A

PROJECT REPORT ON

“Modelling and Analysis of DC Motor”

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A Report submitted to MIT Academy of Engineering, Alandi (D.) submitted in partial fulfilment of the requirement for Fifth Semester of BACHELOR OF TECHNOLOGY in School of Electrical Engineering.

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CERTIFICATE

This is certify that,

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of T. Y. B. Tech. have submitted a Report on,

Modelling and Analysis of DC Motor

The said work is completed by putting the requirement of hours as per prescribed curriculum during the academic year 2019-20. The report is submitted in the partial fulfilment of the requirements for the course **Control Systems** in the Fifth Semester of Degree of Engineering in School of Electrical Engineering, MIT Academy of Engineering.

Date : 2 / 12 / 2019

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ABSTRACT

Modeling and Analysis of DC Motor of some form is the most widely used actuating mechanism in many manufacturing systems. Typical examples may include accurate positioning in robotic welding or accurate velocity in conveyor systems. There are numerous applications wherein, one encounters the need to control mechanical motions.

The DC motor has been a workhorse in the industry as it provides good torque at all speeds and can be manufactured easily and inexpensively.

LIST OF FIGURES

1. INTRODUCTION :

1.1 DC Motor :

A DC motor is any of a class of rotary electrical machines that converts direct current electrical energy into mechanical energy. The most common types rely on the forces produced by magnetic fields. Nearly all types of DC motors have some internal mechanism, either electromechanical or electronic, to periodically change the direction of current flow in part of the motor.

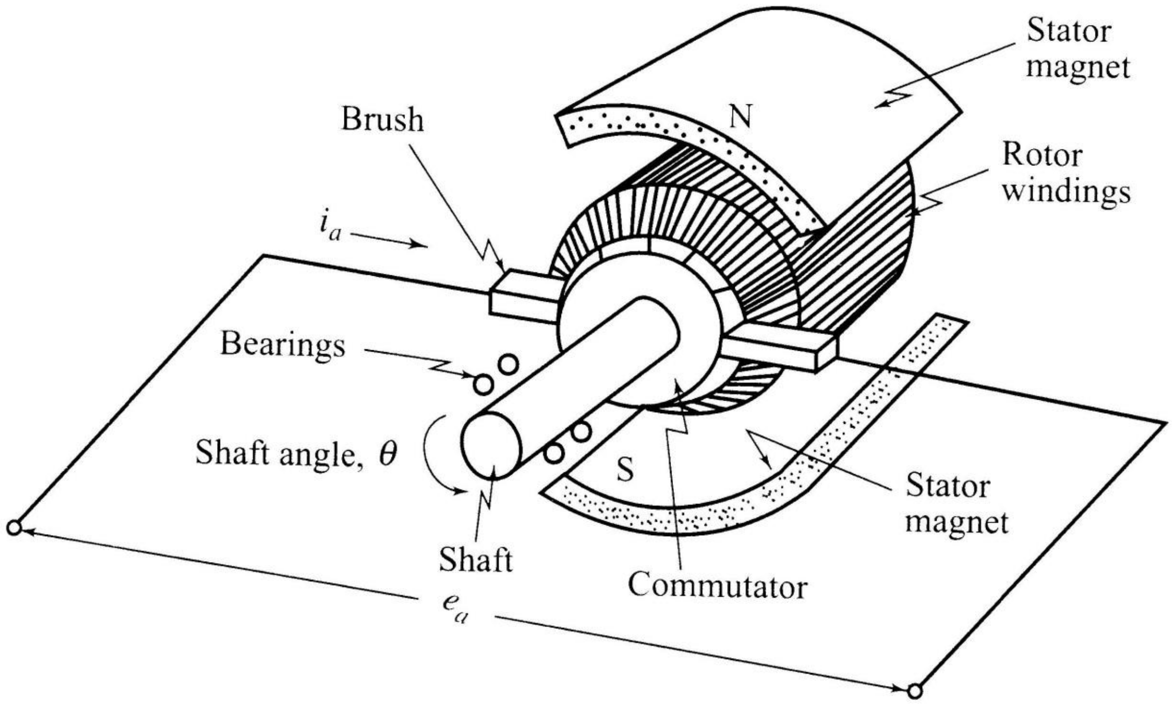


Fig No. 1.1 DC Motor Internal Structure

DC motors were the first form of motor widely used, as they could be powered from existing direct-current lighting power distribution systems. A DC motor's speed can be controlled over a wide range, using either a variable supply voltage or by changing the strength of current in its field windings. Small DC motors are used in tools, toys, and appliances. The universal motor can operate on direct current but is a lightweight brushed motor used for portable power tools.

1.2 Working of DC Motor :

A PM motor does not have a field winding on the stator frame, instead relying on PMs to provide the magnetic field against which the rotor field interacts to produce torque. Compensating windings in series with the armature may be used on large motors to improve commutation under load. Because this field is fixed, it cannot be adjusted for speed control. PM fields (stators) are convenient in miniature motors to eliminate the power consumption of the field winding. Most larger DC motors are of the "dynamo" type, which have stator windings. Historically, PMs could not be made to retain high flux if they were disassembled; field windings were more practical to obtain the needed amount of flux. However, large PMs are costly, as well as dangerous and difficult to assemble; this favours wound fields for large machines.

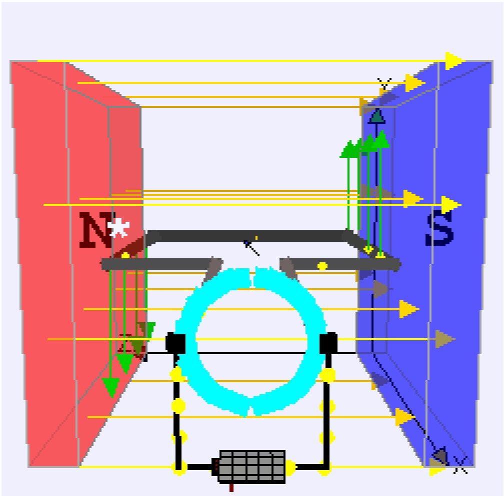


Fig No. 1.2 DC Motor Working

To minimize overall weight and size, miniature PM motors may use high energy magnets made with neodymium or other strategic elements; most such are neodymium-iron-boron alloy. With their higher flux density, electric machines with high-energy PMs are at least competitive with all optimally designed singly fed synchronous and induction electric machines. Miniature motors resemble the structure in the illustration, except that they have at least three rotor poles (to ensure starting, regardless of rotor position) and their outer housing is a steel tube that magnetically links the exteriors of the curved field magnets.

2. ELECTRICAL CIRCUIT :

Equivalent Circuit of PMDC Motor:

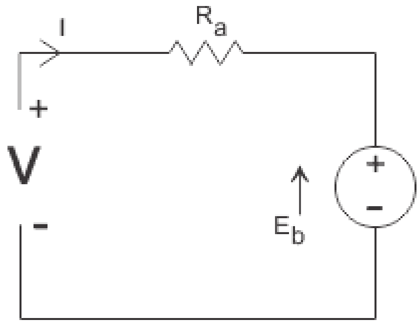


Fig No. 2.1 Circuit Diagram

As in PMDC motor the field is produced by permanent magnet, there is no need of drawing field coils in the equivalent circuit of permanent magnet DC motor.

The supply voltage to the armature will have armature resistance drop and rest of the supply voltage is countered by back emf of the motor. Hence voltage equation of the motor is given by [$V = I*R + E_b$]

Where, I is an armature current and R is armature resistance of the motor.

E_b is the back emf and V is the supply voltage.

Advantages :

1. No need of field excitation arrangement.
2. No input power in consumed for excitation which improve efficiency of DC motor.
3. No field coil hence space for field coil is saved which reduces the overall size of the motor.
4. Cheaper and economical for fractional kW rated applications.

Disadvantages :

1. In this case, the armature reaction of DC motor cannot be compensated hence the magnetic strength of the field may get weak due to demagnetizing effect armature reaction.
2. There is also a chance of getting the poles permanently demagnetized (partial) due to excessive armature current during starting, reversal and overloading condition of the motor.
3. Another major disadvantage of PMDC motor is that, the field in the air gap is fixed and limited and it cannot be controlled externally. Therefore, very efficient speed control of DC motor in this type of motor is difficult.

3. APPLICATION :

PMDC motor is extensively used where small DC motors are required and also very effective control is not required, such as in automobiles starter, toys, wipers, washers, hot blowers, air conditioners, computer disc drives and in many more.

Importance of mathematical modelling and analysis in real life application :

The process of representing real world problems in mathematical terms to find the solutions is called Mathematical modelling. A mathematical model can be considered as a simplification or abstraction of a (complex) real world problem or situation into a mathematical form, thereby converting the real-world problem into a mathematical problem. The mathematical problem can then be solved using whatever known techniques to obtain a mathematical solution. This solution is then interpreted and translated into real terms.

In a mechanical systems, motion can be of different type i.e., Translational, Rotational or Combination of both. The equations governing such motion in mechanical systems are mostly directly or indirectly governed by Newton's laws of motion.

The response of dynamic system to an input may be obtained if these differential equations are solved. Same way electrical systems are governed by Kirchhoff's Law. The mathematical description of the dynamic characteristic of a system is called as mathematical model of a system.

Significance of mathematical modelling :

1. Control systems is the arrangement of physical elements and that physical elements are analyzed to make governing equations
2. Mathematical modelling helps in easy analysis of control systems.
3. As mathematical model is in Laplace domain it is easy to analyze big systems also.

Applications of Mathematical modelling and analysis in real life :

Application 1: Drug Dosage

In prescribing drug dosage, physicians know that residual build-up depends on the time interval between administration of drug doses. They use sub-models for decay rate, assimilation rate and drug concentration with repeated equal doses for prescribing a safe and effective dosage of drug concentration and dose schedule.

Application 2: Time of Death

At the beginning of a murder investigation, a forensic pathologist will go out into the field to examine the scene and then uses Newton's Law of Cooling to approximately determine a victim's time of death.

Application 3: Reactor Risk Assessment

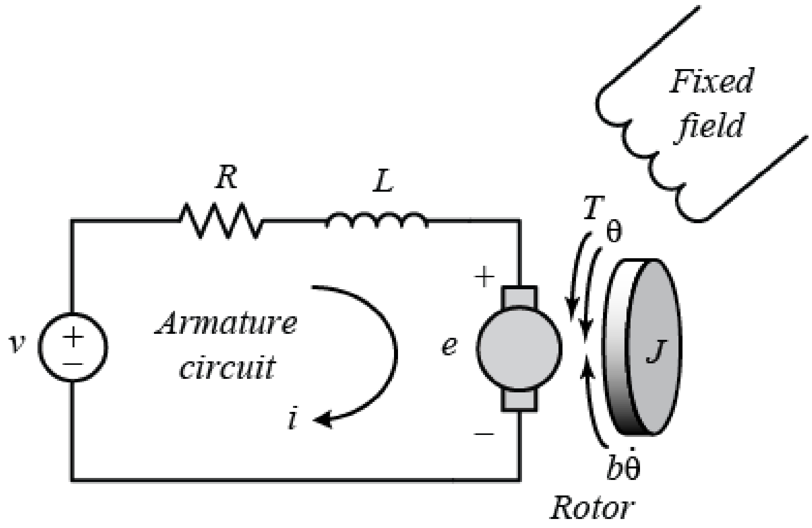
A process operator uses ODEs to analyse the transient response of a reactor in a manufacturing plant (i.e., petrochemical, semiconductor, pharmaceutical) to ensure that it is safe for continuous production without interruption.

4. METHODOLOGY :

Parameters Identification :

DC motors have wide applications in industrial control systems because they are easy to control and model. For analytical control system design and optimization, sometimes a precise model of the DC motor used in a control system may be needed. In this case, the values for reference of the motor parameters given in the motor specifications, usually provided by the motor manufacturer, may not be considered adequate, especially for cheaper DC motors which tend to have relatively large tolerances in their electrical and mechanical parameters. General system identification methods [1–4] can be applied to DC motor model identification. In particular, various methods have been applied to DC motor parameter identification; that is, [5, 6] used the algebraic identification method, [7] used the recursive least square method, [8] applied the inverse theory, [9] used the least square method, and [10] applied the moments method. Identified DC motor models are often subsequently used for controller design and/or optimization

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure.



. Fig No. 4.1 Electric Circuit with Free-body Diagram

We will assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft $\dot{\theta}$. The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

The **physical parameters** for our example are:

(J)	Moment of inertia of the rotor	0.01 kg.m^2
(b)	Motor viscous friction constant	0.1 N.m.s
(Ke)	Electromotive force constant	0.01 V/rad/sec
(Kt)	Motor torque constant	0.01 N.m/amp
(R)	Electric resistance	1 ohm
(L)	Electric inductance	0.5 H

System equations :

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current i by a constant factor K_t as shown in the equation below. This is referred to as an armature-controlled motor.

$$T = K_t i$$

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor K_e.

$$e = K_e \dot{\theta}$$

In SI units, the motor torque and back emf constants are equal, that is, $K_t = K_e$; therefore, we will use K to represent both the motor torque constant and the back emf constant.

From the figure above, we can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law.

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

Transfer Function :

Applying the Laplace transform, the above modelling equations can be expressed in terms of the Laplace variable s.

$$s(Js + b)\Theta(s) = KI(s)$$

$$(Ls + R)I(s) = V(s) - Ks\Theta(s)$$

We arrive at the following open-loop transfer function by eliminating $I(s)$ between the two above equations, where the rotational speed is considered the output and the armature voltage is considered the input.

$$P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \qquad [\frac{rad/sec}{V}]$$

State-Space :

In state-space form, the governing equations above can be expressed by choosing the rotational speed and electric current as the state variables. Again the armature voltage is treated as the input and the rotational speed is chosen as the output

$$\frac{d}{dt}\begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ i \end{bmatrix}$$

5. MATLAB MODELLING :

5.1 Transfer Function :

We can represent the above open-loop transfer function of the motor in MATLAB by defining the parameters and transfer function as follows. Running this code in the command window produces the output shown below :

```
clc;clear all;close all;
J = 0.01;
b = 0.1;
K = 0.01;
R = 1;
L = 0.5;
s = tf('s');
tf_motor = K/((J*s+b)*(L*s+R)+K^2)
```

OUTPUT :

```
tf_motor =

          0.01
  --------------------------
  0.005 s^2 + 0.06 s + 0.1001

Continuous-time transfer function.

>>
```

5.2 Controllability & Observability :

```
clc; clear all; close all;
Num = [0.01];
Den = [0.005 0.06 0.1001];
[A, B, C, D] = tf2ss(Num,Den)
Qc = [B A*B A*A*B]
r  = rank(Qc);
Q0 = [C; C*A; C*A*A]
r1 = rank(Q0);
if r == rank(A)
    disp('your system is contrallable');
else
    disp('your system is not contrallable');
end
if r1 == rank(A)
    disp('your system is observable');
else
    disp('your system is not observable');
end
```

OUTPUT :

```
A =

  -12.0000  -20.0200
    1.0000         0


B =

     1
     0


C =

     0     2


D =

     0


Qc =

    1.0000  -12.0000  123.9800
         0    1.0000  -12.0000


Q0 =

         0    2.0000
    2.0000         0
  -24.0000  -40.0400

your system is contrallable
your system is observable
>> |
```

Fig No. 5.1 Controllability & Observability Output

5.3 Response :

```
clc; clear all; close all;
Num = [0.01];
Den = [0.005 0.06 0.1001];
Tf = tf(Num,Den);
Tf = feedback(Tf,1)
step(Tf)                        % step response
t = 0:0.1:50;
u = t;
[y, t, x] = lsim(Tf,u,t);
figure;
plot(t,y,'b',t,u,'g')           % ramp response
v = 0.5*t.*t;
[z, k] = lsim(Tf,v,t);
figure;
plot(t,z,'b',t,v,'g')           % parabolic response
```

5.3.1 Step Response :

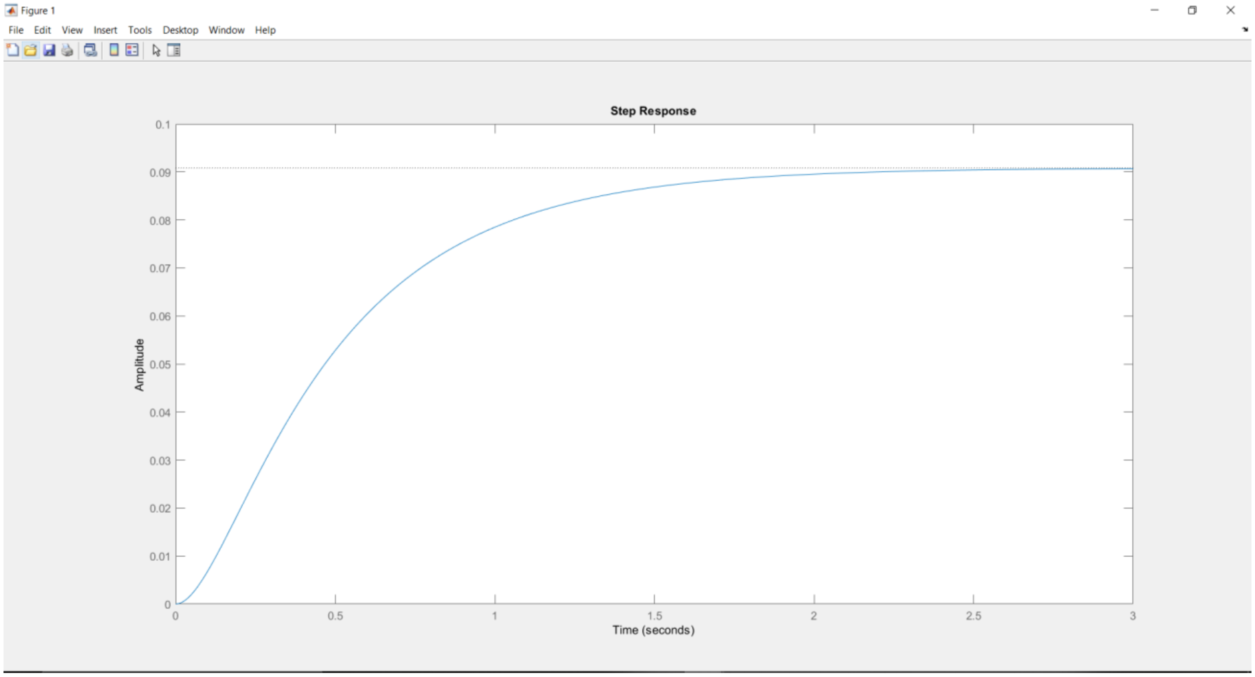


Fig No. 5.2 Step Response Output

5.3.2 Ramp Response :

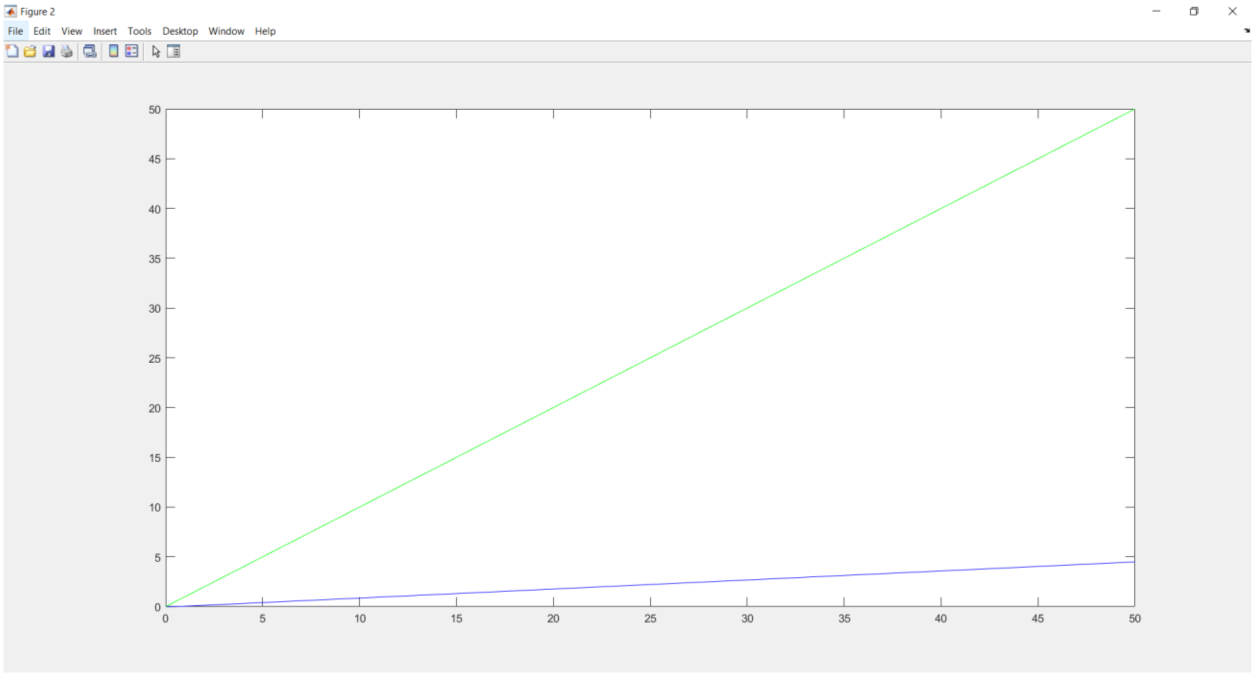


Fig No. 5.3 Ramp Response Output

5.3.3 Parabolic Response :

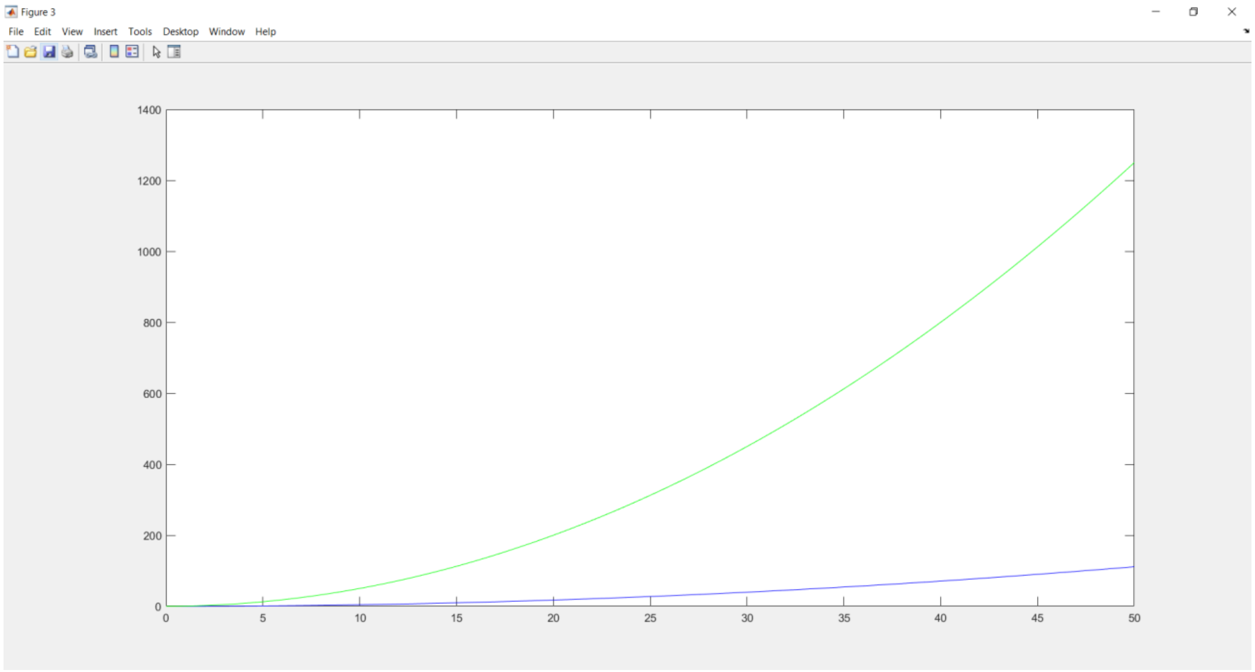


Fig No. 5.4 Parabolic Response Output

5.4 Time Domain Specifications :

```
clc;clear all;close all;
Num = [0.01];
Den = [0.005 0.06 0.1001];
wn = sqrt(Den(1*3))
zeta = (Den(1*2)) / (wn*2)

if ((zeta > 0) && (zeta < 1))
    disp('underdamped');
    td = (1 + 0.7*zeta) / wn
    wd = wn*(sqrt(1 - zeta^2))
    theta = acos(zeta)
    tr = (pi - theta)/wd
    tp = pi/wd
    mp = 100*2.72^((-pi* zeta)/sqrt(1-zeta^2))
    ts = 4/(zeta*wn)
elseif (zeta == 1)
    disp('critically damped');
elseif (zeta > 1)
    disp('overdamped');
end
```

OUTPUT :

```
wn =

    0.3164


zeta =

    0.0948

underdamped
td =

    3.3705


wd =

    0.3150


theta =

    1.4758


tr =

    5.2888


tp =

    9.9746


mp =

   74.1244

ts =

  133.3333

>>
```

Fig No. 5.5 Time Domain Specs Output

5.5 Root Locus :

```
clc; clear all; close all;
H = 1;
Num = [0.01];
Den = [0.005 0.06 0.1001];
ol_tf = tf(Num, Den)
rlocus(ol_tf);
```

OUTPUT :

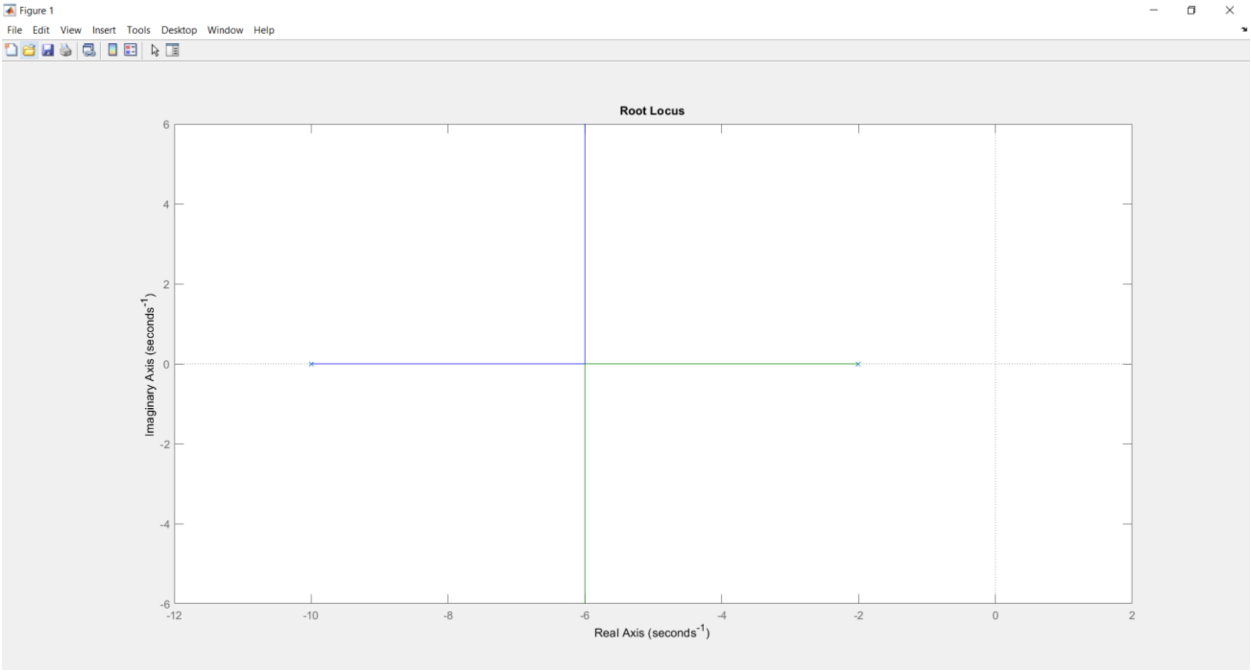


Fig No. 5.6 Root Locus Output

5.6 Bode Plot :

```
clc; clear all; close all;
H = 1;
Num = [0.01];
Den = [0.005 0.06 0.1001];
ol_tf = tf(Num, Den)
bode(ol_tf);
```

OUTPUT :

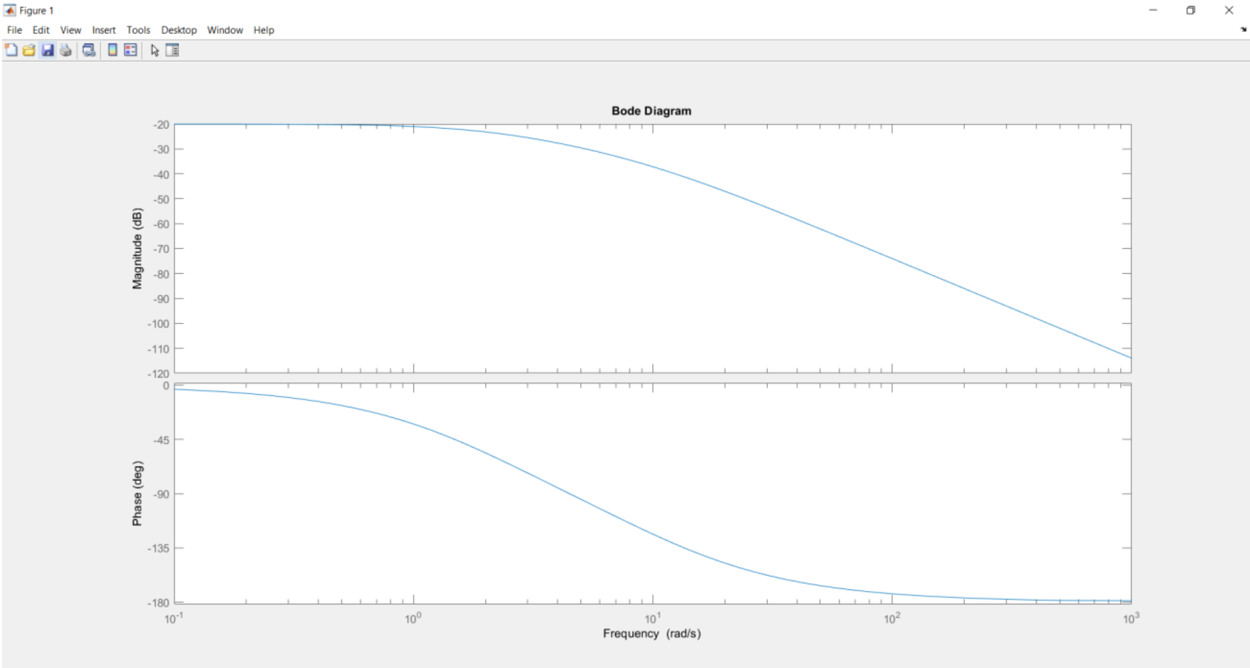


Fig No. 5.7 Bode Plot Output

5.7 PID Control Design :

```
clc; clear all; close all;
H = 1;
Num = [0.01];
Den = [0.005 0.06 0.1001];
ol_tf = tf(Num, Den)
cl_tf = feedback(ol_tf, H)
step(cl_tf);
grid on
hold on
%%
Kp = 24;
Ki = 5;
Kd = 10;
ol_gc = pid(Kp, Ki, Kd);
cl_gc = feedback(ol_gc*ol_tf, H);
step(cl_gc);
```

OUTPUT :

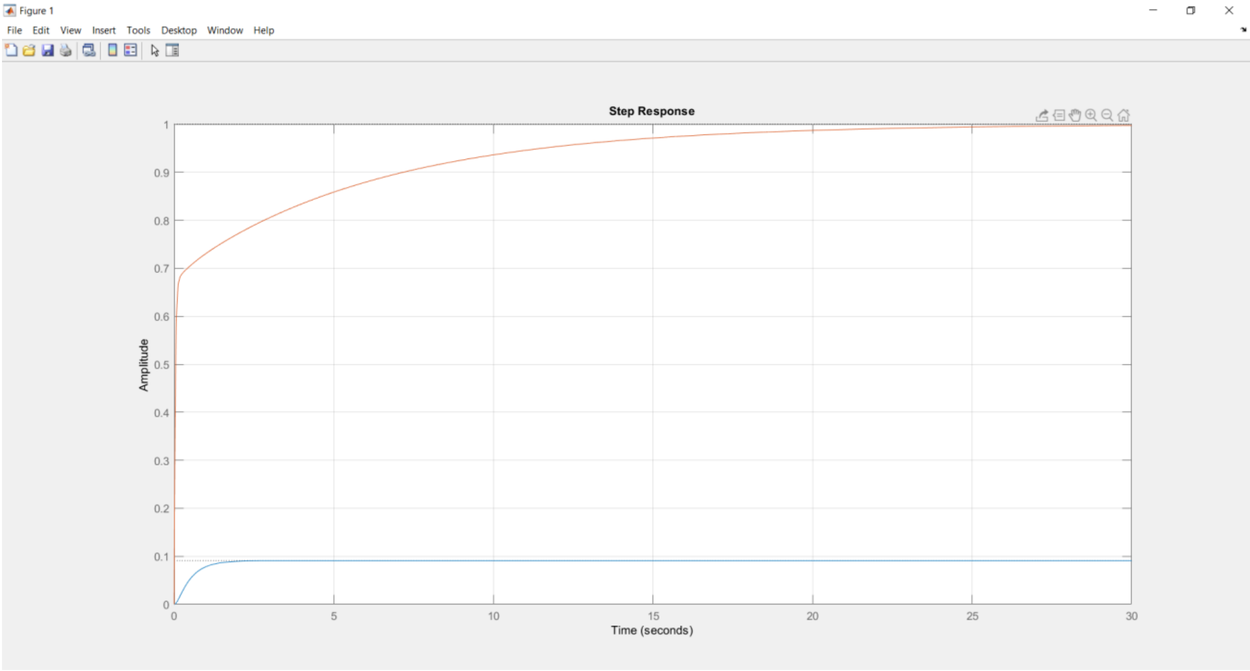


Fig No. 5.8 PID Control Response

6. CONCLUSION :

In real life no system works the way it is intended to. No matter how the accuracy of the system is increased no one can reach the desired state at the point of time it is intended to. So, in order to achieve the results in the best way possible numerous filters are applied to the applications like Rocket thrusters, DC Motor, Steering Wheel, etc. These filters act as a controller to the system as they take feedback from the sensors or external devices and give output correspondingly to correct the error which was generated due to inaccuracy. Since, These errors couldn’t be minimized suddenly or as expected and hence filters like PID controller, Kalman filter, Hybrid controller etc. techniques are used to get desired output in as stable state as possible.

The general block diagram of a filter applied to a system is attached herewith. And these functions require the Transfer Function in order to work and these Transfer Function can only be obtained by knowing all the parameters of the system which in case of DC Motors were Impedance, Inductance, Resistance and Capacitance along with their orientation of interconnection.

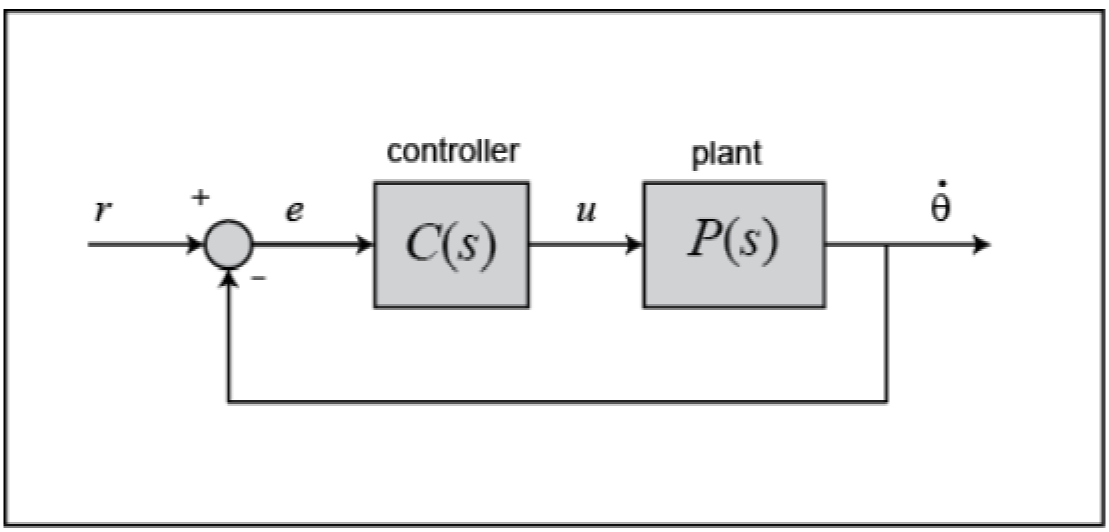


Fig No. 6.1 Controller Block

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