CHECKEDCBOX: Formalizing RLBox in Checked C for Incremental Spatial Memory Safety (Extended Version)

This is an extended version of a paper that appears at the 2022 Computer Security Foundations Symposium.

1. Introduction

Vulnerabilities due to memory corruption, especially spatial memory corruptions, are still a major issue for C programs [3, 42] despite a large body of work that tries to prevent them [39]. Converting these existing programs into safe languages, such as Java, requires considerable effort and is not (yet) feasible for performance-critical environments such as Embedded Operating Systems. Although high-performant safe system languages such as Rust [28] and Go exist, these a too different to constitute a practical target [43], and rewriting in these languages requires considerable effort. Existing automated conversion tools [5, 11, 17] are able to take only small steps (i.e., to unsafe, non-idiomatic Rust) and still *require considerable work from developers*. There are other retrofitting techniques such as CCured [33], Softbound [29], Low Fat pointers [8], and Address Sanitizer (ASAN) [37] aim to enforce spatial safety automatically *without any developer effort*, by analyzing a C program and compiling it to include run-time safety checks. Unfortunately, the resulting *run-time overhead is too high for deployment (between 60%-200%)*, especially in low-powered IoT devices where the usage of C-based system software (e.g., FreeRTOS) is still rampant.

Checked C. Tarditi et al. developed *Checked C*, a safe dialect of C that prevents spatial memory issues (temporal safety is underway [46]) with no overhead. Checked C extends C with *checked pointer types* which are restricted by the compiler to spatially safe uses (temporal safety is underway [46]). Such pointers have one of three possible types, `ptr<`T`>`, `array_ptr<`T`>`, or `nt_array_ptr<`T`>` (*ptr*, *arr*, and *ntarr* for short), representing a pointer to a single element, array of elements, or null-terminated array of elements of type T, respectively. The latter two have an associated *bounds annotation*; e.g., a declaration `array_ptr<`int`> p : count(n)` says that `p` is a pointer to an `int` array whose size is `n`. The compiler uses these bounds annotations to add dynamic checks prior to checked pointer accesses, to prevent spatial safety violations. These run-time checks can often be proved redundant and removed by LLVM, yielding good performance. Specifically, Tarditi et al. [41] reported average run-time overheads of 8.6% on a small benchmark suite, and Duan et al. [7] found essentially *no overhead* when running Checked C-converted portions of the FreeBSD kernel. Furthermore, Checked C is backward compatible as the compiler represents checked pointers as system-level memory words, i.e., without "fattening" metadata, ensuring backward compatibility. The backward compatibility also enables incremental or partial conversion where Checked annotations can be provided to a few pointers. Such partially annotated programs still enjoy the benefits of spatial memory safety on those regions using only Checked pointers (i.e., checked or safe regions). Figure 1 shows the summary of these existing approaches along with Checked C. We argue that *converting to Checked C provides a reasonable and practical approach to adding safety to existing system software written in C*.

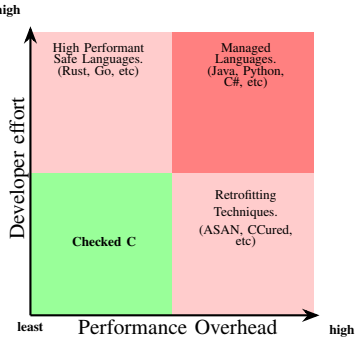

Figure 1: Existing approaches to handle memory corruptions vulnerabilities in C programs along with Checked C.

Converting C to Checked C. The safety guarantees of Checked C come with certain restrictions. For instance, as shown below, Checked C programs cannot use address-taken variables in a bounds expression as the bounds relations may not hold because of possible modifications through pointers.

```
...
array_ptr<int> p : count (n) = NULL;
✖..,&n,.
```

Consequently, converting existing C programs to Checked C might require refactoring, e.g., modifying the above program to not use `&n` expression, which might require considerable effort [7] depending on the program's complexity. Recently, Machiry et al. developed 3C [23] that tries to automatically convert a program to Checked C by adding appropriate pointer annotations. However, as described in 3C, complete automated conversion is infeasible and requires the

developer to convert some code regions manually. Although, the backward compatibility of Checked C helps a partially annotated program to enjoy spatial memory safety on those regions using only Checked pointers (i.e., checked or safe regions).

No safety against unchecked code. But, the unconverted code regions (or unsafe regions) can affect pointers in safe regions and violate certain assumptions leading to vulnerabilities, as demonstrated by cross-language attacks [26]. Although the blameless proof exists [35], it does not state that spatial safety violations cannot happen in Checked regions but rather states that Checked regions *cannot be blamed for any spatial safety violations*. Consider the following example:

```
1  // Checked code
2  int func(array_ptr<char> p : count(5)) {
3  ..p[4]..
4  }
5  // unchecked code
6  ...
7  str = "he";
8  ...
9  assume_bounds_cast<char>(str, 5);
10 ...
11 char ptr[16];
12 ...
13 len <- derived from user input
14 ...
15 memcpy(ptr, buff, len); // buffer overflow
```

Here, the checked function `func` expected a pointer to a buffer of five elements, but unchecked code violated it and invoked the function with a buffer of 2 elements. This results in a spatial safety violation ( ) in the Checked region, but of course, the blame or the root cause is in the unchecked region ( ). Furthermore, since checked and unchecked regions execute in the same address space, spatial memory corruptions in unchecked regions (Line 15) can take down the complete program despite having checked regions. We need *an isolation mechanism to ensure that code executed as part of unchecked regions does not violate the safety guarantees in checked regions*.

This mechanism is called program partitioning [36], and there has been considerable work [1, 4, 21, 22, 40] in the area. Most of these techniques are *data-centric* [21, 22], wherein program data drives the partitioning. E.g., Given sensitive data in a program, the goal is to partition functions into two parts or partitions based on whether a function can access the sensitive data. The performance overhead of these approaches is dominated by marshaling costs and depends on the usage of sensitive data. The overhead of state-of-the-art approaches [21, 22] is prohibitive and varies from 37%-163%. But in our case, we want a low-overhead code-centric partitioning, where the unchecked code (or functions) should be isolated (or partitioned) from checked code. We also want the technique to co-exist and be compatible with Checked C guarantees such that the partition containing checked code should still enjoy its spatial safety.

In this work, we propose a type-directed code-centric program partitioning approach. Specifically, our system, CHECKEDCBOX, extends Checked c using **tainted** (`t_*`) types, which can be used to mark functions and pointers that need to be isolated from the original program. The tainted types long with Checked C allow annotating pointer along two dimensions, i.e., (i) taintedness: a pointer can be either tainted or not (untainted), and (ii) checkedness: it can be either checked or not. The checked types follow the standard Checked C typing rules. However, our type-system disallows all interactions between taint and untainted types. The developer starts by marking desired (unchecked or risky) functions and pointers used in these functions as tainted. Second, CHECKEDCBOX partitions the given program into two partitions ($uc-region$ and $c-region$) of different privileges:

- $uc-region$ (low privilege tainted region): This partition contains tainted types (i.e., functions and pointers) and can only access tainted pointers.
- $c-region$ (high privilege untainted region): This partition contains the remaining (untainted) code and data and has complete access to $c-region$. The functions in $c-region$ can invoke any function in $uc-region$ but not the other way around, except for call-back functions, which we will discuss later.

Finally, during program execution, the $uc-region$ partition will be executed in an existing sandboxed environment (e.g., WASM sandbox), and our compiler will add the necessary instrumentation to facilitate the communication between code in $c-region$ and $uc-region$.

The combination of tainted types and privileged partitions enables us to enforce isolation and provide memory safety without marshaling costs. As functions in the $uc-region$ can only access tainted types, $c-region$ functions should use tainted types to pass pointer arguments to $uc-region$ functions. We avoid marshaling by allocating all tainted pointers (i.e., tainted buffers) in $uc-region$ and thus can be accessed in both partitions. Although memory isolation prevents direct violations, $uc-region$ code can still affect $c-region$ through tainted pointers by confused deputy attacks. Our compiler avoids these attacks by ensuring, either statically or through dynamic checks, that tainted pointers can only point to $uc-region$ address space.

2. Overview and Transcendence

This section discusses the key merge features between Checked-C and RLBox and the problems we are trying to solve.

Maintaining Non-crashing. Previously, the main guarantee of Checked C [20] was the blame theorem, i.e., if there is a crash, the source is from some unchecked regions. For example, at Figure 4 line 31, we call a unchecked function `f` with a checked null-terminated array (NT-array) pointer argument. At line 8, depending on the NT-array size, `free(s[10])` might crash. Even if it does not crash, line 38 is doomed because of the `free` call.

The philosophy is that unchecked regions are written in C and unstable, and programmers will eventually remove them by converting them to Checked C code. The reality

```
1  //in checked region
2
3  int compare_1(nt_array_ptr<char> x: count (0),
4    nt_array_ptr<char> y : count (0)) {
5    int len_x = strlen(x);
6    int len_y = strlen(y);
7    return sum(x,len_x) < sum(y,len_y);
8    }
9  ...
10
11 int stringsort(
12  nt_array_ptr<nt_array_ptr<char>> s : count (n),
13  ptr<(int)(nt_array_ptr<char>,
14   nt_array_ptr<char>)> cmp, int n) {
15   int i, j, gap;
16   int didswap;
17
18   for(gap = n / 2; gap > 0; gap /= 2) {
19     {
20     do  {
21         didswap = 0;
22         for(i = 0; i < n - gap; i++)
23            {
24              j = i + gap;
25              if((*cmp)(s[i], s[j]) > 0)
26              {
27                 int len = strlen(s[i]);
28                 nt_array_ptr<char>
29                   tmp : count (len) = s[i];
30                 s[i] = s[j];
31                 s[j] = tmp;
32                 didswap = 1;
33              }
34            }
35         } while(didswap);
36     }
37   }
38
39   return 0;
40 }
```

Figure 2: Checked stringsort Code

```
1  //in checked region, tainted version
2  int tainted_compare_1(
3    nt_array_tptr<char> x : count (0),
4    nt_array_tptr<char> y : count (0)) {
5   checked (x,y) {
6    int len_x = strlen(x);
7    int len_y = strlen(y);
8    nt_array_ptr<char> tx : count (len_x)
9     = malloc(nt_array<char>, len_x);
10   nt_array_ptr<char> ty : count (len_y)
11    = malloc(nt_array<char>, len_y);
12   safe_memcpy(tx,x,len_x);
13   safe_memcpy(ty,y,len_y);
14   return compare_1(tx,ty);
15  }
16 }
17 ...
18
19 //calling the function turns
20 //an unchecked region to a checked region.
21 int tainted_stringsort(nt_array_tptr
22      <nt_array_tptr<char>> s : count (n),
23  tptr<(int)(nt_array_tptr<char>,
24   nt_array_tptr<char>)> cmp, int n) {
25  checked (s,cmp,n) {
26   int i;
27   nt_array_ptr<nt_array_ptr<char>> p : count (n)
28     = malloc(nt_array<nt_array_ptr<char>>, n);
29   for(i = 0; i < n; i++) {
30     int len = strlen s[i];
31     nt_array_ptr<char> tmp : count (len)
32       = new malloc(nt_array<char>, len);
33     safe_memcpy(tmp,s[i],len);
34     p[i] = tmp;
35   }
36   ptr<(int)(nt_array_ptr<char> : count (0),
37   nt_array_ptr<char> : count (0))>
38     cfun = find_check(cmp);
39
40   return stringsort(p,cfun);
41  }
42 }
```

Figure 3: Tainted stringsort Code

is that users might not want to convert everything. For some insignificant or handy history C code, such as library functions, they might want to keep it as long as no crashing.

The sources of crashing in Checked C are (1) unchecked regions crash themselves; and (2) the misuse of checked pointers in unchecked regions. Enlightened by RLBox [31], we sandbox the unchecked regions and then we utilize the Checked C type system to disallow checked pointers to be used in an unchecked region. To achieve the communication between checked and unchecked regions, we create tainted pointers that can be shared by different regions, whose data are stored in the sandboxed heap. Users are required to copy data to tainted pointers that are shared in unchecked regions. For example, we copy the checked pointer data to the tainted pointer `tp` at Figure 4 line 26, and input the tainted pointer to the unchecked function at line 34. At line 8, even if statement might crash, since tainted pointers are stored in the sandboxed heap, it can be recovered. At line 37, the use of a tainted pointer in a checked region requires a verification on it. This is handled by inserting additional checks and creating exception handling before the use by the CHECKEDCBOX compiler. Thus, the checked pointer `p` is safely used at line 38.

In short, banning checked pointer uses in unchecked regions and providing tainted shared pointers are the solution for crashing. In CHECKEDCBOX, we proved the non-crashing theorem based on the CHECKEDCBOX type system, .i.e, any program execution is stopped in a predictable manner where either the execution terminates or is stopped at a place where an exception handler exists. In fact, our work is *the first formalism of RLBox*, and formally shows the security guarantee that RLBox provides is actually the non-crashing theorem.

```
1  //in unchecked region
2  int f(char ** s, int (*cmp)(char *,char *),
3    int (*sort)(char **, int (*)(char *,char *),
4                int), int n) {
5  ...
6
7    int i = sort(s,cmp,n);
8    free(s[10]);
9  ...
10 }
11
12 int g(int (*cmp)(char *,char *)) {
13 ...
14   int real_addr = derandomize(cmp);
15 ...
16 }
17
18 int main(int n) {
19   nt_array_ptr<nt_array_ptr<char>> p : count(n)
20     = malloc(nt_array<nt_array_ptr<char>>, n);
21
22   nt_array_tptr<nt_array_tptr<char>>
23      tp : count(n) =
24        tmalloc(nt_array<nt_array_tptr<char>>, n);
25 ...
26   safe_memcpy(tp, p, n);
27
28   unchecked {
29     if (BAD) // a flag to call different funs.
30       //input checked pointers
31       f(p, compare_1, stringsort);
32     else
33       //input tainted pointers
34       f(tp, tainted_compare_1,
             tainted_stringsort);
35   }
36
37   if (!BAD) safe_memcpy(p, tp, n);
38   p[10] = "crash?";
39
40   unchecked {
41     if (BAD) g(compare_1)
42       else g(tainted_compare_1);
43   }
44
45   return 0;
46 }
```

Figure 4: Tainted Pointer Usage in Calling Unchecked Fun

Formalism Function Pointers. In C, manipulating function pointers is a major way of implementing high order functions. Previously, we assumed all function calls are through calling by name to a global map. In CHECKEDCBOX, we formalize function pointers and maintain the CHECKEDCBOX type soundness. To the best of our knowledge, this is the first work of formalizing C function pointers with security guarantee.

Figure 2 defines a string sorting algorithm depending on the input function pointer `cmp` defining the generic order for strings, and `compare_1` is an example `cmp` function that adds the ASCII numbers of characters in the two strings and compare the results. In addition, function pointers enable the callback mechanism, i.e., a server sends a function pointer to a client in an unchecked region, and allows the client to access some server resources by calling back the pointer. This is a common usage between a web-browser and untrusted third party libraries. The function call in Figure 4 line 34 is one such usage.

We also utilize CHECKEDCBOX subtyping relation to permit function pointer static auto-casting. Function pointer type information might contain array pointer bound information, for which it is inconvenient to coincide the defined types for a function implementation and the function pointer type. For example, the `cmp` argument in `stringsort` (Figure 2) has type `ptr<(int)(nt_array_ptr<char>, nt_array_ptr<char>)>`, meaning that the function takes two NT-array pointers with arbitrary size and outputs an integer. The function `compare_1`'s pointer has type `ptr<(int)(nt_array_ptr<char> : count (0), nt_array_ptr<char> : count (0))>` To use `compare_1` in `stringsort`, the type is auto-cast to the `cmp`'s type. In general, if function pointer x has type $*(tl \to t)$, and y has $*(tl' \to t')$, in order to use x as y, tl' should be a subtype of tl and t subtypes to t'.

Not Exposing Checked Pointer Addresses. The design for the non-crashing property bans the checked pointer uses in unchecked regions. Thus, there is no reason to permit checked pointer variable assignments in unchecked regions; especially, this might expose a checked pointer address to untrusted parties. For example, the call to function g at Figure 4 line 41 lives in an unchecked region, and g might use some mechanism, such as derandomizing ASLR [38], to achieve the checked pointer address. Thus, it enables a third party to access any checked heap and function data by simple pointer arithmetic.

To prevent the checked pointer address leak, we prevent any unchecked regions acknowledge checked pointer variables. In addition, to facilitate checked function callbacks, the CHECKEDCBOX compiler compiles every checked function with an additional tainted shell function. Users are required to serve unchecked regions with the tainted shell pointer instead of the original checked function pointer. For example, `tainted_compare_1` and `tainted_stringsort` in Figure 3 are the tainted shells of the checked functions `compare_1` and `stringsort`. In the tainted shells, the arguments are tainted versions of the corresponding arguments in the checked functions. Inside the shell body, create checked pointer copies of the tainted arguments, and call the checked functions. In addition, once a checked function returns, if the output is a checked pointer, we also its the data to a new tainted pointer and exit the shell. Figure 4 line 42 is an example of serving the function call living in an unchecked region with a tainted shell pointer argument `tainted_compare_1`. Even if g derndomizes its address (line 14), the shell address is in the sandbox and has no harm, and calling the shell never exposes any checked pointer information outside of the shell.

Conceptually, the shell is run in a checked region. One

Variables: x Integers: $n ::= \mathbb{Z}$

Context Mode:	m	::=	$\texttt{c} \mid \texttt{u}$
Pointer Mode:	ξ	::=	$m \mid \texttt{t}$
Bound:	b	::=	$n \mid x + n$
	β	::=	(b, b)
Word Type:	τ	::=	$\texttt{int} \mid \texttt{ptr}^{\xi}\ \omega$
Type Flag:	κ	::=	$nt \mid \cdot$
Type:	ω	::=	$\tau \mid [\beta\ \tau]_{\kappa} \mid \forall\ \overline{x}.\ \overline{\tau} \to \tau$
Expression:	e	::=	$n : \tau \mid x \mid e + e \mid (\tau)e \mid \langle\tau\rangle e$
		$\mid$	$\texttt{strlen}(x) \mid *e \mid *e = e$
		$\mid$	$\texttt{let}\ x = e\ \texttt{in}\ e \mid \texttt{if}\ (e)\ e\ \texttt{else}\ e$
		$\mid$	$\texttt{malloc}(\xi, \omega) \mid e(\overline{e})$
		$\mid$	$\texttt{unchecked}(\overline{x})\{e\} \mid \texttt{checked}(\overline{x})\{e\}$

Figure 5: CORECHKCBOX Syntax

$$m \vdash \texttt{int} \qquad \frac{\xi \wedge m \vdash \tau \qquad \xi \leq m}{m \vdash \texttt{ptr}^{\xi}\ [\beta\ \tau]_{\kappa}}$$

$$\frac{\xi \wedge m \vdash \tau \qquad \xi \leq m}{m \vdash \texttt{ptr}^{\xi}\ \tau} \qquad \frac{\xi \wedge m \vdash \tau \qquad \xi \leq m \qquad FV(\overline{\tau}) \cup FV(\tau) \subseteq \overline{x}}{m \vdash \texttt{ptr}^{\xi}\ (\forall\ \overline{x}.\ \overline{\tau} \to \tau)}$$

$$\texttt{t} \wedge \texttt{c} = \texttt{u} \qquad \xi \wedge \texttt{u} = \texttt{u} \qquad \texttt{c} \wedge m = m \qquad m_1 \wedge m_2 = m_2 \wedge m_1$$
$$\xi \leq \xi \qquad \texttt{t} \leq \xi$$

Figure 6: Well-formedness for Types

can imagine that a tainted shell is a safe closure that contains a checked block. Once the closure is called, the system is turned to a checked region, so that the checked function call at Figure 3 line 14 is safe, even if it is called by g in Figure 4 , because it lives in the checked region. In the CHECKEDCBOX formalism, we formalize a *checked* block on top of the existing unchecked regions, and the transition of a tainted shell call creates a checked block containing the shell body. We also make sure that no arguments in these tainted shell contains any checked pointers, as well as no output is of a checked type.

3. Formalization

This section describes the formal model of CHECKEDCBOX, called CORECHKCBOX, making precise its syntax, semantics, and type system. It also develops CORECHKCBOX's meta-theory, including type soundness, the non-exposure, and the non-crashing theorem.

3.1. Syntax

The syntax of CORECHKCBOX is given by the expression-based language presented in Fig. 5.

There are two notions of type in CORECHKCBOX. Types τ classify word-sized values including integers and pointers, while types ω classify multi-word values such as arrays, null-terminated arrays, functions, and single-word-size values. Pointer types ($\texttt{ptr}^{m}\ \omega$) include a mode annotation (m) which is either checked (`c`) or unchecked (`u`) and a type (ω) denoting valid values that can be pointed to. Array types include both the type of elements (τ) and a bound (β) comprised of an upper and lower bound on the size of the array ((b_l, b_h)). Bounds b are limited to integer literals n and expressions $x + n$. Whether an array pointer is null terminated or not is determined by annotation κ, which is nt for null-terminated arrays, and $\cdot$ otherwise (we elide the $\cdot$ when writing the type). CHECKEDCBOX function types ($\forall\ \overline{x}.\ \overline{\tau} \to \tau$) reflect its dependent function declrations, where $\overline{x}$ represents a list of `int`type variables in a dependent function header, which bind all bounds appearing in $\overline{\tau}$ and τ. We have a well-formed requirement for a function type, such that all variables in $\overline{\tau}$ and τ are bounded by $\overline{x}$. Here is the corresponding CHECKEDCBOX syntax for these types:

```
   array_tptr<τ> : count(n)   ⇔  ptr^t [(0,n) τ]
 nt_array_ptr<τ> : count(n)   ⇔  ptr^c [(0,n) τ]_nt
tptr<(int)(nt_array_tptr<τ> : count(n),
        nt_array_tptr<τ>)>: count(n))>
⇔ ptr^t (∀ n. ptr^t [(0,n) τ]_nt × ptr^t [(0,n) τ]_nt → int)
```

As a convention we write $\texttt{ptr}^{\texttt{c}}\ [b\ \tau]$ to mean $\texttt{ptr}^{\texttt{c}}\ [(0, b)\ \tau]$, so the above examples could be rewritten $\texttt{ptr}^{\texttt{c}}\ [n\ \tau]$ and $\texttt{ptr}^{\texttt{c}}\ [n\ \tau]_{nt}$, respectively.

CORECHKCBOX expressions include literals ($n : \tau$), variables (x), addition (e_1 `+` e_2), static casts ($(\tau)e$), dynamic casts ($\langle\tau\rangle e$) [1], the `strlen` operation ($\texttt{strlen}(x)$), pointer dereference and assignment ($*e$) and ($*e_1 = e_2$), resp.), let binding ($\texttt{let}\ x = e_1\ \texttt{in}\ e_2$), conditionals ($\texttt{if}\ (e)\ e_1\ \texttt{else}\ e_2$), memory allocation ($\texttt{malloc}(\xi, \omega)$), function calls ($e(\overline{e})$), unchecked blocks ($\texttt{unchecked}(\overline{x})\{e\}$), and checked blocks ($\texttt{checked}(\overline{x})\{e\}$).

Integer literals n are annotated with a type τ which can be either `int`, or $\texttt{ptr}^{m}\ \omega$ in the case n is being used as a heap address (this is useful for the semantics); $0 : \texttt{ptr}^{m}\ \omega$ (for any m and ω) represents the `null` pointer, as usual. The `strlen` expression operates on variables x rather than arbitrary expressions to simplify managing bounds information in the type system; the more general case can be encoded with a `let`. We use a less verbose syntax for dynamic bounds casts; e.g., the following

```
dyn_bounds_cast<array_ptr<τ>>(e, count(n))
```

becomes $\langle\texttt{ptr}^{\texttt{c}}\ [n\ \tau]\rangle e$.

Compared to the former Checked C model [20], there are four differences. First, the CHECKEDCBOX type annotations have well-formed restrictions in Figure 6, for maintaining non-exposure. Mainly, in a nested pointer $\texttt{ptr}^{\xi}\ (...\texttt{ptr}^{\xi'}\ \tau...)$, if the outside mode ξ is `t` or `u`, the inside mode ξ' cannot be `c`. It is worth noting that pointer modes are a three point partial order ($\leq$), where `t` is the infimum, and $\xi \wedge m$ is a special meet operation that projects pointer modes onto context modes, such that a pointer mode `t` is projected as `u`. Second, $\texttt{malloc}(\xi, \omega)$ includes a mode

1. assumed at compile-time and verified at run-time, see Appendix A

$$
\begin{array}{lcl}
\mu & ::= & n{:}\tau \\
e & ::= & \ldots \mid \texttt{ret}(x, \mu, e) \\
r & ::= & e \mid \texttt{null} \mid \texttt{bounds} \\
E & ::= & \Box \mid E + e \mid n{:}\tau + E \mid (\tau)E \mid \langle\tau\rangle E \mid *E \mid *E = e \\
 & & \mid *n{:}\tau = E \mid \texttt{let}\ x = E\ \texttt{in}\ e \mid \texttt{if}\ (E)\ e\ \texttt{else}\ e \\
 & & \mid E(\overline{e}) \mid n{:}\tau(\overline{E}) \mid \texttt{unchecked}(\overline{x})\{E\} \mid \texttt{checked}(\overline{x})\{E\}
\end{array}
$$

$$
\frac{m = \mathit{mode}(E) \qquad e = E[e'] \qquad (\varphi, \mathcal{H}, e') \longrightarrow (\varphi', \mathcal{H}', e'')}{(\varphi, \mathcal{H}, e) \longrightarrow_m (\varphi', \mathcal{H}', E[e''])}
$$

$$
\frac{\texttt{u} = \mathit{mode}(E) \qquad e = E[e'] \qquad \tau = \mathit{type}(e')}{(\varphi, \mathcal{H}, e) \longrightarrow_{\texttt{u}} (\varphi, \mathcal{H}, E[0{:}\tau])}
$$

$$
\begin{array}{l}
\mathit{mode}(E) = \mathit{mode}'(E, \texttt{c}) \\
\mathit{mode}'(\Box, m) = m \\
\mathit{mode}'(\texttt{unchecked}(\overline{x})\{E\}, m) = \mathit{mode}'(E, \texttt{u}) \\
\mathit{mode}'(\texttt{checked}(\overline{x})\{E\}, m) = \mathit{mode}'(E, \texttt{c}) \\
\mathit{mode}'(\alpha(E), m) = \mathit{mode}'(E, m) \quad [\mathit{owise}]
\end{array}
$$

Figure 7: CORECHKCBOX Semantics: Evaluation

flag ξ for allocating different pointers in different heap mode regions. For simplicity, we disallow ω to be a function type ($\forall\ \overline{x}.\ \overline{\tau} \to \tau$), Third, the first expression e in a function call ($e(\overline{e})$) represents a function pointer. Fourth, we update $\texttt{unchecked}(\overline{x})\{e\}$ to specify $\overline{x}$ restricting all free variables appearing in e. Additionally, we create `checked` blocks indicating the context mode change from `u` to `c`. Both the free variables in the `unchecked` and `checked` blocks and the return types cannot be `c` mode for maintaining non-exposure, i.e., no checked pointer can be communicated between `unchecked` and `checked` blocks.

CORECHKCBOX aims to be simple enough to work with, but powerful enough to encode realistic CHECKED-CBOX idioms. For example, mutable local variables can be encoded as immutable locals that point to the heap; the use of `&` can be simulated with `malloc`; and loops can be encoded as recursive function calls. `structs` are not in Fig. 5 for space reasons, but they are actually in our model, and developed in Appendix F. C-style `unions` have no safe typing in Checked C, so we omit them. Additional syntactic descrition is listed in the Checked C formalism paper [20].

3.2. Semantics

The operational semantics for CORECHKCBOX is defined as a small-step transition relation with the judgment $(\varphi, \mathcal{H}, e) \longrightarrow_m (\varphi', \mathcal{H}', r)$. Here, φ is a *stack* mapping from variables to values $n : \tau$ and $\mathcal{H}$ is a *heap* that is partitioned into two parts (`c` and `u` regions), each of which maps addresses (integer literals) to values $n{:}\tau$. If a pointer has mode `c`, it lives in the `c` region; otherwise, it lives in the `u` region. We wrote $\mathcal{H}(m, n)$ to retrieve the n-location heap value in the m region, and $\mathcal{H}(m)[n \mapsto \mu]$ to update location n with the value μ. It is worth noting that CHECKEDCBOX is not a fat-pointer system; thus, in every heap update, the value type annotation remains the same through program executions. Additionally, for both stack and heap, we ensure $FV(\tau) = \emptyset$ for all the value type annotations τ.

While heap bindings can change, stack bindings are immutable—once variable x is bound to $n : \tau$ in φ, that binding will not be updated; we can model mutable stack variables as pointers into the mutable heap. As mentioned, value $0{:}\tau$ represents a `null` pointer when τ is a pointer type; correspondingly, $\mathcal{H}(m, 0)$ should always be undefined. The relation steps to a *result* r, which is either an expression or a `null` or `bounds` failure, representing a null-pointer dereference or out-of-bounds access, respectively. Such failures are a *good* outcome; stuck states (non-value expressions that cannot transition to a result r) characterize undefined behavior. The context mode m indicates whether the stepped redex within e was in a checked (`c`) or unchecked (`u`) region.

The rules for the main operational semantics judgment—*evaluation*—are given at the bottom of Fig. 7. The first rule takes an expression e, decomposes it into an *evaluation context* E and a sub-expression e' (such that replacing the hole $\Box$ in E with e' would yield e), and then evaluates e' according to the *computation* relation $(\varphi, \mathcal{H}, e') \longrightarrow (\varphi, \mathcal{H}, e'')$, whose rules are given in Fig. 8, discussed shortly. The second rule describes the exception handling for possible crashing behaviors in unchecked region. A program in `u` mode can non-deterministically crash and the CHECKED-CBOX sandbox mechanism recorvers the program to a safe point $(0{:}\tau)$ and continues with the existing program state. There are other rules for describing the halts of evaluation to `null` and `bounds` states in Appendix A.

The *mode* function determines the context mode when evaluating e' based on the context E. For any program execution, the top context mode always starts with `c` ($\mathit{mode}(E) = \mathit{mode}'(E, \texttt{c})$). The result context mode is depeneded on where $\Box$ locates. If it occurs within $(\texttt{unchecked}(\overline{x})\{E'\})$ inside E without a surrounding `checked` block, then the mode is `u`; if the exact opposite happens (within $(\texttt{checked}(\overline{x})\{E'\})$ without surrounding `unchecked` blocks), the mode is `c`; the other cases ($\mathit{mode}'(\alpha(E), m) = \mathit{mode}'(E, m)$) are recursively defined by the above base cases (α represents any syntactic categories in Figure 7 other than $\Box$, `unchecked`, and `checked`). Evaluation contexts E define a standard left-to-right evaluation order. (We explain the $\texttt{ret}(x, \mu, e)$ syntax shortly.)

Fig. 8 shows selected rules for the computation relation; we explain them with the help of the example in Figure 2 and Figure 3.

Checked and Tainted Pointer Operations. The rules for pointer related operations—S-DEFC, S-DEFT, S-ASSIGNARRC, S-ASSIGNARRT, S-DEFNULL, and S-CAST. The first five are deference and assignment operations—illustrate how the semantics checks bounds. Rule S-DEFNULL transitions attempted null-pointer dereferences to `null`, whereas S-DEFC dereferences a `c`-mode non-null (single) pointer. When `null` is returned by the computation relation, the evaluation relation halts the entire evaluation with `null` (using a rule not shown in Fig. 7); it does likewise when `bounds` is returned (see Appendix C). S-ASSIGNARRC assigns to an array as long as 0 (the point of

S-DEFC
$$\frac{\mathcal{H}(\mathtt{c}, n) = n_a : \tau_a}{(\varphi, \mathcal{H}, *n : \mathtt{ptr}^{\mathtt{c}}\ \tau) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$$

S-ASSIGNARRC
$$\frac{\mathcal{H}(\mathtt{c}, n) = n_a : \tau_a \qquad 0 \in [n_l, n_h)}{(\varphi, \mathcal{H}, *n : \mathtt{ptr}^{\mathtt{c}}\ [(n_l, n_h)\ \tau]_\kappa = n_1 : \tau_1) \longrightarrow (\varphi, (\mathcal{H}, \mathtt{c})[n \mapsto n_1 : \tau_a], n_1 : \tau)}$$

S-DEFT
$$\frac{\mathcal{H}(\mathtt{u}, n) = n_a : \tau_a \qquad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathtt{u}} n_a : \tau}{(\varphi, \mathcal{H}, *n : \mathtt{ptr}^{\mathtt{t}}\ \tau) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$$

S-ASSIGNARRT
$$\frac{\mathcal{H}(\mathtt{u}, n) = n_a : \tau_a \qquad 0 \in [n_l, n_h) \qquad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathtt{u}} n_1 : \tau}{(\varphi, \mathcal{H}, *n : \mathtt{ptr}^{\mathtt{t}}\ [(n_l, n_h)\ \tau]_\kappa = n_1 : \tau_1) \longrightarrow (\varphi, (\mathcal{H}, \mathtt{u})[n \mapsto n_1 : \tau_a], n_1 : \tau)}$$

S-DEFNULL
$$(\varphi, \mathcal{H}, *0 : \mathtt{ptr}^{\mathtt{c}}\ \omega) \longrightarrow (\varphi, \mathcal{H}, \mathtt{null})$$

S-CAST
$$(\varphi, \mathcal{H}, (\tau) n : \tau') \longrightarrow (\varphi, \mathcal{H}, n : \varphi(\tau))$$

S-RETEND
$$(\varphi, \mathcal{H}, \mathtt{ret}(x, \mu, \mu')) \longrightarrow (\varphi, \mathcal{H}, \mu')$$

S-LET
$$(\varphi, \mathcal{H}, \mathtt{let}\ x = n : \tau\ \mathtt{in}\ e) \longrightarrow (\varphi, \mathcal{H}, \mathtt{ret}(x, n : \tau, e))$$

S-RETCON
$$\frac{\varphi(x) = \mu \qquad (\varphi[x \mapsto \mu'], \mathcal{H}, e) \longrightarrow (\varphi', \mathcal{H}', e')}{(\varphi, \mathcal{H}, \mathtt{ret}(x, \mu', e)) \longrightarrow (\varphi'[x \mapsto \mu], \mathcal{H}', \mathtt{ret}(x, \varphi'(x), e'))}$$

S-UNCHECKED
$$(\varphi, \mathcal{H}, \mathtt{unchecked}(\overline{x})\{\mu\}) \longrightarrow (\varphi, \mathcal{H}, \mu)$$

S-FUNC
$$\frac{\Xi(\mathtt{c}, n) = \tau\ (\overline{x} : \overline{\tau})\ (\mathtt{c}, e)}{(\varphi, \mathcal{H}, n : (\mathtt{ptr}^{\mathtt{c}}\ \tau)(\overline{n_a} : \overline{\tau_a})) \longrightarrow (\varphi, \mathcal{H}, \mathtt{let}\ \overline{x} = \overline{n} : (\overline{\tau}[\overline{n}/\overline{x}])\ \mathtt{in}\ (\tau[\overline{n}/\overline{x}])e)}$$

S-CHECKED
$$(\varphi, \mathcal{H}, \mathtt{checked}(\overline{x})\{\mu\}) \longrightarrow (\varphi, \mathcal{H}, \mu)$$

S-FUNT
$$\frac{\Xi(\mathtt{u}, n) = \tau\ (\overline{x} : \overline{\tau})\ (\mathtt{t}, e) \qquad \emptyset; \mathcal{H}; \emptyset \vdash_{\mathtt{u}} n : \mathtt{ptr}^{\mathtt{t}}\ \tau}{(\varphi, \mathcal{H}, n : (\mathtt{ptr}^{\mathtt{t}}\ \tau)(\overline{n_a} : \overline{\tau_a})) \longrightarrow (\varphi, \mathcal{H}, \mathtt{let}\ \overline{x} = \overline{n} : (\overline{\tau}[\overline{n}/\overline{x}])\ \mathtt{in}\ (\tau[\overline{n}/\overline{x}])e)}$$

Figure 8: CORECHKCBOX Semantics: Computation (Selected Rules)

```
nt_array_ptr<char> safe_strcat
   (nt_array_ptr<char> dst : count(n),
    nt_array_ptr<char> src : count(0), int n) {
  int x = strlen(dst);
  int y = strlen(src);
  nt_array_ptr<char> c : count(n) =
    dyn_bounds_cast
          <nt_array_ptr<char>>(dst,count(n));
    // sets c == dst with bound n (not x)
  if (x+y < n) {
    for (int i = 0; i < y; ++i)
      *(c+x+i) = *(src+i);
    *(c+x+y) = '\0';
    return dst;
  }
  return null;
}
```

Figure 9: Implementation of safe `strcat`

dereference) is within the bounds designated by the pointer's annotation and strictly less than the upper bound.

S-DEFT and S-ASSIGNARRT are similar rules to S-DEFC and S-ASSIGNARRC for tainted pointers. Any dynamic heap use of a tainted pointer requires a verification. Performing such a verification equates to performing a type check for a pointer constant in Figure 12. We explain this shortly in Section 3.3. For now, the verification step, e.g. $\mathtt{u}; \emptyset; \mathcal{H}; \emptyset \vdash n_a : \tau$ in S-DEFC, means we verify that the value n_a has type τ in the u region heap.

Static casts of a literal $n : \tau'$ to a type τ are handled by S-CAST. In a type-correct program, such casts are confirmed safe by the type system no matter if the target is a `t` or `c` pointer. To evaluate a cast, the rule updates the type annotation on n. Before doing so, it must "evaluate" any variables that occur in τ according to their bindings in φ. For example, if τ was $\mathtt{ptr}^{\mathtt{c}}\ [(0, x+3)\ \mathtt{int}]$, then $\varphi(\tau)$ would produce $\mathtt{ptr}^{\mathtt{c}}\ [(0, 5)\ \mathtt{int}]$ if $\varphi(x) = 2$. The full formalism, including `struct` and null-terminated bound widening pointer operations, is given in Appendix A.

Unchecked and Checked Blocks. Semantically, `unchecked` and `checked` blocks (S-UNCHECKED and S-CHECKED) act as a classical C block structures. The interesting part is their type rules in Section 3.3.

Binding and Function Calls. The semantics handles variable scopes using the special `ret` form. S-LET evaluates to a configuration whose stack is φ extended with a binding for x, and whose expression is $\mathtt{ret}(x, n : \tau, e))$ which keeps φ unchanged, but remember x with the new value $n : \tau$ in the scope e. Every time when Evaluation proceeds on e (Rule S-RETCON), we install the current stack value $n : \tau$ for x in φ, after one-step evaluation is completed, we restore the result $\varphi'(x)$ in the result $\mathtt{ret}(x, \varphi'(x), e'))$. This procedure continues until it becomes a literal $n : \tau$, in which case S-RETEND remove the `ret` frame and evaluates to $n : \tau$.

Function calls are handled by S-FUNC and S-FUNT, for `c`and `t`mode function pointers, respectively. A call to function f causes f's definition to be retrieved from Ξ, which maps function names to function data $\tau\ (\overline{x} : \overline{\tau})\ (\xi, e)$, where τ is the return type, $(\overline{x} : \overline{\tau})$ is the parameter list of variables and their types, ξ determines the mode of the function, and e is the function body. Similar to $\mathcal{H}$, the global function store Ξ is also partitioned into two parts (`c` and `u` regions), each of which maps addresses (integer literals) to the function data described above.

The CHECKEDCBOX functions are dependent functions.

Recall that array bounds in types may refer to in-scope variables; e.g., parameter `dst`'s bound `count(n)` refers to parameter `n` on lines 2-3 in Fig. 9. Semantically, the call is expanded into a `let` which binds parameter variables $\overline{x}$ to the actual arguments $\overline{n}$, but annotated with the parameter types $\overline{\tau}$ (this will be safe for type-correct programs). The function body e is wrapped in a static cast $(\tau[\overline{n}/\overline{x}])$ which is the function's return type but with any parameter variables $\overline{x}$ appearing in that type substituted with the call's actual arguments $\overline{n}$. To see why this is needed, suppose that `safe_strcat` in Fig. 9 is defined to return a `nt_array_ptr<int>:count(n)` typed term, and assume that we perform a `safe_strcat` function call as `x=safe_strcat(a,b,10)`. After the evaluation of `safe_strcat`, the function returns a value with type `nt_array_ptr<int>:count(10)` because we substitute bound variable `n` in the defined return type with `10` from the function call's argument list. Note that the S-FUNC and S-FUNT rules replace the annotations $\overline{\tau_a}$ with $\overline{\tau}$ (after instantiation) from the function's signature. Using $\overline{\tau_a}$ when executing the body of the function has no impact on the soundness of CORECHKCBOX, but will violate Theorem 6, which we introduce in Sec. 4.

3.3. Typing

We now turn to the CORECHKCBOX type system. The typing judgment has the form $\Gamma;\Theta \vdash_m e : \tau$, which states that in a type environment Γ (mapping variables to their types) and a predicate environment Θ (mapping integer-typed variables to Boolean predicates), expression e will have type τ if evaluated in mode m. Key rules for this judgment are given in Fig. 10. All remaining rules are given in Appendix B and E.

Pointer Access. Rules T-DEFARR and T-ASSIGNARR type-check array dereference and assignment operations resp., returning the type of pointed-to objects; rules for pointers to single objects are similar. The condition $m \leq m'$ ensures that checked pointers and unchecked pointers can only be deferenced in checked and unchecked regions, respectively; the type rule for The rules do not attempt to reason whether the access is in bounds; this check is deferred to the semantics.

Type Equality and Subtyping and Casting. In CHECKEDCBOX, we define the type equality $\tau =_\Theta \tau'$ is an equivalent relation over the bound equality ($=_\Theta$) of the bounds in (NT-)array types and alpha equivalence of two function types in Figure 11. Two (NT-)array pointers $[\beta\ \tau]_\kappa$ and $[\beta'\ \tau']_\kappa$ are equivalent, if $\beta =_\Theta \beta'$; two function types $\forall\,\overline{x}.\,\overline{\tau} \to \tau$ and $\forall\,\overline{y}.\,\overline{\tau'} \to \tau'$ are equivalent, if we can find a same length variable set $\overline{z}$ that is substituted for $\overline{x}$ and $\overline{y}$ in $\overline{\tau} \to \tau$ and $\overline{\tau'} \to \tau'$, respectively, and the substitution results are equal.

The T-CASTPTR rule permits casting from an expression of type τ' to a checked pointer when $\tau' \sqsubseteq \texttt{ptr}^{\texttt{c}}\ \tau$. This subtyping relation $\sqsubseteq$ is given in Fig. 11 built on the type equality ($\tau =_\Theta \tau' \Rightarrow \tau \sqsubseteq_\Theta \tau'$); the many rules ensure the relation is transitive. Most of the rules handle casting between array pointer types. The second rule $0 \leq b_l \wedge b_h \leq 1 \Rightarrow \texttt{ptr}^m\ \tau \sqsubseteq \texttt{ptr}^m\ [(b_l, b_h)\ \tau]$ permits treating a singleton pointer as an array pointer with $b_h \leq 1$ and $0 \leq b_l$. Two function pointers are subtyped ($\texttt{ptr}^\xi\ \forall\,\overline{x}.\,\overline{\tau} \to \tau \sqsubseteq_\Theta \texttt{ptr}^\xi\ \forall\,\overline{x}.\,\overline{\tau'} \to \tau'$), if the output type are subtyped ($\tau \sqsubseteq_\Theta \tau'$) and the argument types are oppositely subtyped ($\overline{\tau'} \sqsubseteq_\Theta \overline{\tau}$).

Since bounds expressions may contain variables, determining assumptions like $b_l \leq b_l'$ requires reasoning about those variables' possible values. The type system uses Θ to make such reasoning more precise. Θ is a map from variables x to equation predicates P, which have the form $P ::= \texttt{ge\_0} \mid \texttt{eq}\ b$. It maps variables to equations that are recorded along the type checking procedure. If Θ maps x to `ge_0`, that means that $x \geq 0$; `eq` b means that x is equivalent to the bound value b in the current context, such as in the type judgment for e_2 in Rule T-LETINT and T-RET. Appendix D. has an example rule for populating Θ with a `ge_0` predicate.

Constant Validity. Given a constant $n : \tau$, the type judgment $\Theta;\mathcal{H};\sigma \vdash_m n : \tau$ checks in Figure 12 if the constant is valid, where $\mathcal{H}$ is the initial heap that the constant resides on and σ is a set of constant assumed to be checked. A global function store Ξ is also required in checking the validity of a function pointer. A valid function pointer should appear in the right store region (`c` or `u`) and the address stores a function with the right type.

The last rule in Figure 12 describes the validity check for a non-function pointer, where every element in the pointer range ($[0, size(\omega))$) should be well typed. A checked pointer checks validity in type step as rule T-CONSTC, while a tainted/unchecked pointer does not check for such during the type checking. Tainted pointers are verified through the validity check in dynamic execution as we mentioned above.

Unchecked and Checked Blocks. During the type checking, Both `checked`$(\overline{x})\{e\}$ and `unchecked`$(\overline{x})\{e\}$ check all free variables are within $\overline{x}$, and the types for $\overline{x}$ have no checked pointers. A `checked` and `unchecked` block represents the transition stage from a checked to an checked region, or vice versa. We need to make sure no checked pointers are information exposed to unsafe code regions.

Dependent Function Pointers and Let Bindings. Rule T-LET and T-LETINT type a `let` expression, which also admits type dependency. In particular, the result of evaluating a `let` may have a type that refers to one of its bound variables (e.g., if the result is a checked pointer with a variable-defined bound); if so, we must substitute away this variable once it goes out of scope (T-LETINT). Note that we restrict the expression e_1 to syntactically match the structure of a Bounds expression b (see Fig. 5).

Rule T-RET types a `ret` expression, which does not appear in source programs but is introduced by the semantics when evaluating a let binding (rule S-LET in Fig. 8); this rule is needed for the preservation proof.

Rule T-FUN is the standard dependent function call rule. It learns the function pointer type ($\texttt{ptr}^\xi\ \forall\,\overline{x}.\,\overline{\tau} \to \tau$) of e by a recursive type check, type-checks the actual arguments $\overline{e}$ which have types $\overline{\tau'}$, and then confirms that each of

$$\text{T-ConstU}\ \frac{\neg \mathtt{c}(\tau)}{\Gamma;\Theta \vdash_u n{:}\tau : \tau} \qquad \text{T-ConstC}\ \frac{\Theta;\mathcal{H};\emptyset \vdash_c n : \tau}{\Gamma;\Theta \vdash_c n{:}\tau : \tau} \qquad \text{T-Def}\ \frac{\xi \leq m}{\Gamma;\Theta \vdash_m e : \mathtt{ptr}^{\xi}\ \tau}{\Gamma;\Theta \vdash_m *e : \tau}$$

$$\text{T-AssignArr}\ \frac{\Gamma;\Theta \vdash_m e_1 : \mathtt{ptr}^{\xi}\ [\beta\ \tau]_{\kappa} \quad \Gamma;\Theta \vdash_m e_2 : \tau' \quad \tau' \sqsubseteq_{\Theta} \tau \quad \xi \leq m}{\Gamma;\Theta \vdash_m *e_1 = e_2 : \tau} \qquad \text{T-CastPtr}\ \frac{\Gamma;\Theta \vdash_m e : \tau' \quad \tau' \sqsubseteq \mathtt{ptr}^{\xi}\ \tau}{\Gamma;\Theta \vdash_m (\mathtt{ptr}^{\xi}\ \tau)e : \mathtt{ptr}^{\xi}\ \tau}$$

$$\text{T-Let}\ \frac{x \notin FV(\tau') \quad \Gamma;\Theta \vdash_m e_1 : \tau \quad \Gamma[x \mapsto \tau];\Theta \vdash_m e_2 : \tau'}{\Gamma;\Theta \vdash_m \mathtt{let}\ x = e_1\ \mathtt{in}\ e_2 : \tau'} \qquad \text{T-LetInt}\ \frac{x \in FV(\tau') \Rightarrow e_1 \in \text{Bound} \quad \Gamma;\Theta \vdash_m e_1 : \tau \quad \Gamma[x \mapsto \tau];\Theta[x \mapsto \mathtt{eq}\ e_1] \vdash_m e_2 : \tau'}{\Gamma;\Theta \vdash_m \mathtt{let}\ x = e_1\ \mathtt{in}\ e_2 : \tau'[e_1/x]}$$

$$\text{T-Ret}\ \frac{\Gamma[x \mapsto \tau'];\Theta[x \mapsto \mathtt{eq}\ n] \vdash_m e : \tau}{\Gamma;\Theta \vdash_m \mathtt{ret}(x, n{:}\tau', e) : \tau}$$

$$\text{T-Checked}\ \frac{\forall x \in \overline{x}\ .\ \neg \mathtt{c}(\Gamma(x)) \quad FV(e) \in \overline{x} \quad \Gamma;\Theta \vdash_c e : \tau}{\Gamma;\Theta \vdash_m \mathtt{checked}(\overline{x})\{e\} : \tau} \qquad \text{T-Unchecked}\ \frac{\forall x \in \overline{x}\ .\ \neg \mathtt{c}(\Gamma(x)) \quad FV(e) \in \overline{x} \quad \Gamma;\Theta \vdash_u e : \tau}{\Gamma;\Theta \vdash_m \mathtt{unchecked}(\overline{x})\{e\} : \tau}$$

$$\text{T-Fun}\ \frac{\Gamma;\Theta \vdash_m e : \mathtt{ptr}^{\xi}\ \forall\ \overline{x}.\ \overline{\tau} \to \tau \quad \Gamma;\Theta \vdash_m \overline{e} : \overline{\tau'} \quad \overline{e'} = \{e' | (e', \mathtt{int}) \in (\overline{e} : \overline{\tau'})\} \quad \forall e'\ .\ e' \in \overline{e'} \Rightarrow e' \in \text{Bound} \quad \overline{\tau'} \sqsubseteq_{\Theta} \overline{\tau}[\overline{e'}/\overline{x}]}{\Gamma;\Theta \vdash_m e(\overline{e}) : \tau[\overline{e'}/\overline{x}]}$$

$$\mathtt{c}(\mathtt{int}) = \mathtt{false} \qquad \mathtt{c}(\mathtt{ptr}^{\mathtt{c}}\ \omega) = \mathtt{true} \qquad \mathtt{c}(\mathtt{ptr}^{\xi}\ \omega) = \mathtt{false}\ \ [\textit{owise}]$$

Figure 10: Selected type rules

Bound Inequality and Equality:

$$\begin{aligned}
n \leq n' &\Rightarrow n &&\leq_{\Theta} n' \\
n \leq n' &\Rightarrow x + n &&\leq_{\Theta} x + n' \\
n \leq n' \wedge \Theta(x) = \mathtt{ge_0} &\Rightarrow n &&\leq_{\Theta} x + n' \\
\Theta(x) = \mathtt{eq}\ b \wedge b + n \leq_{\Theta} b' &\Rightarrow x + n &&\leq_{\Theta} b' \\
\Theta(x) = \mathtt{eq}\ b \wedge b' \leq_{\Theta} b + n &\Rightarrow b' &&\leq_{\Theta} x + n \\
b \leq_{\Theta} b' \wedge b' \leq_{\Theta} b &\Rightarrow b &&=_{\Theta} b'
\end{aligned}$$

Type Equility:

$$\begin{aligned}
\mathtt{int} &&&=_{\Theta} \mathtt{int} \\
\omega =_{\Theta} \omega' &\Rightarrow \mathtt{ptr}^{\xi}\ \omega &&=_{\Theta} \mathtt{ptr}^{\xi}\ \omega' \\
\beta =_{\Theta} \beta' \wedge \tau =_{\Theta} \tau' &\Rightarrow [\beta\ \tau]_{\kappa} &&=_{\Theta} [\beta'\ \tau']_{\kappa} \\
\mathit{cond}(\overline{x}, \overline{\tau} \to \tau, \overline{y}, \overline{\tau'} \to \tau') &\Rightarrow \forall\ \overline{x}.\ \overline{\tau} \to \tau &&=_{\Theta} \forall\ \overline{y}.\ \overline{\tau'} \to \tau'
\end{aligned}$$

Subtype:

$$\begin{aligned}
\tau =_{\Theta} \tau' &\Rightarrow \tau &&\sqsubseteq_{\Theta} \tau' \\
&\mathtt{ptr}^{\mathtt{t}}\ \omega &&\sqsubseteq_{\Theta} \mathtt{ptr}^{\mathtt{u}}\ \omega' \\
0 \leq_{\Theta} b_l \wedge b_h \leq_{\Theta} 1 &\Rightarrow \mathtt{ptr}^{m}\ \tau &&\sqsubseteq_{\Theta} \mathtt{ptr}^{m}\ [(b_l, b_h)\ \tau] \\
b_l \leq_{\Theta} 0 \wedge 1 \leq_{\Theta} b_h &\Rightarrow \mathtt{ptr}^{m}\ [(b_l, b_h)\ \tau] &&\sqsubseteq_{\Theta} \mathtt{ptr}^{m}\ \tau \\
b_l \leq_{\Theta} 0 \wedge 1 \leq_{\Theta} b_h &\Rightarrow \mathtt{ptr}^{m}\ [(b_l, b_h)\ \tau]_{nt} &&\sqsubseteq_{\Theta} \mathtt{ptr}^{m}\ \tau \\
b_l \leq_{\Theta} b_l' \wedge b_h' \leq_{\Theta} b_h &\Rightarrow \mathtt{ptr}^{m}\ [(b_l, b_h)\ \tau]_{nt} &&\sqsubseteq_{\Theta} \mathtt{ptr}^{m}\ [(b_l', b_h')\ \tau] \\
b_l \leq_{\Theta} b_l' \wedge b_h' \leq_{\Theta} b_h &\Rightarrow \mathtt{ptr}^{m}\ [(b_l, b_h)\ \tau]_{\kappa} &&\sqsubseteq_{\Theta} \mathtt{ptr}^{m}\ [(b_l', b_h')\ \tau]_{\kappa} \\
\overline{\tau'} \sqsubseteq_{\Theta} \overline{\tau} \wedge \tau \sqsubseteq_{\Theta} \tau' &\Rightarrow \mathtt{ptr}^{\xi}\ \forall\ \overline{x}.\ \overline{\tau} \to \tau &&\sqsubseteq_{\Theta} \mathtt{ptr}^{\xi}\ \forall\ \overline{x}.\ \overline{\tau'} \to \tau'
\end{aligned}$$

$$n' + n = \mathit{add}(n', n) \qquad (x + n') + n = x + \mathit{add}(n', n)$$

$$\mathit{cond}(\overline{x}, \tau, \overline{y}, \tau') = \exists \overline{z}\ .\ \overline{x} \cup \overline{z} \wedge \overline{y} \cup \overline{z} \wedge \mathit{size}(\overline{x}) = \mathit{size}(\overline{y}) = \mathit{size}(\overline{z}) \wedge \tau[\overline{z}/\overline{x}] = \tau'[\overline{z}/\overline{x}]$$

Figure 11: Type Equality and Subtyping

these types is a subtype of the declared type of e's corresponding parameter ($\overline{\tau}$). Because functions have dependent types bounded by $\overline{x}$, we substitute each parameter e_i for its corresponding parameter x_i in both the parameter types and the return type. Consider the `safe_strcat` function in Fig. 9; its parameter type for `dst` depends on `n`. The T-Fun rule will substitute `n` with the argument at a call-site.

$$\Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{int} \qquad \Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\mathtt{u}}\ \omega$$

$$\Theta;\mathcal{H};\sigma \vdash_m 0 : \mathtt{ptr}^{\xi}\ \omega \qquad \frac{(n{:}\mathtt{ptr}^{\xi}\ \omega) \in \sigma}{\Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\xi}\ \omega}$$

$$\frac{\mathtt{ptr}^{\xi'}\ \omega' \sqsubseteq_{\Theta} \mathtt{ptr}^{\xi}\ \omega \qquad \Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\xi'}\ \omega'}{\Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\xi}\ \omega}$$

$$\frac{\xi \leq m \quad \Xi(m, n) = \tau\ (\overline{x'} : \overline{\tau})\ (\xi, e) \quad \overline{x} = \{x | (x : \mathtt{int}) \in (\overline{x'} : \overline{\tau})\}}{\Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\xi}\ (\forall\ \overline{x}.\ \overline{\tau} \to \tau)}$$

$$\frac{\neg \mathit{fun\_t}(\omega) \qquad \xi \leq m \qquad \forall i \in [0, \mathit{size}(\omega))\ .\ \Theta;\mathcal{H};(\sigma \cup \{(n : \mathtt{ptr}^{\xi}\ \omega))\} \vdash_m \mathcal{H}(m, n + i)}{\Theta;\mathcal{H};\sigma \vdash_m n : \mathtt{ptr}^{\xi}\ \omega}$$

$$\mathit{fun\_t}(\forall\ \overline{x}.\ \overline{\tau} \to \tau) = \mathtt{true} \qquad \mathit{fun\_t}(\omega) = \mathtt{false}\ \ [\textit{owise}]$$

Figure 12: Verification/Type Rules for Constants

3.4. Type Soundness, Non-exposure, Non-crashing

In this subsection, we focus on our main meta-theoretic results about CoreChkCBox: type soundness (progress and preservation), non-exposure, and non-crashing. These proofs have been carried out in our Coq model.

Type soundness relies on several *well-formedness*:

Definition 1 (Type Environment Well-formedness). A type environment Γ is well-formed iff every variable

mentioned as type bounds in Γ are bounded by `nat` typed variables in Γ.

Definition 2 (Heap Well-formedness). For every m, A heap $\mathcal{H}$ is well-formed iff (i) $\mathcal{H}(m, 0)$ is undefined, and (ii) for all $n : \tau$ in the range of $\mathcal{H}(m)$, type τ contains no free variables.

Definition 3 (Stack Well-formedness). A stack snapshot φ is well-formed iff for all $n : \tau$ in the range of φ, type τ contains no free variables.

We also need to introduce a notion of *consistency*, relating heap environments before and after a reduction step, and type environments, predicate sets, and stack snapshots together.

Definition 4 (Stack Consistency). A type environment Γ, variable predicate set Θ, and stack snapshot φ are consistent—written $\Gamma; \Theta \vdash \varphi$—iff for every variable x, $\Theta(x)$ is defined implies $\Gamma(x) = \tau$ for some τ and $\varphi(x) = n : \tau'$ for some n, τ' where $\tau' \sqsubseteq_\Theta \tau$.

Definition 5 (Checked Stack-Heap Consistency). A stack snapshot φ is consistent with heap $\mathcal{H}$—written $\mathcal{H} \vdash \varphi$—iff for every variable x, $\varphi(x) = n : \tau$ with $\mathit{mode}(\tau) = \mathtt{c}$ implies $\emptyset; \mathcal{H}(\mathtt{c}); \emptyset \vdash_\mathtt{c} n : \tau$.

Definition 6 (Checked Heap-Heap Consistency). A heap $\mathcal{H}'$ is consistent with $\mathcal{H}$—written $\mathcal{H} \triangleright \mathcal{H}'$—iff for every constant n, $\emptyset; \mathcal{H}; \emptyset \vdash_\mathtt{c} n : \tau$ implies $\emptyset; \mathcal{H}'; \emptyset \vdash_\mathtt{c} n : \tau$.

Moreover, as a program evaluates, its expression may contain literals $n : \tau$ where τ is a pointer type, i.e., n is an index in $\mathcal{H}$ (perhaps because n was chosen by `malloc`). The normal type-checking judgment for e is implicitly parameterized by $\mathcal{H}$, and the rules for type-checking literals confirm that pointed-to heap cells are compatible with (subtypes of) the pointer's type annotation; in turn this check may precipitate checking the type consistency of the heap itself. We follow the same approach as Ruef et al. [34], and show the rules in Fig. 12; the judgment $\Theta; \mathcal{H}; \sigma \vdash_m n : \tau$ is used to confirm literal well-typing, where σ is a set of pointer literals already checked in $\mathcal{H}$ (to allow pointer cycles). See Appendix B for further discussion.

Progress states that a CHECKEDCBOX program can always make a move:

Theorem 1 (Progress).
For any CHECKEDCBOX program e, heap $\mathcal{H}$, stack φ, type environment Γ, and variable predicate set Θ that are all are well-formed, consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$) and well typed ($\Gamma; \Theta \vdash_\mathtt{c} e : \tau$ for some τ), one of the following holds:

- e is a value ($n : \tau$).
- there exists φ' $\mathcal{H}'$ r, such that $(\varphi, \mathcal{H}, e) \longrightarrow_m (\varphi', \mathcal{H}', r)$.

There are two forms of preservation regarding the checked and unchecked regions. Checked Preservation states that a reduction step preserves both the type and consistency of the program being reduced.

Theorem 2 (Checked Preservation). For any CHECKEDCBOX program e, heap $\mathcal{H}$, stack φ, type environment Γ, and variable predicate set Θ that are all are well-formed, consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$) and well typed ($\Gamma; \Theta \vdash_\mathtt{c} e : \tau$ for some τ), if there exists φ', $\mathcal{H}'$ and e', such that $(\varphi, \mathcal{H}, e) \longrightarrow_\mathtt{c} (\varphi', \mathcal{H}', e')$, then $\mathcal{H}'$ is checked region consistent with $\mathcal{H}$ ($\mathcal{H} \triangleright \mathcal{H}'$) and there exists Γ', Θ' and τ' that are well formed, checked region consistent ($\Gamma'; \Theta' \vdash \varphi'$ and $\mathcal{H}' \vdash \varphi'$) and well typed ($\Gamma'; \Theta' \vdash_\mathtt{c} e : \tau'$), where $\tau' \sqsubseteq \tau$.

Theorem 3 (Unchecked Preservation). For any CHECKEDCBOX program e, heap $\mathcal{H}$, stack φ, type environment Γ, and variable predicate set Θ that are all are well-formed and well typed ($\Gamma; \Theta \vdash_\mathtt{c} e : \tau$ for some τ), if there exists φ', $\mathcal{H}'$ and e', such that $(\varphi, \mathcal{H}, e) \longrightarrow_\mathtt{u} (\varphi', \mathcal{H}', e')$, then $\mathcal{H}'(\mathtt{c}) = \mathcal{H}(\mathtt{c})$.

Using the above theorem, we first show the non-exposure theorem, where code in unchecked region cannot observe know a valid checked pointer address.

Theorem 4 (Non-Exposure). For any CHECKEDCBOX program e, heap $\mathcal{H}$, stack φ, type environment Γ, and variable predicate set Θ that are all are well-formed and well typed ($\Gamma; \Theta \vdash_\mathtt{c} e : \tau$ for some τ), if there exists φ', $\mathcal{H}'$ and e', such that $(\varphi, \mathcal{H}, e) \longrightarrow_\mathtt{u} (\varphi', \mathcal{H}', e')$ and $e = E[\alpha(x)]$ and $\mathit{mode}(E) = \mathtt{u}$, where $\alpha(x)$ is some expression (not `checked` nor `unchecked`) containing variable x, if x is a checked pointer, it is a null one.

Using these two theorems we can prove our main result, *non-crashing*, which states that a well-typed program can never be *stuck* (expression e is a non-value that cannot take a step[2]).

Theorem 5 (Non-Crashing). For any CHECKEDCBOX program e, heap $\mathcal{H}$, stack φ, type environment Γ, and variable predicate set Θ that are well-formed and consistent ($\Gamma; \Theta \vdash \varphi$ and $\mathcal{H} \vdash \varphi$), if e is well-typed ($\varphi; \Theta \vdash_\mathtt{c} e : \tau$ for some τ) and there exists φ_i, $\mathcal{H}_i$, e_i, and m_i for $i \in [1, k]$, such that $(\varphi, \mathcal{H}, e) \longrightarrow_{m_1} (\varphi_1, \mathcal{H}_1, e_1) \longrightarrow_{m_2} \ldots \longrightarrow_{m_k} (\varphi_k, \mathcal{H}_k, r)$, then r can never be *stuck*.

4. Compilation

The main subtlety of compiling Checked C to Clang/LLVM is to capture the annotations on pointer literals that track array bound information, which is used in premises of rules like S-DEFARRAY and S-ASSIGNARR to prevent spatial safety violations. The Checked C compiler [20] inserted additional pointer checks for verifying pointers being not null and the bounds are within their limits. The latter is done by introducing additional shadow variables for storing (NT-)array pointer bound information.

2. Note that `bounds` and `null` are *not* stuck expressions—they represent a program terminated by a failed run-time check. A program that tries to access $\mathcal{H}n$ but $\mathcal{H}$ is undefined at n will be stuck, and violates spatial safety.

	c	t	u
	Impl / Foml	Impl / Foml	Impl / Foml
c	$*x$ / $*(\mathtt{c}, x)$	$\mathtt{sand\_get}(x)$ / $*(\mathtt{u}, x)$	$\times$
u	$\times$	$*x$ / $*(\mathtt{u}, x)$	$*x$ / $*(\mathtt{u}, x)$

Figure 13: Compiled Targets for Dereference

In CHECKEDCBOX, context and pointer modes determine the particular heap/function store that a pointer points to, i.e., cpointer points to checked regions, while tand upointers point to unchecked regions. Unchecked regions are associated with a sandbox mechanism that permits exception handling of potential memory failures. In the compiled LLVM code, pointer access operations have different syntax when the modes are different. Figure 13 lists the different compiled syntax for a deference operation ($*x$) for the compiler implementation (Impl) and formalism (Foml). The columns represent different pointer modes and the rows represent context modes. For example, when we have a tpointer in a c-mode region, we use the sandbox pointer access function ($\mathtt{sand\_get}(x)$) accessing the data. In formalism, we create a new deference data-structure in LLVM on top of the existing $*x$ operation: $*(m, x)$. if the mode is c, it accesses the checked heap/function store; otherwise, it accesses the unchecked one.

In the compilation, we use a function mode to get the pointer mode of a function learning from the pointer's type. Thus, the compiler can determine the correct compiled syntax for a pointer access operation with the knowledge of the current context mode. We also build a context switching mechanism to change the context mode when we meet checked and unchecked blocks. We describe the mechanism shortly.

This section shows how CHECKEDCBOX deals with pointer modes, mode switching and function pointer compilations, with no loss of expressiveness; as the Checked C contains the erase of annotations in [20] and Appendix G. For the compiler formalism, we present a compilation algorithm that converts from CHECKEDCBOX to COREC, an untyped language without metadata annotations, which represents an intermediate layer we build on LLVM for simplifying compilation. In COREC, the syntax for deference, assignment, malloc, function calls are: $*(m, e)$, $*(m, e) = e$, $\mathtt{malloc}(m, \omega)$, and $(m, e)(\bar{e})$. The algorithm sheds light on how compilation can be implemented in the real Checked C compiler, while eschewing many important details (COREC has many differences with LLVM IR).

Compilation is defined by extending CORECHKCBOX's typing judgment thusly:

$$\Gamma; \Theta; \rho \vdash_m e \gg \dot{e} : \tau$$

There is now a COREC output $\dot{e}$ and an input ρ, which maps each (NT-)array pointer variable to its mode and each variable p to a pair of *shadow variables* that keep p's up-to-date upper and lower bounds; these may differ from the bounds in p's type due to bounds widening.[3]

```
int deref_array(n : int,
     p : ptr^c [(0,n) int]_nt,
     q : ptr^t [(0,n) int]_nt) {
  /* ρ(p) = p_lo,p_hi,p_m */
  /* ρ(q) = q_lo,q_hi,q_m */
    * p;
    * q = 1;
}
...
/* p0 : ptr^c [(0,5) int]_nt */
/* q0 : ptr^t [(0,5) int]_nt */
deref_array(5, p0, q0);
```

→

```
deref_array(int n, int* p, int * q) {
  //m is the current context mode
  let p_lo = 0; let p_hi = n;
  let q_lo = 0; let q_hi = n;
  /* runtime checks */
  assert(p_lo ≤0 && 0 ≤p_hi);
  assert(p != 0);
  *(mode(p) ∧ m,p);
  verify(q, not_null && q_lo ≤0 && 0 ≤q_hi);
  *(mode(q) ∧ m,q)=1;
}
...
deref_array(5, p0, q0);
```

Figure 14: Compilation Example for Dependent Functions

We formalize rules for this judgment in PLT Redex [12], following and extending our Coq development for CORECHKCBOX. To give confidence that compilation is correct, we use Redex's property-based random testing support to show that compiled-to $\dot{e}$ simulates e, for all e.

4.1. Approach

Due to space constraints, we explain the rules for compilation by example, using a C-like syntax; the complete rules are given in Appendix G. Each rule performs up to three tasks: (a) conversion of e to A-normal form; (b) insertion of dynamic checks and bound widening expressions; and (c) generate right pointer accessing expressions based on modes. A-normal form conversion is straightforward: compound expressions are handled by storing results of subexpressions into temporary variables, as in the following example.

```
let y=(x+1)+(6+1) .→  let a=x+1;
                      let b=6+1;
                      let y=a+b
```

This simplifies the management of effects from subexpressions. The next two steps of compilation are more interesting. We state them based on different CHECKEDCBOX operations.

3. Since lower bounds are never widened, the lower-bound shadow variable is unnecessary; we include it for uniformity.

Pointer Accesses and Modes. In every declaration (or the beginning of a function body) of a pointer, if the poniter is an (NT-)array one, we first allocates two *shadow variables* to track the lower and upper bounds potentially changed for pointer arithmetic and NT-array bound widening. We also associated with every `c`-mode pointer variable according to its type; we place bounds and null-pointer checks, such as the line 6 and 7 in Figure 14. In addition, in the formalism, before every use of a tainted pointer (Figure 14 line 9), there is an inserted verification step similar to Figure 12, which checks a pointer is well defined in the heap for every element in its defined range. In implementation, we actually optimize the verification away and substitute it with the bounds and null-pointer checks. Because tainted pointer is checked every time before it is used, so that we only need to check the top pointer is well defined without recursively looking at the sub-terms in a nested pointer case. The modes in compiled deference (`*(mode(p)∧m,p)`) and assignment (`*(mode(q)∧m,q)=1`) operations are computed based on the meet operation ($\wedge$) of the pointer mode (e.g. `mode(p)`) and the current context mode (`m`).

Checked and Unchecked Blocks. In the implementation, `unchecked` and `checked` blocks are compiled as context switching functions provided by the sandbox mechanism. `unchecked`$(\overline{x})\{e\}$ is compiled to `sandbox_call`$(\overline{x}, e)$, where we call the sandbox to execute expression e with the arguments $\overline{x}$. `checked`$(\overline{x})\{e\}$ is compiled to `callback`$(\overline{x}, e)$, where we perform a `callback` to a checked block code e inside a sandbox. In CHECKEDCBOX, We adopt an aggressive execution scheme that directly learns pointer addresses from compiled assembly to make the `callback` happen. In the formalism, we rely on the type system to guarantee the context switching without creating the extra function calls for simplicity.

Function Pointers and Calls. Function pointers are dealt with similarly to normal pointers, but we insert checks to check if the pointer address is not null in the function store and if the type is correctly represented, for both `c` and `t` mode pointers [4]. Fpr example, in compiling the `stringsort` function in Figure 2, we place a check `verify_fun(cmp, not_null && type_match)`, and we place a similar check before Figure 4 line 7 to check the tainted `cmp` when it is used. The compilation of function calls (compiling to $(m, e)(\overline{e})$) is similar to the manipulation of pointer access operations in Figure 13.

For compiling dependent function calls, Figure 14 also shows an example. Notice that the bounds for the array pointer `p` are not passed as arguments. Instead, they are initialized according to `p`'s type—see line 4 of the original CORECHKCBOX program at the top of the figure. Line 3 of the generated code sets the lower bound to `0` and the upper bound to `n`.

4. `c`-mode pointers are checked once in the beginning and `t`-mode pointers are checked every time when use

4.2. Metatheory

We formalize both the compilation procedure and the simulation theorem in the PLT Redex model we developed for CORECHKCBOX (see Sec. 3.1), and then attempt to falsify it via Redex's support for random testing. Redex allows us to specify compilation as logical rules (essentially, an extension of typing), but then execute it algorithmically to automatically test whether simulation holds. This process revealed several bugs in compilation and the theorem statement. We ultimately plan to prove simulation in the Coq model.

We use the notation $\gg$ to indicate the *erasure* of stack and heap—the rhs is the same as the lhs but with type annotations removed:

$$\mathcal{H} \gg \dot{\mathcal{H}}$$
$$\varphi \gg \dot{\varphi}$$

In addition, when $\Gamma; \emptyset \vdash \varphi$ and φ is well-formed, we write $(\varphi, \mathcal{H}, e) \gg_m (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$ to denote $\varphi \gg \dot{\varphi}$, $\mathcal{H} \gg \dot{\mathcal{H}}$ and $\Gamma; \Theta; \emptyset \vdash_m e \gg \dot{e} : \tau$ for some τ respectively. Γ is omitted from the notation since the well-formedness of φ and its consistency with respect to Γ imply that e must be closed under φ, allowing us to recover Γ from φ. Finally, we use $\dot{\rightarrow}^*$ to denote the transitive closure of the reduction relation of COREC. Unlike the CORECHKCBOX, the semantics of COREC does not distinguish checked and unchecked regions.

Fig. 15 gives an overview of the simulation theorem.[5] The simulation theorem is specified in a way that is similar to the one by Merigoux et al. [27].

An ordinary simulation property would replace the middle and bottom parts of the figure with the following:

$$(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1)$$

Instead, we relate two erased configurations using the relation $\sim$, which only requires that the two configurations will eventually reduce to the same state.

Theorem 6 (Simulation ($\sim$)). For CORECHKCBOX expressions e_0, stacks φ_0, φ_1, and heap snapshots $\mathcal{H}_0$, $\mathcal{H}_1$, if $\mathcal{H}_0 \vdash \varphi_0$, $(\varphi_0, \mathcal{H}_0, e_0) \gg_c (\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0)$, and if there exists some r_1 such that $(\varphi_0, \mathcal{H}_0, e_0) \rightarrow_c (\varphi_1, \mathcal{H}_1, r_1)$, then the following facts hold:

- if there exists e_1 such that $r = e_1$ and $(\varphi_1, \mathcal{H}_1, e_1) \gg (\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1)$, then there exists some $\dot{\varphi}$,$\dot{\mathcal{H}}$, $\dot{e}$, such that $(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$ and $(\dot{\varphi}_1, \dot{\mathcal{H}}_1, \dot{e}_1) \dot{\rightarrow}^* (\dot{\varphi}, \dot{\mathcal{H}}, \dot{e})$.
- if $r_1 =$ `bounds` or `null`, then we have $(\dot{\varphi}_0, \dot{\mathcal{H}}_0, \dot{e}_0) \dot{\rightarrow}^* (\dot{\varphi}_1, \dot{\mathcal{H}}_1, r_1)$ where $\varphi_1 \gg \dot{\varphi}_1$, $\mathcal{H}_1 \gg \dot{\mathcal{H}}_1$.

Our random generator (discussed in the next section) never produces `unchecked` expressions (whose behavior could be undefined), so we can only test a the simulation theorem as it applies to checked code. This limitation makes

5. We ellide the possibility of $\dot{e}_1$ evaluating to `bounds` or `null` in the diagram for readability.

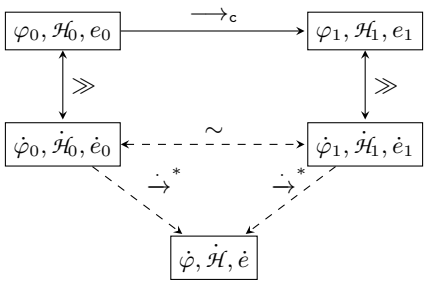

Figure 15: Simulation between CORECHKCBOX and COREC

it unnecessary to state the other direction of the simulation theorem where e_0 is stuck, because Theorem 1 guarantees that e_0 will never enter a stuck state if it is well-typed in checked mode.

The current version of the Redex model has been tested against 20000 expressions with depth less than 10. Each expression can reduce multiple steps, and we test simulation between every two adjacent steps to cover a wider range of programs, particularly the ones that have a non-empty heap.

5. Evaluation

- provide evidence that the CHECKEDCBOX compiler is efficient. Compare the compiler with respect to other work, like RLBox, also the previous Checked C compiler.
- provide user experience of CHECKEDCBOX. We restrict the use of checked pointers compared to previous checked-c compiler. Is the restriction arrangable. We can say that the tainted shells are auto-matically generated, so we have a mechanism for auto-generating tainted pointers if necessary.
- if we have space, we can re-introduce random testing a little, saying that how it helps us to develop the compiler.
- we can then talk about the possible bugs we find in the Checked C compiler for function pointer or the RLBox bugs.

6. Related Work

Our work is most closely related to prior formalizations of C(-like) languages that aim to enforce memory safety, but it also touches on C-language formalization in general.

Formalizing C and Low-level code. A number of prior works have looked at formalizing the semantics of C, including CompCert [2, 19], Ellison and Rosu [10], Kang et al. [16], and Memarian et al. [24, 25]. These works also model pointers as logically coupled with either the bounds of the blocks they point to, or provenance information from which bounds can be derived. None of these is directly concerned with enforcing spatial safety, and that is reflected in the design. For example, memory itself is not be represented as a flat address space, as in our model or real machines, so memory corruption due to spatial safety violations, which Checked C's type system aims to prevent, may not be expressible. That said, these formalizations consider much more of the C language than does CORECHKCBOX, since they are interested in the entire language's behavior.

Spatially Safe C Formalizations. Several prior works formalize C-language transformations or C-language dialects aiming to ensure spatial safety. Hathhorn et al. [14] extends the formalization of Ellison and Rosu [10] to produce a semantics that detects violations of spatial safety (and other forms of undefinedness). It uses a CompCert-style memory model, but "fattens" logical pointer representations to facilitate adding side conditions similar to CORECHKCBOX's. Its concern is bug finding, not compiling programs to use this semantics.

CCured [32] and Softbound [30] implement spatially safe semantics for normal C via program transformation. Like CORECHKCBOX, both systems' operational semantics annotate pointers with their bounds. CCured's equivalent of array pointers are compiled to be "fat," while SoftBound compiles bounds metadata to a separate hashtable, thus retaining binary compatibility at higher checking cost. Checked C uses static type information to enable bounds checks without need of pointer-attached metadata, as we show in Section 4. Neither CCured nor Softbound models null-terminated array pointers, whereas our semantics ensures that such pointers respect the zero-termination invariant, leveraging bounds widening to enhance expressiveness.

Cyclone [13, 15] is a C dialect that aims to ensure memory safety; its pointer types are similar to CCured. Cyclone's formalization [13] focuses on the use of *regions* to ensure temporal safety; it does not formalize arrays or threats to spatial safety. Deputy [6, 45] is another safe-C dialect that aims to avoid fat pointers; it was an initial inspiration for Checked C's design [9], though it provides no specific modeling for null-terminated array pointers. Deputy's formalization [6] defines its semantics directly in terms of compilation, similar in style to what we present in Section 4. Doing so tightly couples typing, compilation, and semantics, which are treated independently in CORECHKCBOX. Separating semantics from compilation isolates meaning from mechanism, easing understandability. Indeed, it was this separation that led us to notice the limitation with Checked C's handling of bounds widening.

The most closely related work is the formalization of Checked C done by Ruef et al. [34]. They present the type system and semantics of a core model of Checked C, mechanized in Coq, and were the first to prove a blame theorem. CORECHKCBOX's Coq-based development (Section 3) substantially extends theirs to include conditionals, dynamically bounded array pointers with dependent types, null-terminated array pointers, dependently typed functions, and subtyping. They postulate that pointer metadata can be erased in a real implementation, but do not show it. Our CORECHKCBOX compiler, formalized and validated in PLT Redex via randomized testing, demonstrates that such metadata *can* be erased; we found that erasure was non-

obvious once null-terminated pointers and bounds widening were considered.

7. Conclusion and Future Work

This paper presented CORECHKCBOX, a formalization of an extended core of the Checked C language which aims to provide spatial memory safety. CORECHKCBOX models dynamically sized and null-terminated arrays with dependently typed bounds that can additionally be widened at runtime. We prove, in Coq, the key safety property of Checked C for our formalization, *blame*: if a mix of checked and unchecked code gives rise to a spatial memory safety violation, then this violation originated in an unchecked part of the code. We also show how programs written in CORECHKCBOX (whose semantics leverage fat pointers) can be compiled to COREC (which does not) while preserving their behavior. We developed a version of CORECHKCBOX written in PLT Redex, and used a custom term generator in conjunction with Redex's randomized testing framework to give confidence that compilation is correct. We also used this framework to cross-check CORECHKCBOX against the Checked C compiler, finding multiple inconsistencies in the process.

As future work, we wish to extend CORECHKCBOX to model more of Checked C, with our Redex-based testing framework guiding the process. The most interesting Checked C feature not yet modeled is *interop types* (itypes), which are used to simplify interactions with unchecked code via function calls. A function whose parameters are itypes can be passed checked or unchecked pointers depending on whether the caller is in a checked region. This feature allows for a more modular C-to-Checked C porting process, but complicates reasoning about blame. A more ambitious next step would be to extend an existing formally verified framework for C, such as CompCert [18] or VeLLVM [44], with Checked C features, towards producing a verified-correct Checked C compiler. We believe that CORECHKCBOX's Coq and Redex models lay the foundation for such a step, but substantial engineering work remains.

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Appendix

1. Differences with the Coq and Redex Models

The Coq and Redex models of CORECHKCBOX may be found at https://github.com/plum-umd/checkedc. The Coq model's syntax is slightly different from that in Fig. 5. In particular, the arguments in a function call are restricted to variables and constants, according to a separate well-formedness condition. A function call `f(e)` can always be written in `let x = e in f(x)` to cope. In addition, conditionals have two syntactic forms: `EIf` is a normal conditional, and `EIfDef` is one whose boolean guard is of the form $*x$. By syntactically distinguishing these two cases, the Coq model does not need the *[prefer]* rule for `if (*x)`... forms as in Fig. 7. The Redex model *does* prioritize such forms but not the same way as in the figure. It uses a variation of the S-VAR rule: The modified rule is equipped with a precondition that is false whenever S-IFNTT is applicable.

The Coq model uses a runtime stack φ as described at the start of Sec. 3.2. The Redex model introduces let bindings during evaluation to simulate a runtime stack. For example, consider the expression $e \equiv$ `let` $x =$ `(5 : int) in` $x +$ `` `x ``. Expression e first steps to `let` $x =$ `(5 : int) in (5:int) +` x, which in turns steps to `let` $x =$ `(5 : int) in (5:int) + (5:int)`. Since the rhs of x is a value, the let binding in e effectively functions as a stack that maps from x to `5 : int`. The let form remains in the expression and lazily replaces the variables in its body. The let form can be removed from the expression only if its body is evaluated to a value, e.g., `let` $x =$ `(5 : int) in (10:int)` steps to `10 : int`. The rule for popping let bindings in this manner corresponds to the S-RET rule in Fig. 8. Leveraging let bindings adds complexity to the semantics but simplifies typing/consistency and term generation during randomized testing.

2. Typing Rules for Literal Pointers

The typing of integer literals, which can also be pointers to the heap, was presented in Sec. 3.4 in Fig. 12. Here we describe these rules further.

The variable type rule (T-VAR) simply checks if a given variable has the defined type in Γ; the constant rule (T-CONST) is slightly more involved. First, it ensures that the type annotation τ does not contain any free variables. More importantly, it ensures that the literal itself is well typed using an auxilliary typing relation $\mathcal{H}; \sigma \vdash n : \tau$.

If the literal's type is an integer, an unchecked pointer, or a null pointer, it is well typed, as shown by the top three rules in Fig. 12. However, if it is a checked pointer `ptr`$^{\mathtt{c}}$ ω, we need to ensure that what it points to in the heap is of the appropriate pointed-to type (ω), and also recursively ensure that any literal pointers reachable this way are also well-typed. This is captured by the bottom rule in the figure, which states that for every location $n + i$ in the pointers' range $[n, n + \mathit{size}(\omega))$, where *size* yields the size of its argument, then the value at the location $\mathcal{H}(n + i)$ is also well-typed. However, as heap snapshots can contain cyclic structures (which would lead to infinite typing deriviations), we use a scope σ to assume that the original pointer is well-typed when checking the types of what it points to. The middle rule then accesses the scope to tie the knot and keep the derivation finite, just like in Ruef et al. [34].

3. Other Semantic Rules

Fig. 16 shows the remaining semantic rules for CORECHKCBOX. We explain a selected few rules in this subsection.

Rule S-VAR loads the value for x in stack φ. Rule S-DEFARRAY dereferences an array pointer, which is similar to the Rule S-DEFNTARRAY in Fig. 8 (dealing with null-terminated array pointers). The only difference is that the range of 0 is at $[n_l, n_h)$ not $[n_l, n_h]$, meaning that one cannot dereference the upper-bound position in an array. Rules DEFARRAYBOUND and DEFNTARRAYBOUND describe an error case for a dereference operation. If we are dereferencing an array/NT-array pointer and the mode is `c`, 0 must be in the range from n_l to n_h (meaning that the dereference is in-bound); if not, the system results in a `bounds` error. Obviously, the dereference of an array/NT-array pointer also experiences a `null` state transition if $n \leq 0$.

Rules S-MALLOC and S-MALLOCBOUND describe the `malloc` semantics. Given a valid type ω_a that contains no free variables, `alloc` function returns an address pointing at the first position of an allocated space whose size is equal to the size of ω_a, and a new heap snapshot $\mathcal{H}'$ that marks the allocated space for the new allocation. The `malloc` is transitioned to the address n with the type `ptr`$^{\mathtt{c}}$ ω_a and new updated heap. It is possible for `malloc` to transition to a `bounds` error if the ω_a is an array/NT-array type $[(n_l, n_h)\ \tau]_\kappa$, and either $n_l \neq 0$ or $n_h \leq 0$. This can happen when the bound variable is evaluated to a bound constant that is not desired.

4. Subtyping for dependent types

The subtyping relation given in Fig. 11 involves dependent bounds, i.e., bounds that may refer to variables. To decide premises $b \leq b'$, we need a decision procedure that accounts for the possible values of these variables. This process considers Θ, tracked by the typing judgment, and φ, the current stack snapshot (when performing subtyping as part of the type preservation proof).

Definition 7 (Inequality).

- $n \leq m$ if n is less than or equal to m.
- $x + n \leq x + m$ if n is less than or equal to m.
- All other cases result in `false`.

To capture bound variables in dependent types, the Checked C subtyping relation ($\sqsubseteq$) is parameterized by a restricted stack snapshot $\varphi|_\rho$ and the predicate map Θ, where φ is a stack and ρ is a set of variables. $\varphi|_\rho$ means to restrict the domain of φ to the variable set ρ. Clearly, we have the

S-VAR
$$(\varphi, \mathcal{H}, x) \longrightarrow (\varphi, \mathcal{H}, \varphi(x))$$

S-DEFARRAY
$$\frac{\mathcal{H}(n) = n_a : \tau_a \qquad 0 \in [n_l, n_h)}{(\varphi, \mathcal{H}, *n : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau]_{nt}) \longrightarrow (\varphi, \mathcal{H}, n_a : \tau)}$$

S-DEFARRAYBOUND
$$\frac{0 \notin [n_l, n_h)}{(\varphi, \mathcal{H}, *n : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau]_{\kappa}) \longrightarrow (\varphi, \mathcal{H}, \texttt{bounds})}$$

S-DEFNTARRAYBOUND
$$\frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, *n : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau]_{nt}) \longrightarrow (\varphi, \mathcal{H}, \texttt{bounds})}$$

S-ASSIGN
$$\frac{\mathcal{H}(n) = n_a : \tau_a}{(\varphi, \mathcal{H}, *n : \texttt{ptr}^{\texttt{c}}\ \tau = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}[n \mapsto n_1 : \tau], n_1 : \tau)}$$

S-ASSIGNNULL
$$(\varphi, \mathcal{H}, *0 : \texttt{ptr}^{\texttt{c}}\ \omega = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}, \texttt{null})$$

S-ASSIGNARRBOUND
$$\frac{0 \notin [n_l, n_h)}{(\varphi, \mathcal{H}, *n : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau]_{\kappa} = n_1 : \tau_1) \longrightarrow (\varphi, \mathcal{H}, \texttt{bounds})}$$

S-MALLOC
$$\frac{\varphi(\omega) = \omega_a \qquad \texttt{alloc}(\mathcal{H}, \omega_a) = (n, \mathcal{H}')}{(\varphi, \mathcal{H}, \texttt{malloc}(\omega,)) \longrightarrow (\varphi, \mathcal{H}', n : \texttt{ptr}^{\texttt{c}}\ \omega_a)}$$

S-MALLOCBOUND
$$\frac{\varphi(\omega) = [(n_l, n_h)\ \tau]_{\kappa} \qquad (n_l \neq 0 \vee n_h \leq 0)}{(\varphi, \mathcal{H}, \texttt{malloc}(\omega,)) \longrightarrow (\varphi, \mathcal{H}', \texttt{bounds})}$$

S-IFT
$$\frac{n \neq 0}{(\varphi, \mathcal{H}, \texttt{if}\ (n : \tau)\ e_1\ \texttt{else}\ e_2) \longrightarrow (\varphi, \mathcal{H}, e_1)}$$

S-IFF
$$(\varphi, \mathcal{H}, \texttt{if}\ (0 : \tau)\ e_1\ \texttt{else}\ e_2) \longrightarrow (\varphi, \mathcal{H}, e_2)$$

S-UNCHECKED
$$(\varphi, \mathcal{H}, \texttt{unchecked}(n : \tau)\{\longrightarrow\}(\varphi, \mathcal{H}, n : \tau)$$

S-STR
$$\frac{0 \in [n_l, n_h] \qquad n_a \leq n_h \qquad \mathcal{H}(n + n_a) = 0 \qquad (\forall i. n \leq i < n + n_a \Rightarrow (\exists n_i\ t_i. \mathcal{H}(n + i) = n_i : \tau_i \wedge n_i \neq 0))}{(\varphi, \mathcal{H}, \texttt{strlen}(n : \texttt{ptr}^{m}\ [(n_l, n_h)\ \tau])) \longrightarrow (\varphi, \mathcal{H}, n_a : \texttt{int})}$$

S-STRBOUNDS
$$\frac{0 \notin [n_l, n_h]}{(\varphi, \mathcal{H}, \texttt{strlen}(n : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau])) \longrightarrow (\varphi, \mathcal{H}, \texttt{bounds})}$$

S-STRNULL
$$(\varphi, \mathcal{H}, \texttt{strlen}(0 : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau])) \longrightarrow (\varphi, \mathcal{H}, \texttt{null})$$

S-ADD
$$\frac{n = n_1 + n_2}{(\varphi, \mathcal{H}, n_1 : \texttt{int} + n_2 : \texttt{int}) \longrightarrow (\varphi, \mathcal{H}, n)}$$

S-ADDARR
$$\frac{n = n_1 + n_2 \qquad n_l' = n_l - n_2 \qquad n_h' = n_h - n_2}{(\varphi, \mathcal{H}, n_1 : \texttt{ptr}^{m}\ [(n_l, n_h)\ \tau]_{\kappa} + n_2 : \texttt{int}) \longrightarrow (\varphi, \mathcal{H}, n : \texttt{ptr}^{m}\ [(n_l', n_h')\ \tau]_{\kappa})}$$

S-ADDARRNULL
$$n(\varphi, \mathcal{H}, 0 : \texttt{ptr}^{\texttt{c}}\ [(n_l, n_h)\ \tau]_{\kappa} + n_2 : \texttt{int}) \longrightarrow (\varphi, \mathcal{H}, \texttt{null})$$

Figure 16: Remaining CORECHKCBOX Semantics Rules (extends Fig. 8)

relation: $\varphi|_\rho \subseteq \varphi$. $\sqsubseteq$ being parameterized by $\varphi|_\rho$ refers to that when we compare two bounds $b \leq b'$, we actually do $\varphi|_\rho(b) \leq \varphi|_\rho(b')$ by interpreting the variables in b and b' with possible values in $\varphi|_\rho$. Let's define a subset relation $\preceq$ for two restricted stack snapshot $\varphi|_\rho$ and $\varphi'|_\rho$:

Definition 8 (Subset of Stack Snapshots). Given two $\varphi|_\rho$ and $\varphi'|_\rho$, $\varphi|_\rho \preceq \varphi'|_\rho$, iff for $x \in \rho$ and y, $(x, y) \in \varphi|_\rho \Rightarrow (x, y) \in \varphi'|_\rho$.

For every two restricted stack snapshots $\varphi|_\rho$ and $\varphi'|_\rho$, such that $\varphi|_\rho \preceq \varphi'|_\rho$, we have the following theorem in Checked C (proved in Coq):

Theorem 7 (Stack Snapshot Theorem). Given two types τ and τ', two restricted stack snapshots $\varphi|_\rho$ and $\varphi'|_\rho$, if $\varphi|_\rho \preceq \varphi'|_\rho$, and $\tau \sqsubseteq \tau'$ under the parameterization of $\varphi|_\rho$, then $\tau \sqsubseteq \tau'$ under the parameterization of $\varphi'|_\rho$.

Clearly, for every $\varphi|_\rho$, we have $\emptyset \preceq \varphi|_\rho$. The type checking stage is a compile-time process, so $\varphi|_\rho$ is $\emptyset$ at the type checking stage. Stack snapshots are needed for proving type preserving, as variables in bounds expressions are evaluated away.

As mentioned in the main text, $\sqsubseteq$ is also parameterized by Θ, which provides the range of allowed values for a bound variable; thus, more $\sqsubseteq$ relation is provable. For example, in Fig. 9, the `strlen` operation in line 4 turns the type of `dst` to be $\texttt{ptr}^{\texttt{c}}\ [(0, x)\ \texttt{int}]_{nt}$ and extends the upper bound to `x`. In the `strlen` type rule, it also inserts a predicate $\texttt{x} \geq 0$ in Θ; thus, the cast operation in line 16 is valid because $\texttt{ptr}^{\texttt{c}}\ [(0, x)\ \texttt{int}]_{nt} \sqsubseteq \texttt{ptr}^{\texttt{c}}\ [(0, 0)\ \texttt{int}]_{nt}$ is provable when we know $\texttt{x} \geq 0$.

Note that if φ and Θ are $\emptyset$, we do only the syntactic $\leq$ comparison; otherwise, we apply φ to both sides of $\sqsubseteq$, and then determine the $\leq$ comparasion based on a Boolean predicate decision procedure on top of Θ. This process allows us to type check both an input expression and the intermediate expression after evaluating an expression.

T-DEF
$$\frac{\Gamma;\Theta \vdash_m e : \texttt{ptr}^{m'}\ \tau \qquad m \le m'}{\Gamma;\Theta \vdash_m *e : \tau}$$

T-MAC
$$\Gamma;\Theta \vdash_m \texttt{malloc}(\omega,:)\texttt{ptr}^{\texttt{c}}\ \omega$$

T-ADD
$$\frac{\Gamma;\Theta \vdash_m e_1 : \texttt{int} \qquad \Gamma;\Theta \vdash_m e_2 : \texttt{int}}{\Gamma;\Theta \vdash_m (e_1 + e_2) : \texttt{int}}$$

T-IND
$$\frac{\Gamma;\Theta \vdash_m e_1 : \texttt{ptr}^{m'}\ [\beta\ \tau]_\kappa \qquad \Gamma;\Theta \vdash_m e_2 : \texttt{int} \qquad m \le m'}{\Gamma;\Theta \vdash_m *(e_1 + e_2) : \tau}$$

T-ASSIGN
$$\frac{\Gamma;\Theta \vdash_m e_1 : \texttt{ptr}^{m'}\ \tau \quad \Gamma;\Theta \vdash_m e_2 : \tau' \qquad \tau' \sqsubseteq \tau \qquad m \le m'}{\Gamma;\Theta \vdash_m *e_1 = e_2 : \tau}$$

T-INDASSIGN
$$\frac{\Gamma;\Theta \vdash_m e_1 : \texttt{ptr}^{m'}\ [\beta\ \tau]_\kappa \quad \Gamma;\Theta \vdash_m e_2 : \texttt{int} \qquad \Gamma;\Theta \vdash_m e_3 : \tau' \qquad \tau' \sqsubseteq \tau \qquad m \le m'}{\Gamma;\sigma \vdash_m *(e_1 + e_2) = e_3 : \tau}$$

Figure 17: Remaining CORECHKCBOX Type Rules (extends Fig. 10)

5. Other Type Rules

Here we show the type rules for other Checked C operations in Fig. 17. Rule T-DEF is for dereferencing a non-array pointer. The statement $m \le m'$ ensures that no unchecked pointers are used in checked regions. Rule T-MAC deals with `malloc` operations. There is a well-formedness check to require that the possible bound variables in ω must be in the domain of Γ (see Fig. 19). This is similar to the well-formedness assumption of the type environment (Definition 1) Rule T-ADD deals with binary operations whose subterms are integer expressions, while rule T-IND serves the case for pointer arithmetic. For simplicity, in the Checked C formalization, we do not allow arbitrary pointer arithmetic. The only pointer arithmetic operations allowed are the forms shown in rules T-IND and T-INDASSIGN in Fig. 17. Rule T-ASSIGN assigns a value to a non-array pointer location. The predicate $\tau' \sqsubseteq \tau$ requires that the value being assigned is a subtype of the pointer type. The T-INDASSIGN rule is an extended assignment operation for handling assignments for array/NT-array pointers with pointer arithmetic. Rule T-UNCHECKED type checks `unchecked` blocks.

6. Struct Pointers

Checked C has `struct` types and `struct` pointers. Fig. 18 contains the syntax of `struct` types as well as new subtyping relations built on the `struct` values. For a `struct` typed value, Checked C has a special operation

Struct Syntax:

Type	$\texttt{struct}\ T$
Structdefs	$D \in T \rightharpoonup fs$
Fields	$fs ::= \tau\ \texttt{f} \mid \tau\ \texttt{f}; fs$

Struct Subtype:

$$D(T) = fs \land fs(0) = \texttt{nat} \Rightarrow \texttt{ptr}^m\ \texttt{struct}\ T \sqsubseteq \texttt{ptr}^m\ \texttt{nat}$$
$$D(T) = fs \land fs(0) = \texttt{nat} \land 0 \le b_l \land b_h \le 1 \Rightarrow \texttt{ptr}^m\ \texttt{struct}\ T \sqsubseteq \texttt{ptr}^m\ [(b_l, b_h)\ \texttt{nat}]$$

Struct Type Rule:

T-STRUCT
$$\frac{\Gamma;\Theta \vdash_m e : \texttt{ptr}^m\ \texttt{struct}\ T \qquad D(T) = fs \qquad fs(f) = \tau_f}{\Gamma;\Theta \vdash_m \&e{\rightarrow}f : \texttt{ptr}^m\ \tau_f}$$

Struct Semantics:

S-STRUCTCHECKED
$$\frac{n > 0 \qquad D(T) = fs \qquad fs(f) = \tau_a \qquad n_a = \texttt{index}(fs, f)}{(\varphi, \mathcal{H}, \&n{:}\texttt{ptr}^{\texttt{c}}\ \texttt{struct}\ T{\rightarrow}f) \longrightarrow (\varphi, \mathcal{H}, n_a{:}\texttt{ptr}^{\texttt{c}}\ \tau_a)}$$

S-STRUCTNULL
$$\frac{n = 0}{(\varphi, \mathcal{H}, \&n{:}\texttt{ptr}^{\texttt{c}}\ \texttt{struct}\ T{\rightarrow}f) \longrightarrow (\varphi, \mathcal{H}, \texttt{null})}$$

S-STRUCTUNCHECKED
$$\frac{D(T) = fs \qquad fs(f) = \tau_a \qquad n_a = \texttt{index}(fs, f)}{(\varphi, \mathcal{H}, \&n{:}\texttt{ptr}^{\texttt{u}}\ \texttt{struct}\ T{\rightarrow}f) \longrightarrow (\varphi, \mathcal{H}, n_a{:}\texttt{ptr}^{\texttt{u}}\ \tau_a)}$$

Figure 18: CORECHKCBOX Struct Definitions

$$\Gamma \vdash n \qquad \frac{x : \texttt{int} \in \Gamma}{\Gamma \vdash x + n} \qquad \frac{\Gamma \vdash b_l \qquad \Gamma \vdash b_h}{\Gamma \vdash (b_l, b_h)} \qquad \Gamma \vdash \texttt{int}$$

$$\frac{\Gamma \vdash \beta \qquad \Gamma \vdash \tau}{\Gamma \vdash \texttt{ptr}^m\ [\beta\ \tau]_\kappa} \qquad \frac{\Gamma \vdash \tau}{\Gamma \vdash \texttt{ptr}^m\ \tau} \qquad \frac{T \in D}{\Gamma \vdash \texttt{ptr}^m\ \texttt{struct}\ T}$$

Figure 19: Well-formedness for Types and Bounds

for it, which is $\&e{\rightarrow}f$. This operation indexes the f-th position `struct` T item, if the expression e is evaluated to a `struct` pointer $\texttt{ptr}^m\ \texttt{struct}\ T$. Rule T-STRUCT in Fig. 18 describes its typing behavior. Rules S-STRUCTCHECKED and S-STRUCTUNCHECKED describe the semantic behaviors of $\&e{\rightarrow}f$ on a given `struct` `checked`/`unchecked` pointers, while rule S-STRUCTNULL describes a `checked` `struct` null-pointer case. In our Coq/Redex formalization, we include the `struct` values and the operation $\&e{\rightarrow}f$. We omit it in the main text due to the paper length limitation.

7. The Compilation Rules

Fig. 23 and Fig. 24 shows the syntax for COREC, the target language for compilation. We syntactically restrict the expressions to be in A-normal form to simplify the presentation of the compilation rules. In the Redex model, we oc-

$$\frac{\Gamma \vdash \overline{x} : \overline{\tau} \qquad \Gamma[\overline{x} \mapsto \overline{\tau}] \vdash \tau \qquad \Gamma[\overline{x} \mapsto \overline{\tau}]; \Theta \vdash_{\mathtt{c}} e : \tau}{\Gamma \vdash \tau\ (\overline{x} : \overline{\tau})\ e} \qquad \Gamma \vdash \cdot$$

$$\frac{\Gamma \vdash \tau \qquad \Gamma[x \mapsto \tau] \vdash \overline{x} : \overline{\tau}}{\Gamma \vdash x : \tau, \overline{x} : \overline{\tau}}$$

Figure 20: Well-formedness for functions

$$\frac{\Gamma \vdash \tau}{\Gamma \vdash \tau\ \mathtt{f}} \qquad \frac{\Gamma \vdash \tau \qquad \Gamma \vdash fs}{\Gamma \vdash \tau\ \mathtt{f}; fs}$$

Figure 21: Well-formedness for structs

$$\frac{\Gamma[\overline{x} \mapsto \overline{\tau}]; \emptyset \vdash e \gg \dot{e} : \tau}{\Gamma \vdash \tau\ (\overline{x} : \overline{\tau})\ e \gg (\overline{x})\ \dot{e}}$$

Figure 22: Compilation Rules for Functions

casionally break this constraint to speed up the performance of random testing by removing unnecessary let bindings. To allow explicit runtime checks, we include `bounds` and `null` as part of CoreC expressions which, once evaluated, result in an corresponding error state. $x = \dot{a}$ is a new syntactic form that modifies the stack variable x with the result of $\dot{a}$. It is essential for bounds widening. $\leq$ and $-$ are introduced to operate on bounds and decide whether we need to halt with a bounds error or widen a null-terminated string.

Atoms	$\dot{a}$	::=	$n \mid x$
C-Expressions	$\dot{c}$	::=	$\dot{a} \mid \mathtt{strlen}(\dot{a}) \mid \mathtt{malloc}(\dot{a}, \mid) f(\overline{\dot{a}})$
		$\mid$	$\mid \dot{a} \circ \dot{a} \mid \mathtt{*}\dot{a}$
		$\mid$	$\mathtt{*}\dot{a} = \dot{a} \mid x = \dot{a} \mid \mathtt{if}\ (\dot{a})\ \dot{e}\ \mathtt{else}\ \dot{e}$
		$\mid$	$\mathtt{bounds} \mid \mathtt{null}$
Expressions	$\dot{e}$	::=	$\dot{c} \mid \mathtt{let}\ x = \dot{c}\ \mathtt{in}\ \dot{e}$
Binops	$\circ$	::=	$+ \mid - \mid \leq$
Closure	$\dot{C}$	::=	$\square \mid \mathtt{let}\ x = \dot{a}\ \mathtt{in}\ \dot{C}$
		$\mid$	$\mathtt{if}\ (\dot{a})\ \dot{e}\ \mathtt{else}\ \dot{C} \mid \mathtt{if}\ (\dot{a})\ \dot{C}\ \mathtt{else}\ \dot{e}$
Bounds Map	ρ	$\in$	$\mathtt{Var} \rightharpoonup \mathtt{Var} \times \mathtt{Var}$

Figure 23: CoreC Syntax

$\dot{\mu}$	::=	$n \mid \bot$
$\dot{c}$	::=	$\ldots \mid \mathtt{ret}(x, \dot{\mu}, \dot{e})$
$\dot{H}$	$\in$	$\mathbb{Z} \rightharpoonup \mathbb{Z}$
$\dot{r}$	::=	$\dot{e} \mid \mathtt{null} \mid \mathtt{bounds}$
$\dot{E}$	::=	$\square \mid \mathtt{let}\ x = \dot{E}\ \mathtt{in}\ \dot{e} \mid \mathtt{ret}(x, i, \dot{E})$
	$\mid$	$\mathtt{if}\ (\dot{E})\ \dot{e}\ \mathtt{else}\ \dot{e} \mid \mathtt{strlen}(\dot{E})$
	$\mid$	$\mathtt{malloc}(\dot{E}, \mid) f(\overline{\dot{E}}) \mid \dot{E} \circ \dot{a} \mid n \circ \dot{E}$
	$\mid$	$\mathtt{*}\dot{E} \mid \mathtt{*}\dot{E} = \dot{a} \mid \mathtt{*}n = \dot{E} \mid x = \dot{E}$
$\overline{\dot{E}}$	::=	$\dot{E} \mid n, \overline{\dot{E}} \mid \overline{\dot{E}}, \dot{a}$

Figure 24: CoreC Semantic Defs

CoreC does not include any annotations. We remove structs from CoreC because we can always statically convert expressions of the form $\&n : \tau \rightarrow f$ into $n + n_f$, where n_f is the statically determined offset of f within the struct. We ellide the semantics of CoreC because it is self-evident and mirrors the semantics CoreChkCBox. The difference is that in CoreC, only `bounds` and `null` can step into an error state. All failed dereferences and assignments would result in a stuck state and therefore we rely on the compiler to explicitly insert checks for checked pointers.

Fig. 27 and Fig. 28 shows the rules for the compilation judgment for expressions,

$$\Gamma; \rho \vdash e \gg \dot{C}, \dot{a}$$

The judgment is presented differently from the one in Sec. 4, which was simplified for presentation purposes. First, we remove Θ and m because these parameters are only used for checking and have no impact on compilation. Second, the judgment includes two outputs, a closure $\dot{C}$ and an atom expression $\dot{a}$, instead of a single CoreC expression $\dot{e}$. $\dot{C}$ can be intuitively understood as a partially constructed program or context. Whereas $\dot{E}$ is used for evaluation, $\dot{C}$ is used purely as a device for compilation. As an example, when compiling $(1 : \mathtt{int}) + (2 : \mathtt{int})$, we would first create a fresh variable x, and then produce two outputs:

$$\dot{C} = \mathtt{let}\ x = 1 + 2\ \mathtt{in}\ \square$$

$$\dot{a} = x$$

To obtain the compiled expression $\dot{e}$, we plug $\dot{a}$ into $\dot{C}$ using the usual notation $\dot{C}[\dot{a}]$. We can also use $\dot{C}$ to represent runtime checks, which usually take the form $\mathtt{let}\ x = \dot{c}\ \mathtt{in}\ \square$, where $\dot{c}$ contains the check whose evaluation must not trigger `bounds` or `null` for the program to continue (see Fig. 26 for the metafunctions that create those checks).

This unconventional output format enables us to separate the evaluation of the term and the computation that relies on the term's evaluated result. Since effects and reduction (except for variables) happen only within closures, we can precisely control the order in which effects and evaluation happen by composing the contexts in a specific order. Given two closures $\dot{C}_1$ and $\dot{C}_2$, we write $\dot{C}_1[\dot{C}_2]$ to denote the meta operation of plugging $\dot{C}_2$ into $\dot{C}_1$. We also use $\dot{C}_{a;b;c}$ as a shorthand for $\dot{C}_a[\dot{C}_b[\dot{C}_c]]$. In the C-Ind rule, we first evaluate the expressions that correspond to e_1 and e_2 through $\dot{C}_1$ and $\dot{C}_2$, and then perform a null check and an addition through $\dot{C}_n$ and $\dot{C}_3$. Finally, we dereference the result through $\dot{C}_4$ before returning the pair $\dot{C}_4, \dot{x}_4$, propagating the flexibility to the compilation rule that recursively calls C-Ind.

Fig. 26 shows the metafunctions that create closures representing dynamic checks. These functions first examine whether the pointer is a checked. If the pointer is unchecked, an empty closure $\square$ will be returned, because there is no need to perform a check. For bounds checking, there is a special case for NT-array pointers, where the bounds are retrived from the shadow variables (found by looking up ρ) on the stack rather than using the bounds specified in the type annotation. This is how we achieve the same precise runtime behavior as CoreChkCBox in our compiled expressions.

Fig. 25 shows the metafunctions related to bounds widening. $\vdash_{extend}$ takes ρ, a checked NT-array pointer variable x, and its bounds (b_l, b_h) as inputs, and returns an extended ρ' that maps x to two fresh variables x_l, x_h, together with a closure $\dot{C}$ that initializes x_l and x_h to b_l and b_h respectively. This function is used in the C-LET rule to extend ρ before compiling the body of the `let` binding. The updated ρ' can be used for generating precise bounds checks, and for inserting expressions that can potentially widen the upper bounds, as seen in the $\vdash_{widenstr}$ metafunction used in the C-STR compilation rule.

$$\frac{x_l, x_h = \texttt{fresh} \qquad \rho' = \rho[x \mapsto (x_l, x_h)] \qquad \dot{C} = \texttt{let } x_l = b_l \texttt{ in let } x_h = b_h \texttt{ in } \square}{\dot{C}, \rho' = \ \vdash_{extend} \rho, x, \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_{nt}}$$

$$\frac{x_l, x_h = \rho(x) \qquad x_w = \texttt{fresh} \qquad \dot{C} = \texttt{let } x_w = \texttt{if } (x_h)\ 0 \texttt{ else } x_h = 1 \texttt{ in } \square}{\dot{C} = \ \vdash_{widenderef} \rho, x, \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_{nt}} \qquad \frac{e \notin dom(\rho)}{\square = \ \vdash_{widenstr} \rho, e, \dot{a}, \texttt{ptr}^m\ [\beta\ \tau]_{nt}}$$

$$\frac{x_l, x_h = \rho(e) \qquad x_a = \texttt{fresh} \qquad \dot{C} = \texttt{let } x_a = \texttt{if } (\dot{a} \leq x_h)\ 0 \texttt{ else } x_h = \dot{a} \texttt{ in } \square}{\dot{C} = \ \vdash_{widenstr} \rho, e, \dot{a}, \texttt{ptr}^c\ [\beta\ \tau]_{nt}}$$

Figure 25: Metafunctions for Widening

$$\frac{x = \texttt{fresh} \qquad \dot{C} = \texttt{let } x = \texttt{if } (\dot{a})\ 0 \texttt{ else null in } \square}{\dot{C} = \ \vdash_{null} \dot{a}, c} \qquad \square = \ \vdash_{null} \dot{a}, u$$

$$\square = \ \vdash_{boundsR} \rho, e, \texttt{ptr}^u\ [\beta\ \tau]_\kappa, \dot{a}$$

$$\frac{\begin{array}{c} x_l, x_h = \rho(e) \\ x_{cl}, x_{ch} = \texttt{fresh} \qquad \dot{C}_{cl} = \texttt{let } x_{cl} = \texttt{if } (x_l \leq \dot{a})\ 0 \texttt{ else bounds in } \square \qquad \dot{C}_{ch} = \texttt{let } x_{ch} = \texttt{if } (\dot{a} \leq x_h)\ 0 \texttt{ else bounds in } \square \end{array}}{\dot{C}_{cl;ch} = \ \vdash_{boundsR} \rho, e, \texttt{ptr}^c\ [\beta\ \tau]_\kappa, \dot{a}}$$

$$\frac{\begin{array}{c} e \notin dom(\rho) \qquad x_l, x_h, x_{cl}, x_{ch} = \texttt{fresh} \qquad \dot{C}_l = \texttt{let } x_l = b_l \texttt{ in } \square \qquad \dot{C}_h = \texttt{let } x_h = b_h \texttt{ in } \square \\ \dot{C}_{cl} = \texttt{let } x_{cl} = \texttt{if } (x_l \leq \dot{a})\ 0 \texttt{ else bounds in } \square \qquad \dot{C}_{ch} = \texttt{let } x_{ch} = \texttt{if } (\dot{a} \leq x_h)\ 0 \texttt{ else bounds in } \square \end{array}}{\dot{C}_{l;h;cl;ch} = \ \vdash_{boundsR} \rho, e, \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_{nt}, \dot{a}}$$

$$\frac{\begin{array}{c} e \notin dom(\rho) \qquad x_l, x_h, x_{cl}, x_{ch} = \texttt{fresh} \qquad \dot{C}_l = \texttt{let } x_l = b_l \texttt{ in } \square \qquad \dot{C}_h = \texttt{let } x_h = b_h \texttt{ in } \square \\ \dot{C}_{cl} = \texttt{let } x_{cl} = \texttt{if } (x_l \leq \dot{a})\ 0 \texttt{ else bounds in } \square \qquad \dot{C}_{ch} = \texttt{let } x_{ch} = \texttt{if } (x_h \leq \dot{a}) \texttt{ bounds else } 0 \texttt{ in } \square \end{array}}{\dot{C}_{l;h;cl;ch} = \ \vdash_{boundsR} \rho, e, \texttt{ptr}^c\ [(b_l, b_h)\ \tau], \dot{a}}$$

$$\square = \ \vdash_{boundsW} \rho, e, \texttt{ptr}^u\ [\beta\ \tau]_\kappa, \dot{a}$$

$$\frac{\begin{array}{c} x_l, x_h = \rho(e) \\ x_{cl}, x_{ch} = \texttt{fresh} \qquad \dot{C}_{cl} = \texttt{let } x_{cl} = \texttt{if } (x_l \leq \dot{a})\ 0 \texttt{ else bounds in } \square \qquad \dot{C}_{ch} = \texttt{let } x_{ch} = \texttt{if } (\dot{a} \leq x_h)\ 0 \texttt{ else bounds in } \square \end{array}}{\dot{C}_{cl;ch} = \ \vdash_{boundsW} \rho, e, \texttt{ptr}^c\ [\beta\ \tau]_\kappa, \dot{a}}$$

$$\frac{\begin{array}{c} e \notin dom(\rho) \qquad x_l, x_h, x_{cl}, x_{ch} = \texttt{fresh} \qquad \dot{C}_l = \texttt{let } x_l = b_l \texttt{ in } \square \qquad \dot{C}_h = \texttt{let } x_h = b_h \texttt{ in } \square \\ \dot{C}_{cl} = \texttt{let } x_{cl} = \texttt{if } (x_l \leq \dot{a})\ 0 \texttt{ else bounds in } \square \qquad \dot{C}_{ch} = \texttt{let } x_{ch} = \texttt{if } (x_h \leq \dot{a}) \texttt{ bounds else } 0 \texttt{ in } \square \end{array}}{\dot{C}_{l;h;cl;ch} = \ \vdash_{boundsW} \rho, e, \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_\kappa, \dot{a}}$$

$$\frac{\begin{array}{c} e \notin dom(\rho) \qquad x_l, x_l', x_h, x_h' = \texttt{fresh} \qquad \dot{C}_1 = \texttt{let } x_l = b_l \texttt{ in let } x_h = b_h \texttt{ in } \square \\ \dot{C}_2 = \texttt{let } x_l' = b_l' \texttt{ in let } x_h' = b_h' \texttt{ in } \square \qquad \dot{C}_3 = \texttt{if } (x_l' \leq x_l)\ \square \texttt{ else bounds} \qquad \dot{C}_4 = \texttt{if } (x_h \leq x_h')\ \square \texttt{ else bounds} \end{array}}{\dot{C}_{1;2;3;4} = \ \vdash_{boundsD} \rho, e, \texttt{ptr}^m\ [(b_l, b_h)\ \tau]_\kappa, \texttt{ptr}^m\ [(b_l', b_h')\ \tau]_\kappa}$$

$$\frac{\begin{array}{c} x_l', x_h' = \rho(e) \qquad x_l, x_h = \texttt{fresh} \\ \dot{C}_1 = \texttt{let } x_l = b_l \texttt{ in let } x_h = b_h \texttt{ in } \square \qquad \dot{C}_2 = \texttt{if } (x_l' \leq x_l)\ \square \texttt{ else bounds} \qquad \dot{C}_3 = \texttt{if } (x_h \leq x_h')\ \square \texttt{ else bounds} \end{array}}{\dot{C}_{1;2;3} = \ \vdash_{boundsD} \rho, e, \texttt{ptr}^m\ [(b_l, b_h)\ \tau]_\kappa, \texttt{ptr}^m\ [(b_l', b_h')\ \tau]_\kappa}$$

Figure 26: Metafunctions for Dynamic Checks

C-CONST
$$\Gamma; \rho \vdash n : \tau \gg \square, n : \tau$$

C-VAR
$$\frac{x : \tau \in \Gamma}{\Gamma; \rho \vdash x \gg \square, x : \tau}$$

C-CAST
$$\frac{\Gamma; \rho \vdash e \gg \dot{C}, \dot{a} : \tau'}{\Gamma; \rho \vdash (\tau)e \gg \dot{C}, \dot{a} : \tau}$$

C-DYNCAST
$$\frac{\Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a} : \texttt{ptr}^m\ [\beta'\ \tau]_\kappa \qquad \dot{C}_b = \ \vdash_{boundsD}\ \rho, e, \texttt{ptr}^m\ [\beta\ \tau]_\kappa, \texttt{ptr}^m\ [\beta'\ \tau]_\kappa}{\Gamma; \rho \vdash \langle \texttt{ptr}^m\ [\beta\ \tau]_\kappa \rangle e \gg \dot{C}_{1;b}, \dot{a} : \texttt{ptr}^m\ [\beta\ \tau]_\kappa}$$

C-STR
$$\frac{\Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ [\beta\ \tau_a]_{nt} \qquad \dot{C}_n = \ \vdash_{null}\ \dot{a}_1, m \qquad \dot{C}_b = \ \vdash_{boundsR}\ \rho, \dot{a}_1, \texttt{ptr}^m\ [\beta\ \tau_a]_{nt}, 0 \qquad x_2 = \texttt{fresh} \qquad \dot{C}_2 = \texttt{let}\ x_2 = \texttt{strlen}(\dot{a}_1)\ \texttt{in}\ \square \qquad \dot{C}_w = \ \vdash_{widenstr}\ \rho, e, \dot{a}_1, \texttt{ptr}^m\ [\beta\ \tau_a]_{nt}}{\Gamma; \rho \vdash \texttt{strlen}(e) \gg \dot{C}_{1;n;b;2;w}, x_2 : \texttt{int}}$$

C-LETSTR
$$\frac{\Gamma(y) = \texttt{ptr}^c\ [(b_l, b_h)\ \tau_a]_{nt} \qquad x \notin FV(\tau) \qquad \Gamma; \rho \vdash \texttt{strlen}(y) \gg \dot{C}_1, \dot{a}_1 : \texttt{int} \qquad \dot{C}_2 = \texttt{let}\ x = \dot{a}_1\ \texttt{in}\ \square \qquad \Gamma[x \mapsto \texttt{int}, y \mapsto [\texttt{ptr}^c\ [(b_l, x)\ \tau_a]_{nt}]]; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau}{\Gamma; \rho \vdash \texttt{let}\ x = \texttt{strlen}(y)\ \texttt{in}\ e \gg \dot{C}_{1;2;3}, \dot{a}_3 : \tau}$$

C-IF
$$\frac{\Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \tau \qquad \Gamma; \rho \vdash e_1 \gg \dot{C}_2, \dot{a}_2 : \tau_2 \qquad \Gamma; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau_3 \qquad x_4 = \texttt{fresh} \qquad \dot{C}_4 = \texttt{let}\ x_4 = \texttt{if}\ (\dot{a}_1)\ \dot{C}_2[\dot{a}_2]\ \texttt{else}\ \dot{C}_3[\dot{a}_3]\ \texttt{in}\ \square}{\Gamma; \rho \vdash \texttt{if}\ (e_1)\ e_2\ \texttt{else}\ e_3 \gg \dot{C}_{1;4}, x_4 : \tau_2 \sqcup \tau_3}$$

C-IFNT
$$\frac{\Gamma; \rho \vdash x : \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_{nt} \qquad b_h = 0 \Rightarrow \Gamma' = \Gamma[x \mapsto \texttt{ptr}^c\ [(b_l, 1)\ \tau]_{nt}] \qquad b_h \neq 0 \Rightarrow \Gamma' = \Gamma \qquad \Gamma; \rho \vdash *x \gg \dot{C}_1, \dot{a}_1 : \tau_1 \qquad \Gamma'; \rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau_2 \qquad \Gamma; \rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau_3 \qquad \dot{C}_w = \ \vdash_{widenderef}\ \rho, x, \texttt{ptr}^c\ [(b_l, b_h)\ \tau]_{nt} \qquad x_4 = \texttt{fresh} \qquad \dot{C}_4 = \texttt{let}\ x_4 = \texttt{if}\ (\dot{a}_1)\ \dot{C}_{2;w}[\dot{a}_2]\ \texttt{else}\ \dot{C}_3[\dot{a}_3]\ \texttt{in}\ \square}{\Gamma; \rho \vdash \texttt{if}\ (*x)\ e_1\ \texttt{else}\ e_2 \gg \dot{C}_{1;4}, x_4 : \tau_1 \sqcup \tau_2}$$

C-LET
$$\frac{(x \in FV(\tau') \Rightarrow e_1 \in Bound) \qquad \Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \tau_1 \qquad \dot{C}_2, \rho' = \ \vdash_{extend}\ \rho, x, \tau_1 \qquad \dot{C}_3 = \texttt{let}\ x = \dot{a}_1\ \texttt{in}\ \square \qquad \Gamma[x \mapsto \tau]; \rho' \vdash e_4 \gg \dot{C}_4, \dot{a}_4 : \tau_4}{\Gamma; \rho' \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_4 \gg \dot{C}_{1;2;3;4}, \dot{a}_4 : \tau_4[\tau_1 = \texttt{int} \Rightarrow x \mapsto e_1]}$$

C-RET
$$\frac{\Gamma(x) \neq \bot \qquad \Gamma; \rho \vdash e \gg \dot{C}_1, \dot{a}_1 : \tau \qquad x_2 = \texttt{fresh} \qquad \mu \gg \dot{\mu} \qquad \dot{C}_2 = \texttt{let}\ x_2 = \texttt{ret}(x, \dot{\mu}, \dot{C}_1[\dot{a}_1])\ \texttt{in}\ \square}{\Gamma; \rho \vdash \texttt{ret}(x, \mu, e) \gg \dot{C}_2, x_2 : \tau}$$

C-FUN
$$\frac{\Xi(f) = \tau\ (\overline{x} : \overline{\tau})\ e \qquad (\forall e_i \in \overline{e}\ \ \tau_i \in \overline{\tau}\ .\ \Gamma; \rho \vdash e_i \gg \dot{C}_i, \dot{a}_i : \qquad \tau_i' \wedge \tau_i' \sqsubseteq \tau_i[\overline{e}/\overline{x}]) \qquad x_f = \texttt{fresh} \qquad \dot{C}_f = \texttt{let}\ x_f = f(\overline{a})\ \texttt{in}\ \square}{\Gamma; \rho \vdash f(\overline{e}) \gg \overline{\dot{C}}[\dot{C}_f], x_f : \tau[\overline{e}/\overline{x}]}$$

C-DEF
$$\frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ \tau \qquad \dot{C}_n = \ \vdash_{null}\ \dot{a}_1, m \qquad x_2 = \texttt{fresh} \qquad \dot{C}_2 = \texttt{let}\ x_2 = *\dot{a}_1\ \texttt{in}\ \square}{\Gamma; \rho \vdash *e_1 \gg \dot{C}_{1;n;2}, x_2 : \tau}$$

C-DEFARR
$$\frac{\Gamma; \rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ [(b_l, b_h)\ \tau]_\kappa \qquad \dot{C}_n = \ \vdash_{null}\ \dot{a}_1, m \qquad \dot{C}_b = \ \vdash_{boundsR}\ \rho, e_1, \texttt{ptr}^m\ [(b_l, b_h)\ \tau]_\kappa, 0 \qquad x_2 = \texttt{fresh} \qquad \dot{C}_2 = \texttt{let}\ x_2 = *\dot{a}_1\ \texttt{in}\ \square}{\Gamma; \rho \vdash *e_1 \gg \dot{C}_{1;n;b;2}, x_2 : \tau}$$

C-MAC
$$\frac{\dot{C}_1, \dot{a}_1 = \texttt{sizeof}(\omega) \qquad x_2 = \texttt{fresh} \qquad \dot{C}_2 = \texttt{let}\ x_2 = \texttt{malloc}(\dot{a}_1, \ )\texttt{in}\ \square}{\Gamma; \rho \vdash \texttt{malloc}(\omega, \gg)\dot{C}_{1;2}, x_2 : \texttt{ptr}^c\ \omega}$$

Figure 27: Compilation

C-ADD

$$\frac{\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{int} \qquad \Gamma;\rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \texttt{int} \qquad x_3 = \texttt{fresh} \qquad \dot{C}_3 = \texttt{let } x_3 \texttt{ = } \dot{a}_1 \texttt{ + } \dot{a}_2 \texttt{ in } \square}{\Gamma;\rho \vdash \dot{C}_3, x_3 : \texttt{int}}$$

C-IND

$$\frac{\begin{array}{c}\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ [\beta\ \tau]_\kappa \qquad \Gamma;\rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \texttt{int} \qquad \dot{C}_n = \ \vdash_{null} \dot{a}_1, m \\ \dot{C}_b = \ \vdash_{boundsR} \rho, e_1, \texttt{ptr}^m\ [\beta\ \tau]_\kappa, \dot{a}_2 \qquad x_3, x_4 = \texttt{fresh} \qquad \dot{C}_3 = \texttt{let } x_3 \texttt{ = } \dot{a}_1 \texttt{ + } \dot{a}_2 \texttt{ in } \square \qquad \dot{C}_4 = \texttt{let } x_4 \texttt{ = } \texttt{*}x_3 \texttt{ in } \square\end{array}}{\Gamma;\rho \vdash \texttt{*}(e_1 \texttt{ + } e_2) \gg \dot{C}_{1;2;n;3;b;4}, x_4 : \tau}$$

C-ASSIGN

$$\frac{\begin{array}{c}\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^{m'}\ \tau \\ \dot{C}_n = \ \vdash_{null} \dot{a}_1, m \qquad \Gamma;\rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau' \qquad \tau' \sqsubseteq \tau \qquad x_3 = \texttt{fresh} \qquad \dot{C}_3 = \texttt{let } x_3 \texttt{ = } \texttt{*}\dot{a}_1 \texttt{ = } \dot{a}_2 \texttt{ in } \square\end{array}}{\Gamma;\rho \vdash \texttt{*}e_1 \texttt{ = } e_2 \gg \dot{C}_{1;2;n;3}, x_3 : \tau}$$

C-ASSIGNARR

$$\frac{\begin{array}{c}\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^{m'}\ [\beta\ \tau]_\kappa \qquad \dot{C}_n = \ \vdash_{null} \dot{a}_1, m \qquad \dot{C}_b = \ \vdash_{boundsW} \rho, e_1, \texttt{ptr}^m\ [(b_l, b_h)\ \tau]_\kappa, 0 \\ \Gamma;\rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \tau' \qquad x_3 = \texttt{fresh} \qquad \dot{C}_3 = \texttt{let } x_3 \texttt{ = } \texttt{*}\dot{a}_1 \texttt{ = } \dot{a}_2 \texttt{ in } \square \qquad \tau' \sqsubseteq \tau\end{array}}{\Gamma;\rho \vdash \texttt{*}e_1 \texttt{ = } e_2 \gg \dot{C}_{1;2;n;b;3}, x_3 : \tau}$$

C-INDASSIGN

$$\frac{\begin{array}{c}\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ [\beta\ \tau]_\kappa \\ \Gamma;\rho \vdash e_2 \gg \dot{C}_2, \dot{a}_2 : \texttt{int} \qquad \dot{C}_n = \ \vdash_{null} \dot{a}_1, m \qquad \dot{C}_b = \ \vdash_{boundsW} \rho, e_1, \texttt{ptr}^m\ [\beta\ \tau]_\kappa, \dot{a}_2 \\ \Gamma;\rho \vdash e_3 \gg \dot{C}_3, \dot{a}_3 : \tau' \qquad x_4, x_5 = \texttt{fresh} \qquad \dot{C}_4 = \texttt{let } x_4 \texttt{ = } \dot{a}_1 \texttt{ + } \dot{a}_2 \texttt{ in } \square \qquad \dot{C}_5 = \texttt{let } x_5 \texttt{ = } \texttt{*}x_4 \texttt{ = } x_3 \square \texttt{ in } \tau' \sqsubseteq \tau\end{array}}{\Gamma;\rho \vdash \texttt{*}(e_1 \texttt{ + } e_2) \texttt{ = } e_3 \gg \dot{C}_{1;2;n;3;4;b;5} : \tau}$$

C-STRUCT

$$\frac{\begin{array}{c}\Gamma;\rho \vdash e_1 \gg \dot{C}_1, \dot{a}_1 : \texttt{ptr}^m\ \texttt{struct}\ T \\ D(T) = \tau_0\ f_0 \ldots; \tau_j\ f; \ldots \qquad \dot{C}_n = \ \vdash_{null} \dot{a}_1, m \qquad x_2 = \texttt{fresh} \qquad \dot{C}_2 = \texttt{let } x_2 \texttt{ = } \dot{a}_1 \texttt{ + } \dot{j} \texttt{ in } \square\end{array}}{\Gamma;\rho \vdash \&e_1 \rightarrow f \gg \dot{C}_2, x_2 : \texttt{ptr}^m\ \tau_f}$$

C-UNCHECKED

$$\frac{\Gamma;\rho \vdash e \gg \dot{C}, \dot{a} : \tau}{\Gamma;\rho \vdash \texttt{unchecked}(e)\{\gg\}\dot{C}, \dot{a} : \tau}$$

Figure 28: Compilation (Continued)