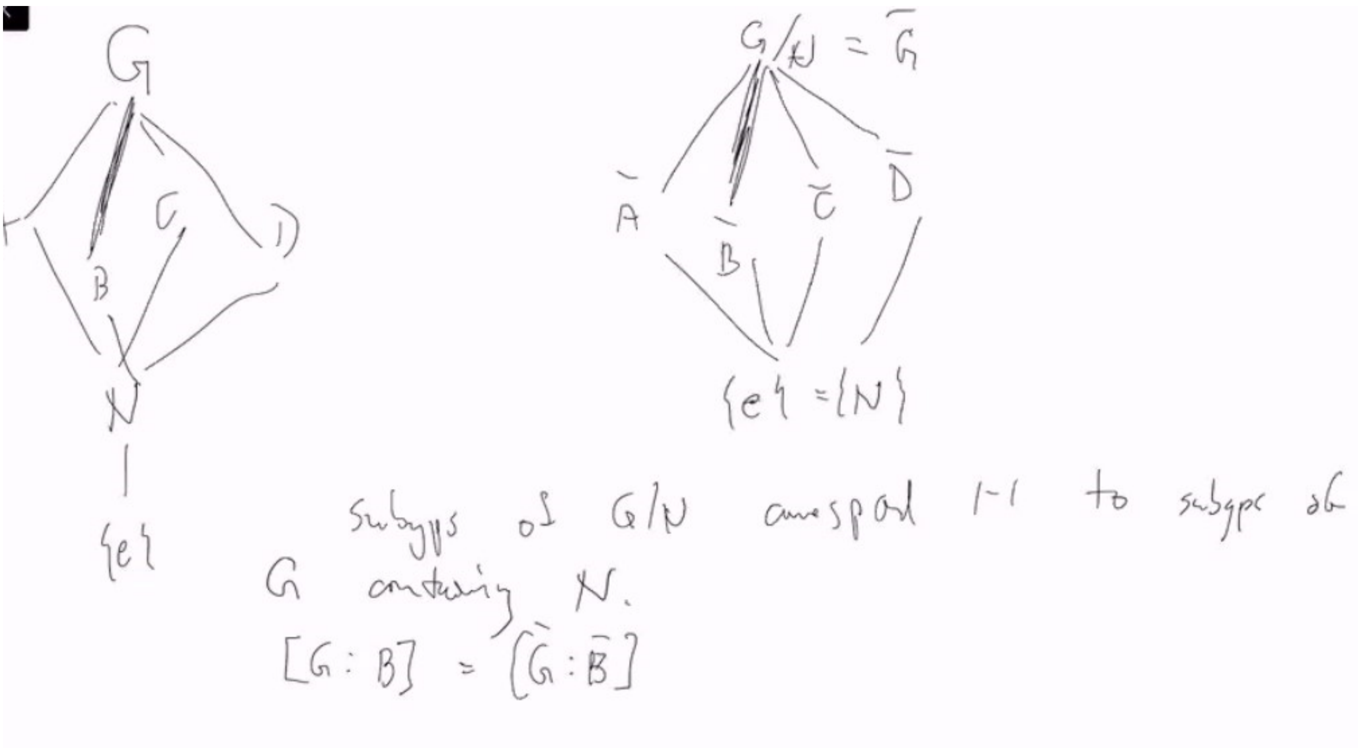
Lec 10

Quotient is subtle

Recall things on tutorial : 4th Isomorphism Theorem:

between lattices



- recall this G has to be finite
- $[G : B] = [\bar{G} : \bar{B}]$
- $\bar{A \cap C} = \bar{A} \cap \bar{C}$
- $\bar{A} \triangleleft \bar{B}$ iff $A \trianglelefteq B$
- $<\bar{A},\bar{B}> = \bar{<A, B>}$
- this is an isomorphism between lattices of subgroups
- the lattice of subgroup of G/N and of G containing N are "the same"

Propositions:

- Suppose G is abelian and $p | ord(G)$, some prime p
  - then G contains an element of order p
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- this theorem is not true in general
- to some extent this is the converse of lagrange theorem
- proof. (complete) induction
  - Given a group G, assume the result holds for all groups H with size less than $|G|$

- we will use that to prove the result for G
    - Suppose $x \in G$, if $p | ord(x)$ then $|x| = pk$ i.e.
        - $x^{pk} = e$, so x^k is the generator for a prime order group
    - Suppose $p \not | G$, let $N = \trianglelefteq G$, (since G is abeilean)
        - and it is a proper subset since $p | ord(G)$
        - consider G/N, thus $p | ord(G/N)$ and p divides it and then we can apply induction hypothesis
            - thus there exists $y \in G/N$ s.t. $y^p = e$
            - we translate back y into G, that $(y'N) ^ p = N$ by abelian, $y'^p \in N$ but $y'^k \not \in N$ for $1 \le k \le p$
            - if $y'^p = e$. we are done.
            - otherwise $|y'| > p$. Write $|y'| = pm$
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