Your name is: ______________________ **Your group is:** ________

1	2	3	4	5	6	7	8	9	$\sum$
/1.1	/1.1	/1.1	/1.1	/1.1	/1.1	/1.1	/1.1	/1.2	/10

1. Let $\mathbb{R}^3$ be an oriented Euclidean space; let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$ be such that $\|\mathbf{a}\| = 1$, $\|\mathbf{b}\| = 3$, and $\angle\widehat{\mathbf{a}, \mathbf{b}} = \frac{\pi}{6}$. Then, *find* $\|(-2 \cdot \mathbf{a} + 3 \cdot \mathbf{b}) \times (3 \cdot \mathbf{a} - \mathbf{b})\|$.

2. Let $\mathbb{R}^3$ be a 3-dimensional Euclidean space; let $\mathcal{A}$ be an orthonormal basis for $\mathbb{R}^3$; let

$$M_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \left\langle \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix} \right\rangle \quad \text{and} \quad M_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \left\langle \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\rangle$$

be two linear varieties in $\mathbb{R}^3$ (all coordinates are with respect to $\mathcal{A}$). Then, *find the distance between M_1 and M_2.*

3. Let $\mathbb{V}$ be a 3-dimensional vector space over the field of reals, $\mathbb{R}$; let $\mathcal{A}$ be an ordered basis for $\mathbb{V}$; let a quadratic form $q \in \mathcal{Q}(\mathbb{V})$ be specified by the following equality

$$q(\mathbf{x}) = x_1^2 + 6 \cdot x_2^2 + 4 \cdot x_3^2 + 4 \cdot x_1x_2 - 2 \cdot x_1x_3,$$

where $[x_1\ x_2\ x_3]^{\mathrm{T}} = [\mathbf{x}]_{\mathcal{A}}$. Then, *is q positive definite?*

4. Let $\mathbb{V}$ be a 2-dimensional vector space the field of reals, $\mathbb{R}$; let $\mathcal{A}$ be an ordered basis for $\mathbb{V}$; let a linear operator $\varphi \in \mathcal{L}(\mathbb{V})$ be specified by its coordinate matrix $T(\varphi, \mathcal{A}) = \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix}$. Then, *is φ diagonalizable?*

5. Let $\mathbb{V}$ be a 4-dimensional Euclidean space; let $\mathcal{A} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4)$ be an orthonormal basis for $\mathbb{V}$; let $\mathbf{a} = 2 \cdot \mathbf{e}_1 + \mathbf{e}_2 - 3 \cdot \mathbf{e}_3 - 2 \cdot \mathbf{e}_4$ and $\mathbf{a} = \mathbf{e}_2 + \mathbf{e}_3 + 2 \cdot \mathbf{e}_4$. Then, *find the 2-volume of the 2-parallelotope $P(\mathbf{a}, \mathbf{b})$.*

6. Let $\mathbb{V}$ be a 3-dimensional vector space over the field of reals, $\mathbb{R}$; let $\mathcal{A} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be an ordered basis for $\mathbb{V}$; let

$$\langle \mathbf{x}, \mathbf{y} \rangle = x_1y_1 + 7 \cdot x_2y_2 + 2 \cdot x_3y_3 + x_2y_3 + x_3y_2,$$

where $[x_1\ x_2\ x_3]^{\mathrm{T}} = [\mathbf{x}]_{\mathcal{A}}$ and $[y_1\ y_2\ y_3]^{\mathrm{T}} = [\mathbf{y}]_{\mathcal{A}}$, be the scalar product on $\mathbb{V}$; let $\mathbf{a} = \mathbf{e}_1 - \mathbf{e}_2 + 2 \cdot \mathbf{e}_3$ and $\mathbf{b} = 2 \cdot \mathbf{e}_1 - \mathbf{e}_3$. Then, *find the angle between $\mathbf{a}$ and $\mathbf{b}$.*

7. Let $\mathbb{V}$ be a 3-dimensional Euclidean space; let $\mathcal{A} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be an orthonormal basis for $\mathbb{V}$; let S be the linear span of the vectors $\mathbf{a} = -\mathbf{e}_2 + \mathbf{e}_3$ and $\mathbf{b} = \mathbf{e}_1 + \mathbf{e}_2$; let $\mathbf{x} = -2 \cdot \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3$. Then, *find* $\mathrm{rj}_S(\mathbf{x})$.

8. Let $\mathbb{R}^4$ be a 4-dimensional Euclidean space; Let $\mathcal{A}$ be an orthonormal basis for $\mathbb{R}^4$; let

$$M = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} + \left\langle \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix} \right\rangle$$

be a linear variety in $\mathbb{R}^4$ (all coordinates are with respect to $\mathcal{A}$). Then, *find a system of linear equations whose solution set equals M.*

9. Let $\mathbb{V}$ be a 3-dimensional vector space over the field of reals, $\mathbb{R}$; let $\mathcal{A}$ be an ordered basis for $\mathbb{V}$; let a quadratic form $q \in \mathcal{Q}(\mathbb{V})$ be specified by the following equality

$$q(\mathbf{x}) = x_1x_2 + 2 \cdot x_1x_3,$$

where $[x_1\ x_2\ x_3]^{\mathrm{T}} = [\mathbf{x}]_{\mathcal{A}}$. Then, *find a normal basis for q.*