Coherence as Constraint Satisfaction

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Thagard & Verbeurgt (1998)
"Coherence as constraint satisfaction"

<u>Feedback on discussion points</u>

(1) My compliments: Interesting discussions!
(2) Also compliments for good quality responses to questions, and including help and corrections where possible.
(3) Reminder: the course is about cognitive psychology, not cognitive engineering.
(4) Some interesting properties and problems were discovered, some of which we will discuss in today's class.

Formalization and analysis of Coherence as Constraint Satisfaction

Thagard & Verbeurgt (1998)
"Coherence as constraint satisfaction"

Overview of class

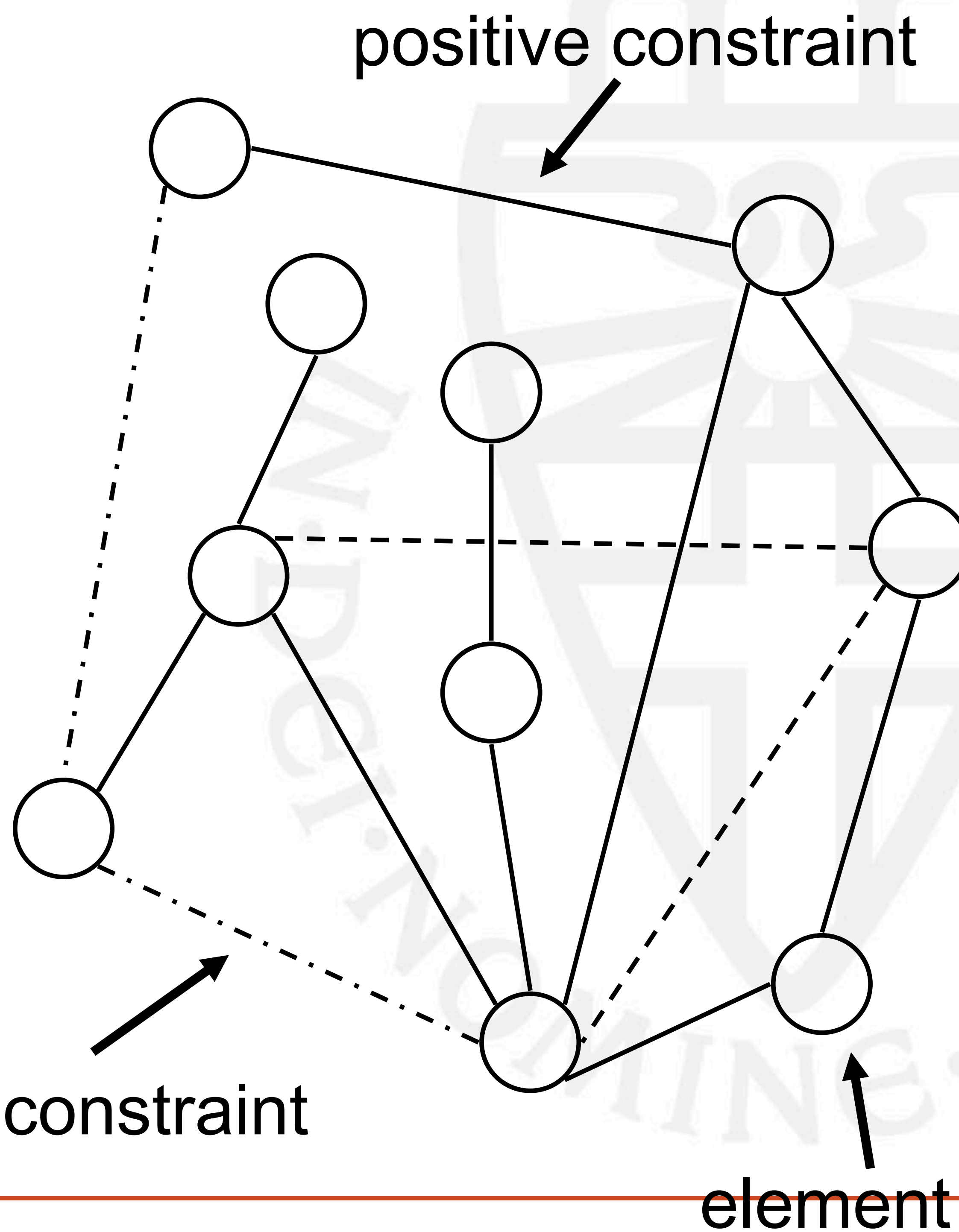
(1) From informal to formal coherence model
(2) Analysis of model properties
(3) Exercises

Coherence as constraint satisfaction

COHERENCE (informal)
Input: Networks of interconnected representations, where links represent coherence and incoherence relations between representations.
Output: Accept and reject representations in a way that is maximally coherent.

Coherence as constraint satisfaction

COHERENCE (pre-formal)
Input: A network of elements, connected by positive and negative constraints.
Output: A partition of the set of elements into sets *A* (accepted) and *R* (rejected) elements such that number of satisfied constraints is maximized.



Coherence as constraint satisfaction

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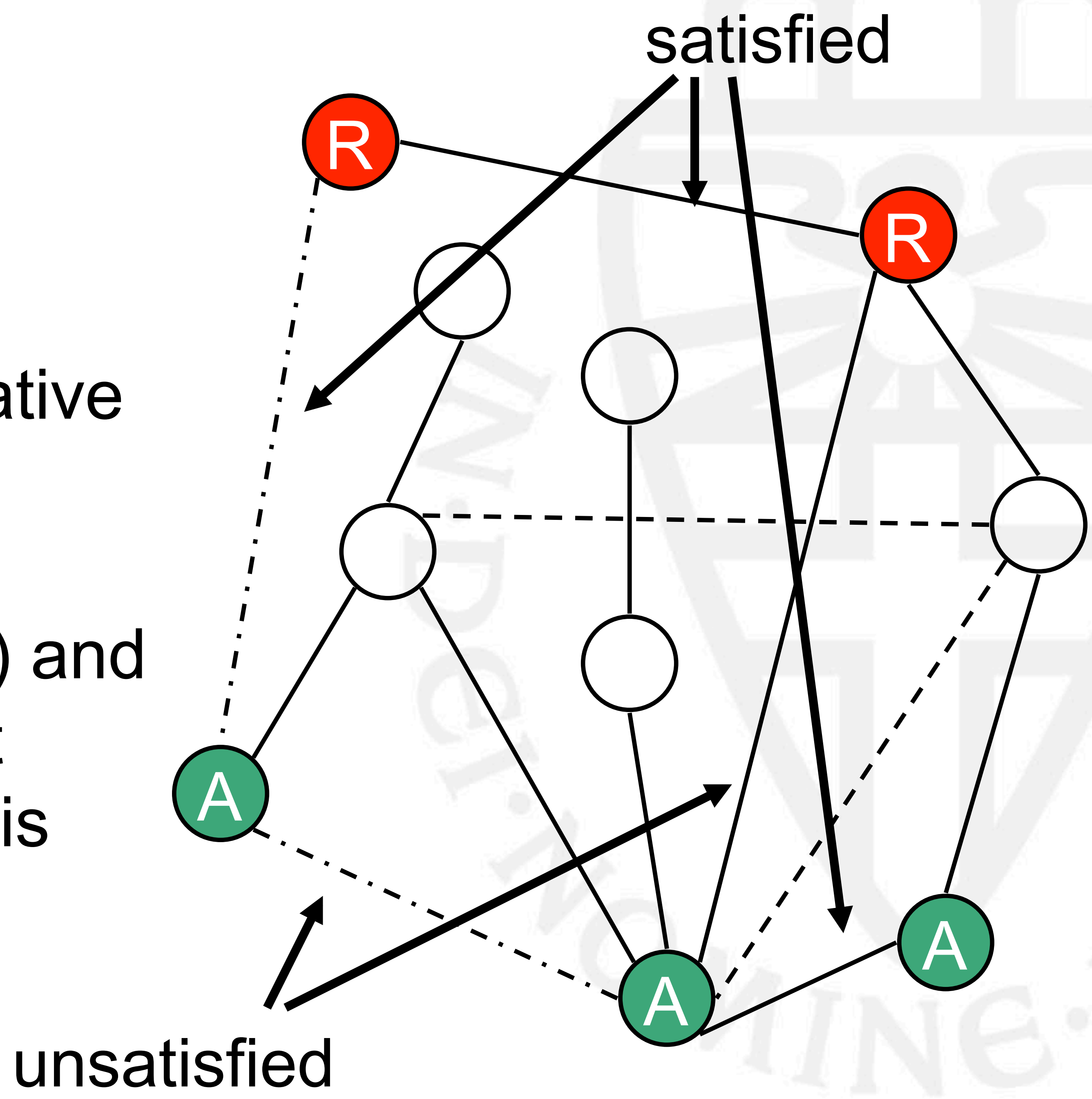

Coherence as constraint satisfaction

COHERENCE (formalization 1)

Input: A graph $G = (V, E)$, with a weight $w_{ij} > 0$ for each edge $(v_i, v_j) \in E = C^+ \cup C^- \subseteq V \times V$ (where $C^+ \cap C^- = \emptyset$).

Output: A partition of the vertices V into sets A and R such that $\mathrm{Coh}(A, R) =$

$$\sum_{(v_i, v_j) \in C^+ \text{ and } v_i, v_j \in A \text{ or } v_i, v_j \in R} w_{ij} \; +$$

$$\sum_{(v_i, v_j) \in C^- \text{ and } v_i \in A, v_j \in R \text{ or } v_j \in A, v_i \in R} w_{ij}$$

is maximized.

Coherence as constraint satisfaction

COHERENCE (formalization 2)

Input: A graph $G = (V, E)$, with a weight $w_{ij} > 0$ for each edge $(v_i, v_j) \in E = C^+ \cup C^- \subseteq V \times V$ (where $C^+ \cap C^- = \varnothing$).

Output: A truth assignment $T : V \rightarrow$ {true, false} such that

$$\mathrm{Coh}(T) = \sum_{(v_i, v_j) \in C^+ \text{ and } T(v_i) = T(v_j)} w_{ij} + \sum_{(v_i, v_j) \in C^- \text{ and } T(v_i) \neq T(v_j)} w_{ij}$$

is maximized.

Coherence as constraint satisfaction

COHERENCE (formalization 3)

Input: A Hopfield network $G = (V, E)$, with a weight $-1 < w_{ij} < +1$ for each connection $(v_i, v_j) \in E = V \times V$.

Output: An activation pattern $A : V \to \{-1, 1\}$ such that harmony $H(A) =$

$$\sum_i \sum_j w_{ij} A(v_i) A(v_j)$$

is maximized.

Coherence as constraint satisfaction

COHERENCE (formalization 3)
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Exercise: Proof that Formalization 3 is equivalent to 1 or 2.

Properties of Coherence as computational-level theory

Symmetry
Truth-conductivity
Foundational versus discriminating coherence
Computationally intractable (NP-hard)
Unexplained (“miracle”) parts of the process

Symmetry
Coh(A,R) = Coh(R,A)

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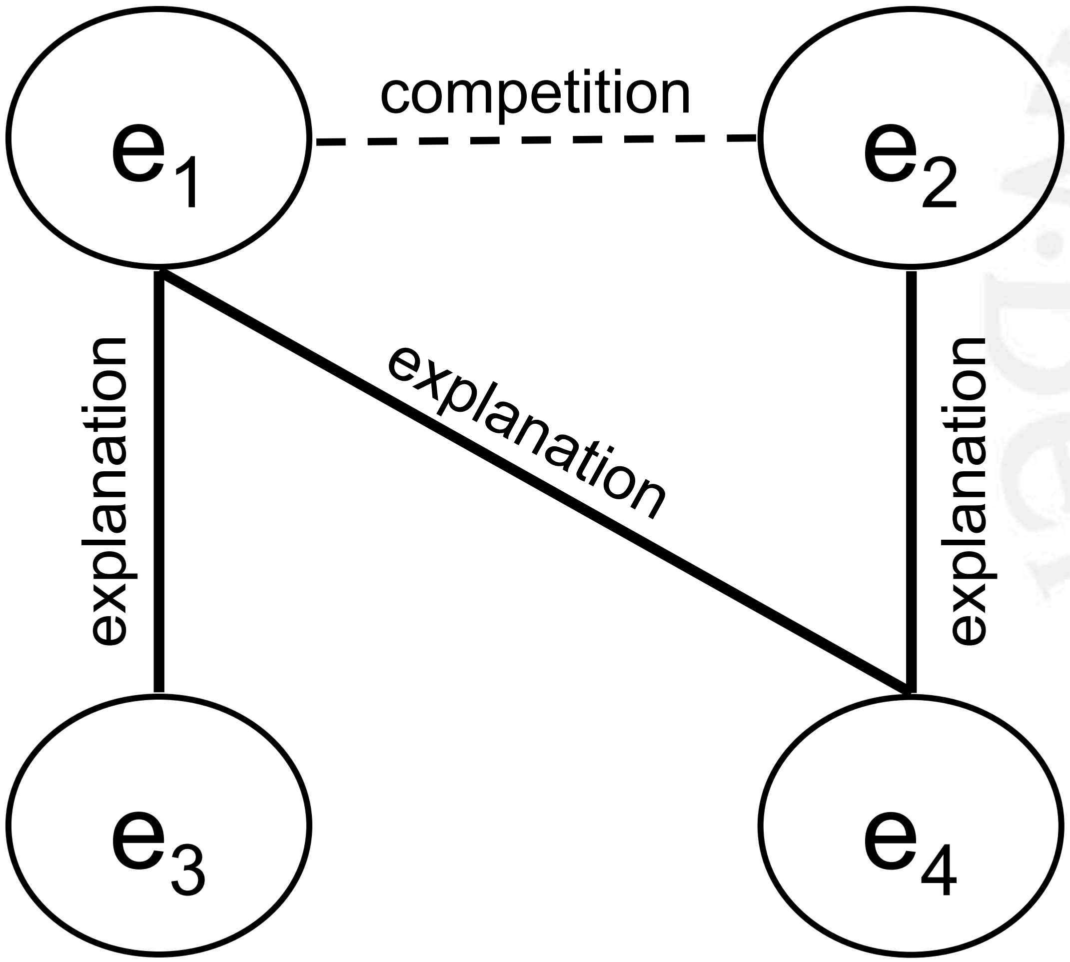


Data priority principle

FOUNDATIONAL COHERENCE

Input: A graph $G = (V, E)$, with a weight $w_{ij} > 0$ for each edge $(v_i, v_j) \in E = C^+ \cup C^- \subseteq V \times V$ (where $C^+ \cap C^- = \emptyset$) and some special type of (data) elements $D \subseteq V$.

Output: A truth assignment $T : V \to \{\text{true, false}\}$ such that $T(d)$ = true for all $d \in D$ and Coh(T) =

$$\sum_{(v_i, v_j) \in C^+ \text{ and } T(v_i) = T(v_j)} w_{ij}$$

$$+ \sum_{(v_i, v_j) \in C^- \text{ and } T(v_i) \neq T(v_j)} w_{ij}$$

is maximized.

Data priority principle

DISCRIMINATING COHERENCE

Input: A graph $G = (V, E)$, with a weight $w_{ij} > 0$ for each edge $(v_i, v_j) \in E = C^+ \cup C^- \subseteq V \times V$ (where $C^+ \cap C^- = \varnothing$) and a weight w_d for each $d \in D \subseteq V$.

Output: A truth assignment $T : V \to \{\text{true, false}\}$ such that Coh(T) =

$$\sum_{(v_i, v_j) \in C^+ \text{ and } T(v_i) = T(v_j)} w_{ij} + \sum_{(v_i, v_j) \in C^- \text{ and } T(v_i) \neq T(v_j)} w_{ij}$$

$$+ \sum_{d \in D, T(d) = true} w_d$$

is maximized.

Data priority principle

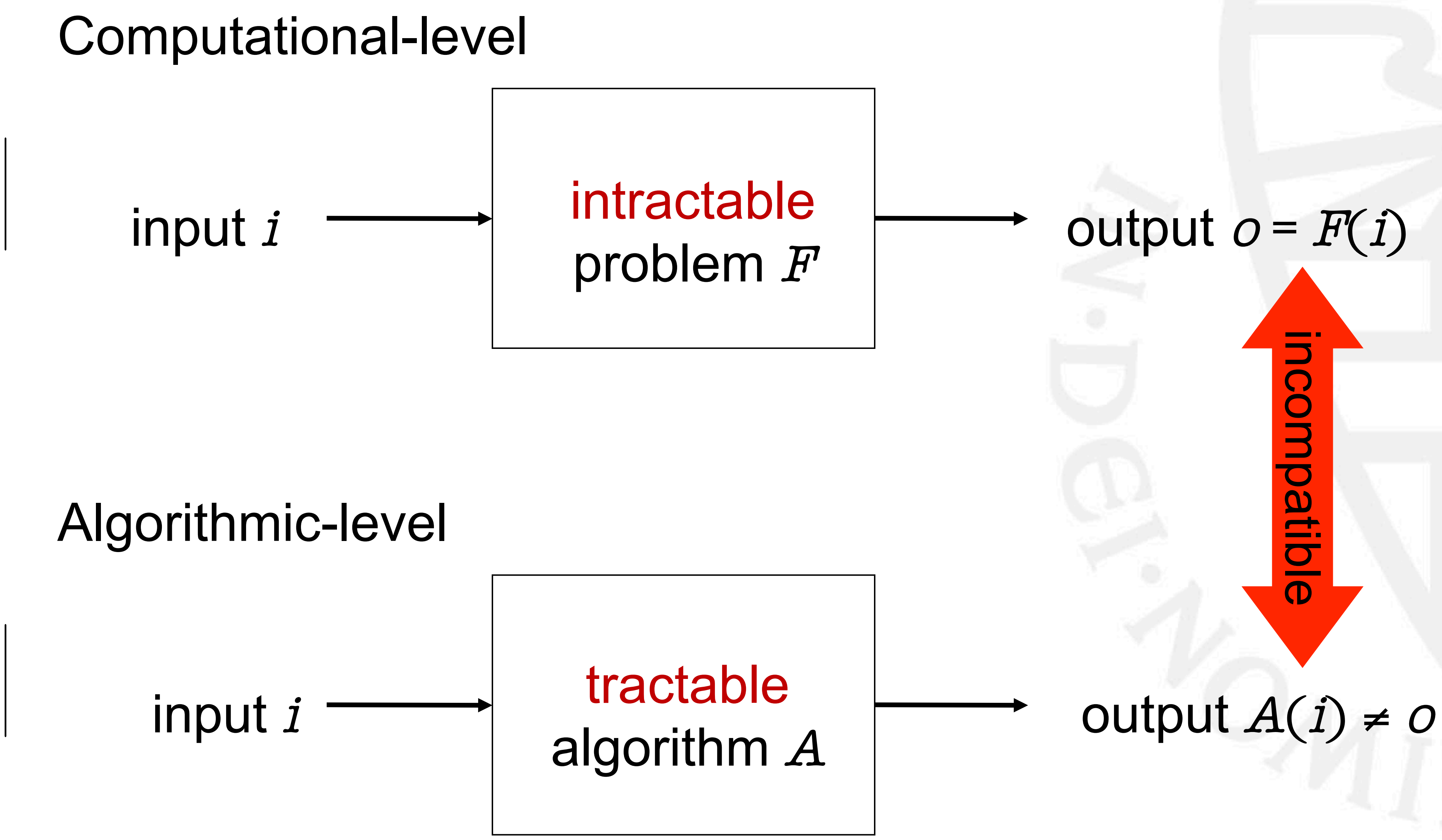
Exercise:

(1) Do the Foundational (F-) and Discriminating (D-) Coherence problems always have unique solutions?

(2) Can ‘symmetry’ also occur under the F- and D-Coherence formalizations?

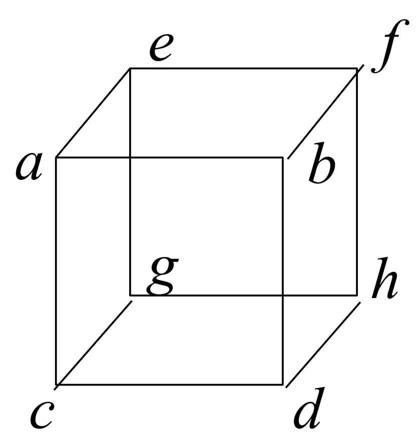
(3) If the answer to (1) or (2) is Yes, what implications does this have for the model as a model of truth or justified belief.

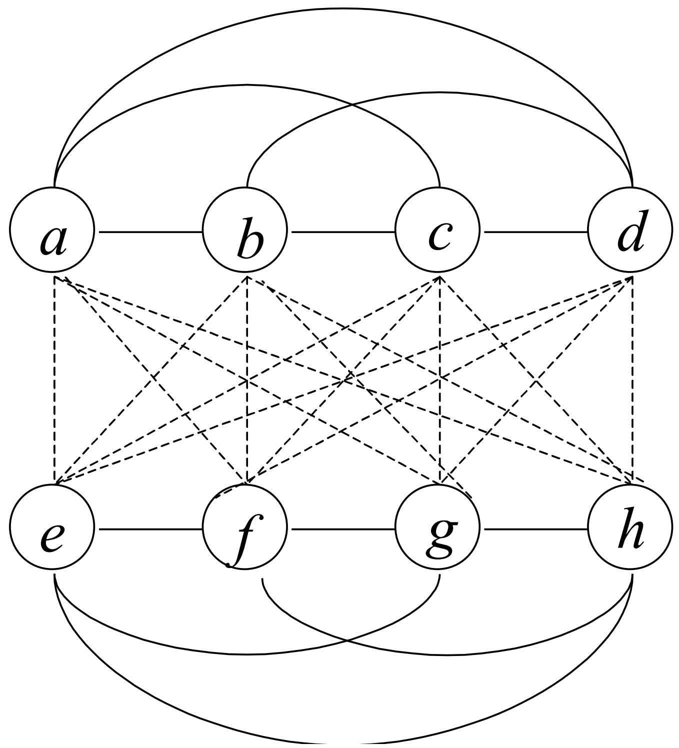
Computational intractability

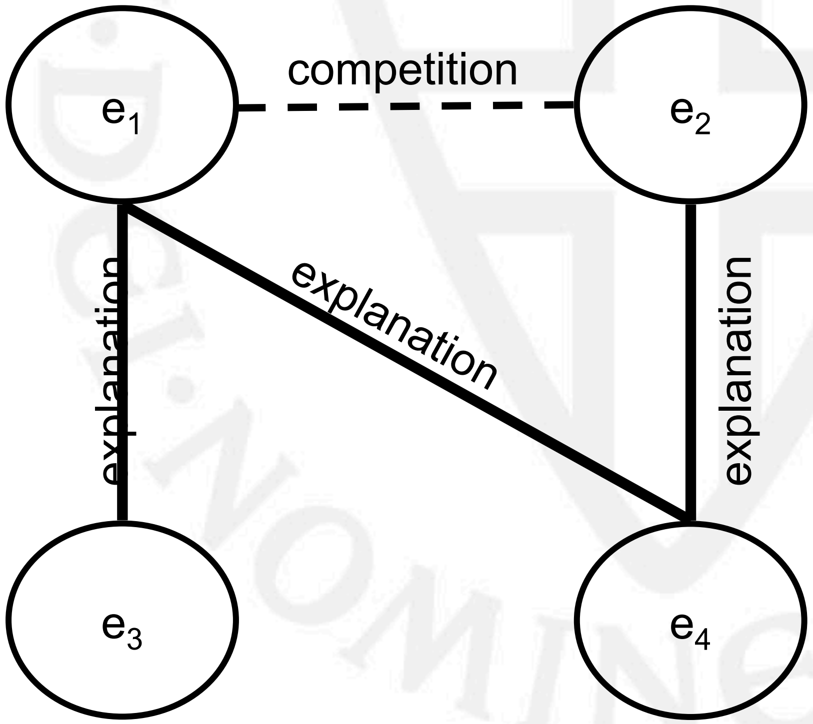


Computational intractability

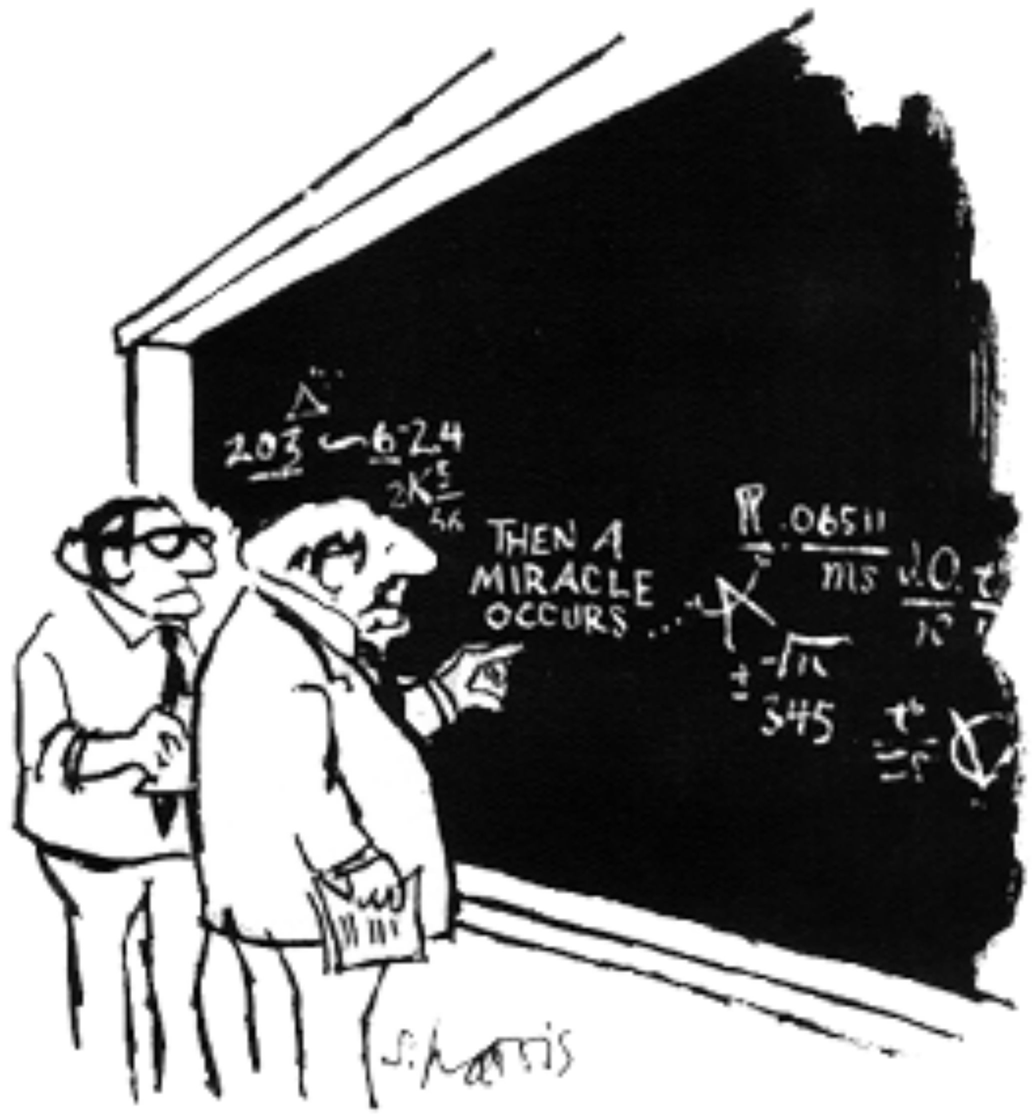
Computational intractability of COHERENCE does not imply coherence maximization is unfeasible for <u>cognitively relevant</u> belief networks.





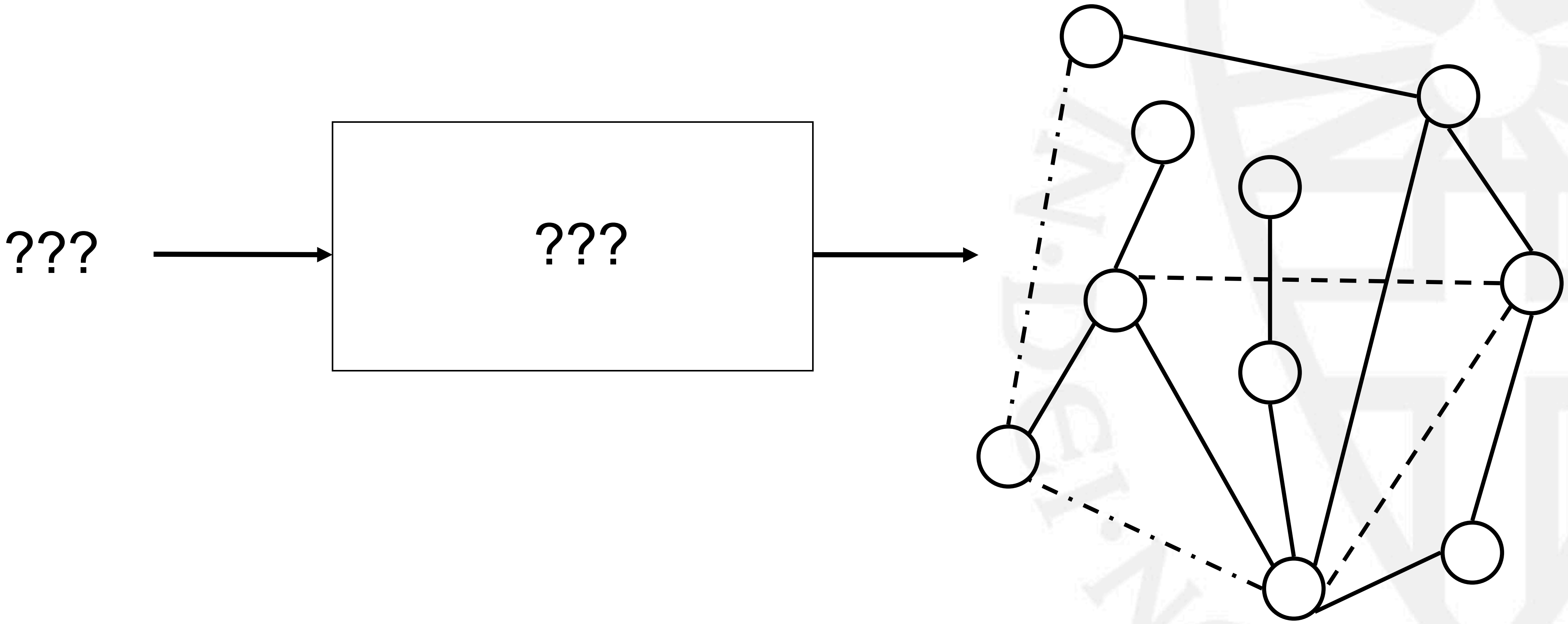


A “miracle” aspect of the model

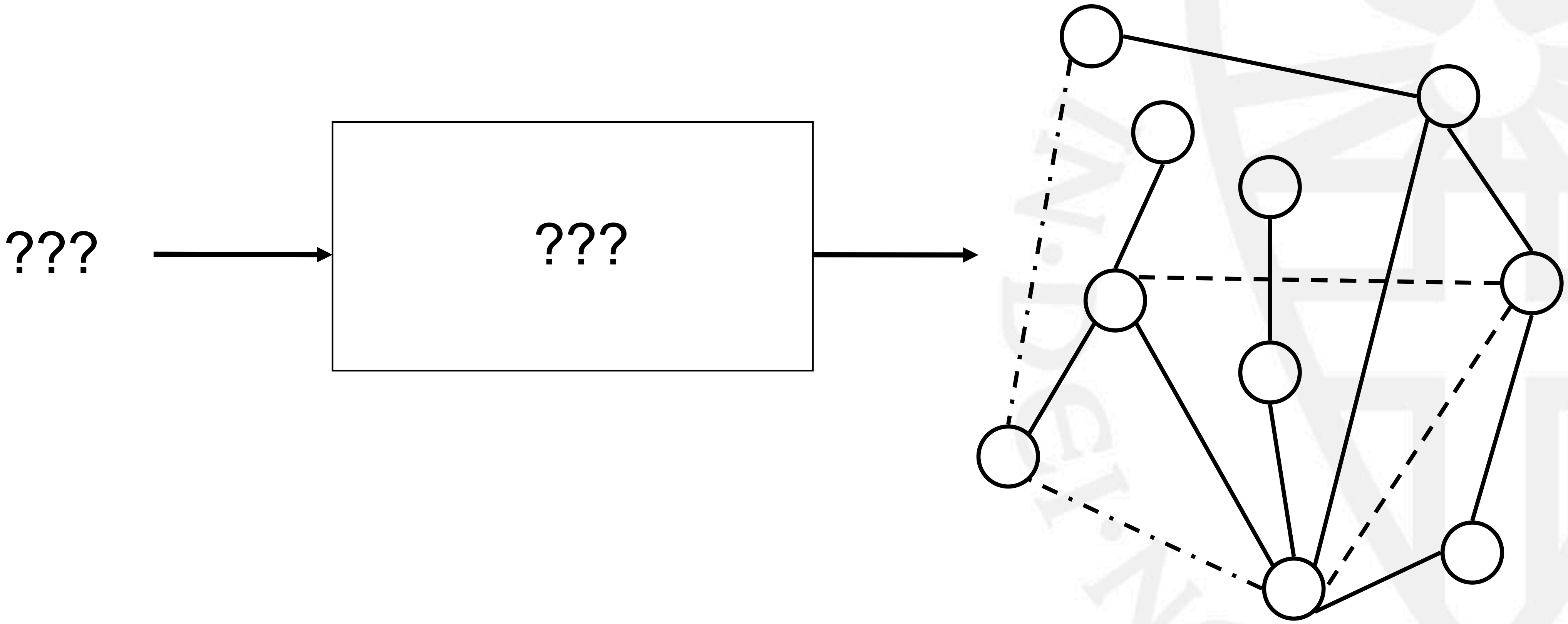


"I think you should be more explicit here in step two."

A “miracle” aspect of the model



A “miracle” aspect of the model



Exercise: What could the “miracle” process be?