CS 344: Design & Analysis of Algorithms

Lecture 1

Sep 3, 2019

Staff

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TAs:

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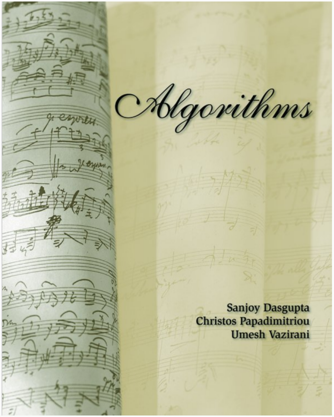
Recitations

04	Tue 8:25 PM - 9:20 PM, SEC-205 (Yuwei)
05	Thu 8:55 AM - 9:50 AM, BE-250 (Zelong)
06	Thu 8:25 PM - 9:20 PM, SEC-209 (Jianchao)
07	Tue 8:25 PM - 9:20 PM, SEC-207 (Chengguizi)

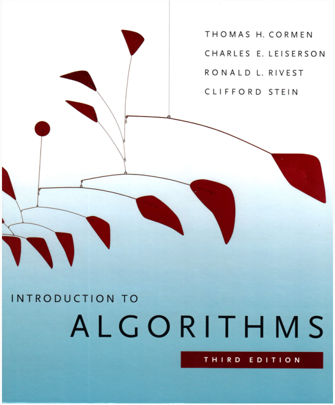
Homeworks

- 5–6 written homeworks
- 1–2 weeks for each
- small groups are allowed (≤ 3 people)

Textbooks



DPV



CLRS

Exams

- Midterm 1: Oct. 17
- Midterm 2: Nov. 14
- Final: Dec. 17 (8 pm – 11 pm)

Grading (tentative)

Homework	20%
Midterm 1	25%
Midterm 2	25%
Final exam	30%

Topics (tentative)

- Big-O notation, asymptotics
- Divide and conquer
- Dynamic programming
- Greedy algorithms
- Sorting
- Graph algorithms
- Linear programming
- Number theoretic algorithms
- Computational geometry
- String matching
- NP-completeness
- Approximation algorithms

Why algorithms?

Fibonacci numbers: $\{0, 1, 1, 2, 3, 5, 8, 13, 21, ...\}$

$$F_n = F_{n-1} + F_{n-2}$$

This grows quickly:

$$F_n \approx 2^{0.649n}$$

```python
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

Three questions:

- Is it correct?
- How much time does it take?
- Can we do better?

For $n \leq 1$, 1 operation.

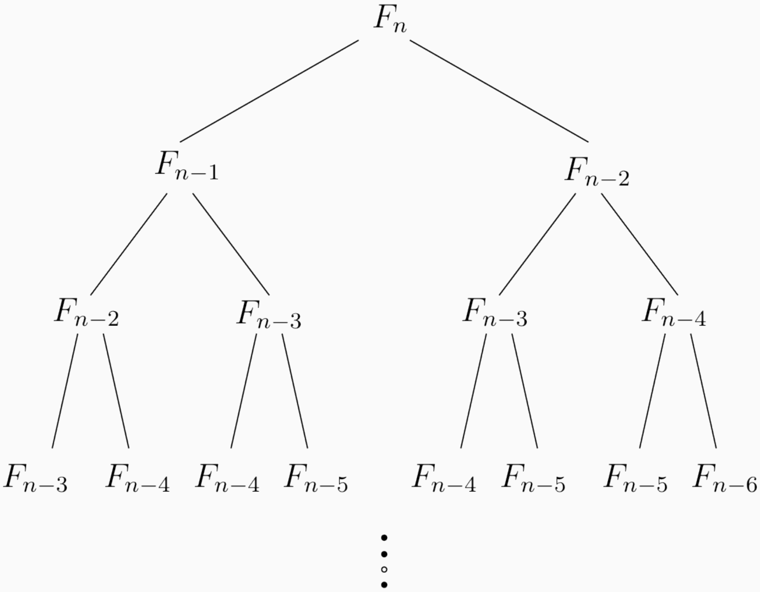
For $n > 1$,

$$T(n) = T(n-1) + T(n-2) + 3$$

$$T(n) = T(n-1) + T(n-2) + 3$$

$$F_n = F_{n-1} + F_{n-2}$$

$$F_n \approx 2^{0.649n}$$


F_n
F_{n-1}
F_{n-2}
F_{n-2}
F_{n-3}
F_{n-3}
F_{n-4}
F_{n-3}
F_{n-4}
F_{n-4}
F_{n-5}
F_{n-4}
F_{n-5}
F_{n-5}
F_{n-6}

```python
def fib2(n):
    fibs = [0, 1]
    for i in range(2, n+1):
        fibs.append(fibs[i-1] + fibs[i-2])
    return fibs[n]
```

Sum numbers 1 through n:

```
sum = 0
for i in range(1, n+1):
    sum += i
```

- Is it correct?
- How long does it take?
- Can we do better?

Sum numbers 1 through *n*:

```
sum = n * (n + 1) / 2
```

- Is it correct?
- How long does it take?
- Can we do better?

Base case ($n = 0$):

$$\sum_{i=1}^{0} i = 0 = \frac{0(0+1)}{2}$$

Inductive step:

Assume that

$$\sum_{i=1}^{n-1} i = \frac{(n-1)((n-1)+1)}{2} = \frac{(n-1)n}{2}$$

and prove that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$
\begin{aligned}
\sum_{i=1}^{n} i &= \sum_{i=1}^{n-1} i + n \\
&= \frac{(n-1)n}{2} + n \\
&= \frac{(n-1)n + 2n}{2} \\
&= \frac{n^2 - n + 2n}{2} \\
&= \frac{n^2 + n}{2} \\
&= \frac{n(n+1)}{2}
\end{aligned}
$$

How long does it take?

- One addition
- One multiplication
- One division

We generally want to abstract out certain details:

- Per-operation cost
- Specialized CPU instructions
- Compiler optimizations
- Memory layout
- Cache effects

This goes from exponential to polynomial, but is it that important?

We have Moore's law...

- How many new Fibonacci numbers could we compute?
- Exponential algorithm $\Rightarrow$ polynomial benefit
- Polynomial algorithm $\Rightarrow$ exponential benefit

We care mainly about the growth of the running time, in terms of the input size, n.

- $42n^2 + 3n + 4$
- $7n + 2$
- $2^{0.649n} + n^4 + 2n^2$

And we care mainly about the dominant factor in the running time.

- $42n^2 + 3n + 4 \rightarrow 42n^2$
- $7n + 2 \rightarrow 7n$
- $2^{0.649n} + n^4 + 2n^2 \rightarrow 2^{0.649n}$

Finally, we want to eliminate constant coefficients.

- $42n^2 + 3n + 4 \to n^2$
- $7n + 2 \to n$
- $2^{0.649n} + n^4 + 2n^2 \to 2^n$

Big-O notation:

- $42n^2 + 3n + 4 = O(n^2)$
- $7n + 2 = O(n)$
- $2^{0.649n} + n^4 + 2n^2 = O(2^n)$

Aside: "equality" in big-O notation

- $O(n) = O(n^2)$
- $O(n^2) \neq O(n)$

Aside: "equality" in big-O notation

- $O(n) \in O(n^2)$
- $O(n^2) \notin O(n)$

What does $f(n) = O(g(n))$ mean?

- $g(n)$ is a kind of upper bound on $f(n)$
- for sufficiently large n

What does $f(n) = O(g(n))$ mean?

Attempt 1:

For all n, $f(n) \leq g(n)$.

What does $f(n) = O(g(n))$ mean?

Attempt 2:

There exists some constant c, such that

for all n, $f(n) \leq c \cdot g(n)$.

What does $f(n) = O(g(n))$ mean?

There exists some constants c and N such that,

for all $n > N$, $f(n) \leq c \cdot g(n)$.

$$\exists c, N \; \forall n > N, f(n) \leq c \cdot g(n)$$

$$42n^2 + 3n + 4 = O(n^2)$$

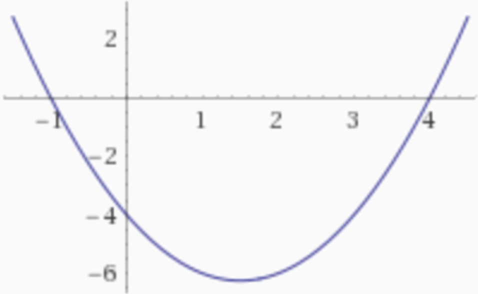
means $42n^2 + 3n + 4 \leq c \cdot n^2$ for $n > N$.

$42n^2 + 3n + 4 \leq c \cdot n^2$ for $n > N$.

Let $c = 43$. When is $42n^2 + 3n + 4 \leq 43n^2$?

$$\begin{aligned} 42n^2 + 3n + 4 &\leq 43n^2 \\ 3n + 4 &\leq n^2 \\ 0 &\leq n^2 - 3n - 4 \end{aligned}$$

$$n^2 - 3n - 4$$



$$42n^2 + 3n + 4 = O(n^2)$$

since for all $n > 4$,

$$42n^2 + 3n + 4 \leq 43n^2$$

Rules of thumb:

- Constants can be omitted: $7n \to n$
- Higher exponents dominate: $n^3 + n^2 \to n^3$
- Exponentials dominate polynomials: $2^n + n^2 \to 2^n$
- Polynomials dominate logarithms: $n + \log n \to n$

We also need to consider different inputs, since they may cause the algorithm to perform more or less work.

- Worst case
- Best case
- Average case

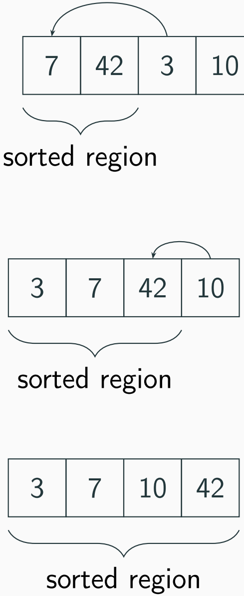
Insertion sort: work from left to right, sorting as we go, and “insert” a value where it belongs in the already-sorted region.

- Say we’re looking at element `i`
- Copy `arr[i]` to a temporary variable
- Find where that value belongs and insert it

Insertion sort:

```
for each position i:
- walk backwards from i,
  shifting values as you go,
  until you find a value less than a[i]
- put a[i] into this new position
```

$$\underbrace{\boxed{7}}_{\text{sorted region}}\boxed{42}\boxed{3}\boxed{10}$$


7 42 3 10
sorted region
3 7 42 10
sorted region
3 7 10 42
sorted region

Insertion sort:

- Worst case: $O(n^2)$
- Best case: $O(n)$

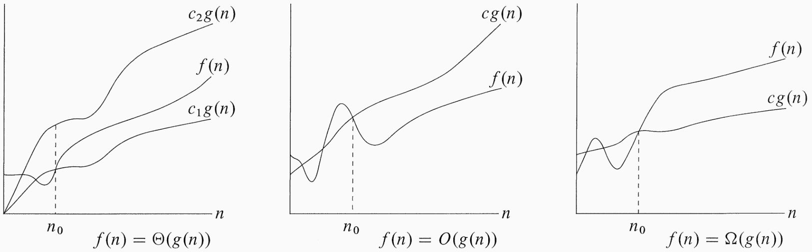
Big-O notation is similar to a “less-than” relation.

What about “greater-than” or “equal”?

- $O(\cdot)$: "less-than"
- $\Omega(\cdot)$: "greater-than"
- $\Theta(\cdot)$: "equal"

$$f(n) = \Omega(g(n)) \text{ means } g(n) = O(f(n))$$

$$\begin{aligned} f(n) = \Theta(g(n)) \text{ means } & f(n) = O(g(n)) \\ \text{and } & g(n) = O(f(n)) \end{aligned}$$


$c_2g(n)$
$f(n)$
$c_1g(n)$
n
n_0
$f(n) = \Theta(g(n))$
$cg(n)$
$f(n)$
n
n_0
$f(n) = O(g(n))$
$f(n)$
$cg(n)$
n
n_0
$f(n) = \Omega(g(n))$

- $7n^2 + 3 = O(n^2)$
- $7n^2 + 3 = O(n^3)$
- $7n^2 + 3 = \Omega(n)$
- $7n^2 + 3 = \Theta(n^2)$