaft 12mm diameter is within design limits. Comparing the to the yield strength of 350MPa, there is a factor of safety of 2.85 compared to the von Mises stress value of 122.80MPa.

For Steering Column Shaft:

Free Body Diagram (FBD) of Lower Shaft Column:



Calculate Maximum Bending Moment:

Mass of column shaft:

- Density, ρ : $8 g/cm^3 = 8000 kg/m^3$
- Shaft diameter = 23mm.
- Length shaft = 353.81mm.
- Volume shaft =

Mass, M: $M = \rho \times V$

$= 8000 \times 1.45 \times 10^{-4}$

$\approx 1.16 kg$

$V = \pi \left(\frac{d}{2}\right)^2 L$

$= \pi \left(\frac{0.023}{2}\right)^2 \times 0.35381$

$= 1.45 \times 10^{-4} m^3$

Moment at A - ↻ $\Sigma M_A = 0$:

$-628.4N(65 \times 10^{-3} m) - Mg\left(\frac{353.81}{2} \times 10^{-3} m\right)\cos 71.67 + B_y \cdot (353.81 \times 10^{-3} m) \times \cos 71.67 = 0$ (1)

$B_y = \frac{628.4(65 \times 10^{-3}) + 11.38(353.81/2 \times 10^{-3} m)}{353.81 \times 10^{-3} m \cdot \cos 71.67°}$

$= 385.18N$

Sub into (1):

$M_A \Rightarrow -84.34Nm = \underline{84.34Nm}$ (↺).

Moment at B - ↺ $\Sigma M_B = 0$:

$-628.4N[(353.81 - 65) \times 10^{-3}] - Mg \times \left(\frac{353.81}{2} \times 10^{-3} m\right) \times \cos 71.67° + R_{Ax} \cos 71.67 (353.81 \times 10^{-3}) = 0$ (2)

Find R_{Ax}:

$+\uparrow \Sigma F_y = 0$:

$+R_{Ax} - Mg + B_y - 628.4 \cos 71.67 = 0$

$= +R_{Ax} - 11.3 + 386.18 - (-197.63)$

$R_{Ax} = 176.35N$ (↓).

Sub into (2):

$M_B = -182.12N + 176.35 \cos 71.67 (353.81 \times 10^{-3})$.

$= -162.5\ Nm \Rightarrow \underline{162.5Nm}$ (↻).

Scanned with CamScanner

(Figure - Calculation 2 – Bending moment)

Referring the moment calculations above, neglecting the collective weight of the column shaft, pinion and U joint, the maximum moment occurs at around point B of 162.5Nm.

Calculate Bending Stress:

$$\sigma_{b=}\frac{M \times y}{I}$$

Where y is the distance from neutral axis to the outer fibre, and I is the moment of inertia of a solid shaft, πd^4/ 64. Let the motor shaft diameter d be 20mm.

$$\sigma_{b=}\frac{162.5 \text{ Nm} \times (20 \times 10^{-3}/2)}{I}=206.90\ MPa$$

Since the $206.90MPa$ does not satisfy the allowable stress of 175MPa, a 20mm shaft diameter is not sufficient in diameter. Therefore, the current diameter of 15mm column shaft is not adequate.

Let the column shaft diameter be 25mm:

$$\sigma_{b=}\frac{162.5 \text{ Nm} \times (25 \times 10^{-3}/2)}{I}= 105.93\ MPa$$

As 105.93 MPa is below the yield strength of 350 MPa for steel by a factor of 3.3, 25mm size shaft will be able to safely resist the bending moment.

The minimum suitable shaft diameter using general steel is 22mm:

$$\sigma_{b=}\frac{162.5 \text{ Nm} \times (22 \times 10^{-3}/2)}{I}=155.45\ MPa$$

As $155.45\ MPa$ is under the allowable shear stress of 175MPa, a diameter of 22mm is suitable for use.

Through selecting to improve the material's properties, particularly yield strength through several methods such as heat treatment, or replacing the general steel with a steel that contains higher yield strength, a smaller diameter shaft can be used.

Using AISI 4140 heated treated to 870 degrees Celsius, it has a yield strength of 655MPa. (MtWeb, n.d) With a factor of safety of 6.18 and 4.21 compared to the shaft's bending stress of the 25mm and 23mm shafts respectively, both scenarios surpasses the allowable yield strength of this particular 4140 steel at 327.5 MPa.

The minimum shaft diameter to satisfy the allowable yield strength (or FOS of 2), using 4140 steel for bending only is the size of 17mm:

$$\sigma_{b=}\frac{162.5 \text{ Nm} \times ((17 \times 10^{-3})/2)}{I}=336.90\ MPa$$

Thus, a value of 336.90 MPa has a FOS of 2.06 from the heat treated 4140 yield strength of 655MPa.

Neglecting Axial loads, using the Von Mises Stresses formula(s) for 22mm shaft (general steel):

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2}$$

$$\sigma = (155.45^2 \text{ MPa} + 3 \times 17.86^2)^{1/2}$$

= 158.5 MPa

The shaft of 22mm diameter does not satisfy the allowable yield stress of 175MPa, with a FOS of 2.2, compared the von mises stress value of 158.5 MPa to the yield strength of 350MPa.

Recalculating torsion for the shaft 17mm diameter for the selected 4140 steel:

Shear Stress:

$$\tau = \frac{37.5 \cdot \left(\frac{0.017}{2}\right)}{J} \approx 38.87 \text{ MPa}$$

Recalculating Von Mises Stresses:

$$\boldsymbol{\sigma} = (336.90^2 \text{ MPa} + 3 \times 38.87^2)^{1/2}$$

= 343.56.02 MPa

As the allowable shear stress of 4140 Steel is 343.56.02 MPa, the 17mm shaft diameter does not satisfies the 2 times factor of safety of the allowable shear stress. However, the use of the 17mm shaft may be considered.

Verify if 18mm 4140 is feasible:

Shear Stress:

$$\tau = \frac{37.5 \cdot \left(\frac{0.018}{2}\right)}{J} \approx 32.75\ MPa$$

Bending moment:

$$\sigma_b = \frac{162.5 \text{ Nm} \times ((18 \times 10^{-3})/2)}{I} = 283.81\ MPa$$

Von Mises Stresses:

$$\boldsymbol{\sigma}=(283.81^2 \text{ MPa} + 3 \times 32.75^2)^{1/2}$$

$$= 289.42 \text{ MPa}$$

As the value of 289.42MPa is under 327.5MPa of the allowable shear stress, a shaft of 18mm of 4140 steel suitable.

Axial Forces

Axial loads are usually comparatively small at critical locations where bending and torsions forces occur, therefore axial forces of both of the motor and column shafts have been omitted out of the equations., as the

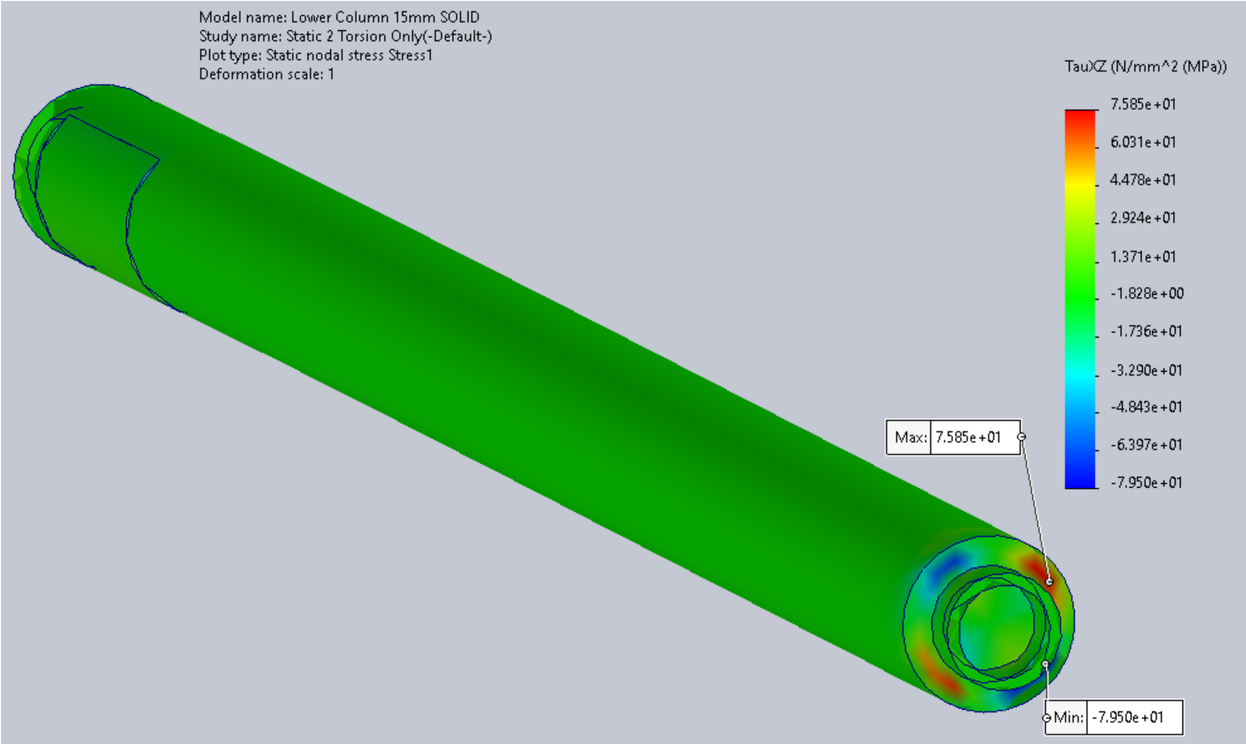
Finite Element Analysis Simulation

Using Solidworks, Finite Element Analysis (FEA) simulation was performed on the steering column shaft.

Steering Column Shaft – Torsion forces modeled only

15mm diameter general steel (1020):

Modelling for torsion as the only modelled force, at the driver pulley's location, which from the centre of the belt is approximately 65mm to the centre of the steering rack's pinion on our physical prototype's model, which is about 27mm from the bottom shaft's edge. on the current 15mm diameter shaft (modified from hollow to solid)



(Figure - Screenshot of FEA Simulation – Shear Stress – Default mesh density)

(Screenshot of FEA Simulation – Shear Stress - Lower side of Column 15mm Shaft)

(Screenshot of FEA Simulation – FOS - Upper side of Column 15mm Shaft)

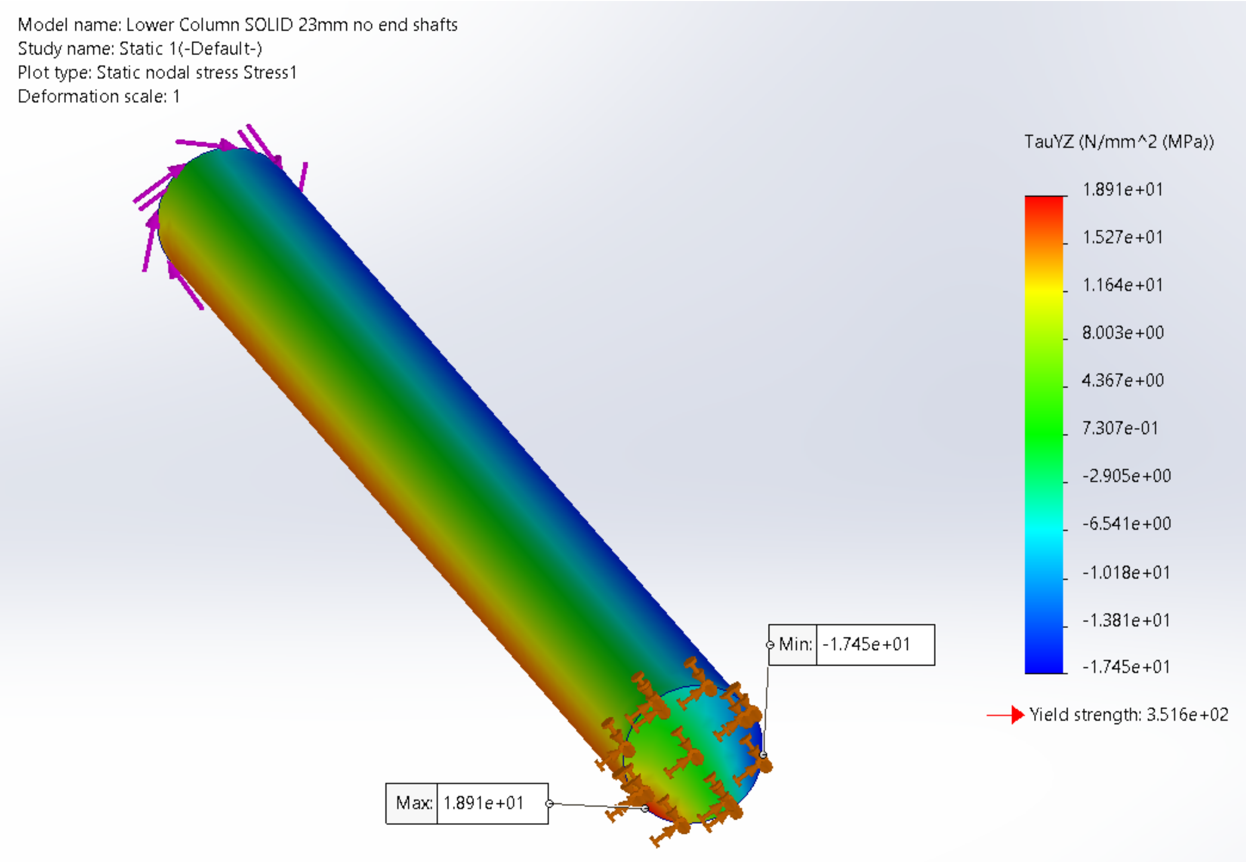
(Screenshot of FEA Simulation – FOS - Lower side of Column 15mm Shaft)

23mm diameter general steel (1020):

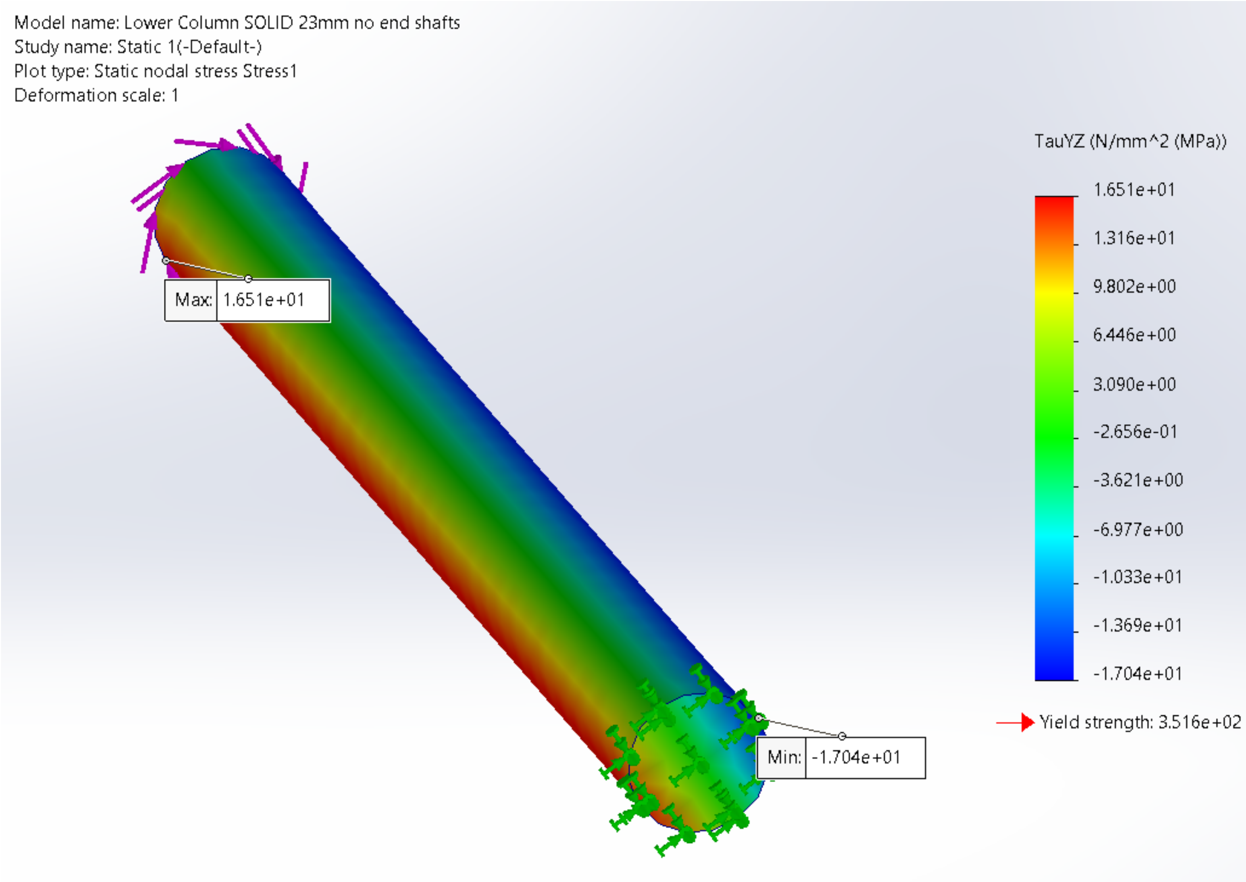
To more accurately simulate the torsion only along the main diameter of the shaft instead of with the smaller shaft extrusions on both ends, simulations on terms of shear stress along the length of the shaft was conducted with a plain 23mm shaft, of various mesh densities, on the coarsest setting, one other on the default setting, whilst the other one was conducted on the finest setting.

From the Figures below, it can be seen that the finest mesh simulation, depicted the closest results out of the three simulations, which is almost identical results of a maximum shear stress of 15.83 MPa compared to the hand-calculated value of 15.70MPa.

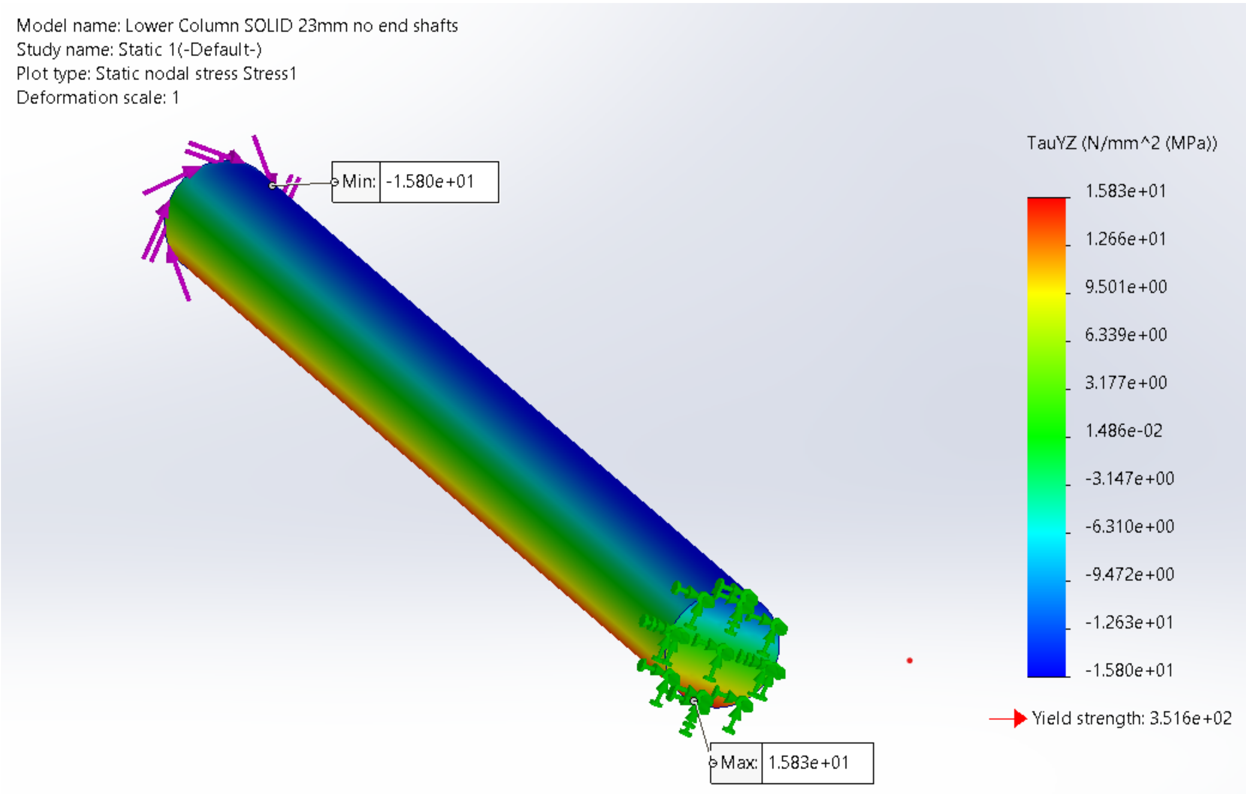
Surprisingly, the simulation with the coarsest density achieved more accurate results than the default computation (roughly set to half way between the coarse/fine slider.)



(Figure - Screenshot of FEA Simulation – Shear Stress – Default mesh density)

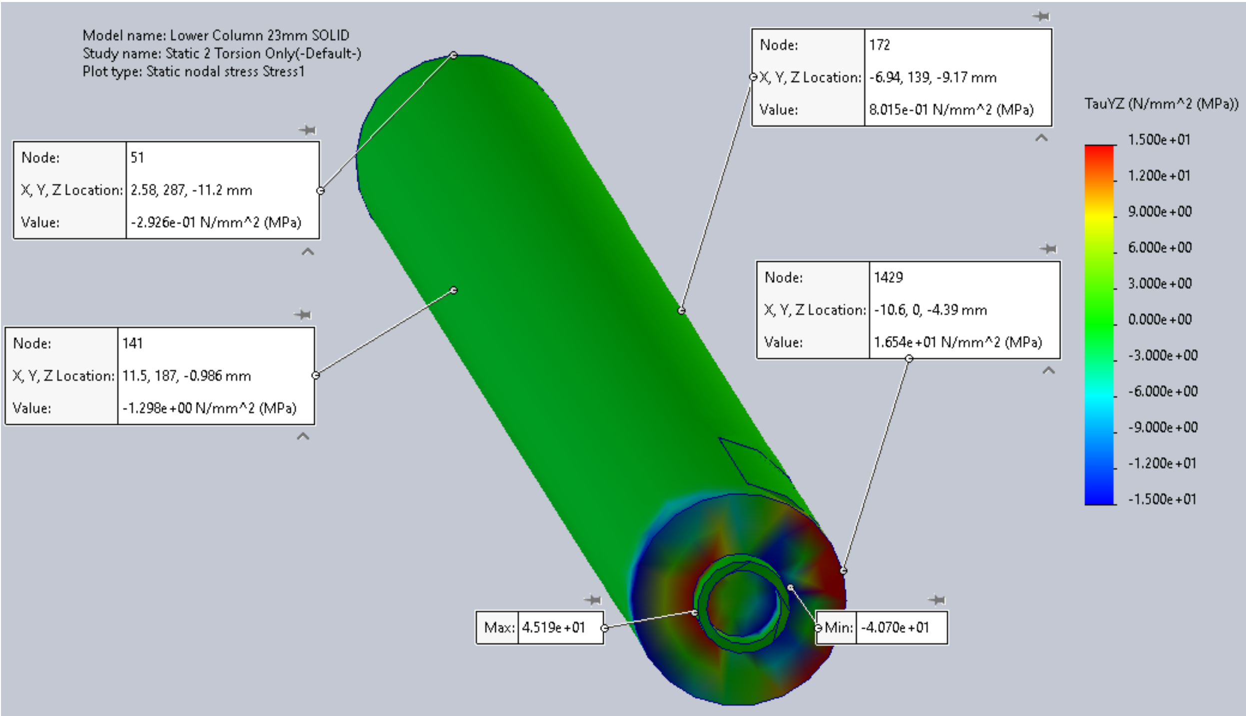


(Figure - Screenshot of FEA Simulation – Shear Stress – Coarsest mesh density)



(Figure - Screenshot of FEA Simulation – Shear Stress – Finest mesh density)

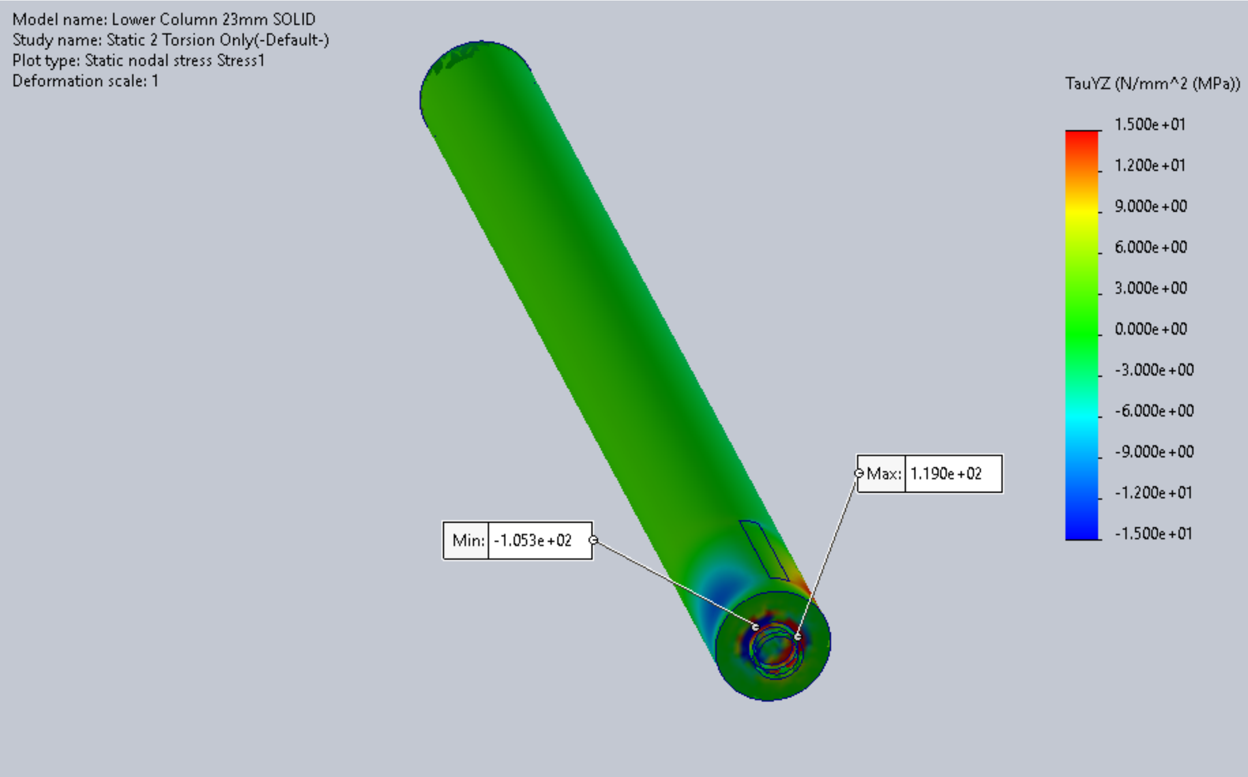
Shear stress simulation of the actual 23mm shaft model presented major variation of the stress distribution along the shaft. The Figure shown below portrayed a manual driving condition where the steering force from the wheels are exerted into the shaft, transferring to the shaft's main face and along the surface area of the extruded side shaft.



(Figure - Screenshot of FEA Simulation – Shear Stress – Coarsest mesh density)

Steering Column Shaft – All Calculated forces modeled

23mm diameter general steel (1020):



(Screenshot of FEA Simulation – Shear Stress - Upper side of Column 15mm Shaft)