

CHECKEDCBOX: Formalizing RLBox in Checked C for Incremental Spatial Memory Safety (Extended Version)

This is an extended version of a paper that appears at the 2022 Computer Security Foundations Symposium.

1. Introduction

Vulnerabilities due to memory corruption, especially spatial memory corruptions, are still a major issue for C programs [2, 41] despite a large body of work that tries to prevent them [38]. Vulnerabilities due to memory corruption, especially spatial memory corruptions, are still a major issue for C programs [2, 41] despite a large body of work that tries to prevent them [38]. One way to handle this is to convert these programs into safe languages such as Java. However, it requires considerable development effort. Furthermore, the performance overhead prohibits them from being used in performance-critical settings such as Operating Systems.

With the increased support for high-performant safe system languages such as Rust and Go, we can have tools that convert C to a memory-safe language, e.g., Rust, a promising memory-safe systems language [26]. Unfortunately, Rust is too different from C to constitute a practical target [43], at least for now: the (best of breed) `c2rust` tool [3, 17] transliterates C to *unsafe, non-idiomatic* Rust, and recent work [9] is able to take only small steps to close the safety gap and still requires considerable work from developers.

There are other retrofitting techniques such as CCured [31], Softbound [27], Low Fat pointers [6], and Address Sanitizer (ASAN) [36] aim to enforce spatial safety automatically *without any developer effort*, by analyzing a C program and compiling it to include run-time safety checks. Unfortunately, the resulting run-time overhead is too high for deployment (between 60%-200%), especially in low-powered IoT devices where the usage of C-based system software (e.g., FreeRTOS) is still rampant. Figure ?? shows the summary of these existing approaches.

Checked C [40] as a practical target to which to convert legacy/active C code. Checked C extends C with *checked pointer types* which are restricted by the compiler to spatially safe uses (temporal safety is underway [46]). Such pointers have one of three possible types, `ptr<T>`, `array_ptr<T>`, or `nt_array_ptr<T>` (*ptr*, *arr*, and *ntarr* for short), representing a pointer to a single element, array of elements, or null-terminated array of elements of type *T*, respectively. The latter two have an associated *bounds annotation*; e.g., a declaration `array_ptr<int> p : count(n)` says that *p* is a pointer to an `int` array whose

size is *n*. Checked C’s Clang/LLVM-based compiler represents checked pointers as system-level memory words, i.e., without “fattening” metadata, ensuring backward compatibility. The compiler uses these bounds annotations to add dynamic checks prior to checked pointer accesses, to prevent spatial safety violations. These run-time checks can often be proved redundant and removed by LLVM, yielding good performance. Tarditi et al. [40] reported average run-time overheads of 8.6% on a small benchmark suite, and [5] found essentially no overhead when running Checked C-converted portions of the FreeBSD kernel.

However, converting existing C programs to Checked C requires considerable effort [5]. Recently, Machiry et al. developed 3C [21] that tries to automatically convert a program to Checked C by adding appropriate pointer annotations. However, as described in 3C, complete automated conversion is infeasible and consequently requires the developer to convert some code regions manually. Thanks to the backward compatibility of Checked C, the partially annotated program still enjoys the benefits of spatial memory safety on those regions using only Checked pointers (i.e., checked or safe regions). But, the unconverted code regions (or unsafe regions) can affect pointers in safe regions and violate certain assumptions leading to vulnerabilities, as demonstrated by cross-language attacks [24]. Although the blameless proof exists [35], it does not state that spatial safety violations cannot happen in Checked regions but rather states that Checked regions *cannot be blamed for any spatial safety violations*. For instance, a checked function expects a pointer to a buffer of five elements, but an unchecked region can call this function with a pointer to a buffer of three elements. This results in a spatial safety violation in the Checked region, but of course, the blame or the root cause is in the unchecked region. Furthermore, since checked and unchecked regions execute in the same address space, spatial memory corruptions in unchecked regions can take down the complete program despite having checked regions. We need an isolation mechanism to ensure that code executed as part of unchecked regions does not violate the safety guarantees in checked regions. Such an isolation mechanism should have low/no overhead and be backward compatible (i.e., pointers should have no additional metadata), preserving the advantages of Checked C.

In this work, we propose CHECKEDCBOX, an isolation mechanism that enables unchecked regions or, in general, any arbitrary set of functions to be executed in an isolated address space (*uc - region*). So that checked regions can

still enjoy true spatial memory safety even in the presence of unchecked or unsafe regions. CHECKEDCBOX extends Checked c using **tainted** (`t_*`) types, representing the code and data that belong to the *uc-region*.

Automated Partitioning. A pointer (checked or unchecked) can be marked as tainted; similarly, a function can be marked as tainted. Along with enforcing spatial safety constraints of checked tainted pointers, the CHECKEDCBOX compiler will bundle all the tainted types (functions and pointers) so that they remain in an isolated sandbox (*uc-region*), and non-tainted types will remain in the process address space (*c-region*).

Memory Access Restrictions. The functions in the *uc-region* can only access tainted pointers, i.e., memory within the corresponding sandbox region. However, the functions in *c-region* have no-such restrictions and can access both *c-* and *uc-region*'s memory. Similarly, the functions in *c-region* can invoke any function in *uc-region* but not the other way around, except for call-back functions, which we will discuss later.

Marshaling free communication. As functions in the *uc-region* can only access tainted types, *c-region* functions should use tainted types to pass pointer arguments to *uc-region* functions. We avoid marshaling as all tainted buffers (i.e., regions pointed by tainted pointers) are allocated in *uc-region* and thus can be accessed in both *c-* and *uc-region*.

Additional checks. Although memory isolation prevents direct violations, *uc-region* code can still affect *c-region* through tainted pointers by confused deputy attacks. However, our compiler avoids these attacks by ensuring, either statically or through dynamic checks, that tainted pointers in *c-region* can only point to *uc-region* address space, and our type system enforces that tainted pointers cannot be assigned to non-tainted pointers.

2. Overview and Transcendence

This section discusses the key merge features between Checked-C and RLBox and the problems we are trying to solve.

Maintaining Non-crashing. Previously, the main guarantee of Checked C [20] was the blame theorem, i.e., if there is a crash, the source is from some unchecked regions. For example, at Figure 3 line 31, we call a unchecked function `f` with a checked null-terminated array (NT-array) pointer argument. At line 8, depending on the NT-array size, `free(s[10])` might crash. Even if it does not crash, line 38 is doomed because of the `free` call.

The philosophy is that unchecked regions are written in C and unstable, and programmers will eventually remove them by converting them to Checked C code. The reality is that users might not want to convert everything. For some insignificant or handy history C code, such as library functions, they might want to keep it as long as no crashing.

The sources of crashing in Checked C are (1) unchecked regions crash themselves; and (2) the misuse of checked pointers in unchecked regions. Enlightened by RLBox [29],

```

1 //in checked region
2
3 int compare_1(nt_array_ptr<char> x: count (0),
4   nt_array_ptr<char> y : count (0)) {
5   int len_x = strlen(x);
6   int len_y = strlen(y);
7   return sum(x,len_x) < sum(y,len_y);
8 }
9 ...
10
11 int stringsort(
12   nt_array_ptr<nt_array_ptr<char>> s : count (n),
13   ptr<(int)(nt_array_ptr<char>,
14   nt_array_ptr<char>>> cmp, int n) {
15   int i, j, gap;
16   int didswap;
17
18   for(gap = n / 2; gap > 0; gap /= 2) {
19     {
20       do {
21         didswap = 0;
22         for(i = 0; i < n - gap; i++)
23           {
24             j = i + gap;
25             if((*cmp)(s[i], s[j]) > 0)
26               {
27                 int len = strlen(s[i]);
28                 nt_array_ptr<char>
29                   tmp : count (len) = s[i];
30                 s[i] = s[j];
31                 s[j] = tmp;
32                 didswap = 1;
33               }
34           }
35       } while(didswap);
36     }
37   }
38   return 0;
39 }
40

```

Figure 1: Checked stringsort Code

we sandbox the unchecked regions and then we utilize the Checked C type system to disallow checked pointers to be used in an unchecked region. To achieve the communication between checked and unchecked regions, we create tainted pointers that can be shared by different regions, whose data are stored in the sandboxed heap. Users are required to copy data to tainted pointers that are shared in unchecked regions. For example, we copy the checked pointer data to the tainted pointer `tp` at Figure 3 line 26, and input the tainted pointer to the unchecked function at line 34. At line 8, even if statement might crash, since tainted pointers are stored in the sandboxed heap, it can be recovered. At line 37, the use of a tainted pointer in a checked region requires a verification on it. This is handled by inserting additional checks and creating exception handling before the use by the CHECKEDCBOX compiler. Thus, the checked pointer `p` is safely used at line 38.

```

1 //in checked region, tainted version
2 int tainted_compare_1(
3     nt_array_tptr<char> x : count (0),
4     nt_array_tptr<char> y : count (0)) {
5     checked (x,y) {
6         int len_x = strlen(x);
7         int len_y = strlen(y);
8         nt_array_ptr<char> tx : count (len_x)
9         = malloc(nt_array<char>, len_x);
10        nt_array_ptr<char> ty : count (len_y)
11        = malloc(nt_array<char>, len_y);
12        safe_memcpy(tx,x,len_x);
13        safe_memcpy(ty,y,len_y);
14        return compare_1(tx,ty);
15    }
16 }
17 ...
18
19 //calling the function turns
20 //an unchecked region to a checked region.
21 int tainted_stringsort(nt_array_tptr
22     <nt_array_tptr<char>> s : count (n),
23     tptr<(int)(nt_array_tptr<char>,
24     nt_array_tptr<char>>> cmp, int n) {
25     checked (s,cmp,n) {
26         int i;
27         nt_array_ptr<nt_array_ptr<char>> p : count (n)
28         = malloc(nt_array<nt_array_ptr<char>>, n);
29         for(i = 0; i < n; i++) {
30             int len = strlen s[i];
31             nt_array_ptr<char> tmp : count (len)
32             = new malloc(nt_array<char>, len);
33             safe_memcpy(tmp,s[i],len);
34             p[i] = tmp;
35         }
36         ptr<(int)(nt_array_ptr<char> : count (0),
37         nt_array_ptr<char> : count (0))>
38         cfun = find_check(cmp);
39
40         return stringsort(p,cfun);
41     }
42 }

```

Figure 2: Tainted stringsort Code

In short, banning checked pointer uses in unchecked regions and providing tainted shared pointers are the solution for crashing. In CHECKEDCBOX, we proved the non-crashing theorem based on the CHECKEDCBOX type system, i.e, any program execution is stopped in a predictable manner where either the execution terminates or is stopped at a place where an exception handler exists. In fact, our work is *the first formalism of RLBox*, and formally shows the security guarantee that RLBox provides is actually the non-crashing theorem.

Formalism Function Pointers. In C, manipulating function pointers is a major way of implementing high order functions. Previously, we assumed all function calls are through calling by name to a global map. In CHECKEDCBOX, we formalize function pointers and maintain the CHECKEDCBOX type soundness. To the best of our knowl-

```

1 //in unchecked region
2 int f(char ** s, int (*cmp)(char *,char *),
3     int (*sort)(char **, int (*)(char *,char *),
4         int), int n) {
5     ...
6
7     int i = sort(s,cmp,n);
8     free(s[10]);
9     ...
10 }
11
12 int g(int (*cmp)(char *,char *)) {
13     ...
14     int real_addr = derandomize(cmp);
15     ...
16 }
17
18 int main(int n) {
19     nt_array_ptr<nt_array_ptr<char>> p : count(n)
20     = malloc(nt_array<nt_array_ptr<char>>, n);
21
22     nt_array_tptr<nt_array_ptr<char>>
23     tp : count(n) =
24     tmalloc(nt_array<nt_array_ptr<char>>, n);
25     ...
26     safe_memcpy(tp, p, n);
27
28     unchecked {
29         if (BAD) // a flag to call different funs.
30             //input checked pointers
31             f(p, compare_1, stringsort);
32         else
33             //input tainted pointers
34             f(tp, tainted_compare_1,
35                 tainted_stringsort);
36     }
37
38     if (!BAD) safe_memcpy(p, tp, n);
39     p[10] = "crash?";
40
41     unchecked {
42         if (BAD) g(compare_1)
43         else g(tainted_compare_1);
44     }
45
46     return 0;
47 }

```

Figure 3: Tainted Pointer Usage in Calling Unchecked Fun

edge, this is the first work of formalizing C function pointers with security guarantee.

Figure 1 defines a string sorting algorithm depending on the input function pointer `cmp` defining the generic order for strings, and `compare_1` is an example `cmp` function that adds the ASCII numbers of characters in the two strings and compare the results. In addition, function pointers enable the callback mechanism, i.e., a server sends a function pointer to a client in an unchecked region, and allows the client to access some server resources by calling back the pointer. This is a common usage between a web-browser and

untrusted third party libraries. The function call in Figure 3 line 34 is one such usage.

We also utilize CHECKEDCBOX subtyping relation to permit function pointer static auto-casting. Function pointer type information might contain array pointer bound information, for which it is inconvenient to coincide the defined types for a function implementation and the function pointer type. For example, the `cmp` argument in `stringsort` (Figure 1) has type `ptr<(int)(nt_array_ptr<char>, nt_array_ptr<char>>>`, meaning that the function takes two NT-array pointers with arbitrary size and outputs an integer. The function `compare_1`'s pointer has type `ptr<(int)(nt_array_ptr<char> : count(0), nt_array_ptr<char> : count(0))>`. To use `compare_1` in `stringsort`, the type is auto-cast to the `cmp`'s type. In general, if function pointer x has type $*(tl \rightarrow t)$, and y has $*(tl' \rightarrow t')$, in order to use x as y , tl' should be a subtype of tl and t subtypes to t' .

Not Exposing Checked Pointer Addresses. The design for the non-crashing property bans the checked pointer uses in unchecked regions. Thus, there is no reason to permit checked pointer variable assignments in unchecked regions; especially, this might expose a checked pointer address to untrusted parties. For example, the call to function g at Figure 3 line 41 lives in an unchecked region, and g might use some mechanism, such as derandomizing ASLR [37], to achieve the checked pointer address. Thus, it enables a third party to access any checked heap and function data by simple pointer arithmetic.

To prevent the checked pointer address leak, we prevent any unchecked regions acknowledge checked pointer variables. In addition, to facilitate checked function callbacks, the CHECKEDCBOX compiler compiles every checked function with an additional tainted shell function. Users are required to serve unchecked regions with the tainted shell pointer instead of the original checked function pointer. For example, `tainted_compare_1` and `tainted_stringsort` in Figure 2 are the tainted shells of the checked functions `compare_1` and `stringsort`. In the tainted shells, the arguments are tainted versions of the corresponding arguments in the checked functions. Inside the shell body, create checked pointer copies of the tainted arguments, and call the checked functions. In addition, once a checked function returns, if the output is a checked pointer, we also its the data to a new tainted pointer and exit the shell. Figure 3 line 42 is an example of serving the function call living in an unchecked region with a tainted shell pointer argument `tainted_compare_1`. Even if g derandomizes its address (line 14), the shell address is in the sandbox and has no harm, and calling the shell never exposes any checked pointer information outside of the shell.

Conceptually, the shell is run in a checked region. One can imagine that a tainted shell is a safe closure that contains a checked block. Once the closure is called, the system is turned to a checked region, so that the checked function call at Figure 2 line 14 is safe, even if it is called by g in Figure 3, because it lives in the checked region. In the CHECKEDCBOX formalism, we formalize a *checked* block

Variables: x	Integers: $n ::= \mathbb{Z}$
Context Mode: m	$::= c \mid u$
Pointer Mode: ξ	$::= m \mid t$
Bound: b	$::= n \mid x + n$
	$\beta ::= (b, b)$
Word Type: τ	$::= \text{int} \mid \text{ptr}^\xi \omega$
Type Flag: κ	$::= nt \mid \cdot$
Type: ω	$::= \tau \mid [\beta \tau]_\kappa \mid \forall \bar{x}. \bar{\tau} \rightarrow \tau$
Expression: e	$::= n : \tau \mid x \mid e + e \mid (\tau)e \mid \langle \tau \rangle e$ $\mid \text{strlen}(x) \mid *e \mid *e = e$ $\mid \text{let } x = e \text{ in } e \mid \text{if } (e) e \text{ else } e$ $\mid \text{malloc}(\xi, \omega) \mid e(\bar{e})$ $\mid \text{unchecked}(\bar{x})\{e\} \mid \text{checked}(\bar{x})\{e\}$

Figure 4: CORECHKCBOX Syntax

$$\begin{array}{c}
m \vdash \text{int} \qquad \frac{\xi \wedge m \vdash \tau \quad \xi \leq m}{m \vdash \text{ptr}^\xi [\beta \tau]_\kappa} \\
\\
\frac{\xi \wedge m \vdash \tau \quad \xi \leq m}{m \vdash \text{ptr}^\xi \tau} \qquad \frac{\xi \wedge m \vdash \tau \quad \xi \leq m \quad FV(\bar{\tau}) \cup FV(\tau) \subseteq \bar{x}}{m \vdash \text{ptr}^\xi (\forall \bar{x}. \bar{\tau} \rightarrow \tau)} \\
\\
t \wedge c = u \quad \xi \wedge u = u \quad c \wedge m = m \quad m_1 \wedge m_2 = m_2 \wedge m_1 \\
\xi \leq \xi \quad t \leq \xi
\end{array}$$

Figure 5: Well-formedness for Types

on top of the existing unchecked regions, and the transition of a tainted shell call creates a checked block containing the shell body. We also make sure that no arguments in these tainted shell contains any checked pointers, as well as no output is of a checked type.

3. Formalization

This section describes the formal model of CHECKEDCBOX, called CORECHKCBOX, making precise its syntax, semantics, and type system. It also develops CORECHKCBOX's meta-theory, including type soundness, the non-exposure, and the non-crashing theorem.

3.1. Syntax

The syntax of CORECHKCBOX is given by the expression-based language presented in Fig. 4.

There are two notions of type in CORECHKCBOX. Types τ classify word-sized values including integers and pointers, while types ω classify multi-word values such as arrays, null-terminated arrays, functions, and single-word-size values. Pointer types ($\text{ptr}^m \omega$) include a mode annotation (m) which is either checked (c) or unchecked (u) and a type (ω) denoting valid values that can be pointed to. Array

types include both the type of elements (τ) and a bound (β) comprised of an upper and lower bound on the size of the array $((b_l, b_h))$. Bounds b are limited to integer literals n and expressions $x + n$. Whether an array pointer is null terminated or not is determined by annotation κ , which is nt for null-terminated arrays, and \cdot otherwise (we elide the \cdot when writing the type). CHECKEDCBOX function types $(\forall \bar{x}. \bar{\tau} \rightarrow \tau)$ reflect its dependent function declarations, where \bar{x} represents a list of `int` type variables in a dependent function header, which bind all bounds appearing in $\bar{\tau}$ and τ . We have a well-formed requirement for a function type, such that all variables in $\bar{\tau}$ and τ are bounded by \bar{x} . Here is the corresponding CHECKEDCBOX syntax for these types:

```
array_ptr< $\tau$ > : count( $n$ )  $\Leftrightarrow$  ptrt [(0,  $n$ )  $\tau$ ]
nt_array_ptr< $\tau$ > : count( $n$ )  $\Leftrightarrow$  ptrc [(0,  $n$ )  $\tau$ ]nt
tptr<(int)(nt_array_ptr< $\tau$ > : count( $n$ ),
nt_array_ptr< $\tau$ >)> : count( $n$ )>>
 $\Leftrightarrow$  ptrt ( $\forall n$ . ptrt [(0,  $n$ )  $\tau$ ]nt  $\times$  ptrt [(0,  $n$ )  $\tau$ ]nt  $\rightarrow$  int)
```

As a convention we write $\text{ptr}^c[b \tau]$ to mean $\text{ptr}^c[(0, b) \tau]$, so the above examples could be rewritten $\text{ptr}^c[n \tau]$ and $\text{ptr}^c[n \tau]_{nt}$, respectively.

CORECHKCBOX expressions include literals $(n : \tau)$, variables (x) , addition $(e_1 + e_2)$, static casts $((\tau)e)$, dynamic casts $(\langle \tau \rangle e)$ ¹, the `strlen` operation (`strlen(x)`), pointer dereference and assignment $(*e)$ and $(*e_1 = e_2)$, resp.), let binding (`let $x = e_1$ in e_2`), conditionals (`if (e) e_1 else e_2`), memory allocation (`malloc(ξ, ω)`), function calls $(e(\bar{e}))$, unchecked blocks $(\text{unchecked}(\bar{x})\{e\})$, and checked blocks $(\text{checked}(\bar{x})\{e\})$.

Integer literals n are annotated with a type τ which can be either `int`, or $\text{ptr}^m \omega$ in the case n is being used as a heap address (this is useful for the semantics); $0 : \text{ptr}^m \omega$ (for any m and ω) represents the null pointer, as usual. The `strlen` expression operates on variables x rather than arbitrary expressions to simplify managing bounds information in the type system; the more general case can be encoded with a `let`. We use a less verbose syntax for dynamic bounds casts; e.g., the following

```
dyn_bounds_cast<array_ptr< $\tau$ >>( $e$ , count( $n$ ))
becomes  $\langle \text{ptr}^c[n \tau] \rangle e$ .
```

Compared to the former Checked C model [20], there are four differences. First, the CHECKEDCBOX type annotations have well-formed restrictions in Figure 5, for maintaining non-exposure. Mainly, in a nested pointer $\text{ptr}^\xi(\dots \text{ptr}^{\xi'} \tau \dots)$, if the outside mode ξ is `t` or `u`, the inside mode ξ' cannot be `c`. It is worth noting that pointer modes are a three point partial order (\leq), where `t` is the infimum, and $\xi \wedge m$ is a special meet operation that projects pointer modes onto context modes, such that a pointer mode `t` is projected as `u`. Second, `malloc(ξ, ω)` includes a mode flag ξ for allocating different pointers in different heap mode regions. For simplicity, we disallow ω to be a function type $(\forall \bar{x}. \bar{\tau} \rightarrow \tau)$. Third, the first expression e in a function call $(e(\bar{e}))$ represents a function pointer. Fourth, we update $\text{unchecked}(\bar{x})\{e\}$ to specify \bar{x} restricting all free variables

```
 $\mu ::= n : \tau$ 
 $e ::= \dots \mid \text{ret}(x, \mu, e)$ 
 $r ::= e \mid \text{null} \mid \text{bounds}$ 
 $E ::= \square \mid E + e \mid n : \tau + E \mid (\tau)E \mid \langle \tau \rangle E \mid *E \mid *E = e$ 
 $\quad \mid *n : \tau = E \mid \text{let } x = E \text{ in } e \mid \text{if } (E) e \text{ else } e$ 
 $\quad \mid E(\bar{e}) \mid n : \tau(\bar{E}) \mid \text{unchecked}(\bar{x})\{E\} \mid \text{checked}(\bar{x})\{E\}$ 
```

$$\frac{m = \text{mode}(E) \quad e = E[e'] \quad (\varphi, \mathcal{H}, e') \longrightarrow (\varphi', \mathcal{H}', e'')}{(\varphi, \mathcal{H}, e) \longrightarrow_m (\varphi', \mathcal{H}', E[e''])}$$

$$\frac{u = \text{mode}(E) \quad e = E[e'] \quad \tau = \text{type}(e')}{(\varphi, \mathcal{H}, e) \longrightarrow_u (\varphi, \mathcal{H}, E[0 : \tau])}$$

```
mode( $E$ ) = mode'( $E, c$ )
mode'( $\square, m$ ) =  $m$ 
mode'( $\text{unchecked}(\bar{x})\{E\}, m$ ) = mode'( $E, u$ )
mode'( $\text{checked}(\bar{x})\{E\}, m$ ) = mode'( $E, c$ )
mode'( $\alpha(E), m$ ) = mode'( $E, m$ ) [otherwise]
```

Figure 6: CORECHKCBOX Semantics: Evaluation

appearing in e . Additionally, we create checked blocks indicating the context mode change from `u` to `c`. Both the free variables in the unchecked and checked blocks and the return types cannot be `c` mode for maintaining non-exposure, i.e., no checked pointer can be communicated between unchecked and checked blocks.

CORECHKCBOX aims to be simple enough to work with, but powerful enough to encode realistic CHECKEDCBOX idioms. For example, mutable local variables can be encoded as immutable locals that point to the heap; the use of `&` can be simulated with `malloc`; and loops can be encoded as recursive function calls. `structs` are not in Fig. 4 for space reasons, but they are actually in our model, and developed in Appendix F. C-style `unions` have no safe typing in Checked C, so we omit them. Additional syntactic description is listed in the Checked C formalism paper [20].

3.2. Semantics

The operational semantics for CORECHKCBOX is defined as a small-step transition relation with the judgment $(\varphi, \mathcal{H}, e) \longrightarrow_m (\varphi', \mathcal{H}', r)$. Here, φ is a *stack* mapping from variables to values $n : \tau$ and \mathcal{H} is a *heap* that is partitioned into two parts (`c` and `u` regions), each of which maps addresses (integer literals) to values $n : \tau$. If a pointer has mode `c`, it lives in the `c` region; otherwise, it lives in the `u` region. We wrote $\mathcal{H}(m, n)$ to retrieve the n -location heap value in the m region, and $\mathcal{H}(m)[n \mapsto \mu]$ to update location n with the value μ . It is worth noting that CHECKEDCBOX is not a fat-pointer system; thus, in every heap update, the value type annotation remains the same through program executions. Additionally, for both stack and heap, we ensure $FV(\tau) = \emptyset$ for all the value type annotations τ .

While heap bindings can change, stack bindings are immutable—once variable x is bound to $n : \tau$ in φ , that binding will not be updated; we can model mutable stack

1. assumed at compile-time and verified at run-time, see Appendix A

variables as pointers into the mutable heap. As mentioned, value $0:\tau$ represents a null pointer when τ is a pointer type; correspondingly, $\mathcal{H}(m, 0)$ should always be undefined. The relation steps to a *result* r , which is either an expression or a null or bounds failure, representing a null-pointer dereference or out-of-bounds access, respectively. Such failures are a *good* outcome; stuck states (non-value expressions that cannot transition to a result r) characterize undefined behavior. The context mode m indicates whether the stepped redex within e was in a checked (c) or unchecked (u) region.

The rules for the main operational semantics judgment—*evaluation*—are given at the bottom of Fig. 6. The first rule takes an expression e , decomposes it into an *evaluation context* E and a sub-expression e' (such that replacing the hole \square in E with e' would yield e), and then evaluates e' according to the *computation* relation $(\varphi, \mathcal{H}, e') \rightarrow (\varphi, \mathcal{H}, e'')$, whose rules are given in Fig. 7, discussed shortly. The second rule describes the exception handling for possible crashing behaviors in unchecked region. A program in u mode can non-deterministically crash and the CHECKED-CBOX sandbox mechanism recovers the program to a safe point ($0:\tau$) and continues with the existing program state. There are other rules for describing the halts of evaluation to null and bounds states in Appendix A.

The *mode* function determines the context mode when evaluating e' based on the context E . For any program execution, the top context mode always starts with c ($mode(E) = mode'(E, c)$). The result context mode is depended on where \square locates. If it occurs within ($unchecked(\bar{x})\{E'\}$) inside E without a surrounding checked block, then the mode is u; if the exact opposite happens (within ($checked(\bar{x})\{E'\}$) without surrounding unchecked blocks), the mode is c; the other cases ($mode'(\alpha(E), m) = mode'(E, m)$) are recursively defined by the above base cases (α represents any syntactic categories in Figure 6 other than \square , unchecked, and checked). Evaluation contexts E define a standard left-to-right evaluation order. (We explain the $ret(x, \mu, e)$ syntax shortly.)

Fig. 7 shows selected rules for the computation relation; we explain them with the help of the example in Figure 1 and Figure 2.

Checked and Tainted Pointer Operations. The rules for pointer related operations—S-DEFC, S-DEFT, S-ASSIGNARRC, S-ASSIGNARRT, S-DEFNULL, and S-CAST. The first five are deference and assignment operations—illustrate how the semantics checks bounds. Rule S-DEFNULL transitions attempted null-pointer dereferences to null, whereas S-DEFC dereferences a c-mode non-null (single) pointer. When null is returned by the computation relation, the evaluation relation halts the entire evaluation with null (using a rule not shown in Fig. 6); it does likewise when bounds is returned (see Appendix C). S-ASSIGNARRC assigns to an array as long as 0 (the point of dereference) is within the bounds designated by the pointer’s annotation and strictly less than the upper bound.

S-DEFT and S-ASSIGNARRT are similar rules to S-DEFC and S-ASSIGNARRC for tainted pointers. Any dynamic heap use of a tainted pointer requires a verification.

Performing such a verification equates to performing a type check for a pointer constant in Figure 11. We explain this shortly in Section 3.3. For now, the verification step, e.g. $u; \emptyset; \mathcal{H}; \emptyset \vdash n_a : \tau$ in S-DEFC, means we verify that the value n_a has type τ in the u region heap.

Static casts of a literal $n : \tau'$ to a type τ are handled by S-CAST. In a type-correct program, such casts are confirmed safe by the type system no matter if the target is a t or c pointer. To evaluate a cast, the rule updates the type annotation on n . Before doing so, it must “evaluate” any variables that occur in τ according to their bindings in φ . For example, if τ was $ptr^c [(0, x + 3) \text{ int}]$, then $\varphi(\tau)$ would produce $ptr^c [(0, 5) \text{ int}]$ if $\varphi(x) = 2$. The full formalism, including struct and null-terminated bound widening pointer operations, is given in Appendix A.

Unchecked and Checked Blocks. Semantically, unchecked and checked blocks (S-UNCHECKED and S-CHECKED) act as a classical C block structures. The interesting part is their type rules in Section 3.3.

Binding and Function Calls. The semantics handles variable scopes using the special `ret` form. S-LET evaluates to a configuration whose stack is φ extended with a binding for x , and whose expression is $ret(x, n:\tau, e)$ which keeps φ unchanged, but remember x with the new value $n:\tau$ in the scope e . Every time when Evaluation proceeds on e (Rule S-RETCN), we install the current stack value $n:\tau$ for x in φ , after one-step evaluation is completed, we restore the result $\varphi'(x)$ in the result $ret(x, \varphi'(x), e')$. This procedure continues until it becomes a literal $n:\tau$, in which case S-RETEND remove the `ret` frame and evaluates to $n:\tau$.

Function calls are handled by S-FUNC and S-FUNT, for `cand` and `tmode` function pointers, respectively. A call to function f causes f ’s definition to be retrieved from Ξ , which maps function names to function data $\tau(\bar{x}:\bar{\tau})(\xi, e)$, where τ is the return type, $(\bar{x}:\bar{\tau})$ is the parameter list of variables and their types, ξ determines the mode of the function, and e is the function body. Similar to \mathcal{H} , the global function store Ξ is also partitioned into two parts (c and u regions), each of which maps addresses (integer literals) to the function data described above.

The CHECKEDCBOX functions are dependent functions. Recall that array bounds in types may refer to in-scope variables; e.g., parameter `dst`’s bound `count(n)` refers to parameter `n` on lines 2-3 in Fig. 8. Semantically, the call is expanded into a `let` which binds parameter variables \bar{x} to the actual arguments \bar{n} , but annotated with the parameter types $\bar{\tau}$ (this will be safe for type-correct programs). The function body e is wrapped in a static cast $(\tau[\bar{n}/\bar{x}])$ which is the function’s return type but with any parameter variables \bar{x} appearing in that type substituted with the call’s actual arguments \bar{n} . To see why this is needed, suppose that `safe_strcat` in Fig. 8 is defined to return a `nt_array_ptr<int>:count(n)` typed term, and assume that we perform a `safe_strcat` function call as `x=safe_strcat(a,b,10)`. After the evaluation of `safe_strcat`, the function returns a value with type `nt_array_ptr<int>:count(10)` because we substitute bound variable `n` in the defined return type with 10

$\text{S-DEFC} \quad \frac{\mathcal{H}(\mathbf{c}, n) = n_a : \tau_a}{(\varphi, \mathcal{H}, *n : \mathbf{ptr}^c \tau) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$	$\text{S-ASSIGNARRC} \quad \frac{\mathcal{H}(\mathbf{c}, n) = n_a : \tau_a \quad 0 \in [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \mathbf{ptr}^c [(n_l, n_h) \tau]_\kappa = n_1 : \tau_1) \longrightarrow (\varphi, (\mathcal{H}, \mathbf{c})[n \mapsto n_1 : \tau_a], n_1 : \tau)}$	
$\text{S-DEFT} \quad \frac{\mathcal{H}(\mathbf{u}, n) = n_a : \tau_a \quad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathbf{u}} n_a : \tau}{(\varphi, \mathcal{H}, *n : \mathbf{ptr}^t \tau) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$	$\text{S-ASSIGNARRT} \quad \frac{\mathcal{H}(\mathbf{u}, n) = n_a : \tau_a \quad 0 \in [n_l, n_h] \quad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathbf{u}} n_1 : \tau}{(\varphi, \mathcal{H}, *n : \mathbf{ptr}^t [(n_l, n_h) \tau]_\kappa = n_1 : \tau_1) \longrightarrow (\varphi, (\mathcal{H}, \mathbf{u})[n \mapsto n_1 : \tau_a], n_1 : \tau)}$	
$\text{S-DEFNULL} \quad (\varphi, \mathcal{H}, *0 : \mathbf{ptr}^c \omega) \longrightarrow (\varphi, \mathcal{H}, \mathbf{null})$	$\text{S-CAST} \quad (\varphi, \mathcal{H}, (\tau)n : \tau') \longrightarrow (\varphi, \mathcal{H}, n : \varphi(\tau))$	$\text{S-RETEnd} \quad (\varphi, \mathcal{H}, \mathbf{ret}(x, \mu, \mu')) \longrightarrow (\varphi, \mathcal{H}, \mu')$
$\text{S-LET} \quad (\varphi, \mathcal{H}, \mathbf{let} \ x = n : \tau \ \mathbf{in} \ e) \longrightarrow (\varphi, \mathcal{H}, \mathbf{ret}(x, n : \tau, e))$	$\text{S-RETCON} \quad \frac{\varphi(x) = \mu \quad (\varphi[x \mapsto \mu'], \mathcal{H}, e) \longrightarrow (\varphi', \mathcal{H}', e')}{(\varphi, \mathcal{H}, \mathbf{ret}(x, \mu', e)) \longrightarrow (\varphi'[x \mapsto \mu], \mathcal{H}', \mathbf{ret}(x, \varphi'(x), e'))}$	
$\text{S-UNCHECKED} \quad (\varphi, \mathcal{H}, \mathbf{unchecked}(\bar{x})\{\mu\}) \longrightarrow (\varphi, \mathcal{H}, \mu)$	$\text{S-FUNC} \quad \frac{\Xi(\mathbf{c}, n) = \tau \ (\bar{x} : \bar{\tau}) \ (\mathbf{c}, e)}{(\varphi, \mathcal{H}, n : (\mathbf{ptr}^c \tau)(\bar{n}_a : \bar{\tau}_a)) \longrightarrow (\varphi, \mathcal{H}, \mathbf{let} \ \bar{x} = \bar{n} : (\bar{\tau}[\bar{n}/\bar{x}]) \ \mathbf{in} \ (\tau[\bar{n}/\bar{x}])e)}$	
$\text{S-CHECKED} \quad (\varphi, \mathcal{H}, \mathbf{checked}(\bar{x})\{\mu\}) \longrightarrow (\varphi, \mathcal{H}, \mu)$	$\text{S-FUNT} \quad \frac{\Xi(\mathbf{u}, n) = \tau \ (\bar{x} : \bar{\tau}) \ (\mathbf{t}, e) \quad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathbf{u}} n : \mathbf{ptr}^t \tau}{(\varphi, \mathcal{H}, n : (\mathbf{ptr}^t \tau)(\bar{n}_a : \bar{\tau}_a)) \longrightarrow (\varphi, \mathcal{H}, \mathbf{let} \ \bar{x} = \bar{n} : (\bar{\tau}[\bar{n}/\bar{x}]) \ \mathbf{in} \ (\tau[\bar{n}/\bar{x}])e)}$	

Figure 7: CORECHKCBOX Semantics: Computation (Selected Rules)

```

1  nt_array_ptr<char> safe_strcat
2    (nt_array_ptr<char> dst : count(n),
3     nt_array_ptr<char> src : count(0), int n) {
4     int x = strlen(dst);
5     int y = strlen(src);
6     nt_array_ptr<char> c : count(n) =
7       dyn_bounds_cast
8         <nt_array_ptr<char>>(dst, count(n));
9     // sets c == dst with bound n (not x)
10    if (x+y < n) {
11      for (int i = 0; i < y; ++i)
12        *(c+x+i) = *(src+i);
13      *(c+x+y) = '\0';
14      return dst;
15    }
16    return null;
17  }

```

Figure 8: Implementation of safe `strcat`

from the function call's argument list. Note that the S-FUNC and S-FUNT rules replace the annotations $\bar{\tau}_a$ with $\bar{\tau}$ (after instantiation) from the function's signature. Using $\bar{\tau}_a$ when executing the body of the function has no impact on the soundness of CORECHKCBOX, but will violate Theorem 6, which we introduce in Sec. 4.

3.3. Typing

We now turn to the CORECHKCBOX type system. The typing judgment has the form $\Gamma; \Theta \vdash_m e : \tau$, which states that in a type environment Γ (mapping variables to their types) and a predicate environment Θ (mapping integer-

typed variables to Boolean predicates), expression e will have type τ if evaluated in mode m . Key rules for this judgment are given in Fig. 9. All remaining rules are given in Appendix B and E.

Pointer Access. Rules T-DEFARR and T-ASSIGNARR type-check array dereference and assignment operations resp., returning the type of pointed-to objects; rules for pointers to single objects are similar. The condition $m \leq m'$ ensures that checked pointers and unchecked pointers can only be dereferenced in checked and unchecked regions, respectively; the type rule for The rules do not attempt to reason whether the access is in bounds; this check is deferred to the semantics.

Type Equality and Subtyping and Casting. In CHECKEDCBOX, we define the type equality $\tau =_{\Theta} \tau'$ is an equivalent relation over the bound equality ($=_{\Theta}$) of the bounds in (NT-)array types and alpha equivalence of two function types in Figure 10. Two (NT-)array pointers $[\beta \tau]_{\kappa}$ and $[\beta' \tau']_{\kappa}$ are equivalent, if $\beta =_{\Theta} \beta'$; two function types $\forall \bar{x}. \bar{\tau} \rightarrow \tau$ and $\forall \bar{y}. \bar{\tau}' \rightarrow \tau'$ are equivalent, if we can find a same length variable set \bar{z} that is substituted for \bar{x} and \bar{y} in $\bar{\tau} \rightarrow \tau$ and $\bar{\tau}' \rightarrow \tau'$, respectively, and the substitution results are equal.

The T-CASTPTR rule permits casting from an expression of type τ' to a checked pointer when $\tau' \sqsubseteq \mathbf{ptr}^c \tau$. This subtyping relation \sqsubseteq is given in Fig. 10 built on the type equality ($\tau =_{\Theta} \tau' \Rightarrow \tau \sqsubseteq_{\Theta} \tau'$); the many rules ensure the relation is transitive. Most of the rules handle casting between array pointer types. The second rule $0 \leq b_l \wedge b_h \leq 1 \Rightarrow \mathbf{ptr}^m \tau \sqsubseteq \mathbf{ptr}^m [(b_l, b_h) \tau]$ permits treating a singleton pointer as an array pointer with $b_h \leq 1$ and $0 \leq b_l$. Two function pointers are subtyped

$\frac{}{\Gamma; \Theta \vdash_c u : \tau : \tau}$	$\frac{\text{T-CONSTC} \quad \Theta; \mathcal{H}; \emptyset \vdash_c n : \tau}{\Gamma; \Theta \vdash_c n : \tau : \tau}$	$\frac{\text{T-DEF} \quad m' \leq m \quad \Gamma; \Theta \vdash_m e : \mathbf{ptr}^{m'} \tau}{\Gamma; \Theta \vdash_m *e : \tau}$	$\frac{\text{T-ASSIGNARR} \quad \Gamma; \Theta \vdash_m e_1 : \mathbf{ptr}^{m'} [\beta \tau]_\kappa \quad \Gamma; \Theta \vdash_m e_2 : \tau' \quad \tau' \sqsubseteq_\Theta \tau \quad m' \leq m}{\Gamma; \Theta \vdash_m *e_1 = e_2 : \tau}$
$\frac{\text{T-LET} \quad x \notin FV(\tau') \quad \Gamma; \Theta \vdash_m e_1 : \tau \quad \Gamma[x \mapsto \tau]; \Theta \vdash_m e_2 : \tau'}{\Gamma; \Theta \vdash_m \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau'}$	$\frac{\text{T-LETINT} \quad x \in FV(\tau') \Rightarrow e_1 \in \mathbf{Bound} \quad \Gamma; \Theta \vdash_m e_1 : \tau \quad \Gamma[x \mapsto \tau]; \Theta[x \mapsto \mathbf{eq} \ e_1] \vdash_m e_2 : \tau'}{\Gamma; \Theta \vdash_m \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau'[e_1/x]}$	$\frac{\text{T-RET} \quad m; \emptyset; \Gamma; \emptyset \vdash \mu : \tau' \quad \Gamma[x \mapsto \tau']; \Theta[x \mapsto \mathbf{eq} \ n] \vdash_m e : \tau}{\Gamma; \Theta \vdash_m \mathbf{ret}(x, n : \tau', e) : \tau}$	
$\frac{\text{T-CHECKED} \quad \forall x \in \bar{x}. \neg c(\Gamma(x)) \quad FV(e) \in \bar{x} \quad \Gamma; \Theta \vdash_u e : \tau}{\Gamma; \Theta \vdash_m \mathbf{checked}(\bar{x})\{e\} : \tau}$	$\frac{\text{T-UNCHECKED} \quad \forall x \in \bar{x}. \neg c(\Gamma(x)) \quad FV(e) \in \bar{x} \quad \Gamma; \Theta \vdash_c e : \tau}{\Gamma; \Theta \vdash_m \mathbf{unchecked}(\bar{x})\{e\} : \tau}$	$\frac{\text{T-FUN} \quad \Gamma; \Theta \vdash_m e : \mathbf{ptr}^\xi \forall \bar{x}. \bar{\tau} \rightarrow \tau \quad \Gamma; \Theta \vdash_m \bar{e} : \bar{\tau}' \quad \bar{e}' = \{e' \mid (e', \mathbf{int}) \in (\bar{e} : \bar{\tau}')\} \quad \forall e'. e' \in \bar{e}' \Rightarrow e' \in \mathbf{Bound} \quad \bar{\tau}' \sqsubseteq_\Theta \bar{\tau}[\bar{e}'/\bar{x}]}{\Gamma; \Theta \vdash_m e(\bar{e}) : \tau[\bar{e}'/\bar{x}]}$	
$c(\mathbf{int}) = \mathbf{false} \quad c(\mathbf{ptr}^c \omega) = \mathbf{true} \quad c(\mathbf{ptr}^\xi \omega) = \mathbf{false} \text{ [otherwise]}$			

Figure 9: Selected type rules

($\mathbf{ptr}^\xi \forall \bar{x}. \bar{\tau} \rightarrow \tau \sqsubseteq_\Theta \mathbf{ptr}^\xi \forall \bar{x}. \bar{\tau}' \rightarrow \tau'$), if the output type are subtyped ($\tau \sqsubseteq_\Theta \tau'$) and the argument types are oppositely subtyped ($\bar{\tau}' \sqsubseteq_\Theta \bar{\tau}$).

Since bounds expressions may contain variables, determining assumptions like $b_l \leq b'_l$ requires reasoning about those variables' possible values. The type system uses Θ to make such reasoning more precise. Θ is a map from variables x to equation predicates P , which have the form $P ::= \mathbf{ge_0} \mid \mathbf{eq } b$. It maps variables to equations that are recorded along the type checking procedure. If Θ maps x to $\mathbf{ge_0}$, that means that $x \geq 0$; $\mathbf{eq } b$ means that x is equivalent to the bound value b in the current context, such as in the type judgment for e_2 in Rule T-LETINT and T-RET. Appendix D. has an example rule for populating Θ with a $\mathbf{ge_0}$ predicate.

Constant Validity. Given a constant $n : \tau$, the type judgment $\Theta; \mathcal{H}; \sigma \vdash_m n : \tau$ checks in Figure 11 if the constant is valid, where \mathcal{H} is the initial heap that the constant resides on and σ is a set of constant assumed to be checked. A global function store Ξ is also required in checking the validity of a function pointer. A valid function pointer should appear in the right store region (c or u) and the address stores a function with the right type.

The last rule in Figure 11 describes the validity check for a non-function pointer, where every element in the pointer range ($[0, \mathbf{size}(\omega))$) should be well typed. A checked pointer checks validity in type step as rule T-CONSTC, while a tainted/unchecked pointer does not check for such during the type checking. Tainted pointers are verified through the validity check in dynamic execution as we mentioned above.

Unchecked and Checked Blocks. During the type checking, Both $\mathbf{checked}(\bar{x})\{e\}$ and $\mathbf{unchecked}(\bar{x})\{e\}$ check all free variables are within \bar{x} , and the types for \bar{x} have no checked pointers. A checked and unchecked block

Bound Inequality and Equality:

$$\begin{aligned}
n \leq n' &\Rightarrow n \leq_\Theta n' \\
n \leq n' &\Rightarrow x + n \leq_\Theta x + n' \\
n \leq n' \wedge \Theta(x) = \mathbf{ge_0} &\Rightarrow n \leq_\Theta x + n' \\
\Theta(x) = \mathbf{eq } b \wedge b + n \leq_\Theta b' &\Rightarrow x + n \leq_\Theta b' \\
\Theta(x) = \mathbf{eq } b \wedge b' \leq_\Theta b + n &\Rightarrow b' \leq_\Theta x + n \\
b \leq_\Theta b' \wedge b' \leq_\Theta b &\Rightarrow b =_\Theta b'
\end{aligned}$$

Type Equility:

$$\begin{aligned}
&\mathbf{int} =_\Theta \mathbf{int} \\
\omega =_\Theta \omega' &\Rightarrow \mathbf{ptr}^\xi \omega =_\Theta \mathbf{ptr}^\xi \omega' \\
\beta =_\Theta \beta' \wedge \tau =_\Theta \tau' &\Rightarrow [\beta \tau]_\kappa =_\Theta [\beta' \tau']_\kappa \\
\mathbf{cond}(\bar{x}, \bar{\tau} \rightarrow \tau, \bar{y}, \bar{\tau}' \rightarrow \tau') &\Rightarrow \forall \bar{x}. \bar{\tau} \rightarrow \tau =_\Theta \forall \bar{y}. \bar{\tau}' \rightarrow \tau'
\end{aligned}$$

Subtype:

$$\begin{aligned}
\tau =_\Theta \tau' &\Rightarrow \tau \sqsubseteq_\Theta \tau' \\
&\mathbf{ptr}^c \omega \sqsubseteq_\Theta \mathbf{ptr}^u \omega' \\
0 \leq_\Theta b_l \wedge b_h \leq_\Theta 1 &\Rightarrow \mathbf{ptr}^m \tau \sqsubseteq_\Theta \mathbf{ptr}^m [(b_l, b_h) \tau] \\
b_l \leq_\Theta 0 \wedge 1 \leq_\Theta b_h &\Rightarrow \mathbf{ptr}^m [(b_l, b_h) \tau] \sqsubseteq_\Theta \mathbf{ptr}^m \tau \\
b_l \leq_\Theta 0 \wedge 1 \leq_\Theta b_h &\Rightarrow \mathbf{ptr}^m [(b_l, b_h) \tau]_{nt} \sqsubseteq_\Theta \mathbf{ptr}^m \tau \\
b_l \leq_\Theta b'_l \wedge b'_h \leq_\Theta b_h &\Rightarrow \mathbf{ptr}^m [(b_l, b_h) \tau]_{nt} \sqsubseteq_\Theta \mathbf{ptr}^m [(b'_l, b'_h) \tau] \\
b_l \leq_\Theta b'_l \wedge b'_h \leq_\Theta b_h &\Rightarrow \mathbf{ptr}^m [(b_l, b_h) \tau]_\kappa \sqsubseteq_\Theta \mathbf{ptr}^m [(b'_l, b'_h) \tau]_\kappa \\
\bar{\tau}' \sqsubseteq_\Theta \bar{\tau} \wedge \tau \sqsubseteq_\Theta \tau' &\Rightarrow \mathbf{ptr}^\xi \forall \bar{x}. \bar{\tau} \rightarrow \tau \sqsubseteq_\Theta \mathbf{ptr}^\xi \forall \bar{x}. \bar{\tau}' \rightarrow \tau' \\
n' + n = \mathbf{add}(n', n) &\quad (x + n') + n = x + \mathbf{add}(n', n) \\
\mathbf{cond}(\bar{x}, \tau, \bar{y}, \tau') = \exists \bar{z}. \bar{x} \cup \bar{z} \wedge \bar{y} \cup \bar{z} \wedge \mathbf{size}(\bar{x}) = \mathbf{size}(\bar{y}) = \mathbf{size}(\bar{z}) &\quad \wedge \tau[\bar{z}/\bar{x}] = \tau'[\bar{z}/\bar{y}]
\end{aligned}$$

Figure 10: Type Equality and Subtyping

represents the transition stage from a checked to an checked region, or vice versa. We need to make sure no checked pointers are information exposed to unsafe code regions.

Dependent Function Pointers and Let Bindings. Rule T-LET and T-LETINT type a let expression, which also admits type dependency. In particular, the result of eval-

$$\begin{array}{c}
\Theta; \mathcal{H}; \sigma \vdash_m n : \text{int} \qquad \Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^u \omega \\
\\
\Theta; \mathcal{H}; \sigma \vdash_m 0 : \text{ptr}^\xi \omega \quad \frac{(n : \text{ptr}^\xi \omega) \in \sigma}{\Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^\xi \omega} \\
\\
\frac{\text{ptr}^{\xi'} \omega' \sqsubseteq_\Theta \text{ptr}^\xi \omega \quad \Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^{\xi'} \omega'}{\Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^\xi \omega} \\
\\
\frac{\Xi(m, n) = \tau \ (\bar{x}' : \bar{\tau}) \ (\xi, e) \quad \xi \leq m \quad \bar{x} = \{x \mid (x : \text{int}) \in (\bar{x}' : \bar{\tau})\}}{\Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^\xi (\forall \bar{x}. \bar{\tau} \rightarrow \tau)} \\
\\
\frac{\neg \text{fun_l}(\omega) \quad \xi \leq m \quad \forall i \in [0, \text{size}(\omega)) . \Theta; \mathcal{H}; (\sigma \cup \{(n : \text{ptr}^\xi \omega)\}) \vdash_m \mathcal{H}(m, n + i)}{\Theta; \mathcal{H}; \sigma \vdash_m n : \text{ptr}^\xi \omega} \\
\\
\text{fun_l}(\forall \bar{x}. \bar{\tau} \rightarrow \tau) = \text{true} \quad \text{fun_l}(\omega) = \text{false} \ [\text{otherwise}]
\end{array}$$

Figure 11: Verification/Type Rules for Constants

uating a let may have a type that refers to one of its bound variables (e.g., if the result is a checked pointer with a variable-defined bound); if so, we must substitute away this variable once it goes out of scope (T-LETINT). Note that we restrict the expression e_1 to syntactically match the structure of a Bounds expression b (see Fig. 4).

Rule T-RET types a ret expression, which does not appear in source programs but is introduced by the semantics when evaluating a let binding (rule S-LET in Fig. 7); this rule is needed for the preservation proof.

Rule T-FUN is the standard dependent function call rule. It learns the function pointer type $(\text{ptr}^\xi \forall \bar{x}. \bar{\tau} \rightarrow \tau)$ of e by a recursive type check, type-checks the actual arguments \bar{e} which have types $\bar{\tau}'$, and then confirms that each of these types is a subtype of the declared type of e 's corresponding parameter $(\bar{\tau})$. Because functions have dependent types bounded by \bar{x} , we substitute each parameter e_i for its corresponding parameter x_i in both the parameter types and the return type. Consider the `safe_strcat` function in Fig. 8; its parameter type for `dst` depends on `n`. The T-FUN rule will substitute `n` with the argument at a call-site.

3.4. Type Soundness, Non-exposure, Non-crashing

In this subsection, we focus on our main meta-theoretic results about CORECHKCBOX: type soundness (progress and preservation), non-exposure, and non-crashing. These proofs have been carried out in our Coq model.

Type soundness relies on several *well-formedness*:

Definition 1 (Type Environment Well-formedness). A type environment Γ is well-formed iff every variable mentioned as type bounds in Γ are bounded by nat typed variables in Γ .

Definition 2 (Heap Well-formedness). For every m , A heap \mathcal{H} is well-formed iff (i) $\mathcal{H}(m, 0)$ is undefined, and (ii)

for all $n : \tau$ in the range of $\mathcal{H}(m)$, type τ contains no free variables.

Definition 3 (Stack Well-formedness). A stack snapshot φ is well-formed iff for all $n : \tau$ in the range of φ , type τ contains no free variables.

We also need to introduce a notion of *consistency*, relating heap environments before and after a reduction step, and type environments, predicate sets, and stack snapshots together.

Definition 4 (Stack Consistency). A type environment Γ , variable predicate set Θ , and stack snapshot φ are consistent—written $\Gamma; \Theta \vdash \varphi$ —iff for every variable x , $\Theta(x)$ is defined implies $\Gamma(x) = \tau$ for some τ and $\varphi(x) = n : \tau'$ for some n, τ' where $\tau' \sqsubseteq_\Theta \tau$.

Definition 5 (Checked Stack-Heap Consistency). A stack snapshot φ is consistent with heap \mathcal{H} —written $\mathcal{H} \vdash \varphi$ —iff for every variable x , $\varphi(x) = n : \tau$ with $\text{mode}(\tau) = c$ implies $\emptyset; \mathcal{H}(c); \emptyset \vdash_c n : \tau$.

Definition 6 (Checked Heap-Heap Consistency). A heap \mathcal{H}' is consistent with \mathcal{H} —written $\mathcal{H} \triangleright \mathcal{H}'$ —iff for every constant n , $\emptyset; \mathcal{H}; \emptyset \vdash_c n : \tau$ implies $\emptyset; \mathcal{H}'; \emptyset \vdash_c n : \tau$.

Moreover, as a program evaluates, its expression may contain literals $n : \tau$ where τ is a pointer type, i.e., n is an index in \mathcal{H} (perhaps because n was chosen by `malloc`). The normal type-checking judgment for e is implicitly parameterized by \mathcal{H} , and the rules for type-checking literals confirm that pointed-to heap cells are compatible with (subtypes of) the pointer's type annotation; in turn this check may precipitate checking the type consistency of the heap itself. We follow the same approach as Ruef et al. [34], and show the rules in Fig. 11; the judgment $\Theta; \mathcal{H}; \sigma \vdash_m n : \tau$ is used to confirm literal well-typing, where σ is a set of pointer literals already checked in \mathcal{H} (to allow pointer cycles). See Appendix B for further discussion.

Progress states that a CHECKEDCBOX program can always make a move:

Theorem 1 (Progress).

For any CHECKEDCBOX program e , heap \mathcal{H} , stack φ , type environment Γ , and variable predicate set Θ that are all are well-formed, consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$) and well typed ($\Gamma; \Theta \vdash_c e : \tau$ for some τ), one of the following holds:

- e is a value $(n : \tau)$.
- there exists $\varphi' \mathcal{H}' r$, such that $(\varphi, \mathcal{H}, e) \rightarrow_m (\varphi', \mathcal{H}', r)$.

There are two forms of preservation regarding the checked and unchecked regions. Checked Preservation states that a reduction step preserves both the type and consistency of the program being reduced.

Theorem 2 (Checked Preservation). For any CHECKEDCBOX program e , heap \mathcal{H} , stack φ , type environment Γ , and variable predicate set Θ that are all are well-formed, consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$) and well typed ($\Gamma; \Theta \vdash_c e : \tau$ for some τ), if there exists φ' ,

\mathcal{H}' and e' , such that $(\varphi, \mathcal{H}, e) \rightarrow_c (\varphi', \mathcal{H}', e')$, then \mathcal{H}' is checked region consistent with \mathcal{H} ($\mathcal{H} \triangleright \mathcal{H}'$) and there exists Γ' , Θ' and τ' that are well formed, checked region consistent ($\Gamma'; \Theta' \vdash \varphi'$ and $\mathcal{H}' \vdash \varphi'$) and well typed ($\Gamma'; \Theta' \vdash_c e : \tau'$), where $\tau' \sqsubseteq \tau$.

Theorem 3 (Unchecked Preservation). For any CHECKEDCBOX program e , heap \mathcal{H} , stack φ , type environment Γ , and variable predicate set Θ that are all are well-formed and well typed ($\Gamma; \Theta \vdash_c e : \tau$ for some τ), if there exists φ' , \mathcal{H}' and e' , such that $(\varphi, \mathcal{H}, e) \rightarrow_u (\varphi', \mathcal{H}', e')$, then $\mathcal{H}'(c) = \mathcal{H}(c)$.

Using the above theorem, we first show the non-exposure theorem, where code in unchecked region cannot observe know a valid checked pointer address.

Theorem 4 (Non-Exposure). For any CHECKEDCBOX program e , heap \mathcal{H} , stack φ , type environment Γ , and variable predicate set Θ that are all are well-formed and well typed ($\Gamma; \Theta \vdash_c e : \tau$ for some τ), if there exists φ' , \mathcal{H}' and e' , such that $(\varphi, \mathcal{H}, e) \rightarrow_u (\varphi', \mathcal{H}', e')$ and $e = E[\alpha(x)]$ and $mode(E) = u$, where $\alpha(x)$ is some expression (not checked nor unchecked) containing variable x , if x is a checked pointer, it is a null one.

Using these two theorems we can prove our main result, *non-crashing*, which states that a well-typed program can never be *stuck* (expression e is a non-value that cannot take a step²).

Theorem 5 (Non-Crashing). For any CHECKEDCBOX program e , heap \mathcal{H} , stack φ , type environment Γ , and variable predicate set Θ that are well-formed and consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$), if e is well-typed ($\varphi; \Theta \vdash_c e : \tau$ for some τ) and there exists φ_i , \mathcal{H}_i , e_i , and m_i for $i \in [1, k]$, such that $(\varphi, \mathcal{H}, e) \rightarrow_{m_1} (\varphi_1, \mathcal{H}_1, e_1) \rightarrow_{m_2} \dots \rightarrow_{m_k} (\varphi_k, \mathcal{H}_k, r)$, then r can never be *stuck*.

4. Compilation

The main subtlety of compiling Checked C to Clang/LLVM is to capture the annotations on pointer literals that track array bound information, which is used in premises of rules like S-DEFARRAY and S-ASSIGNARR to prevent spatial safety violations. The Checked C compiler [20] inserted additional pointer checks for verifying pointers being not null and the bounds are within their limits. The latter is done by introducing additional shadow variables for storing (NT-)array pointer bound information.

In CHECKEDCBOX, context and pointer modes determine the particular heap/function store that a pointer points to, i.e., cpointer points to checked regions, while t and upointers point to unchecked regions. Unchecked regions are associated with a sandbox mechanism that permits exception handling of potential memory failures. In the compiled

	c	t	u
	Impl / Foml	Impl / Foml	Impl / Foml
c	$*x / *(c, x)$	$sand_get(x) / *(u, x)$	\times
u	\times	$*x / *(u, x)$	$*x / *(u, x)$

Figure 12: Compiled Targets for Dereference

LLVM code, pointer access operations have different syntax when the modes are different. Figure 12 lists the different compiled syntax for a dereference operation ($*x$) for the compiler implementation (Impl) and formalism (Foml). The columns represent different pointer modes and the rows represent context modes. For example, when we have a tpointer in a c-mode region, we use the sandbox pointer access function ($sand_get(x)$) accessing the data. In formalism, we create a new dereference data-structure in LLVM on top of the existing $*x$ operation: $*(m, x)$. if the mode is c, it accesses the checked heap/function store; otherwise, it accesses the unchecked one.

In the compilation, we use a function `mode` to get the pointer mode of a function learning from the pointer's type. Thus, the compiler can determine the correct compiled syntax for a pointer access operation with the knowledge of the current context mode. We also build a context switching mechanism to change the context mode when we meet checked and unchecked blocks. We describe the mechanism shortly.

This section shows how CHECKEDCBOX deals with pointer modes, mode switching and function pointer compilations, with no loss of expressiveness; as the Checked C contains the erase of annotations in [20] and Appendix G. For the compiler formalism, we present a compilation algorithm that converts from CHECKEDCBOX to COREC, an untyped language without metadata annotations, which represents an intermediate layer we build on LLVM for simplifying compilation. In COREC, the syntax for dereference, assignment, malloc, function calls are: $*(m, e)$, $*(m, e) = e$, $malloc(m, \omega)$, and $(m, e)(\bar{e})$. The algorithm sheds light on how compilation can be implemented in the real Checked C compiler, while eschewing many important details (COREC has many differences with LLVM IR).

Compilation is defined by extending CORECHKCBOX's typing judgment thusly:

$$\Gamma; \Theta; \rho \vdash_m e \gg \dot{e} : \tau$$

There is now a COREC output \dot{e} and an input ρ , which maps each (NT-)array pointer variable to its mode and each variable p to a pair of *shadow variables* that keep p 's up-to-date upper and lower bounds; these may differ from the bounds in p 's type due to bounds widening.³

We formalize rules for this judgment in PLT Redex [10], following and extending our Coq development for CORECHKCBOX. To give confidence that compilation is correct, we use Redex's property-based random testing support to show that compiled-to \dot{e} simulates e , for all e .


2. Note that bounds and null are *not* stuck expressions—they represent a program terminated by a failed run-time check. A program that tries to access $\mathcal{H}n$ but \mathcal{H} is undefined at n will be stuck, and violates spatial safety.

3. Since lower bounds are never widened, the lower-bound shadow variable is unnecessary; we include it for uniformity.

```

1  int deref_array(n : int,
2      p : ptrc [(0, n) int]nt,
3      q : ptrt [(0, n) int]nt) {
4      /* ρ(p) = p_lo, p_hi, p_m */
5      /* ρ(q) = q_lo, q_hi, q_m */
6      * p;
7      * q = 1;
8  }
9  ...
10 /* p0 : ptrc [(0, 5) int]nt */
11 /* q0 : ptrt [(0, 5) int]nt */
12 deref_array(5, p0, q0);
.

```



```

1  deref_array(int n, int* p, int * q) {
2      //m is the current context mode
3      let p_lo = 0; let p_hi = n;
4      let q_lo = 0; let q_hi = n;
5      /* runtime checks */
6      assert(p_lo ≤ 0 && 0 ≤ p_hi);
7      assert(p != 0);
8      *(mode(p) ∧ m, p);
9      verify(q, not_null && q_lo ≤ 0 && 0 ≤ q_hi);
10     *(mode(q) ∧ m, q)=1;
11 }
12 ...
13 deref_array(5, p0, q0);


```

Figure 13: Compilation Example for Dependent Functions

4.1. Approach

Due to space constraints, we explain the rules for compilation by example, using a C-like syntax; the complete rules are given in Appendix G. Each rule performs up to three tasks: (a) conversion of e to A-normal form; (b) insertion of dynamic checks and bound widening expressions; and (c) generate right pointer accessing expressions based on modes. A-normal form conversion is straightforward: compound expressions are handled by storing results of subexpressions into temporary variables, as in the following example.

```

let y=(x+1)+(6+1) . 
let a=x+1;
let b=6+1;
let y=a+b

```

This simplifies the management of effects from subexpressions. The next two steps of compilation are more interesting. We state them based on different CHECKEDCBOX operations.

Pointer Accesses and Modes. In every declaration (or the beginning of a function body) of a pointer, if the pointer is an (NT)-array one, we first allocate two *shadow variables* to track the lower and upper bounds potentially changed for pointer arithmetic and NT-array bound widening. We also associated with every c-mode pointer variable according to its type; we place bounds and null-pointer checks, such as the line 6 and 7 in Figure 13. In addition, in the formalism, before every use of a tainted pointer (Figure 13 line 9), there is an inserted verification step similar to Figure 11,

which checks a pointer is well defined in the heap for every element in its defined range. In implementation, we actually optimize the verification away and substitute it with the bounds and null-pointer checks. Because tainted pointer is checked every time before it is used, so that we only need to check the top pointer is well defined without recursively looking at the sub-terms in a nested pointer case. The modes in compiled deference $(*(mode(p) \wedge m, p))$ and assignment $(*(mode(q) \wedge m, q)=1)$ operations are computed based on the meet operation (\wedge) of the pointer mode (e.g. $mode(p)$) and the current context mode (m).

Checked and Unchecked Blocks. In the implementation, unchecked and checked blocks are compiled as context switching functions provided by the sandbox mechanism. $unchecked(\bar{x})\{e\}$ is compiled to $sandbox_call(\bar{x}, e)$, where we call the sandbox to execute expression e with the arguments \bar{x} . $checked(\bar{x})\{e\}$ is compiled to $callback(\bar{x}, e)$, where we perform a callback to a checked block code e inside a sandbox. In CHECKEDCBOX, We adopt an aggressive execution scheme that directly learns pointer addresses from compiled assembly to make the callback happen. In the formalism, we rely on the type system to guarantee the context switching without creating the extra function calls for simplicity.

Function Pointers and Calls. Function pointers are dealt with similarly to normal pointers, but we insert checks to check if the pointer address is not null in the function store and if the type is correctly represented, for both c and t mode pointers⁴. For example, in compiling the `stringsort` function in Figure 1, we place a check `verify_fun(cmp, not_null && type_match)`, and we place a similar check before Figure 3 line 7 to check the tainted `cmp` when it is used. The compilation of function calls (compiling to $(m, e)(\bar{e})$) is similar to the manipulation of pointer access operations in Figure 12.

For compiling dependent function calls, Figure 13 also shows an example. Notice that the bounds for the array pointer `p` are not passed as arguments. Instead, they are initialized according to `p`'s type—see line 4 of the original CORECHKCBOX program at the top of the figure. Line 3 of the generated code sets the lower bound to 0 and the upper bound to `n`.

4.2. Metatheory

We formalize both the compilation procedure and the simulation theorem in the PLT Redex model we developed for CORECHKCBOX (see Sec. 3.1), and then attempt to falsify it via Redex's support for random testing. Redex allows us to specify compilation as logical rules (essentially, an extension of typing), but then execute it algorithmically to automatically test whether simulation holds. This process revealed several bugs in compilation and the theorem statement. We ultimately plan to prove simulation in the Coq model.

4. c-mode pointers are checked once in the beginning and t-mode pointers are checked every time when use

We use the notation \gg to indicate the *erasure* of stack and heap—the rhs is the same as the lhs but with type annotations removed:

$$\begin{aligned}\mathcal{H} &\gg \dot{\mathcal{H}} \\ \varphi &\gg \dot{\varphi}\end{aligned}$$

In addition, when $\Gamma; \emptyset \vdash \varphi$ and φ is well-formed, we write $(\varphi, \mathcal{H}, e) \gg_m (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$ to denote $\varphi \gg \dot{\varphi}$, $\mathcal{H} \gg \dot{\mathcal{H}}$ and $\Gamma; \emptyset; \emptyset \vdash_m e \gg \dot{e} : \tau$ for some τ respectively. Γ is omitted from the notation since the well-formedness of φ and its consistency with respect to Γ imply that e must be closed under φ , allowing us to recover Γ from φ . Finally, we use $\dot{\rightarrow}^*$ to denote the transitive closure of the reduction relation of COREC. Unlike the CORECHKCBOX, the semantics of COREC does not distinguish checked and unchecked regions.

Fig. 14 gives an overview of the simulation theorem.⁵ The simulation theorem is specified in a way that is similar to the one by Merigoux et al. [25].

An ordinary simulation property would replace the middle and bottom parts of the figure with the following:

$$(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1)$$

Instead, we relate two erased configurations using the relation \sim , which only requires that the two configurations will eventually reduce to the same state.

Theorem 6 (Simulation (\sim)). For CORECHKCBOX expressions e_0 , stacks φ_0, φ_1 , and heap snapshots $\mathcal{H}_0, \mathcal{H}_1$, if $\mathcal{H}_0 \vdash \varphi_0$, $(\varphi_0, \mathcal{H}_0, e_0) \gg_c (\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0)$, and if there exists some r_1 such that $(\varphi_0, \mathcal{H}_0, e_0) \rightarrow_c (\varphi_1, \mathcal{H}_1, r_1)$, then the following facts hold:

- if there exists e_1 such that $r = e_1$ and $(\varphi_1, \mathcal{H}_1, e_1) \gg (\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1)$, then there exists some $\dot{\varphi}, \dot{\mathcal{H}}, \dot{e}$, such that $(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$ and $(\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1) \dot{\rightarrow}^* (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$.
- if $r_1 = \text{bounds}$ or null , then we have $(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}_1, \dot{\mathcal{H}}_1, r_1)$ where $\varphi_1 \gg \dot{\varphi}_1$, $\mathcal{H}_1 \gg \dot{\mathcal{H}}_1$.

Our random generator (discussed in the next section) never produces unchecked expressions (whose behavior could be undefined), so we can only test the simulation theorem as it applies to checked code. This limitation makes it unnecessary to state the other direction of the simulation theorem where e_0 is stuck, because Theorem 1 guarantees that e_0 will never enter a stuck state if it is well-typed in checked mode.

The current version of the Redex model has been tested against 20000 expressions with depth less than 10. Each expression can reduce multiple steps, and we test simulation between every two adjacent steps to cover a wider range of programs, particularly the ones that have a non-empty heap.

5. We ellide the possibility of \dot{e}_1 evaluating to `bounds` or `null` in the diagram for readability.

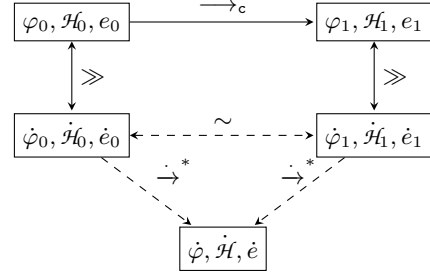


Figure 14: Simulation between CORECHKCBOX and COREC

5. Evaluation

- provide evidence that the CHECKEDCBOX compiler is efficient. Compare the compiler with respect to other work, like RLBox, also the previous Checked C compiler.
- provide user experience of CHECKEDCBOX. We restrict the use of checked pointers compared to previous checked-c compiler. Is the restriction arrngable. We can say that the tainted shells are auto-matically generated, so we have a mechanism for auto-generating tainted pointers if necessary.
- if we have space, we can re-introduce random testing a little, saying that how it helps us to develop the compiler.
- we can then talk about the possible bugs we find in the Checked C compiler for function pointer or the RLBox bugs.

6. Related Work

Our work is most closely related to prior formalizations of C(-like) languages that aim to enforce memory safety, but it also touches on C-language formalization in general.

Formalizing C and Low-level code. A number of prior works have looked at formalizing the semantics of C, including CompCerts [1, 19], Ellison and Rosu [8], Kang et al. [15], and Memarian et al. [22, 23]. These works also model pointers as logically coupled with either the bounds of the blocks they point to, or provenance information from which bounds can be derived. None of these is directly concerned with enforcing spatial safety, and that is reflected in the design. For example, memory itself is not be represented as a flat address space, as in our model or real machines, so memory corruption due to spatial safety violations, which Checked C's type system aims to prevent, may not be expressible. That said, these formalizations consider much more of the C language than does CORECHKCBOX, since they are interested in the entire language's behavior.

Spatially Safe C Formalizations. Several prior works formalize C-language transformations or C-language dialects aiming to ensure spatial safety. Hathhorn et al. [13] extends the formalization of Ellison and Rosu [8] to produce a semantics that detects violations of spatial safety

(and other forms of undefinedness). It uses a CompCert-style memory model, but “fattens” logical pointer representations to facilitate adding side conditions similar to CORECHKCBOX’s. Its concern is bug finding, not compiling programs to use this semantics.

CCured [30] and Softbound [28] implement spatially safe semantics for normal C via program transformation. Like CORECHKCBOX, both systems’ operational semantics annotate pointers with their bounds. CCured’s equivalent of array pointers are compiled to be “fat,” while SoftBound compiles bounds metadata to a separate hashtable, thus retaining binary compatibility at higher checking cost. Checked C uses static type information to enable bounds checks without need of pointer-attached metadata, as we show in Section 4. Neither CCured nor Softbound models null-terminated array pointers, whereas our semantics ensures that such pointers respect the zero-termination invariant, leveraging bounds widening to enhance expressiveness.

Cyclone [11, 14] is a C dialect that aims to ensure memory safety; its pointer types are similar to CCured. Cyclone’s formalization [11] focuses on the use of *regions* to ensure temporal safety; it does not formalize arrays or threats to spatial safety. Deputy [4, 45] is another safe-C dialect that aims to avoid fat pointers; it was an initial inspiration for Checked C’s design [7], though it provides no specific modeling for null-terminated array pointers. Deputy’s formalization [4] defines its semantics directly in terms of compilation, similar in style to what we present in Section 4. Doing so tightly couples typing, compilation, and semantics, which are treated independently in CORECHKCBOX. Separating semantics from compilation isolates meaning from mechanism, easing understandability. Indeed, it was this separation that led us to notice the limitation with Checked C’s handling of bounds widening.

The most closely related work is the formalization of Checked C done by Ruef et al. [34]. They present the type system and semantics of a core model of Checked C, mechanized in Coq, and were the first to prove a blame theorem. CORECHKCBOX’s Coq-based development (Section 3) substantially extends theirs to include conditionals, dynamically bounded array pointers with dependent types, null-terminated array pointers, dependently typed functions, and subtyping. They postulate that pointer metadata can be erased in a real implementation, but do not show it. Our CORECHKCBOX compiler, formalized and validated in PLT Redex via randomized testing, demonstrates that such metadata *can* be erased; we found that erasure was non-obvious once null-terminated pointers and bounds widening were considered.

7. Conclusion and Future Work

This paper presented CORECHKCBOX, a formalization of an extended core of the Checked C language which aims to provide spatial memory safety. CORECHKCBOX models dynamically sized and null-terminated arrays with dependently typed bounds that can additionally be widened at runtime. We prove, in Coq, the key safety property

of Checked C for our formalization, *blame*: if a mix of checked and unchecked code gives rise to a spatial memory safety violation, then this violation originated in an unchecked part of the code. We also show how programs written in CORECHKCBOX (whose semantics leverage fat pointers) can be compiled to COREC (which does not) while preserving their behavior. We developed a version of CORECHKCBOX written in PLT Redex, and used a custom term generator in conjunction with Redex’s randomized testing framework to give confidence that compilation is correct. We also used this framework to cross-check CORECHKCBOX against the Checked C compiler, finding multiple inconsistencies in the process.

As future work, we wish to extend CORECHKCBOX to model more of Checked C, with our Redex-based testing framework guiding the process. The most interesting Checked C feature not yet modeled is *interop types* (itypes), which are used to simplify interactions with unchecked code via function calls. A function whose parameters are itypes can be passed checked or unchecked pointers depending on whether the caller is in a checked region. This feature allows for a more modular C-to-Checked C porting process, but complicates reasoning about blame. A more ambitious next step would be to extend an existing formally verified framework for C, such as CompCert [18] or VeLLVM [44], with Checked C features, towards producing a verified-correct Checked C compiler. We believe that CORECHKCBOX’s Coq and Redex models lay the foundation for such a step, but substantial engineering work remains.

References

- [1] Sandrine Blazy and Xavier Leroy. Mechanized Semantics for the Clight Subset of the C Language. *Journal of Automated Reasoning*, 43(3):263–288, 2009. ISSN 1573-0670. doi: 10.1007/s10817-009-9148-3. URL <http://dx.doi.org/10.1007/s10817-009-9148-3>.
- [2] BlueHat. Memory corruption is still the most prevalent security vulnerability. <https://www.zdnet.com/article/microsoft-70-percent-of-all-security-bugs-are-memory-safety-issues/>, 2019. Accessed: 2020-02-11.
- [3] c2rust. C to rust translation, refactoring, and cross-checking. <https://c2rust.com/>, 2018.
- [4] Jeremy Condit, Matthew Harren, Zachary Anderson, David Gay, and George C. Necula. Dependent Types for Low-Level Programming. In *Proceedings of European Symposium on Programming (ESOP ’07)*, 2007.
- [5] Junhan Duan, Yudi Yang, Jie Zhou, and John Criswell. Refactoring the FreeBSD kernel with Checked C. In *Proceedings of the 2020 IEEE Cybersecurity Development Conference (SecDev)*, 2020.
- [6] Gregory J Duck and Roland HC Yap. Heap bounds protection with low fat pointers. In *Proceedings of the 25th International Conference on Compiler Construction*, pages 132–142. ACM, 2016.
- [7] Archibald Samuel Elliott, Andrew Ruef, Michael Hicks, and David Tarditi. Checked C: Making C Safe by Extension. In *2018 IEEE Cybersecurity Development (SecDev)*, pages 53–60, 2018. doi: 10.1109/SecDev.2018.00015.

- [8] Chucky Ellison and Grigore Rosu. An Executable Formal Semantics of C with Applications. In *Proceedings of the 39th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '12, pages 533–544, New York, NY, USA, 2012. ACM. ISBN 978-1-4503-1083-3. doi: 10.1145/2103656.2103719. URL <http://doi.acm.org/10.1145/2103656.2103719>.
- [9] Mehmet Emre, Kyle Dewey, Ryan Schroeder, and Ben Harek. Translating C to safer Rust. In *Proceedings of the 2021 ACM on Programming Languages (PACMPL)*, 5(OOPSLA), 2021.
- [10] Matthias Felleisen, Robert Bruce Findler, and Matthew Flatt. *Semantics Engineering with PLT Redex*. The MIT Press, 1st edition, 2009. ISBN 0262062755.
- [11] Dan Grossman, Greg Morrisett, Trevor Jim, Michael Hicks, Yanling Wang, and James Cheney. Region-based Memory Management in Cyclone. In *PLDI*, 2002.
- [12] Arjun Guha, Claudiu Saftoiu, and Shriram Krishnamurthi. The Essence of Javascript. In *Proceedings of the 24th European Conference on Object-Oriented Programming, ECOOP'10*, page 126–150, Berlin, Heidelberg, 2010. Springer-Verlag. ISBN 3642141064.
- [13] Chris Hathhorn, Chucky Ellison, and Grigore Rosu. Defining the Undefinedness of C. *SIGPLAN Not.*, 50(6):336–345, June 2015. ISSN 0362-1340. doi: 10.1145/2813885.2737979. URL <https://doi.org/10.1145/2813885.2737979>.
- [14] Trevor Jim, Greg Morrisett, Dan Grossman, Michael Hicks, James Cheney, and Yanling Wang. Cyclone: A Safe Dialect of C. In *USENIX Annual Technical Conference*, pages 275–288, Monterey, CA, 2002. USENIX.
- [15] Jeehoon Kang, Chung-Kil Hur, William Mansky, Dmitri Garbuzov, Steve Zdancewic, and Viktor Vafeiadis. A Formal C Memory Model Supporting Integer-pointer Casts. *SIGPLAN Not.*, 50(6):326–335, June 2015. ISSN 0362-1340. doi: 10.1145/2813885.2738005. URL <http://doi.acm.org/10.1145/2813885.2738005>.
- [16] Leonidas Lampropoulos and Benjamin C. Pierce. *QuickChick: Property-Based Testing in Coq*. Software Foundations series, volume 4. Electronic textbook, August 2018. Version 1.0. <http://www.cis.upenn.edu/~bcpierce/sf>.
- [17] Per Larson. Migrating legacy code to Rust. RustConf 2018 talk, August 2018.
- [18] Xavier Leroy. Formal verification of a realistic compiler. *Communications of the ACM*, 52(7):107–115, July 2009. ISSN 0001-0782. doi: 10.1145/1538788.1538814. URL <http://doi.acm.org/10.1145/1538788.1538814>.
- [19] Xavier Leroy, Andrew W. Appel, Sandrine Blazy, and Gordon Stewart. The CompCert Memory Model, Version 2. Research Report RR-7987, INRIA, June 2012. URL <https://hal.inria.fr/hal-00703441>.
- [20] Liyi Li, Yiyun Liu, Deena L. Postol, Leonidas Lampropoulos, David Van Horn, and Michael Hicks. A formal model of Checked C. In *Proceedings of the Computer Security Foundations Symposium (CSF)*, August 2022.
- [21] Aravind Machiry, John Kastner, Matt McCutchen, Aaron Eline, Kyle Headley, and Michael Hicks. C to checked c by 3c. *Proceedings of the ACM on Programming Languages*, 6(OOPSLA1):1–29, 2022.
- [22] Kayvan Memarian, Justus Matthiesen, James Lingard, Kyn-dylan Nienhuis, David Chisnall, Robert N. M. Watson, and Peter Sewell. Into the Depths of C: Elaborating the de Facto Standards. *SIGPLAN Not.*, 51(6):1–15, June 2016. ISSN 0362-1340. doi: 10.1145/2980983.2908081. URL <https://doi.org/10.1145/2980983.2908081>.
- [23] Kayvan Memarian, Victor B. F. Gomes, Brooks Davis, Stephen Kell, Alexander Richardson, Robert N. M. Watson, and Peter Sewell. Exploring C Semantics and Pointer Provenance. *Proc. ACM Program. Lang.*, 3(POPL):67:1–67:32, January 2019. ISSN 2475-1421. doi: 10.1145/3290380. URL <http://doi.acm.org/10.1145/3290380>.
- [24] Samuel Mergendahl, Nathan Burow, and Hamed Okhravi. Cross-language attacks. 2022.
- [25] Denis Merigoux, Nicolas Chataing, and Jonathan Protzenko. Catala: A Programming Language for the Law. *arXiv preprint arXiv:2103.03198*, 2021.
- [26] Mozilla. Rust Programming Language. <https://www.rust-lang.org/>, 2021.
- [27] Santosh Nagarakatte, Jianzhou Zhao, Milo MK Martin, and Steve Zdancewic. Softbound: Highly compatible and complete spatial memory safety for c. *ACM Sigplan Notices*, 44(6):245–258, 2009.
- [28] Santosh Nagarakatte, Jianzhou Zhao, Milo M.K. Martin, and Steve Zdancewic. SoftBound: Highly Compatible and Complete Spatial Memory Safety for C. In *Proceedings of the 30th ACM SIGPLAN Conference on Programming Language Design and Implementation, PLDI '09*, page 245–258, New York, NY, USA, 2009. Association for Computing Machinery. ISBN 9781605583921. doi: 10.1145/1542476.1542504. URL <https://doi.org/10.1145/1542476.1542504>.
- [29] Shravan Narayan, Craig Disselkoen, Tal Garfinkel, Nathan Froyd, Eric Rahm, Sorin Lerner, Hovav Shacham, and Deian Stefan. Retrofitting fine grain isolation in the firefox renderer. In *29th USENIX Security Symposium (USENIX Security 20)*, pages 699–716. USENIX Association, August 2020. ISBN 978-1-939133-17-5. URL <https://www.usenix.org/conference/usenixsecurity20/presentation/narayan>.
- [30] George C. Necula, Jeremy Condit, Matthew Harren, Scott McPeak, and Westley Weimer. CCured: Type-Safe Retrofitting of Legacy Software. *ACM Transactions on Programming Languages and Systems (TOPLAS)*, 27(3), 2005.
- [31] George C Necula, Jeremy Condit, Matthew Harren, Scott McPeak, and Westley Weimer. Ccured: type-safe retrofitting of legacy software. *ACM Transactions on Programming Languages and Systems (TOPLAS)*, 27(3):477–526, 2005.
- [32] Michał H. Pałka, Koen Claessen, Alejandro Russo, and John Hughes. Testing an Optimising Compiler by Generating Random Lambda Terms. In *Proceedings of the 6th International Workshop on Automation of Software Test, AST '11*, pages 91–97, New York, NY, USA, 2011. ACM. ISBN 978-1-4503-0592-1. doi: 10.1145/1982595.1982615. URL <http://doi.acm.org/10.1145/1982595.1982615>.
- [33] Ricardo Peña. An Introduction to Liquid Haskell. *Electronic Proceedings in Theoretical Computer Science*, 237:68–80, Jan 2017. ISSN 2075-2180. doi: 10.4204/eptcs.237.5. URL <http://dx.doi.org/10.4204/EPTCS.237.5>.
- [34] Andrew Ruef, Leonidas Lampropoulos, Ian Sweet, David Tarditi, and Michael Hicks. Achieving Safety Incrementally with Checked C. In Flemming Nielson and David Sands, editors, *Principles of Security and Trust*, pages 76–98, Cham, 2019. Springer International Publishing. ISBN 978-3-030-17138-4.
- [35] Andrew Ruef, Leonidas Lampropoulos, Ian Sweet, David Tarditi, and Michael Hicks. Achieving safety incrementally with checked c. In *International Conference on Principles of Security and Trust*, pages 76–98. Springer, Cham, 2019.
- [36] Konstantin Serebryany, Derek Bruening, Alexander Potapenko, and Dmitriy Vyukov. Addresssanitizer: A fast address sanity checker. In *USENIX Annual Technical Conference*, pages 309–318, 2012.

- [37] Hovav Shacham, Matthew Page, Ben Pfaff, Eu-Jin Goh, Nagendra Modadugu, and Dan Boneh. On the effectiveness of address-space randomization. In *CCS*. ACM, 2004.
- [38] Dokyung Song, Julian Lettner, Prabhu Rajasekaran, Yeoul Na, Stijn Volckaert, Per Larsen, and Michael Franz. SoK: Sanitizing for security. In *Proceedings of the 2019 IEEE Symposium on Security and Privacy (S&P)*, 2019.
- [39] David Tarditi. Extending C with Bounds Safety and Improved Type Safety, 2021. URL <https://github.com/secure-sw-dev/checkedc/>.
- [40] David Tarditi, Archibald Samuel Elliott, Andrew Ruef, and Michael Hicks. Checked C: Making C safe by extension. In *IEEE Cybersecurity Development Conference 2018 (SecDev)*, September 2018.
- [41] CVE Trends. Cve trends. <https://www.cvedetails.com/vulnerabilities-by-types.php>, 2021. Accessed: 2020-10-11.
- [42] Niki Vazou, Eric L. Seidel, Ranjit Jhala, Dimitrios Vytiniotis, and Simon Peyton-Jones. Refinement Types for Haskell. *SIGPLAN Not.*, 49(9):269–282, August 2014. ISSN 0362-1340. doi: 10.1145/2692915.2628161. URL <https://doi.org/10.1145/2692915.2628161>.
- [43] Anna Zeng and Will Crichton. Identifying barriers to adoption for rust through online discourse. *arXiv preprint arXiv:1901.01001*, 2019.
- [44] Jianzhou Zhao, Santosh Nagarakatte, Milo M.K. Martin, and Steve Zdancewic. Formalizing the LLVM Intermediate Representation for Verified Program Transformations. *SIGPLAN Not.*, 47(1):427–440, January 2012. ISSN 0362-1340. doi: 10.1145/2103621.2103709. URL <http://doi.acm.org/10.1145/2103621.2103709>.
- [45] Feng Zhou, Jeremy Condit, Zachary Anderson, Ilya Bagrak, Rob Ennals, Matthew Harren, George Necula, and Eric Brewer. SafeDrive: Safe and recoverable extensions using language-based techniques. In *7th Symposium on Operating System Design and Implementation (OSDI’06)*, Seattle, Washington, 2006. USENIX Association.
- [46] Jie Zhou. The benefits and costs of using fat pointers for temporal memory safety. Poster presentation at PLDI 2021 student research competition (silver medalist), June 2021.

Appendix

1. Differences with the Coq and Redex Models

The Coq and Redex models of CORECHKCBOX may be found at <https://github.com/plum-umd/checkedc>. The Coq model's syntax is slightly different from that in Fig. 4. In particular, the arguments in a function call are restricted to variables and constants, according to a separate well-formedness condition. A function call $f(e)$ can always be written in $\text{let } x = e \text{ in } f(x)$ to cope. In addition, conditionals have two syntactic forms: **Elif** is a normal conditional, and **EIfDef** is one whose boolean guard is of the form $*x$. By syntactically distinguishing these two cases, the Coq model does not need the *[prefer]* rule for $\text{if } (*x) \dots$ forms as in Fig. 6. The Redex model *does* prioritize such forms but not the same way as in the figure. It uses a variation of the S-VAR rule: The modified rule is equipped with a precondition that is false whenever S-INTT is applicable.

The Coq model uses a runtime stack φ as described at the start of Sec. 3.2. The Redex model introduces let bindings during evaluation to simulate a runtime stack. For example, consider the expression $e \equiv \text{let } x = (5 : \text{int}) \text{ in } x + x$. Expression e first steps to $\text{let } x = (5 : \text{int}) \text{ in } (5 : \text{int}) + x$, which in turns steps to $\text{let } x = (5 : \text{int}) \text{ in } (5 : \text{int}) + (5 : \text{int})$. Since the rhs of x is a value, the let binding in e effectively functions as a stack that maps from x to $5 : \text{int}$. The let form remains in the expression and lazily replaces the variables in its body. The let form can be removed from the expression only if its body is evaluated to a value, e.g., $\text{let } x = (5 : \text{int}) \text{ in } (10 : \text{int})$ steps to $10 : \text{int}$. The rule for popping let bindings in this manner corresponds to the S-RET rule in Fig. 7. Leveraging let bindings adds complexity to the semantics but simplifies typing/consistency and term generation during randomized testing.

2. Typing Rules for Literal Pointers

The typing of integer literals, which can also be pointers to the heap, was presented in Sec. 3.4 in Fig. 11. Here we describe these rules further.

The variable type rule (T-VAR) simply checks if a given variable has the defined type in Γ ; the constant rule (T-CONST) is slightly more involved. First, it ensures that the type annotation τ does not contain any free variables. More importantly, it ensures that the literal itself is well typed using an auxiliary typing relation $\mathcal{H}; \sigma \vdash n : \tau$.

If the literal's type is an integer, an unchecked pointer, or a null pointer, it is well typed, as shown by the top three rules in Fig. 11. However, if it is a checked pointer $\text{ptr}^c \omega$, we need to ensure that what it points to in the heap is of the appropriate pointed-to type (ω), and also recursively ensure that any literal pointers reachable this way are also well-typed. This is captured by the bottom rule in the figure, which states that for every location $n + i$ in the pointers' range $[n, n + \text{size}(\omega))$, where size yields the size of its

argument, then the value at the location $\mathcal{H}(n + i)$ is also well-typed. However, as heap snapshots can contain cyclic structures (which would lead to infinite typing derivations), we use a scope σ to assume that the original pointer is well-typed when checking the types of what it points to. The middle rule then accesses the scope to tie the knot and keep the derivation finite, just like in Ruef et al. [34].

3. Other Semantic Rules

Fig. 15 shows the remaining semantic rules for CORECHKCBOX. We explain a selected few rules in this subsection.

Rule S-VAR loads the value for x in stack φ . Rule S-DEFARRAY dereferences an array pointer, which is similar to the Rule S-DEFNTARRAY in Fig. 7 (dealing with null-terminated array pointers). The only difference is that the range of 0 is at $[n_l, n_h)$ not $[n_l, n_h]$, meaning that one cannot dereference the upper-bound position in an array. Rules DEFARRAYBOUND and DEFNTARRAYBOUND describe an error case for a dereference operation. If we are dereferencing an array/NT-array pointer and the mode is c , 0 must be in the range from n_l to n_h (meaning that the dereference is in-bound); if not, the system results in a bounds error. Obviously, the dereference of an array/NT-array pointer also experiences a null state transition if $n \leq 0$.

Rules S-MALLOC and S-MALLOCBOUND describe the malloc semantics. Given a valid type ω_a that contains no free variables, `alloc` function returns an address pointing at the first position of an allocated space whose size is equal to the size of ω_a , and a new heap snapshot \mathcal{H}' that marks the allocated space for the new allocation. The `malloc` is transitioned to the address n with the type $\text{ptr}^c \omega_a$ and new updated heap. It is possible for `malloc` to transition to a bounds error if the ω_a is an array/NT-array type $[(n_l, n_h) \tau]_\kappa$, and either $n_l \neq 0$ or $n_h \leq 0$. This can happen when the bound variable is evaluated to a bound constant that is not desired.

4. Subtyping for dependent types

The subtyping relation given in Fig. 10 involves dependent bounds, i.e., bounds that may refer to variables. To decide premises $b \leq b'$, we need a decision procedure that accounts for the possible values of these variables. This process considers Θ , tracked by the typing judgment, and φ , the current stack snapshot (when performing subtyping as part of the type preservation proof).

Definition 7 (Inequality).

- $n \leq m$ if n is less than or equal to m .
- $x + n \leq x + m$ if n is less than or equal to m .
- All other cases result in **false**.

To capture bound variables in dependent types, the Checked C subtyping relation (\sqsubseteq) is parameterized by a restricted stack snapshot $\varphi|_\rho$ and the predicate map Θ , where φ is a stack and ρ is a set of variables. $\varphi|_\rho$ means to restrict the domain of φ to the variable set ρ . Clearly, we have the

$\text{S-VAR} \quad \frac{}{(\varphi, \mathcal{H}, x) \longrightarrow (\varphi, \mathcal{H}', \varphi(x))}$	$\text{S-DEFARRAY} \quad \frac{\mathcal{H}(n) = n_a : \tau_a \quad 0 \in [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \text{ptr}^c [(n_l, n_h) \tau]_{nt}) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$
$\text{S-DEFARRAYBOUND} \quad \frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \text{ptr}^c [(n_l, n_h) \tau]_{\kappa}) \longrightarrow (\varphi, \mathcal{H}, \text{bounds})}$	$\text{S-DEFNTARRAYBOUND} \quad \frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \text{ptr}^c [(n_l, n_h) \tau]_{nt}) \longrightarrow (\varphi, \mathcal{H}, \text{bounds})}$
$\text{S-ASSIGN} \quad \frac{\mathcal{H}(n) = n_a : \tau_a}{(\varphi, \mathcal{H}, *n : \text{ptr}^c \tau = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}[n \mapsto n_1 : \tau], n_1 : \tau)}$	$\text{S-ASSIGNNULL} \quad \frac{}{(\varphi, \mathcal{H}, *0 : \text{ptr}^c \omega = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}, \text{null})}$
$\text{S-ASSIGNARRBOUND} \quad \frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \text{ptr}^c [(n_l, n_h) \tau]_{\kappa} = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}, \text{bounds})}$	$\text{S-MALLOC} \quad \frac{\varphi(\omega) = \omega_a \quad \text{alloc}(\mathcal{H}, \omega_a) = (n, \mathcal{H}')}{(\varphi, \mathcal{H}, \text{malloc}(\omega,)) \longrightarrow (\varphi, \mathcal{H}', n : \text{ptr}^c \omega_a)}$
$\text{S-MALLOCBOUND} \quad \frac{\varphi(\omega) = [(n_l, n_h) \tau]_{\kappa} \quad (n_l \neq 0 \vee n_h \leq 0)}{(\varphi, \mathcal{H}, \text{malloc}(\omega,)) \longrightarrow (\varphi, \mathcal{H}', \text{bounds})}$	$\text{S-IFT} \quad \frac{n \neq 0}{(\varphi, \mathcal{H}, \text{if } (n : \tau) \ e_1 \ \text{else } e_2) \longrightarrow (\varphi, \mathcal{H}, e_1)}$
$\text{S-IFF} \quad \frac{}{(\varphi, \mathcal{H}, \text{if } (0 : \tau) \ e_1 \ \text{else } e_2) \longrightarrow (\varphi, \mathcal{H}, e_2)}$	$\text{S-UNCHECKED} \quad \frac{}{(\varphi, \mathcal{H}, \text{unchecked}(n : \tau) \{ \longrightarrow \}) \longrightarrow (\varphi, \mathcal{H}, n : \tau)}$
$\text{S-STR} \quad \frac{0 \in [n_l, n_h] \quad n_a \leq n_h \quad \mathcal{H}(n + n_a) = 0 \quad (\forall i. n \leq i < n + n_a \Rightarrow (\exists n_i \ t_i. \mathcal{H}(n + i) = n_i : \tau_i \wedge n_i \neq 0))}{(\varphi, \mathcal{H}, \text{strlen}(n : \text{ptr}^m [(n_l, n_h) \tau])) \longrightarrow (\varphi, \mathcal{H}, n_a : \text{int})}$	
$\text{S-STRBOUNDS} \quad \frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, \text{strlen}(n : \text{ptr}^c [(n_l, n_h) \tau])) \longrightarrow (\varphi, \mathcal{H}, \text{bounds})}$	$\text{S-STRNULL} \quad \frac{}{(\varphi, \mathcal{H}, \text{strlen}(0 : \text{ptr}^c [(n_l, n_h) \tau])) \longrightarrow (\varphi, \mathcal{H}, \text{null})}$
$\text{S-ADD} \quad \frac{n = n_1 + n_2}{(\varphi, \mathcal{H}, n_1 : \text{int} + n_2 : \text{int}) \longrightarrow (\varphi, \mathcal{H}, n)}$	$\text{S-ADDARR} \quad \frac{n = n_1 + n_2 \quad n'_l = n_l - n_2 \quad n'_h = n_h - n_2}{(\varphi, \mathcal{H}, n_1 : \text{ptr}^m [(n_l, n_h) \tau]_{\kappa} + n_2 : \text{int}) \longrightarrow (\varphi, \mathcal{H}, n : \text{ptr}^m [(n'_l, n'_h) \tau]_{\kappa})}$
$\text{S-ADDARRNULL} \quad \frac{}{n(\varphi, \mathcal{H}, 0 : \text{ptr}^c [(n_l, n_h) \tau]_{\kappa} + n_2 : \text{int}) \longrightarrow (\varphi, \mathcal{H}, \text{null})}$	

Figure 15: Remaining CORECHKCBOX Semantics Rules (extends Fig. 7)

relation: $\varphi|_{\rho} \subseteq \varphi$. \sqsubseteq being parameterized by $\varphi|_{\rho}$ refers to that when we compare two bounds $b \leq b'$, we actually do $\varphi|_{\rho}(b) \leq \varphi|_{\rho}(b')$ by interpreting the variables in b and b' with possible values in $\varphi|_{\rho}$. Let's define a subset relation \preceq for two restricted stack snapshot $\varphi|_{\rho}$ and $\varphi'|_{\rho}$:

Definition 8 (Subset of Stack Snapshots). Given two $\varphi|_{\rho}$ and $\varphi'|_{\rho}$, $\varphi|_{\rho} \preceq \varphi'|_{\rho}$, iff for $x \in \rho$ and y , $(x, y) \in \varphi|_{\rho} \Rightarrow (x, y) \in \varphi'|_{\rho}$.

For every two restricted stack snapshots $\varphi|_{\rho}$ and $\varphi'|_{\rho}$, such that $\varphi|_{\rho} \preceq \varphi'|_{\rho}$, we have the following theorem in Checked C (proved in Coq):

Theorem 7 (Stack Snapshot Theorem). Given two types τ and τ' , two restricted stack snapshots $\varphi|_{\rho}$ and $\varphi'|_{\rho}$, if $\varphi|_{\rho} \preceq \varphi'|_{\rho}$, and $\tau \sqsubseteq \tau'$ under the parameterization of $\varphi|_{\rho}$, then $\tau \sqsubseteq \tau'$ under the parameterization of $\varphi'|_{\rho}$.

Clearly, for every $\varphi|_{\rho}$, we have $\emptyset \preceq \varphi|_{\rho}$. The type checking stage is a compile-time process, so $\varphi|_{\rho}$ is \emptyset at

the type checking stage. Stack snapshots are needed for proving type preserving, as variables in bounds expressions are evaluated away.

As mentioned in the main text, \sqsubseteq is also parameterized by Θ , which provides the range of allowed values for a bound variable; thus, more \sqsubseteq relation is provable. For example, in Fig. 8, the **strlen** operation in line 4 turns the type of **dst** to be $\text{ptr}^c [(0, x) \text{int}]_{nt}$ and extends the upper bound to **x**. In the **strlen** type rule, it also inserts a predicate **x** \geq 0 in Θ ; thus, the cast operation in line 16 is valid because $\text{ptr}^c [(0, x) \text{int}]_{nt} \sqsubseteq \text{ptr}^c [(0, 0) \text{int}]_{nt}$ is provable when we know **x** \geq 0.

Note that if φ and Θ are \emptyset , we do only the syntactic \leq comparison; otherwise, we apply φ to both sides of \sqsubseteq , and then determine the \leq comparison based on a Boolean predicate decision procedure on top of Θ . This process allows us to type check both an input expression and the intermediate expression after evaluating an expression.

Struct Syntax:

Type **struct** T
 Structdefs $D \in T \rightarrow fs$
 Fields $fs ::= \tau \ f \mid \tau \ f; fs$

Struct Subtype:

$D(T) = fs \wedge fs(0) = \text{nat} \Rightarrow \text{ptr}^m \text{ struct } T \sqsubseteq \text{ptr}^m \text{ nat}$
 $D(T) = fs \wedge fs(0) = \text{nat} \wedge 0 \leq b_l \wedge b_h \leq 1$
 $\Rightarrow \text{ptr}^m \text{ struct } T \sqsubseteq \text{ptr}^m [(b_l, b_h) \text{ nat}]$

Struct Type Rule:

T-STRUCT
 $\frac{\Gamma; \Theta \vdash_m e : \text{ptr}^m \text{ struct } T \quad D(T) = fs \quad fs(f) = \tau_f}{\Gamma; \Theta \vdash_m \&e \rightarrow f : \text{ptr}^m \tau_f}$

Struct Semantics:

S-STRUCTCHECKED
 $\frac{n > 0 \quad D(T) = fs \quad fs(f) = \tau_a \quad n_a = \text{index}(fs, f)}{(\varphi, \mathcal{H}, \&n : \text{ptr}^c \text{ struct } T \rightarrow f) \longrightarrow (\varphi, \mathcal{H}, n_a : \text{ptr}^c \tau_a)}$

S-STRUCTNULL
 $\frac{n = 0}{(\varphi, \mathcal{H}, \&n : \text{ptr}^c \text{ struct } T \rightarrow f) \longrightarrow (\varphi, \mathcal{H}, \text{null})}$

S-STRUCTUNCHECKED
 $\frac{D(T) = fs \quad fs(f) = \tau_a \quad n_a = \text{index}(fs, f)}{(\varphi, \mathcal{H}, \&n : \text{ptr}^u \text{ struct } T \rightarrow f) \longrightarrow (\varphi, \mathcal{H}, n_a : \text{ptr}^u \tau_a)}$

Figure 17: CORECHKCBOX Struct Definitions

$\frac{\Gamma \vdash n \quad \frac{x : \text{int} \in \Gamma}{\Gamma \vdash x + n} \quad \frac{\Gamma \vdash b_l \quad \Gamma \vdash b_h}{\Gamma \vdash (b_l, b_h)} \quad \Gamma \vdash \text{int}}{\Gamma \vdash \text{int}}$
 $\frac{\Gamma \vdash \beta \quad \Gamma \vdash \tau}{\Gamma \vdash \text{ptr}^m [\beta \ \tau]_\kappa} \quad \frac{\Gamma \vdash \tau}{\Gamma \vdash \text{ptr}^m \tau} \quad \frac{T \in D}{\Gamma \vdash \text{ptr}^m \text{ struct } T}$

Figure 18: Well-formedness for Types and Bounds

T-DEF
 $\frac{\Gamma; \Theta \vdash_m e : \text{ptr}^{m'} \tau \quad m \leq m'}{\Gamma; \Theta \vdash_m *e : \tau}$

T-MAC
 $\Gamma; \Theta \vdash_m \text{malloc}(\omega, :) \text{ptr}^c \omega$

T-ADD
 $\frac{\Gamma; \Theta \vdash_m e_1 : \text{int} \quad \Gamma; \Theta \vdash_m e_2 : \text{int}}{\Gamma; \Theta \vdash_m (e_1 + e_2) : \text{int}}$

T-IND
 $\frac{\Gamma; \Theta \vdash_m e_1 : \text{ptr}^{m'} [\beta \ \tau]_\kappa \quad \Gamma; \Theta \vdash_m e_2 : \text{int} \quad m \leq m'}{\Gamma; \Theta \vdash_m *(e_1 + e_2) : \tau}$

T-ASSIGN
 $\frac{\Gamma; \Theta \vdash_m e_1 : \text{ptr}^{m'} \tau \quad \Gamma; \Theta \vdash_m e_2 : \tau' \quad \tau' \sqsubseteq \tau \quad m \leq m'}{\Gamma; \Theta \vdash_m *e_1 = e_2 : \tau}$

T-INDASSIGN
 $\frac{\Gamma; \Theta \vdash_m e_1 : \text{ptr}^{m'} [\beta \ \tau]_\kappa \quad \Gamma; \Theta \vdash_m e_2 : \text{int} \quad \Gamma; \Theta \vdash_m e_3 : \tau' \quad \tau' \sqsubseteq \tau \quad m \leq m'}{\Gamma; \sigma \vdash_m *(e_1 + e_2) = e_3 : \tau}$

5. Other Type Rules

Here we show the type rules for other Checked C operations in Fig. 16. Rule T-DEF is for dereferencing a non-array pointer. The statement $m \leq m'$ ensures that no unchecked pointers are used in checked regions. Rule T-MAC deals with malloc operations. There is a well-formedness check to require that the possible bound variables in ω must be in the domain of Γ (see Fig. 18). This is similar to the well-formedness assumption of the type environment (Definition 1) Rule T-ADD deals with binary operations whose subterms are integer expressions, while rule T-IND serves the case for pointer arithmetic. For simplicity, in the Checked C formalization, we do not allow arbitrary pointer arithmetic. The only pointer arithmetic operations allowed are the forms shown in rules T-IND and T-INDASSIGN in Fig. 16. Rule T-ASSIGN assigns a value to a non-array pointer location. The predicate $\tau' \sqsubseteq \tau$ requires that the value being assigned is a subtype of the pointer type. The T-INDASSIGN rule is an extended assignment operation for handling assignments for array/NT-array pointers with pointer arithmetic. Rule T-UNCHECKED type checks `unchecked` blocks.

6. Struct Pointers

Checked C has `struct` types and `struct` pointers. Fig. 17 contains the syntax of struct types as well as new subtyping relations built on the struct values. For a struct typed value, Checked C has a special operation for it, which is $\&e \rightarrow f$. This operation indexes the f -th position struct T item, if the expression e is evaluated to a struct pointer $\text{ptr}^m \text{ struct } T$. Rule T-STRUCT in Fig. 17 describes its typing behavior. Rules S-STRUCTCHECKED and S-STRUCTUNCHECKED describe the semantic behaviors of $\&e \rightarrow f$ on a given struct `checked/unchecked` pointers, while rule S-STRUCTNULL describes a `checked` struct null-pointer case. In our Coq/Redex formalization, we include the struct values and the operation $\&e \rightarrow f$. We omit it in the main text due to the paper length limitation.

7. The Compilation Rules

Fig. 22 and Fig. 23 shows the syntax for COREC, the target language for compilation. We syntactically restrict the expressions to be in A-normal form to simplify the presentation of the compilation rules. In the Redex model, we occasionally break this constraint to speed up the performance

$$\begin{array}{c}
\frac{\Gamma \vdash \bar{x} : \bar{\tau} \quad \Gamma[\bar{x} \mapsto \bar{\tau}] \vdash \tau \quad \Gamma[\bar{x} \mapsto \bar{\tau}]; \Theta \vdash_c e : \tau}{\Gamma \vdash \tau (\bar{x} : \bar{\tau}) e} \quad \Gamma \vdash \cdot \\
\\
\frac{\Gamma \vdash \tau \quad \Gamma[x \mapsto \tau] \vdash \bar{x} : \bar{\tau}}{\Gamma \vdash x : \tau, \bar{x} : \bar{\tau}}
\end{array}$$

Figure 19: Well-formedness for functions

$$\frac{\Gamma \vdash \tau}{\Gamma \vdash \tau f} \quad \frac{\Gamma \vdash \tau \quad \Gamma \vdash fs}{\Gamma \vdash \tau f; fs}$$

Figure 20: Well-formedness for structs

$$\frac{\Gamma[\bar{x} \mapsto \bar{\tau}]; \emptyset \vdash e \gg \dot{e} : \tau}{\Gamma \vdash \tau (\bar{x} : \bar{\tau}) e \gg (\bar{x}) \dot{e}}$$

Figure 21: Compilation Rules for Functions

of random testing by removing unnecessary let bindings. To allow explicit runtime checks, we include bounds and null as part of COREC expressions which, once evaluated, result in an corresponding error state. $x = \dot{a}$ is a new syntactic form that modifies the stack variable x with the result of \dot{a} . It is essential for bounds widening. \leq and $-$ are introduced to operate on bounds and decide whether we need to halt with a bounds error or widen a null-terminated string.

Atoms	\dot{a}	::=	$n \mid x$
C-Expressions	\dot{c}	::=	$\dot{a} \mid \text{strlen}(\dot{a}) \mid \text{malloc}(\dot{a},)f(\bar{a})$ $\mid \dot{a} \circ \dot{a} \mid * \dot{a}$ $\mid * \dot{a} = \dot{a} \mid x = \dot{a} \mid \text{if } (\dot{a}) \dot{e} \text{ else } \dot{e}$ $\mid \text{bounds} \mid \text{null}$
Expressions	\dot{e}	::=	$\dot{c} \mid \text{let } x = \dot{c} \text{ in } \dot{e}$
Binops	\circ	::=	$+ \mid - \mid \leq$
Closure	\dot{C}	::=	$\square \mid \text{let } x = \dot{a} \text{ in } \dot{C}$ $\mid \text{if } (\dot{a}) \dot{e} \text{ else } \dot{C} \mid \text{if } (\dot{a}) \dot{C} \text{ else } \dot{e}$
Bounds Map	ρ	\in	$\text{Var} \rightarrow \text{Var} \times \text{Var}$

Figure 22: COREC Syntax

$$\begin{array}{ll}
\dot{\mu} & ::= n \mid \perp \\
\dot{c} & ::= \dots \mid \text{ret}(x, \dot{\mu}, \dot{e}) \\
\dot{H} & \in \mathbb{Z} \rightarrow \mathbb{Z} \\
\dot{r} & ::= \dot{e} \mid \text{null} \mid \text{bounds} \\
\dot{E} & ::= \square \mid \text{let } x = \dot{E} \text{ in } \dot{e} \mid \text{ret}(x, i, \dot{E}) \\
& \mid \text{if } (\dot{E}) \dot{e} \text{ else } \dot{e} \mid \text{strlen}(\dot{E}) \\
& \mid \text{malloc}(\dot{E}, |)f(\bar{E}) \mid \dot{E} \circ \dot{a} \mid n \circ \dot{E} \\
& \mid * \dot{E} \mid * \dot{E} = \dot{a} \mid * n = \dot{E} \mid x = \dot{E} \\
\bar{\dot{E}} & ::= \dot{E} \mid n, \bar{\dot{E}} \mid \bar{\dot{E}}, \dot{a}
\end{array}$$

Figure 23: COREC Semantic Defs

COREC does not include any annotations. We remove structs from COREC because we can always statically convert expressions of the form $\&n : \tau \rightarrow f$ into $n + n_f$, where

n_f is the statically determined offset of f within the struct. We elide the semantics of COREC because it is self-evident and mirrors the semantics CORECHKCBOX. The difference is that in COREC, only bounds and null can step into an error state. All failed dereferences and assignments would result in a stuck state and therefore we rely on the compiler to explicitly insert checks for checked pointers.

Fig. 26 and Fig. 27 shows the rules for the compilation judgment for expressions,

$$\Gamma; \rho \vdash e \gg \dot{C}, \dot{a}$$

The judgment is presented differently from the one in Sec. 4, which was simplified for presentation purposes. First, we remove Θ and m because these parameters are only used for checking and have no impact on compilation. Second, the judgment includes two outputs, a closure \dot{C} and an atom expression \dot{a} , instead of a single COREC expression \dot{e} . \dot{C} can be intuitively understood as a partially constructed program or context. Whereas \dot{E} is used for evaluation, \dot{C} is used purely as a device for compilation. As an example, when compiling $(1 : \text{int}) + (2 : \text{int})$, we would first create a fresh variable x , and then produce two outputs:

$$\dot{C} = \text{let } x = 1 + 2 \text{ in } \square$$

$$\dot{a} = x$$

To obtain the compiled expression \dot{e} , we plug \dot{a} into \dot{C} using the usual notation $\dot{C}[\dot{a}]$. We can also use \dot{C} to represent runtime checks, which usually take the form $\text{let } x = \dot{c} \text{ in } \square$, where \dot{c} contains the check whose evaluation must not trigger bounds or null for the program to continue (see Fig. 25 for the metafunctions that create those checks).

This unconventional output format enables us to separate the evaluation of the term and the computation that relies on the term's evaluated result. Since effects and reduction (except for variables) happen only within closures, we can precisely control the order in which effects and evaluation happen by composing the contexts in a specific order. Given two closures \dot{C}_1 and \dot{C}_2 , we write $\dot{C}_1[\dot{C}_2]$ to denote the meta operation of plugging \dot{C}_2 into \dot{C}_1 . We also use $\dot{C}_{a;b;c}$ as a shorthand for $\dot{C}_a[\dot{C}_b[\dot{C}_c]]$. In the C-IND rule, we first evaluate the expressions that correspond to e_1 and e_2 through \dot{C}_1 and \dot{C}_2 , and then perform a null check and an addition through \dot{C}_n and \dot{C}_3 . Finally, we dereference the result through \dot{C}_4 before returning the pair \dot{C}_4, \dot{x}_4 , propagating the flexibility to the compilation rule that recursively calls C-IND.

Fig. 25 shows the metafunctions that create closures representing dynamic checks. These functions first examine whether the pointer is a checked. If the pointer is unchecked, an empty closure \square will be returned, because there is no need to perform a check. For bounds checking, there is a special case for NT-array pointers, where the bounds are retrieved from the **shadow** variables (found by looking up ρ) on the stack rather than using the bounds specified in the type annotation. This is how we achieve the same precise runtime behavior as CORECHKCBOX in our compiled expressions.

Fig. 24 shows the metafunctions related to bounds widening. \vdash_{extend} takes ρ , a checked NT-array pointer variable x , and its bounds (b_l, b_h) as inputs, and returns an extended ρ' that maps x to two fresh variables x_l, x_h , together with a closure \tilde{C} that initializes x_l and x_h to b_l and b_h respectively. This function is used in the C-LET rule to

extend ρ before compiling the body of the `let` binding. The updated ρ' can be used for generating precise bounds checks, and for inserting expressions that can potentially widen the upper bounds, as seen in the \vdash_{widenstr} metafunction used in the C-STR compilation rule.

$$\begin{array}{c}
\frac{x_l, x_h = \mathbf{fresh} \quad \rho' = \rho[x \mapsto (x_l, x_h)] \quad \dot{C} = \mathbf{let } x_l = b_l \mathbf{ in let } x_h = b_h \mathbf{ in } \square}{\dot{C}, \rho' = \vdash_{\text{extend}} \rho, x, \mathbf{ptr}^c [(b_l, b_h) \tau]_{nt}} \\
\\
\frac{x_l, x_h = \rho(x) \quad x_w = \mathbf{fresh} \quad \dot{C} = \mathbf{let } x_w = \mathbf{if } (x_h) 0 \mathbf{ else } x_h = 1 \mathbf{ in } \square}{\dot{C} = \vdash_{\text{widenderef}} \rho, x, \mathbf{ptr}^c [(b_l, b_h) \tau]_{nt}} \quad \frac{e \notin \text{dom}(\rho)}{\square = \vdash_{\text{widenstr}} \rho, e, \dot{a}, \mathbf{ptr}^m [\beta \tau]_{nt}} \\
\\
\frac{x_l, x_h = \rho(e) \quad x_a = \mathbf{fresh} \quad \dot{C} = \mathbf{let } x_a = \mathbf{if } (\dot{a} \leq x_h) 0 \mathbf{ else } x_h = \dot{a} \mathbf{ in } \square}{\dot{C} = \vdash_{\text{widenstr}} \rho, e, \dot{a}, \mathbf{ptr}^c [\beta \tau]_{nt}}
\end{array}$$

Figure 24: Metafunctions for Widening

$$\begin{array}{c}
\frac{x = \mathbf{fresh} \quad \dot{C} = \mathbf{let } x = \mathbf{if } (\dot{a}) 0 \mathbf{ else null in } \square}{\dot{C} = \vdash_{\text{null}} \dot{a}, c} \quad \square = \vdash_{\text{null}} \dot{a}, u \\
\\
\square = \vdash_{\text{boundsR}} \rho, e, \mathbf{ptr}^u [\beta \tau]_{\kappa}, \dot{a} \\
\\
\frac{x_l, x_h = \rho(e) \quad \dot{C}_{cl} = \mathbf{let } x_{cl} = \mathbf{if } (x_l \leq \dot{a}) 0 \mathbf{ else bounds in } \square \quad \dot{C}_{ch} = \mathbf{let } x_{ch} = \mathbf{if } (\dot{a} \leq x_h) 0 \mathbf{ else bounds in } \square}{\dot{C}_{cl;ch} = \vdash_{\text{boundsR}} \rho, e, \mathbf{ptr}^c [\beta \tau]_{\kappa}, \dot{a}} \\
\\
\frac{e \notin \text{dom}(\rho) \quad x_l, x_h, x_{cl}, x_{ch} = \mathbf{fresh} \quad \dot{C}_l = \mathbf{let } x_l = b_l \mathbf{ in } \square \quad \dot{C}_h = \mathbf{let } x_h = b_h \mathbf{ in } \square}{\dot{C}_{cl} = \mathbf{let } x_{cl} = \mathbf{if } (x_l \leq \dot{a}) 0 \mathbf{ else bounds in } \square \quad \dot{C}_{ch} = \mathbf{let } x_{ch} = \mathbf{if } (\dot{a} \leq x_h) 0 \mathbf{ else bounds in } \square} \\
\dot{C}_{l;h;cl;ch} = \vdash_{\text{boundsR}} \rho, e, \mathbf{ptr}^c [(b_l, b_h) \tau]_{nt}, \dot{a} \\
\\
\frac{e \notin \text{dom}(\rho) \quad x_l, x_h, x_{cl}, x_{ch} = \mathbf{fresh} \quad \dot{C}_l = \mathbf{let } x_l = b_l \mathbf{ in } \square \quad \dot{C}_h = \mathbf{let } x_h = b_h \mathbf{ in } \square}{\dot{C}_{cl} = \mathbf{let } x_{cl} = \mathbf{if } (x_l \leq \dot{a}) 0 \mathbf{ else bounds in } \square \quad \dot{C}_{ch} = \mathbf{let } x_{ch} = \mathbf{if } (x_h \leq \dot{a}) \mathbf{ bounds else 0 in } \square} \\
\dot{C}_{l;h;cl;ch} = \vdash_{\text{boundsR}} \rho, e, \mathbf{ptr}^c [(b_l, b_h) \tau]_{\kappa}, \dot{a} \\
\\
\square = \vdash_{\text{boundsW}} \rho, e, \mathbf{ptr}^u [\beta \tau]_{\kappa}, \dot{a} \\
\\
\frac{x_l, x_h = \rho(e) \quad \dot{C}_{cl} = \mathbf{let } x_{cl} = \mathbf{if } (x_l \leq \dot{a}) 0 \mathbf{ else bounds in } \square \quad \dot{C}_{ch} = \mathbf{let } x_{ch} = \mathbf{if } (\dot{a} \leq x_h) 0 \mathbf{ else bounds in } \square}{\dot{C}_{cl;ch} = \vdash_{\text{boundsW}} \rho, e, \mathbf{ptr}^c [\beta \tau]_{\kappa}, \dot{a}} \\
\\
\frac{e \notin \text{dom}(\rho) \quad x_l, x_h, x_{cl}, x_{ch} = \mathbf{fresh} \quad \dot{C}_l = \mathbf{let } x_l = b_l \mathbf{ in } \square \quad \dot{C}_h = \mathbf{let } x_h = b_h \mathbf{ in } \square}{\dot{C}_{cl} = \mathbf{let } x_{cl} = \mathbf{if } (x_l \leq \dot{a}) 0 \mathbf{ else bounds in } \square \quad \dot{C}_{ch} = \mathbf{let } x_{ch} = \mathbf{if } (x_h \leq \dot{a}) \mathbf{ bounds else 0 in } \square} \\
\dot{C}_{l;h;cl;ch} = \vdash_{\text{boundsW}} \rho, e, \mathbf{ptr}^c [(b_l, b_h) \tau]_{\kappa}, \dot{a} \\
\\
\frac{e \notin \text{dom}(\rho) \quad x_l, x'_l, x_h, x'_h = \mathbf{fresh} \quad \dot{C}_1 = \mathbf{let } x_l = b_l \mathbf{ in let } x_h = b_h \mathbf{ in } \square}{\dot{C}_2 = \mathbf{let } x'_l = b'_l \mathbf{ in let } x'_h = b'_h \mathbf{ in } \square \quad \dot{C}_3 = \mathbf{if } (x'_l \leq x_l) \square \mathbf{ else bounds} \quad \dot{C}_4 = \mathbf{if } (x_h \leq x'_h) \square \mathbf{ else bounds}} \\
\dot{C}_{1;2;3;4} = \vdash_{\text{boundsD}} \rho, e, \mathbf{ptr}^m [(b_l, b_h) \tau]_{\kappa}, \mathbf{ptr}^m [(b'_l, b'_h) \tau]_{\kappa} \\
\\
\frac{x'_l, x'_h = \rho(e) \quad x_l, x_h = \mathbf{fresh} \quad \dot{C}_1 = \mathbf{let } x_l = b_l \mathbf{ in let } x_h = b_h \mathbf{ in } \square \quad \dot{C}_2 = \mathbf{if } (x'_l \leq x_l) \square \mathbf{ else bounds} \quad \dot{C}_3 = \mathbf{if } (x_h \leq x'_h) \square \mathbf{ else bounds}}{\dot{C}_{1;2;3} = \vdash_{\text{boundsD}} \rho, e, \mathbf{ptr}^m [(b_l, b_h) \tau]_{\kappa}, \mathbf{ptr}^m [(b'_l, b'_h) \tau]_{\kappa}}
\end{array}$$

Figure 25: Metafunctions for Dynamic Checks

$$\begin{array}{c}
\text{C-CONST} \\
\frac{}{\Gamma; \rho \vdash n : \tau \gg \square, n : \tau}
\end{array}
\qquad
\begin{array}{c}
\text{C-VAR} \\
\frac{x : \tau \in \Gamma}{\Gamma; \rho \vdash x \gg \square, x : \tau}
\end{array}
\qquad
\begin{array}{c}
\text{C-CAST} \\
\frac{\Gamma; \rho \vdash e \gg \dot{C}, \dot{a} : \tau'}{\Gamma; \rho \vdash (\tau)e \gg \dot{C}, \dot{a} : \tau}
\end{array}$$

$$\begin{array}{c}
\text{C-DYNCAST} \\
\frac{\Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a} : \mathbf{ptr}^m [\beta' \tau]_\kappa \quad \dot{C}_b = \vdash_{\text{bounds}D} \rho, e, \mathbf{ptr}^m [\beta \tau]_\kappa, \mathbf{ptr}^m [\beta' \tau]_\kappa}{\Gamma; \rho \vdash \langle \mathbf{ptr}^m [\beta \tau]_\kappa \rangle e \gg \dot{C}_{1;b}, \dot{a} : \mathbf{ptr}^m [\beta \tau]_\kappa}
\end{array}$$

$$\begin{array}{c}
\text{C-STR} \\
\frac{\Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \mathbf{ptr}^m [\beta \tau_a]_{nt} \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \dot{C}_b = \vdash_{\text{bounds}R} \rho, \dot{a}_1, \mathbf{ptr}^m [\beta \tau_a]_{nt}, 0 \\
x_2 = \text{fresh} \quad \dot{C}_2 = \text{let } x_2 = \text{strlen}(\dot{a}_1) \text{ in } \square \quad \dot{C}_w = \vdash_{\text{widenstr}} \rho, e, \dot{a}_1, \mathbf{ptr}^m [\beta \tau_a]_{nt}}{\Gamma; \rho \vdash \text{strlen}(e) \gg \dot{C}_{1;n;b;2;w}, x_2 : \text{int}}
\end{array}$$

$$\begin{array}{c}
\text{C-LETSTR} \\
\frac{\Gamma(y) = \mathbf{ptr}^c [(b_l, b_h) \tau_a]_{nt} \quad x \notin FV(\tau) \quad \Gamma; \rho \vdash \text{strlen}(y) \gg \dot{C}_1, \dot{a}_1 : \text{int} \quad \dot{C}_2 = \text{let } x = \dot{a}_1 \text{ in } \square \quad \Gamma[x \mapsto \text{int}, y \mapsto [\mathbf{ptr}^c [(b_l, x) \tau_a]_{nt}]]; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau}{\Gamma; \rho \vdash \text{let } x = \text{strlen}(y) \text{ in } e \gg \dot{C}_{1;2;3}, \dot{a}_3 : \tau}
\end{array}$$

$$\begin{array}{c}
\text{C-IF} \\
\frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_2, \dot{a}_2 : \tau_2 \quad \Gamma; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau_3 \quad \Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \tau \quad x_4 = \text{fresh} \quad \dot{C}_4 = \text{let } x_4 = \text{if } (\dot{a}_1) \dot{C}_2[\dot{a}_2] \text{ else } \dot{C}_3[\dot{a}_3] \text{ in } \square}{\Gamma; \rho \vdash \text{if } (e_1) e_2 \text{ else } e_3 \gg \dot{C}_{1;4}, x_4 : \tau_2 \sqcup \tau_3}
\end{array}$$

$$\begin{array}{c}
\text{C-IFNT} \\
\frac{\Gamma; \rho \vdash x : \mathbf{ptr}^c [(b_l, b_h) \tau]_{nt} \quad b_h = 0 \Rightarrow \Gamma' = \Gamma[x \mapsto \mathbf{ptr}^c [(b_l, 1) \tau]_{nt}] \\
b_h \neq 0 \Rightarrow \Gamma' = \Gamma \quad \Gamma; \rho \vdash *x \gg \dot{C}_1, \dot{a}_1 : \tau_1 \quad \Gamma'; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau_2 \quad \Gamma; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau_3 \\
\dot{C}_w = \vdash_{\text{widenderef}} \rho, x, \mathbf{ptr}^c [(b_l, b_h) \tau]_{nt} \quad x_4 = \text{fresh} \quad \dot{C}_4 = \text{let } x_4 = \text{if } (\dot{a}_1) \dot{C}_2[\dot{a}_2] \text{ else } \dot{C}_3[\dot{a}_3] \text{ in } \square}{\Gamma; \rho \vdash \text{if } (*x) e_1 \text{ else } e_2 \gg \dot{C}_{1;4}, x_4 : \tau_1 \sqcup \tau_2}
\end{array}$$

$$\begin{array}{c}
\text{C-LET} \\
\frac{(x \in FV(\tau') \Rightarrow e_1 \in \text{Bound}) \quad \Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \tau_1 \quad \dot{C}_2, \rho' = \vdash_{\text{extend}} \rho, x, \tau_1 \quad \dot{C}_3 = \text{let } x = \dot{a}_1 \text{ in } \square \quad \Gamma[x \mapsto \tau]; \rho' \vdash e_4 \gg \dot{C}_4, \dot{a}_4 : \tau_4}{\Gamma; \rho' \vdash \text{let } x = e_1 \text{ in } e_4 \gg \dot{C}_{1;2;3;4}, \dot{a}_4 : \tau_4[\tau_1 = \text{int} \Rightarrow x \mapsto e_1]}
\end{array}$$

$$\begin{array}{c}
\text{C-RET} \\
\frac{\Gamma(x) \neq \perp \quad \Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \tau \quad x_2 = \text{fresh} \quad \mu \gg \dot{\mu} \quad \dot{C}_2 = \text{let } x_2 = \text{ret}(x, \dot{\mu}, \dot{C}_1[\dot{a}_1]) \text{ in } \square}{\Gamma; \rho \vdash \text{ret}(x, \mu, e) \gg \dot{C}_2, x_2 : \tau}
\end{array}$$

$$\begin{array}{c}
\text{C-FUN} \\
\frac{\Xi(f) = \tau(\bar{x} : \bar{\tau}) e \quad (\forall e_i \in \bar{e} \quad \tau_i \in \bar{\tau} . \Gamma; \rho \vdash e_i \gg \dot{C}_i, \dot{a}_i : \tau'_i \wedge \tau'_i \sqsubseteq \tau_i[\bar{e}/\bar{x}]) \quad x_f = \text{fresh} \quad \dot{C}_f = \text{let } x_f = f(\bar{a}) \text{ in } \square}{\Gamma; \rho \vdash f(\bar{e}) \gg \dot{C}[\dot{C}_f], x_f : \tau[\bar{e}/\bar{x}]}
\end{array}$$

$$\begin{array}{c}
\text{C-DEF} \\
\frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \mathbf{ptr}^m \tau \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad x_2 = \text{fresh} \quad \dot{C}_2 = \text{let } x_2 = * \dot{a}_1 \text{ in } \square}{\Gamma; \rho \vdash *e_1 \gg \dot{C}_{1;n;2}, x_2 : \tau}
\end{array}$$

$$\begin{array}{c}
\text{C-DEFARR} \\
\frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \mathbf{ptr}^m [(b_l, b_h) \tau]_\kappa \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \dot{C}_b = \vdash_{\text{bounds}R} \rho, e_1, \mathbf{ptr}^m [(b_l, b_h) \tau]_\kappa, 0 \quad x_2 = \text{fresh} \quad \dot{C}_2 = \text{let } x_2 = * \dot{a}_1 \text{ in } \square}{\Gamma; \rho \vdash *e_1 \gg \dot{C}_{1;n;b;2}, x_2 : \tau}
\end{array}$$

$$\begin{array}{c}
\text{C-MAC} \\
\frac{\dot{C}_1, \dot{a}_1 = \text{sizeof}(\omega) \quad x_2 = \text{fresh} \quad \dot{C}_2 = \text{let } x_2 = \text{malloc}(\dot{a}_1,) \text{ in } \square}{\Gamma; \rho \vdash \text{malloc}(\omega, \gg) \dot{C}_{1;2}, x_2 : \mathbf{ptr}^c \omega}
\end{array}$$

Figure 26: Compilation

$$\text{C-ADD} \quad \frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{int} \quad \Gamma; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \text{int} \quad x_3 = \text{fresh} \quad \dot{C}_3 = \text{let } x_3 = \dot{a}_1 + \dot{a}_2 \text{ in } \square}{\Gamma; \rho \vdash \dot{C}_3, x_3 : \text{int}}$$

$$\text{C-IND} \quad \frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{ptr}^m [\beta \tau]_\kappa \quad \Gamma; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \text{int} \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \dot{C}_b = \vdash_{\text{boundsR}} \rho, e_1, \text{ptr}^m [\beta \tau]_\kappa, \dot{a}_2 \quad x_3, x_4 = \text{fresh} \quad \dot{C}_3 = \text{let } x_3 = \dot{a}_1 + \dot{a}_2 \text{ in } \square \quad \dot{C}_4 = \text{let } x_4 = *x_3 \text{ in } \square}{\Gamma; \rho \vdash *(e_1 + e_2) \gg \dot{C}_{1;2;n;3;b;4}, x_4 : \tau}$$

$$\text{C-ASSIGN} \quad \frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{ptr}^{m'} \tau \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \Gamma; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau' \quad \tau' \sqsubseteq \tau \quad x_3 = \text{fresh} \quad \dot{C}_3 = \text{let } x_3 = *\dot{a}_1 = \dot{a}_2 \text{ in } \square}{\Gamma; \rho \vdash *e_1 = e_2 \gg \dot{C}_{1;2;n;3}, x_3 : \tau}$$

$$\text{C-ASSIGNARR} \quad \frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{ptr}^{m'} [\beta \tau]_\kappa \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \dot{C}_b = \vdash_{\text{boundsW}} \rho, e_1, \text{ptr}^m [(b_l, b_h) \tau]_\kappa, 0 \quad \Gamma; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau' \quad x_3 = \text{fresh} \quad \dot{C}_3 = \text{let } x_3 = *\dot{a}_1 = \dot{a}_2 \text{ in } \square \quad \tau' \sqsubseteq \tau}{\Gamma; \rho \vdash *e_1 = e_2 \gg \dot{C}_{1;2;n;b;3}, x_3 : \tau}$$

$$\text{C-INDASSIGN} \quad \frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{ptr}^m [\beta \tau]_\kappa \quad \Gamma; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \text{int} \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad \dot{C}_b = \vdash_{\text{boundsW}} \rho, e_1, \text{ptr}^m [\beta \tau]_\kappa, \dot{a}_2 \quad \Gamma; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau' \quad x_4, x_5 = \text{fresh} \quad \dot{C}_4 = \text{let } x_4 = \dot{a}_1 + \dot{a}_2 \text{ in } \square \quad \dot{C}_5 = \text{let } x_5 = *x_4 = x_3 \text{ in } \tau' \sqsubseteq \tau}{\Gamma; \rho \vdash *(e_1 + e_2) = e_3 \gg \dot{C}_{1;2;n;3;4;b;5} : \tau}$$

$$\text{C-STRUCT} \quad \frac{D(T) = \tau_0 \ f_0 \dots; \tau_j \ f; \dots \quad \Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \text{ptr}^m \text{ struct } T \quad \dot{C}_n = \vdash_{\text{null}} \dot{a}_1, m \quad x_2 = \text{fresh} \quad \dot{C}_2 = \text{let } x_2 = \dot{a}_1 + j \text{ in } \square}{\Gamma; \rho \vdash \&e_1 \rightarrow f \gg \dot{C}_2, x_2 : \text{ptr}^m \tau_f}$$

$$\text{C-UNCHECKED} \quad \frac{\Gamma; \rho \vdash e \gg \dot{C}, \dot{a} : \tau}{\Gamma; \rho \vdash \text{unchecked}(e) \{ \gg \} \dot{C}, \dot{a} : \tau}$$

Figure 27: Compilation (Continued)