

Relationship between IQ and symptom severity in ASD

To investigate the relationship between various aspects of IQ and symptom severity in children with ASD data from 72 children were collected. Each children came 6 times for data collection however only data from visit 1 are used in the rest of paper. In this sample of children there were 61 boys and 11 girls with average age of 26.26 months ($sd=7.34$). There were 34 autistic children and 38 typically developing children. The symptom severity was measured on ADOS scale from 0 to 21. 3 aspects of IQ were measured: verbal, nonverbal and social.

All the analysis reported below was performed in bayesian framework and the models were implemented in R (1) using RStudio (1) and package rethinking (3). For the visualizations R packages ggplot2 (4), plot3D (5) and bayesplot (6) were used. For stilling of all plots R package ggthemes (7) was used.

1. The relation between symptom severity and IQ modeled as bivariate regression

In the first part we modeled the relation between symptom severity measured as ADOS and each aspect of IQ as a separate bivariate regression. Thus 3 models are reported. In order to interpret and compare the results better all measurements of IQ were standardized so that mean of the variables equals 0 and standard deviation 1. Each of the models is therefore in following form;

$$\text{ADOS} \sim \alpha + \beta \cdot \text{IQ}$$

Furthermore in this part of analysis only data from first visit and children with ASD were used. That means that our sample had. 34 children with mean age of 32.77 ($sd=5.27$). From the 34 children 29 were boys.

ASD and Verbal IQ

The first relationship modeled was between ADOS and verbal IQ. That means that the model's equation was:

$$\text{ADOS} \sim \alpha + \beta \cdot \text{VerbalIQ}$$

PRIORS

As this model is a bayesian linear model motivated priors for all parameters of the model were defined. They are reported in table 1.

The parameter alpha (α) defines the mean of the dependent variable, ADOS in our case. As we have only children with ASD we would expect the mean to be above the 50% of the ADOS scale but not by much. Therefore the prior for alpha is a normal distribution with mean at 14. To define a normal distribution we need also standard deviation. As the prior is defined only by logical deduction we need to keep them relatively weak therefore we use standard deviation of 1. Such

Priors	
α	Normal (14, 1)
β	Normal (0, 0.5)
σ	Uniform (0, 5)

Table 1: Priors of model using verbal IQ as predictor

sd would imply that 95% of possible means are in range of 12 to 16 which seems as a reasonable estimate.

The parameter β defines an effect of verbal IQ on ADOS. We want to remain skeptical of the relationship and therefore set the mean of the normal prior at 0. By doing that we weight the possibility of having a negative effect the same as having a positive effect. Next we need a standard deviation, in other words what is the strongest effect we believe is possible. We set the sd at 0.5 which in the most extreme case would imply a change of ADOS by 1 with verbal IQ equaling to 1 on the standardized scale. This is rather broad prior that allows both quite strong effects and weak to no effects.

The last prior is for σ or in other words the standard deviation of ADOS. This prior will be uniform from 0 to 5. From 0 because a negative SD does not exist. The upper limit implies that 95% of ADOS scores are ± 8 units from the mean. Given the prior of mean at 14 this would mean that the score of ADOS is most probably between 6 and 24 which is possible as the maximum of ADOS is 21 and there should not be any children in our sample that have very low ADOS as that would mean a typically developing child.

RESULTS

The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 2. For better understanding of the results implied by these estimates the simulated means (black line), their 89% credibility intervals (dark blue) and 89% credibility intervals of mean combined with σ (light blue) were superimposed on the original data. The resulting plot is reported in figure 1.

The estimate of alpha implies that the mean of ADOS is 13.897 which is very close to the actual mean of ADOS (13.853). The effect of -1.22 means that with increase of verbal IQ by 1 the ADOS score decreases by 1.22. And finally the σ implies a standard deviation of ADOS of 3.8 which is again very close to the SD of our sample (4.646).

In short this model indicates that there is a negative relationship between ADOS and verbal IQ.

	Mean	StdDev	5.5%	94.5%	α	β	sigma
α	13.897	0.546	13.024	14.769	1	0.007	0.015
β	-1.22	0.453	-1.943	-0.497	0.007	1	0.472
sigma	3.801	0.523	2.965	4.636	0.015	0.472	1

Table 2: Parameter estimates of model using verbal IQ as predictor

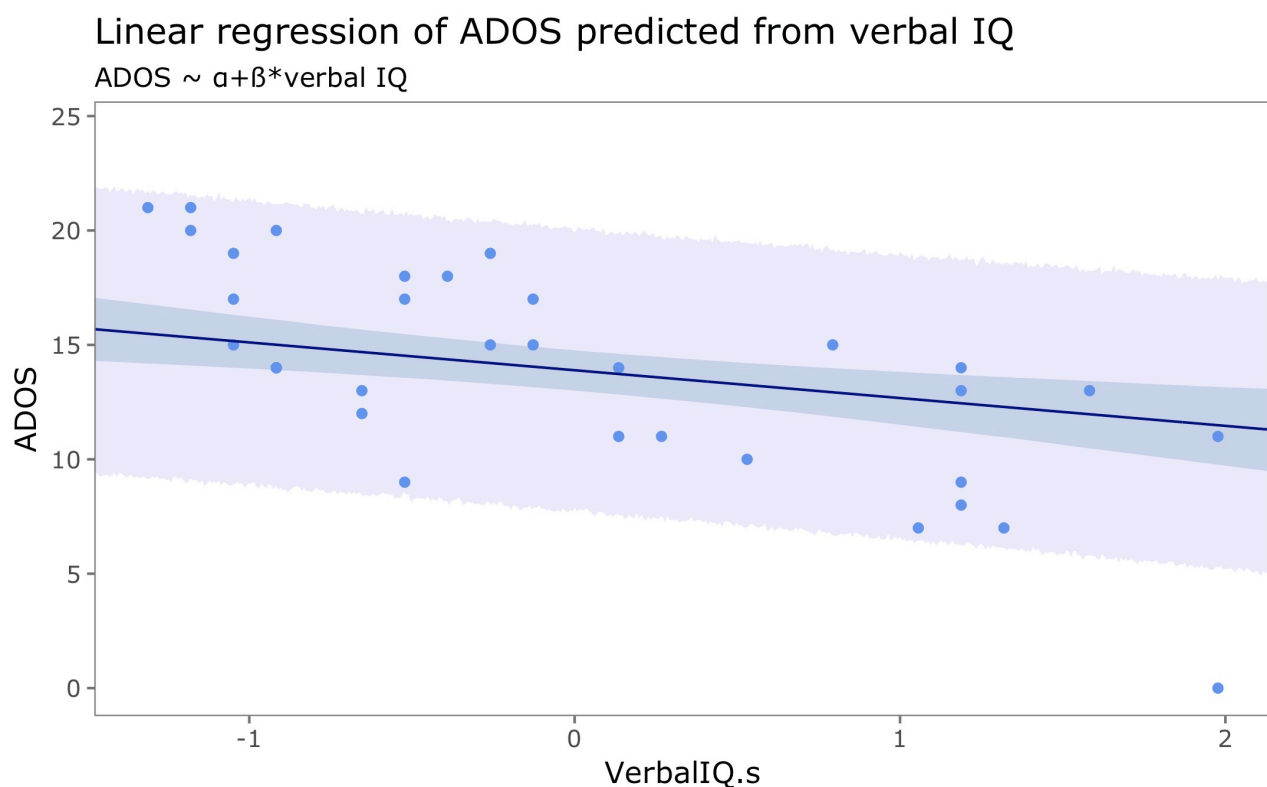


Figure 1: Relationship of verbal IQ and ADOS

QUALITY CHECK

One way of checking the quality of the model is already implied in the plot in figure 1. It's a rather simple check whether our model fits the data correctly. As all the datapoints, except for one, are captured by the 89% credibility interval (light blue) of $\mu + \sigma$, the model seems to be fitted correctly as only one datapoint would not be expected by the model.

Second quality check is visualized in figure 2. One of the assumptions of the model are uncorrelated parameters which is what this plot does, it plots 10 000 samples from posterior of all parameters against each other. None of the parameters is strongly correlated although there is a correlation between β and σ (.472) as seen by the slight elongation of the cloud of points in the plot.

Last quality check could be done by testing how does the choice of priors affect the model.

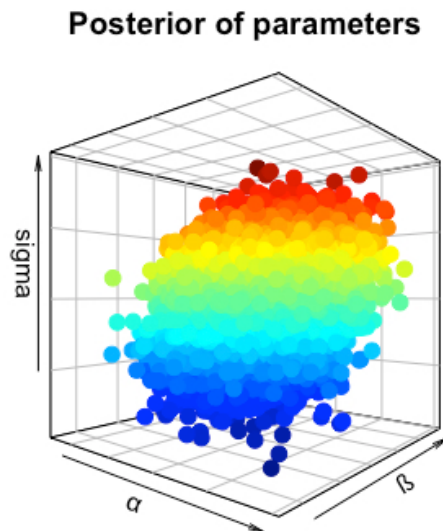


Figure 2: Posterior distribution of regression using verbal IQ as predictor

ASD and Nonverbal IQ

As second the modeled relationship was between ADOS and non-verbal IQ. That means that the model's equation was:

$$\text{ADOS} \sim \alpha + \beta * \text{Non-verbal IQ}$$

PRIORS

Since the dependent variable is still ADOS and the independent variable is standardized we can use the same priors for α and σ .

As for the β the variance of non-verbal IQ is higher than the one of verbal IQ by approximately 1 SD and therefore we can broaden the prior to encompass the possibility of having a stronger effect. The mean of the normal prior stays at 0 but the SD is increased to 0.6. All the priors are reported in table 3.

Priors	
α	Normal (14, 1)
β	Normal (0, 0.6)
σ	Uniform (0, 5)

Table 3: Priors of model using verbal IQ as predictor

RESULTS

The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 4. The results are visualized in the same way as in previous model in figure 3.

The estimate of α implies that the mean of ADOS is 13.901 which is very close to the actual mean of ADOS (13.853). The effect of -1.133 means that with increase of non-verbal IQ by 1 the ADOS score decreases by 1.133. And finally the σ implies a standard deviation of ADOS of 4.036 which is again very close to the SD of our sample (4.646).

In short this model indicates that there is a negative relationship between ADOS and non-verbal IQ.

	Mean	StdDev	5.5%	94.5%	α	β	sigma
α	13.901	0.569	12.991	14.81	1	0.005	0.015
β	-1.133	0.487	-1.911	-0.355	0.005	1	0.348
sigma	4.036	0.522	3.201	4.87	0.015	0.348	1

Table 4: Parameter estimates of model using non-verbal IQ as predictor

Linear regression of ADOS predicted from non-verbal IQ

$$\text{ADOS} \sim \alpha + \beta * \text{NonVerbal IQ}$$

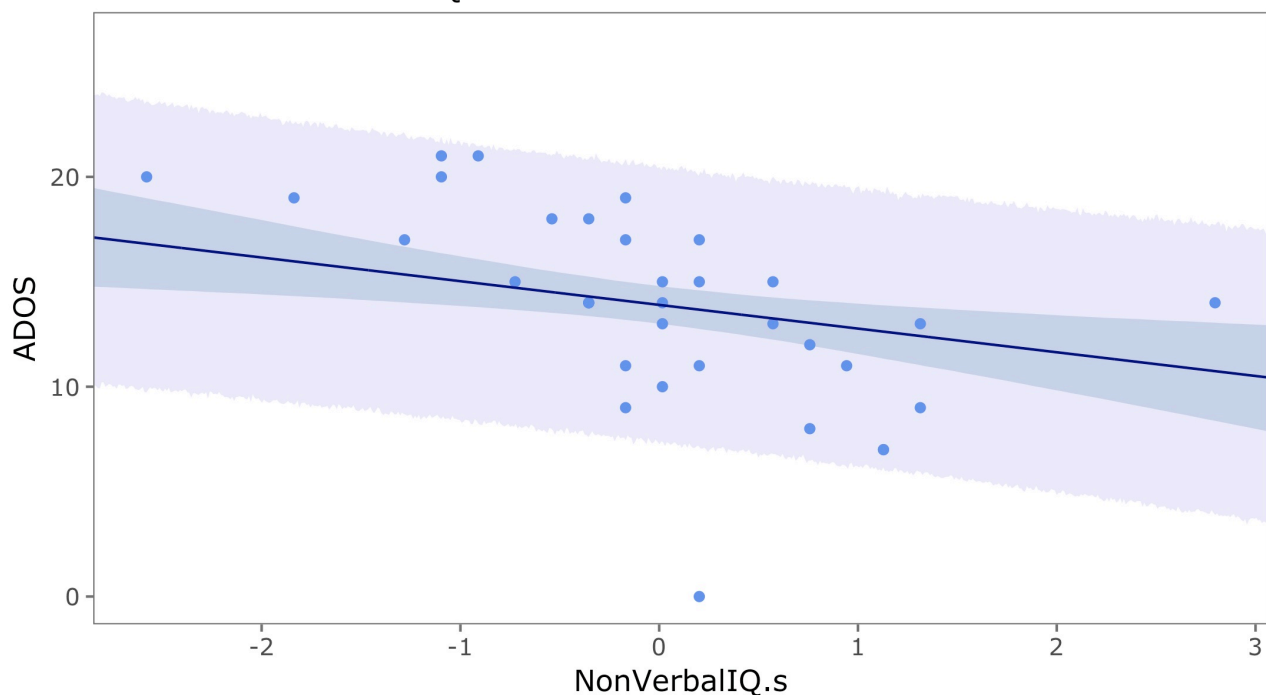


Figure 3: Relationship of non-verbal IQ and ADOS

QUALITY CHECK

We adopt the order of quality checks from the previous model report.

All the datapoints, except for one, are captured by the 89% credibility interval (light blue) of $\mu + \sigma$, the model seems to be fitted correctly as only one datapoint would not be expected by the model. Second quality check is visualized in figure 4. None of the parameters is strongly correlated although there is a correlation between β and σ (.348) as seen by the very small elongation of the cloud of points in the plot.

Posterior of parameters

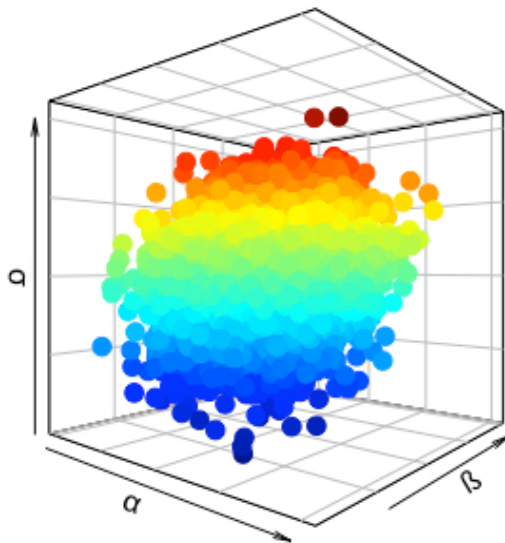


Figure 4: Posterior distribution of regression using non-verbal IQ as predictor

ASD and Social IQ

Finally the relationship between ADOS and social IQ was modeled. That means that the model's equation was:

$$\text{ADOS} \sim \alpha + \beta * \text{SocialIQ}$$

PRIORS

Since the dependent variable is still ADOS and the independent variable is standardized we can use the same priors for α and σ .

As for the β the variance of social IQ is approximately the same as of verbal IQ therefore we can use the prior used before. That is a normal distribution with mean at 0 and standard deviation of 0.5.

RESULTS

The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 5. The results are visualized in the same way as in previous models in figure 5.

The estimate of α implies that the mean of ADOS is 13.899 which is very close to the actual mean of ADOS (13.853). The effect of -1.077 means that with increase of social IQ by 1 the ADOS score decreases by 1.077. And finally the σ implies a standard deviation of ADOS of 3.946 which is again very close to the SD of our sample (4.646).

In short this model indicates that there is a negative relationship between ADOS and social IQ.

	Mean	StdDev	5.5%	94.5%	α	β	sigma
α	13.899	0.561	13.003	14.795	1	0.006	0.015
β	-1.077	0.446	-1.79	-0.364	0.006	1	0.423
sigma	3.946	0.528	3.102	4.79	0.015	0.423	1

Table 5: Parameter estimates of model using social IQ as predictor

Linear regression of ADOS predicted from social IQ

$$\text{ADOS} \sim \alpha + \beta * \text{Social IQ}$$

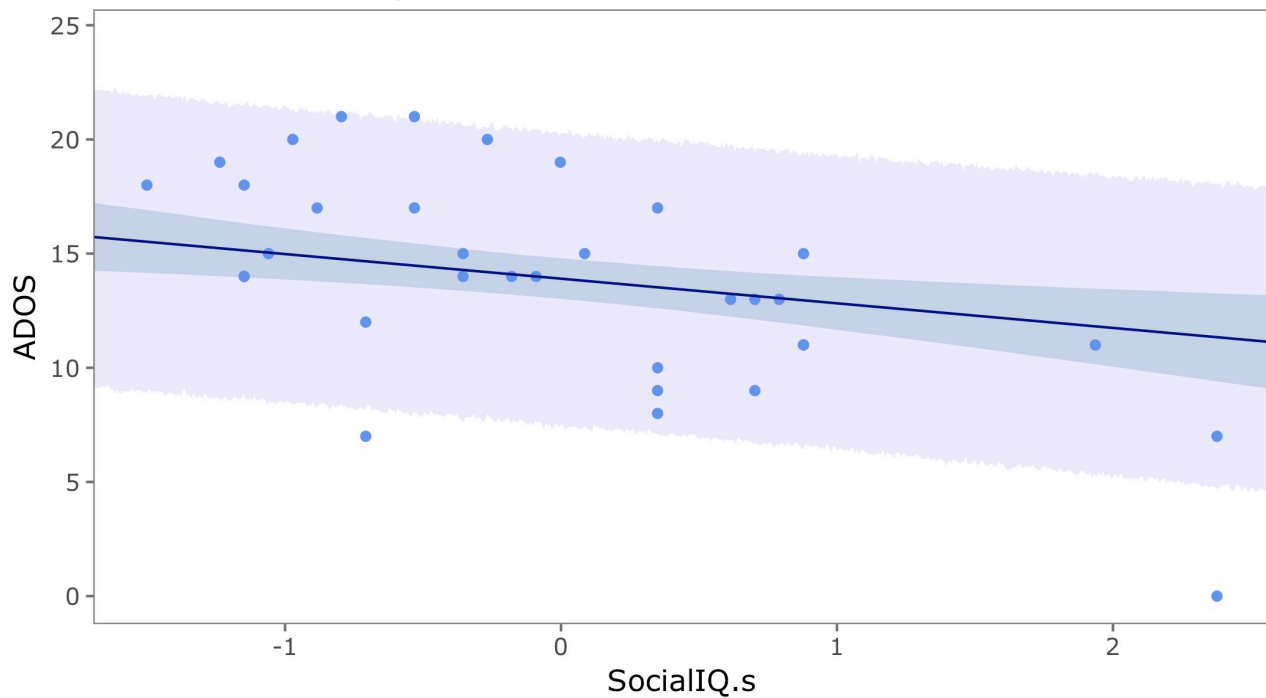


Figure 5: Relationship of social IQ and ADOS

QUALITY CHECK

We adopt the order of quality checks from the previous model report.

3 datapoints are not captured by the 89% credibility interval (light blue) of $\mu + \sigma$, but we can still call the model correctly fitted.

Second quality check is visualized in figure 4 . None of the parameters is strongly correlated although there is a correlation between β and σ (.0.423) as seen by the small elongation of the cloud of points in the plot.

Posterior of parameters

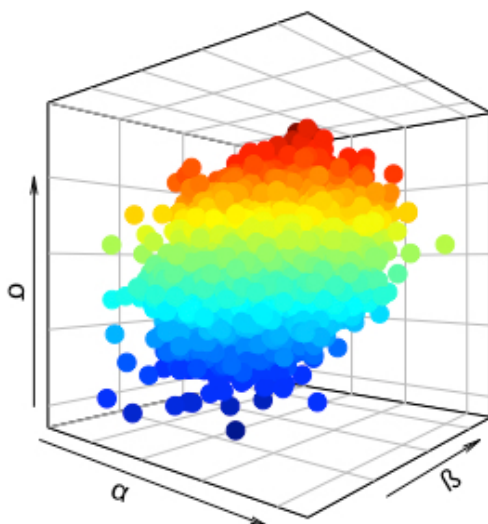


Figure 6: Posterior distribution of regression using social IQ as predictor

2. Do the different aspects of IQ account for different portions of the variance in ADOS?

An obvious way of investigating this question is to build a regression model with all 3 aspects of IQ as predictors of ADOS.

Does it make sense to build such model? Let's assume it does for now and go with that unless we encounter something that would not be compatible with such assumption.

To be able to use all 3 IQs we need to test that they do not share too much variance. Too much shared variance would suggest an existence of general IQ that can describe a person's intelligence without need of additional aspects of IQ. In other words that there are no different aspects of IQ, just one general intelligence quotient.

The correlations between different IQs are reported in table 6. Correlation was tested using linear regression with regularizing priors.

	Mean	StdDev	5.5%	94.5%
Verbal ~ NonVerbal	0.67	0.13	0.46	0.87
Verbal ~ Social	0.60	0.13	0.38	0.81
NonVerbal ~ Social	0.37	0.16	0.11	0.62

Table 6: Correlation among different aspects of IQ

From the table we can see there is no perfect correlation. Values of .6 are quite high however there seems to be still information to learn from each aspect of IQ after knowing the other.

Therefore we can build the above mentioned regression model. We can expect the social IQ to have the largest effect estimate because it is the least correlated of all IQs measures. On the other hand it is possible that estimate of nonverbal IQ will be the lowest and can overlap with zero as its correlation with verbal IQ is very high (almost .9).

We can now move on to building the model with following formula:

$$\text{ADOS} \sim \alpha + \beta \cdot \text{SocialIQ} + \beta \cdot \text{VerbalIQ} + \beta \cdot \text{NonVerbalIQ}$$

The prior of intercept remains a normal distribution with mean 14 and sd 1. For the β parameters regularizing skeptical priors were defined: Normal (0,1). Prior for sigma is: Normal (3,1).

The results of the model are reported in table 7.

	Mean	SD	5.5%	94.5%	a	β Social	β Verbal	β NonVerbal	sigma
a	13.881	0.439	13.18	14.583	1	0.001	0.001	0	0.012
β Social	-1.507	0.512	-2.326	-0.688	0.001	1	-0.398	-0.047	0.094
β Verbal	-1.463	0.581	-2.391	-0.535	0.001	-0.398	1	-0.473	0.071
β NonVerbal	-0.91	0.529	-1.756	-0.064	0	-0.047	-0.473	1	0.01
sigma	2.848	0.335	2.314	3.383	0.012	0.094	0.071	0.01	1

Table 7: Parameter estimates of model using all IQs as predictors

As predicted above the social and verbal IQ estimates are larger than that of nonverbal IQ although they overlap. The estimate of nonverbal IQ is very close to zero.

This model suggests that different aspects of IQ indeed account for different portions of variance of ADOS as there is always a new knowledge to learn for the model from each aspect of IQ after

already knowing the other two. It seems that higher ADOS score is associated with deficiencies in all three measured aspects of IQ.

To assess the model quality we can look at correlation of parameters reported in table 7 above. We can see that none of the parameters is strongly correlated with others. Furthermore a posterior predictive check plot was built that shows simulated distribution of ADOS as predicted by the model. This plot is shown in figure 7. The model seems to be fitted correctly the tails of the outcome. However around the mean the values are both strongly overestimated and underestimated. The model is not performing very good.

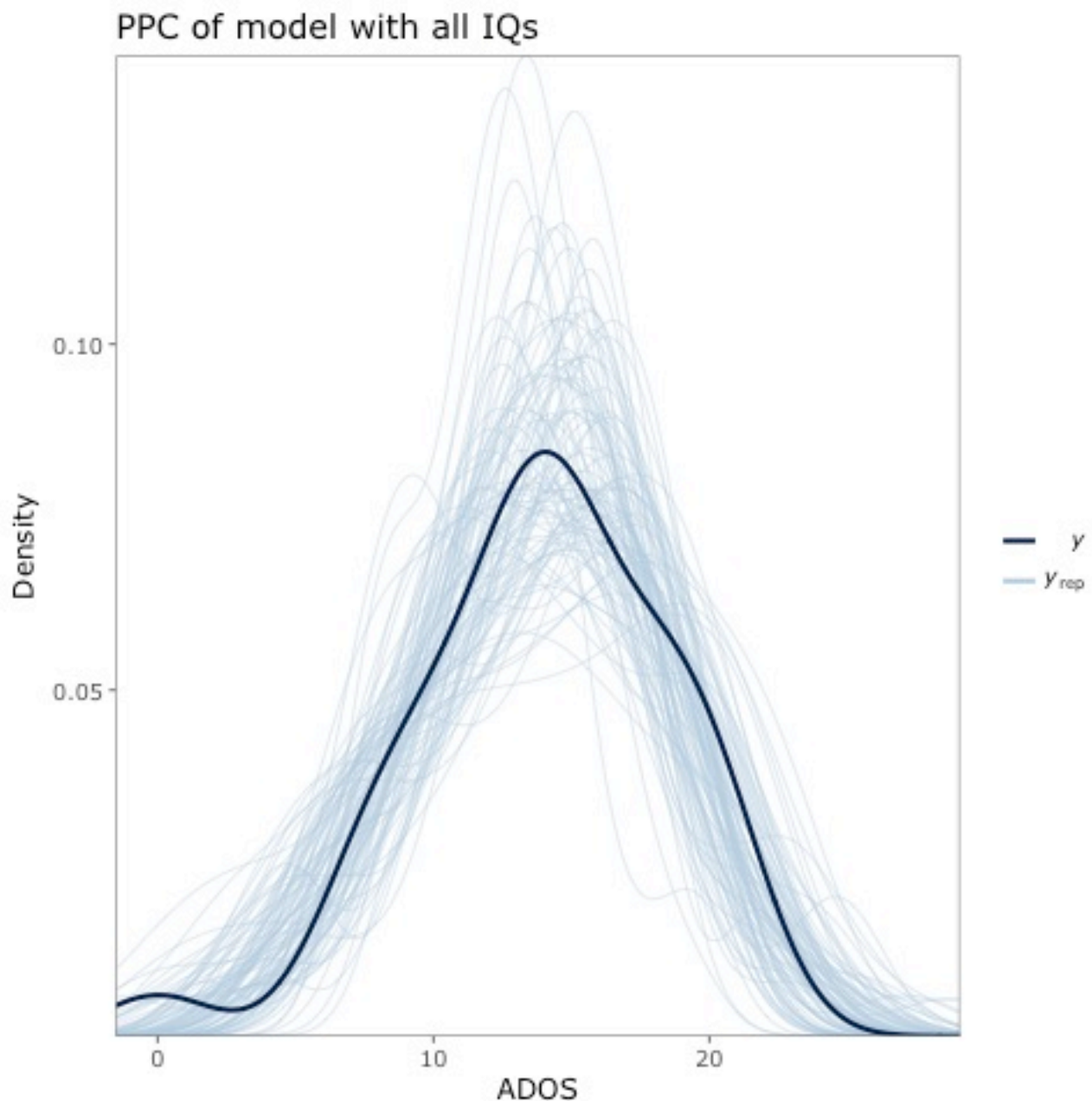


Figure 7: Posterior predictive check plot of ADOS predicted from all 3 aspects of IQ.

3. Including TD children

So far the models were fitted to data of only autistic children. We have additional data from 38 typically developing (TD) children so we can include these in our analysis.

Does it make sense to ask whether IQ and ADOS are related? The answer is both yes and no.

First the yes option; we still expect to see lower IQ values with increase on ADOS scale. However the model will probably perform poorly when predicting TD. The main reason is that although we expect TD children to have higher IQ on average than ASD children there will be remaining variance in IQ that cannot be explained by ADOS. In other words it is possible that some TD children have lower IQ than ASD children due to unmeasured factors such as genetics, education etc. So actually it does not make sense to ask this question when TD children are included. The model will answer the question similarly as if there were only ASD children and will predict quite chaotically in case of the TD children.

There will be an additional problem with fitting the model. When using all collected data the distribution of ADOS is no longer normal. It is bimodal, one mode for each diagnosis. The ADOS of ASD children is normal but the distribution of TD children is exponential. Therefore the model estimated using quadratic approximation is likely to behave strangely as it will try to estimate the distribution of TD children as normal. This problem is visualized in figure 8.

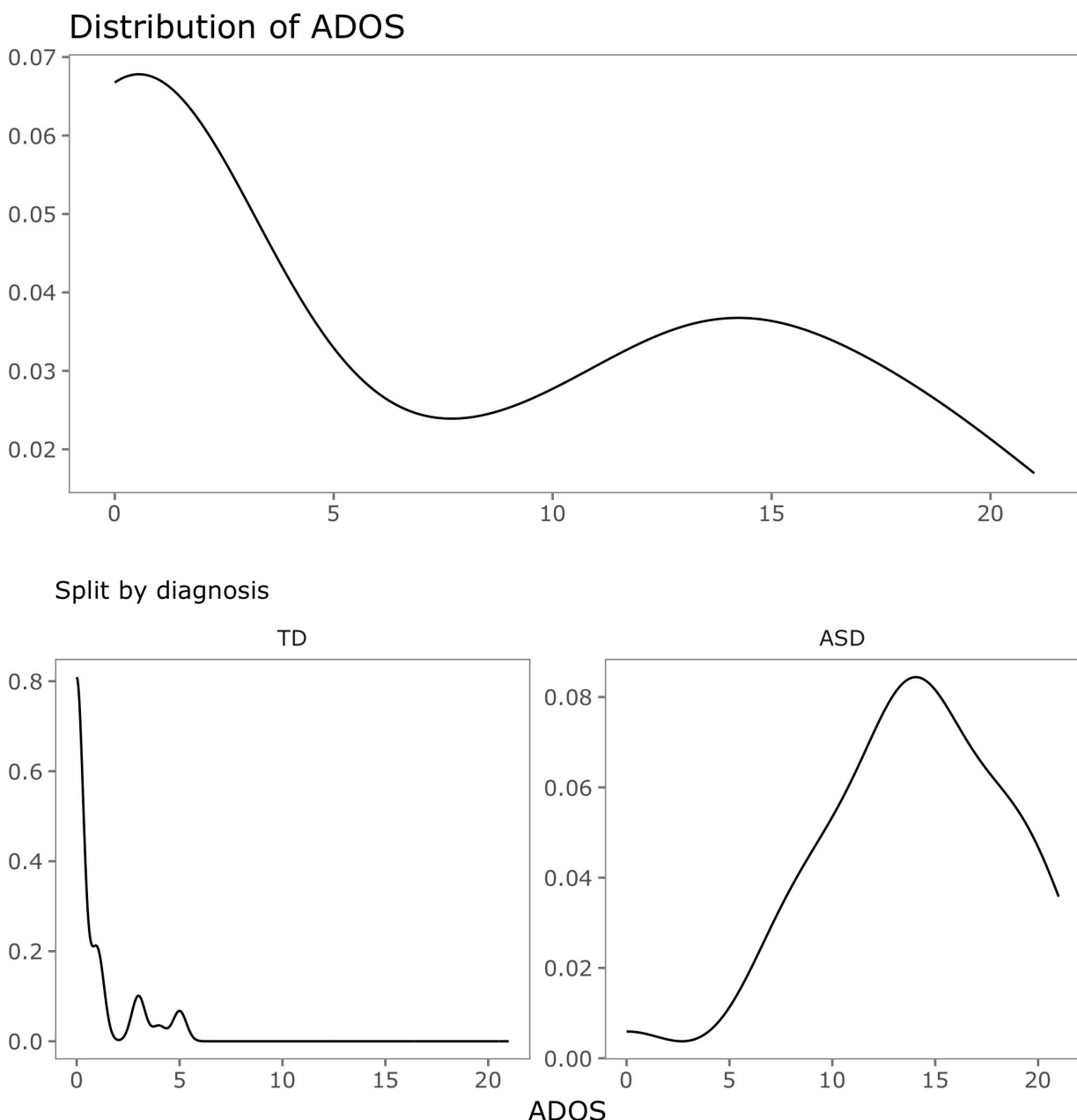


Figure 8: Distribution of ADOS including ASD and TD children (top) and split by diagnosis (bottom).

Let's build the model anyway in following form:

$$\text{ADOS} \sim \alpha + \beta \cdot \text{SocialIQ} + \beta \cdot \text{VerbalIQ} + \beta \cdot \text{NonVerbalIQ} + \beta \cdot \text{ASD}$$

The ASD predictor is a dummy variable indicating autism of the child.

The prior of intercept is a normal distribution with mean 5 and sd 2. This prior was chosen because now we have both ASD and TD children in our data and the mean of ADOS should be considerably lower since TD children should have ADOS of approximately 0. For the β parameters of IQ predictors regularizing skeptical priors were defined: Normal (0,0.5). Prior for ASD is also a skeptical prior in form of normal distribution: Normal (0,2). Prior for sigma is: Normal (3,1).

The results of this model are reported in table 8.

	Mean	SD	5.5%	94.5%	a	β Social	β Verbal	β NonVerbal	β ASD	sigma
a	2.619	0.512	1.801	3.436	1	-0.424	0.025	0.095	-0.758	0.156
β Social	-1.442	0.37	-2.032	-0.851	-0.424	1	-0.233	0.004	0.566	0.11
β Verbal	-0.81	0.325	-1.329	-0.291	0.025	-0.233	1	-0.412	-0.031	0.061
β NonVerbal	-0.484	0.314	-0.987	0.018	0.095	0.004	-0.412	1	-0.124	0.074
β ASD	9.455	0.816	8.150	10.760	-0.758	0.566	-0.031	-0.124	1	-0.175
sigma	2.832	0.265	2.408	3.256	0.156	0.11	0.061	0.074	-0.175	1

Table 8: Parameter estimates of model using all IQs as predictors and including TD children

The model confirms our hypothesis about negative relationship between ADOS and social and verbal IQ. The estimate of effect of non-verbal IQ overlaps with zero and is therefore not very strong. Furthermore the model estimates the children with ASD to have 9.5 higher score on ADOS scale on average than the TD children.

To investigate whether we should trust these results a quality check was performed. From the table 8 we can see that some of the parameters are strongly correlated. For example alpha and β ASD have correlation coefficient of -0.759. Further problems of the model fit can be seen from the predictive posterior check plot reported in figure 9. The first mode of the posterior seems to be fitted quite good although the model is underestimating strongly. Bigger problems come after the first mode where the model is largely overestimating the outcome distribution.

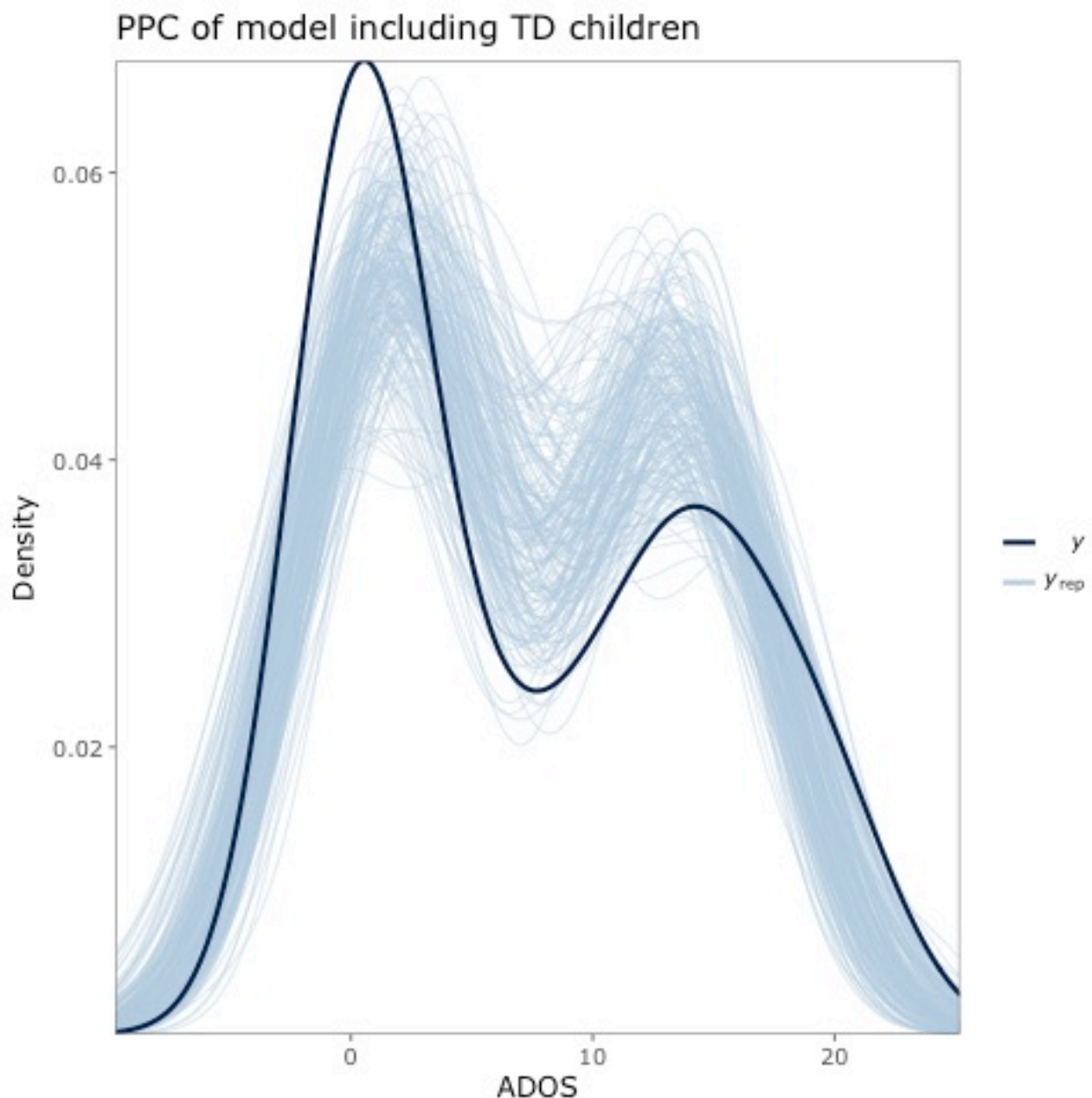


Figure 9: Predictive posterior check of model including TD children

4. Discussion

4.1 Three aspects of IQ and their variance

In the section above we saw that there is some shared variance between verbal, non-verbal and social IQ when looking at autistic children. For better overview the correlation values among the IQs are reported in table 9 below. Furthermore we tested whether the same applies in dataset of typically developing children. These values are also in table 9.

	Verbal - NonVerbal	Verbal - Social	Social - NonVerbal
TD	0.555	0.168	-0.121
ASD	0.676	0.61	0.375

Table 9: Correlation of different aspects of IQ

References

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