Relationship between IQ and symptom severity in ASD

To investigate the relationship between various aspects of IQ and symptom severity in children with ASD data from 72 children were collected. Each children came 6 times for data collection however only data from visit 1 are used in the rest of paper. In this sample of children there were 61 boys and 11 girls with average age of 26.26 months (sd=7.34). There were 34 autistic children and 38 typically developing children. The symptom severity was measured on ADOS scale from 0 to 21. 3 aspects of IQ were measured: verbal, nonverbal and social.
All the analysis reported below was performed in bayesian framework and the models were implemented in R (1) using RStudio (1) and package rethinking (3). For the visualizations R packages ggplot2 (4) and plot3D (5) were used.

The relation between symptom severity and IQ modeled as bivariate regression

In the first part we modeled the relation between symptom severity measured as ADOS and each aspect of IQ as a separate bivariate regression. Thus 3 models are reported. In order to interpret and compare the results better all measurements of IQ were standardized so that mean of the variables equals 0 and standard deviation 1. Each of the models is therefore in following form;

ADOS ~ α+ß*IQ

Furthermore in this part of analysis only data from first visit and children with ASD were used. That means that our sample had. 34 children with mean age of 32.77 (sd=5.27). From the 34 children 29 were boys.

ASD and Verbal IQ

The first relationship modeled was between ADOS and verbal IQ. That means that the model's equation was:

ADOS ~ α+ß*VerbalIQ

PRIORS
As this model is a bayesian linear model motivated priors for all parameters of the model were defined. They are reported in table 1.
The parameter alpha (a) defines the mean of the dependent variable, ADOS in our case. As we have only children with ASD we would expect the mean to be above the 50% of the ADOS scale but not by much. Therefore the prior for alpha is a normal distribution with mean at 14. To define a normal distribution we need also standard deviation. As the prior is defined only by logical deduction we need to keep them relatively weak therefore we use standard deviation of 1. Such sd would imply that 95% of possible means are in range of 12 to 16 which seems as a reasonable estimate.

Priors	
a	Normal (14, 1)
ß	Normal (0, 0.5)
σ	Uniform (0, 5)

Table 1: Priors of model using verbal IQ as predictor

The parameter ß defines an effect of verbal IQ on ADOS. We want to remain skeptical of the relationship and therefore set the mean of the normal prior at 0. By doing that we weight the possibility of having a negative effect the same as having a positive effect. Next we need a standard deviation, in other words what is the strongest effect we believe is possible. We set the sd at 0.5 which in the most extreme case would imply a change of ADOS by 1 with verbal IQ equaling to 1 on the standardized scale. This is rather broad prior that allows both quite strong effects and weak to no effects.
The last prior is for σ or in other words the standard deviation of ADOS. This prior will be uniform from 0 to 5. From 0 because a negative SD does not exist. The upper limit implies that 95% of ADOS scores are ±8 units from the mean. Given the prior of mean at 14 this would mean that the score of ADOS is most probably between 6 and 24 which is possible as the maximum of ADOS is 21 and there should not be any children in our sample that have very low ADOS as that would mean a typically developing child.

RESULTS
The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 2. For better understanding of the results implied by these estimates the simulated means (black line), their 89% credibility intervals (dark blue) and 89% credibility intervals of mean combined with σ (light blue) were superimposed on the original data. The resulting plot is reported in figure 1.
The estimate of alpha implies that the mean of ADOS is 13.897 which is very close to the actual mean of ADOS (13.853). The effect of -1.22 means that with increase of verbal IQ by 1 the ADOS score decreases by 1.22. And finally the σ implies a standard deviation of ADOS of 3.8 which is again very close to the SD of our sample (4.646).
In short this model indicates that there is a negative relationship between ADOS and verbal IQ.

	Mean	StdDev	5.5%	94.5%	a	ß	sigma
a	13.897	0.546	13.024	14.769	1	0.007	0.015
ß	-1.22	0.453	-1.943	-0.497	0.007	1	0.472
sigma	3.801	0.523	2.965	4.636	0.015	0.472	1

Table 2: Parameter estimates of model using verbal IQ as predictor

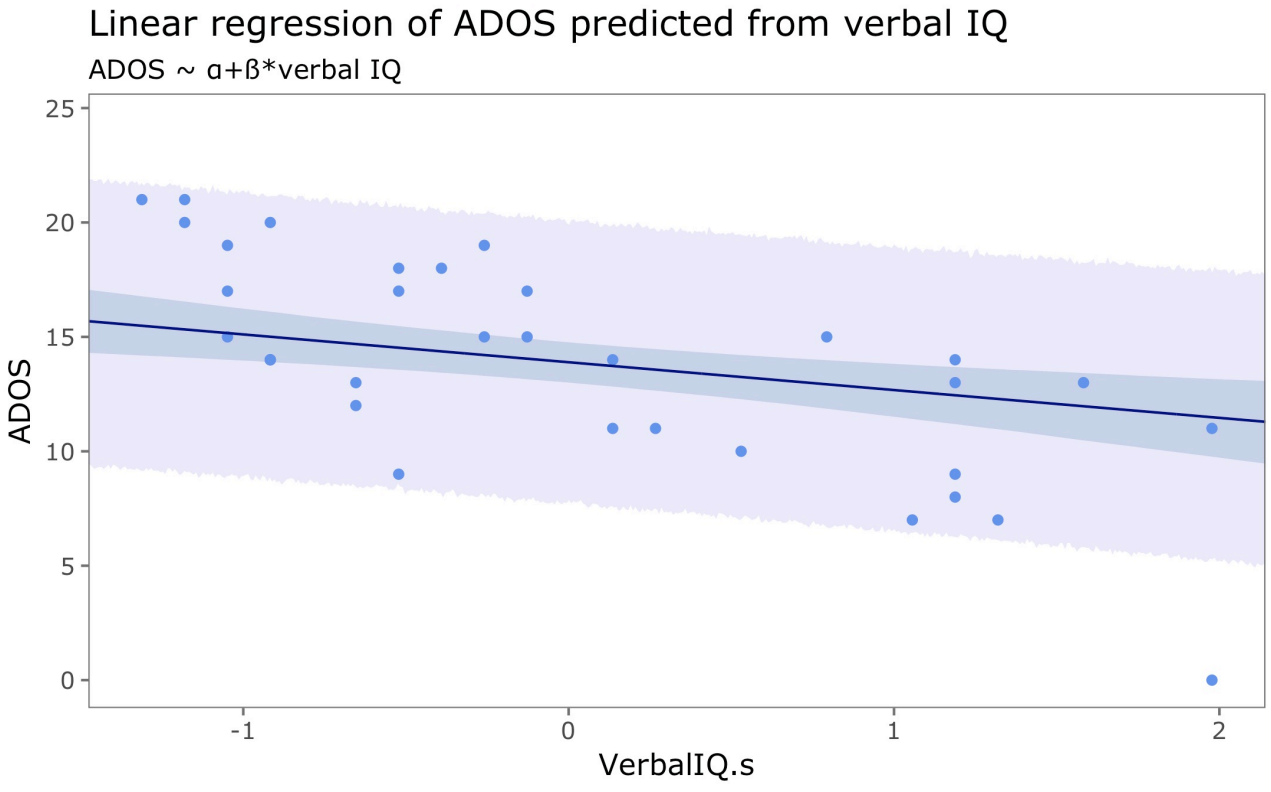

Figure 1: Relationship of verbal IQ and ADOS

QUALITY CHECK
One way of checking the quality of the model is already implied in the plot in figure 1. It's a rather simple check whether our model fits the data correctly. As all the datapoints, except for one, are captured by the 89% credibility interval (light blue) of μ+σ, the model seems to be fitted correctly as only one datapoint would not be expected by the model.
Second quality check is visualized in figure 2. One of the assumptions of the model are uncorrelated parameters which is what this plot does, it plots 10 00 samples from posterior of all parameters against each other. None of the parameters is strongly correlated although there is a correlation between ß and σ (.472) as seen by the slight elongation of the cloud of points in the plot.
Last quality check could be done by testing how does the choice of priors affect the model.

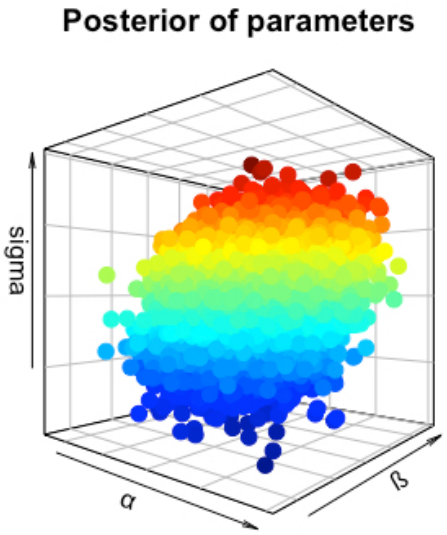

Figure 2: Posterior distribution of regression using verbal IQ as predictor

ASD and Nonverbal IQ

As second the modeled relationship was between ADOS and non-verbal IQ. That means that the model's equation was:

ADOS ~ α+ß*Non-verbalIQ

PRIORS
Since the dependent variable is still ADOS and the independent variable is standardized we can use the same priors for α and σ.
As for the ß the variance of non-verbal IQ is higher than the one of verbal IQ by approximately 1 SD and therefore we can broaden the prior to encompass the possibility of having a stronger effect. The mean of the normal prior stays at 0 but the SD is increased to 0.6. All the priors are reported in table 3.

Priors	
a	Normal (14, 1)
ß	Normal (0, 0.6)
σ	Uniform (0, 5)

Table 3: Priors of model using verbal IQ as predictor

RESULTS

The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 4. The results are visualized in the same way as in previous model in figure 3.

The estimate of alpha implies that the mean of ADOS is 13.901 which is very close to the actual mean of ADOS (13.853). The effect of -1.133 means that with increase of non-verbal IQ by 1 the ADOS score decreases by 1.133. And finally the σ implies a standard deviation of ADOS of 4.036 which is again very close to the SD of our sample (4.646).

In short this model indicates that there is a negative relationship between ADOS and non-verbal IQ.

	Mean	StdDev	5.5%	94.5%	a	ß	sigma
a	13.901	0.569	12.991	14.81	1	0.005	0.015
ß	-1.133	0.487	-1.911	-0.355	0.005	1	0.348
sigma	4.036	0.522	3.201	4.87	0.015	0.348	1

Table 4: Parameter estimates of model using non-verbal IQ as predictor

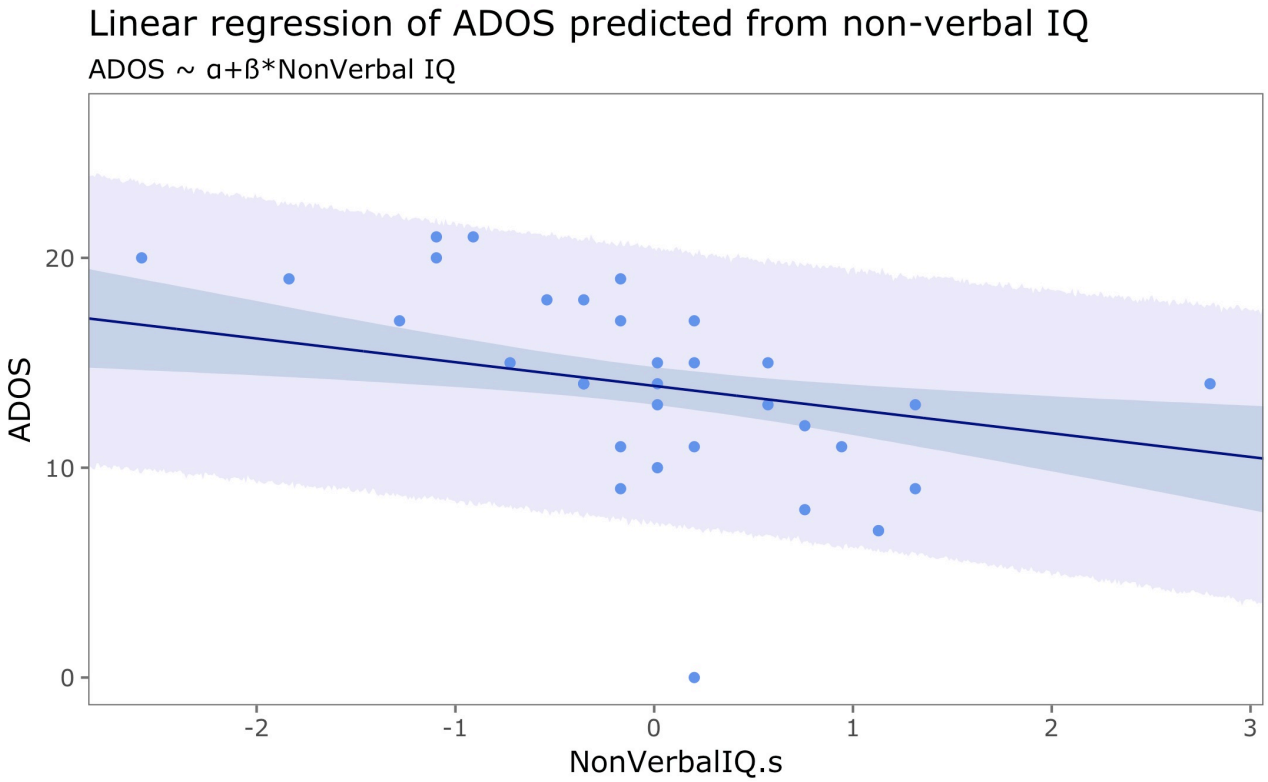

Figure 3: Relationship of non-verbal IQ and ADOS

QUALITY CHECK

We adopt the order of quality checks from the previous model report.

All the datapoints, except for one, are captured by the 89% credibility interval (light blue) of μ+σ, the model seems to be fitted correctly as only one datapoint would not be expected by the model.

Second quality check is visualized in figure 4 . None of the parameters is strongly correlated although there is a correlation between ß and σ (.348) as seen by the very small elongation of the cloud of points in the plot.

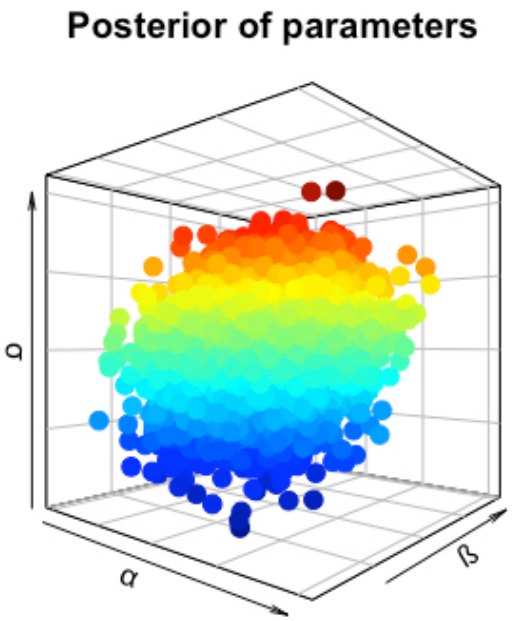

Figure 4: Posterior distribution of regression using non-verbal IQ as predictor

ASD and Social IQ

Finally the relationship between ADOS and social IQ was modeled. That means that the model's equation was:

ADOS ~ α+ß*SocialIQ

PRIORS
Since the dependent variable is still ADOS and the independent variable is standardized we can use the same priors for α and σ.
As for the ß the variance of social IQ is approximately the same as of verbal IQ therefore we can use the prior used before. That is a normal distribution with mean at 0 and standard deviation of 0.5.

RESULTS
The model using the priors described above was estimated using quadratic approximation and the model's output is reported in table 5. The results are visualized in the same way as in previous models in figure 5.
The estimate of alpha implies that the mean of ADOS is 13.899 which is very close to the actual mean of ADOS (13.853). The effect of -1.077 means that with increase of social IQ by 1 the ADOS score decreases by 1.077. And finally the σ implies a standard deviation of ADOS of 3.946 which is again very close to the SD of our sample (4.646).
In short this model indicates that there is a negative relationship between ADOS and social IQ.

	Mean	StdDev	5.5%	94.5%	a	ß	sigma
a	13.899	0.561	13.003	14.795	1	0.006	0.015
ß	-1.077	0.446	-1.79	-0.364	0.006	1	0.423
sigma	3.946	0.528	3.102	4.79	0.015	0.423	1

Table 5: Parameter estimates of model using social IQ as predictor

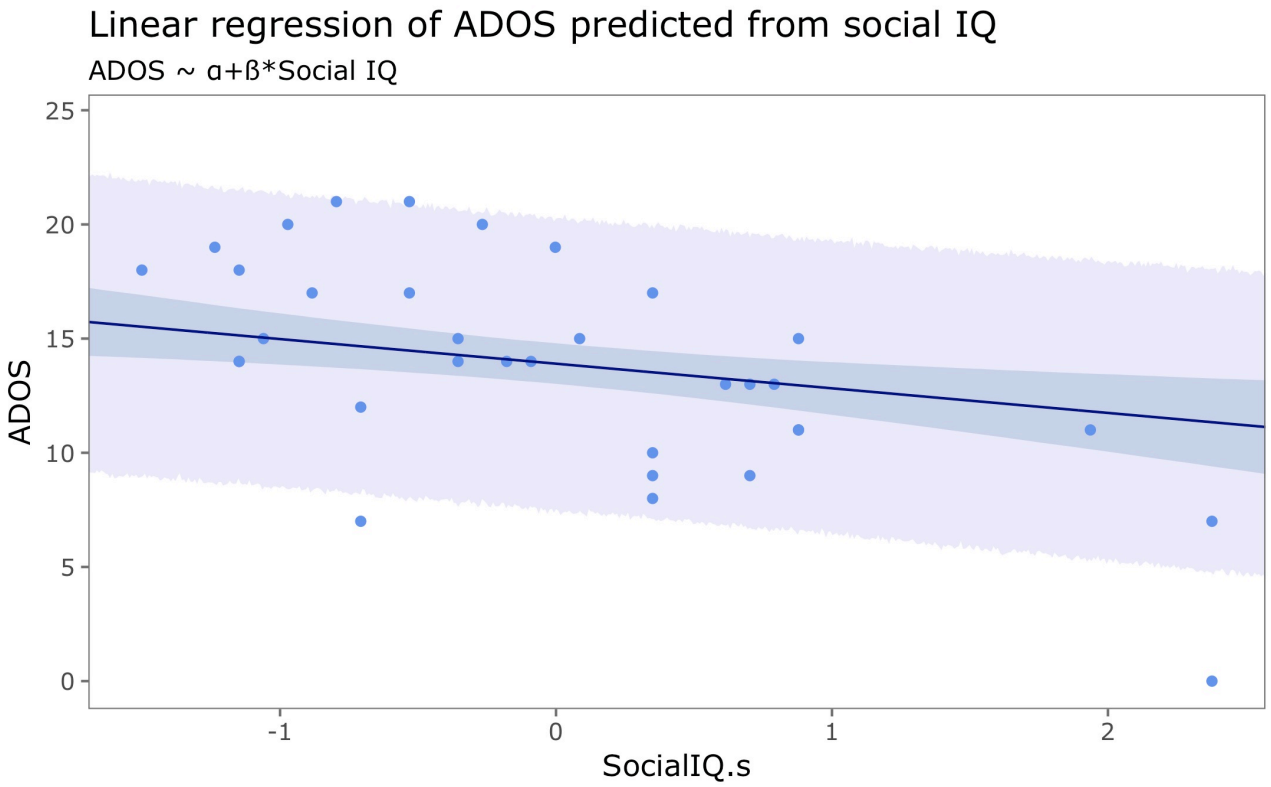

Figure 5: Relationship of social IQ and ADOS

QUALITY CHECK
We adopt the order of quality checks from the previous model report.
3 datapoints are not captured by the 89% credibility interval (light blue) of μ+σ, but we can still call the model correctly fitted.
Second quality check is visualized in figure 4 . None of the parameters is strongly correlated although there is a correlation between ß and σ (.0.423) as seen by the small elongation of the cloud of points in the plot.

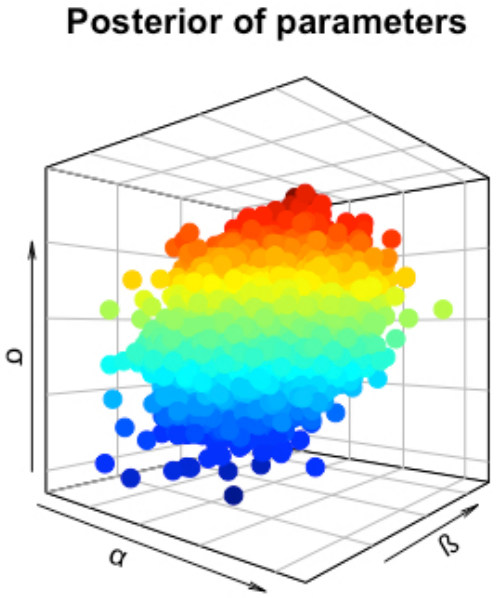

Figure 6: Posterior distribution of regression using social IQ as predictor

References

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