Задание после лекции 9

$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$ – начальное сост. кубита

1. $|\psi_0\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$

$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\cdot\sin\left(\frac{\theta}{2}\right)|1\rangle$

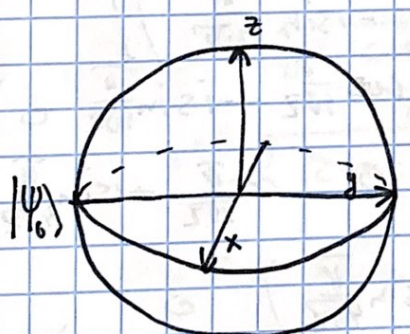
$\begin{cases} \cos\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}} \\ e^{i\varphi}\cdot\sin\left(\frac{\theta}{2}\right) = -\frac{i}{\sqrt{2}} \end{cases}$ $\begin{cases} \theta = \frac{\pi}{2} \\ e^{i\varphi} = -i \Rightarrow \end{cases}$

$\Rightarrow e^{i\varphi} = -i$

$\cos\varphi + i\sin\varphi = -i$

$\begin{cases} \cos\varphi = 0 \\ \sin\varphi = -1 \end{cases} \Rightarrow \underline{\underline{\varphi = \frac{3\pi}{2}}}$

$\theta = \frac{\pi}{2}$; $\varphi = \frac{3\pi}{2}$



2. Дано:

$\hat{n} = (n_x, n_y, n_z) = \left(0, \cos\frac{\pi}{4}, \cos\frac{\pi}{4}\right)$

$\alpha = \frac{\pi}{2}$

Найти:

$\hat{U}(\alpha, \vec{n})$ – ?

Решение:

1) $\hat{U}(\alpha, \vec{n}) = e^{-i\frac{\alpha}{2}(\vec{n}, \vec{\sigma})} = e^{-i\frac{\alpha}{2} n_x \cdot \hat{\sigma}_x} \cdot e^{-i\frac{\alpha}{2} n_y \cdot \hat{\sigma}_y} \cdot$

$\cdot e^{-i\frac{\alpha}{2} n_z \hat{\sigma}_z} = \left(\cos\left(\frac{\alpha}{2} \cdot n_x\right)\hat{I} - i\sin\left(\frac{\alpha}{2} \cdot n_x\right)\right.$

$\left.\cdot \hat{\sigma}_x\right) \cdot \left(\cos\left(\frac{\alpha}{2} \cdot n_y\right) \cdot \hat{I} - i\sin\left(\frac{\alpha}{2} n_y\right)\hat{\sigma}_y\right) \cdot$

$\cdot \left(\cos\left(\frac{\alpha}{2} n_z\right)\hat{I} - i\sin\left(\frac{\alpha}{2} \cdot n_z\right)\hat{\sigma}_z\right) =$

$= \left(\cos(0)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin(0)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right)\left(\cos\left(\frac{\pi}{4} \cdot \cos\frac{\pi}{4}\right)\right.$

$\left.\cdot \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin\left(\frac{\pi}{4} \cdot \cos\frac{\pi}{4}\right)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\right)\left(\cos\left(\frac{\pi}{4}\cos\frac{\pi}{4}\right) \cdot\right.$

$\left.\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin\left(\frac{\pi}{4}\cos\frac{\pi}{4}\right)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right) = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \cdot$

$\cdot \left(\begin{pmatrix}\cos\left(\frac{\pi}{4\sqrt{2}}\right) & 0\\ 0 & \cos\frac{\pi}{4\sqrt{2}}\end{pmatrix} - \begin{pmatrix}0 & \sin\left(\frac{\pi}{4\sqrt{2}}\right)\\ -\sin\left(\frac{\pi}{4\sqrt{2}}\right) & 0\end{pmatrix}\right) \cdot$

$\cdot \left(\begin{pmatrix}\cos\left(\frac{\pi}{4\sqrt{2}}\right) & 0\\ 0 & \cos\frac{\pi}{4\sqrt{2}}\end{pmatrix} - \begin{pmatrix}i\sin\frac{\pi}{4\sqrt{2}} & 0\\ 0 & -i\sin\frac{\pi}{4\sqrt{2}}\end{pmatrix}\right) =$

$= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\cos\frac{\pi}{4\sqrt{2}} & -\sin\frac{\pi}{4\sqrt{2}}\\ \sin\frac{\pi}{4\sqrt{2}} & \cos\left(\frac{\pi}{4\sqrt{2}}\right)\end{pmatrix}\begin{pmatrix}\cos\frac{\pi}{4\sqrt{2}} - i\sin\frac{\pi}{4\sqrt{2}} & 0\\ 0 & \cos\frac{\pi}{4\sqrt{2}} + i\sin\frac{\pi}{4\sqrt{2}}\end{pmatrix}$

$= \begin{pmatrix}\cos\left(\frac{\pi}{4\sqrt{2}}\right) & -\sin\left(\frac{\pi}{4\sqrt{2}}\right)\\ \sin\left(\frac{\pi}{4\sqrt{2}}\right) & \cos\left(\frac{\pi}{4\sqrt{2}}\right)\end{pmatrix}\begin{pmatrix}\left(\cos\frac{\pi}{4\sqrt{2}} - i\sin\frac{\pi}{4\sqrt{2}}\right) & 0\\ 0 & \left(\cos\left(\frac{\pi}{4\sqrt{2}}\right) + i\sin\left(\frac{\pi}{4\sqrt{2}}\right)\right)\end{pmatrix} =$

$= \begin{pmatrix}\left(\cos^2\left(\frac{\pi}{4\sqrt{2}}\right) - i\sin\left(\frac{\pi}{4\sqrt{2}}\right)\cos\left(\frac{\pi}{4\sqrt{2}}\right)\right) & \left(-\left(\sin\left(\frac{\pi}{4\sqrt{2}}\right) \cdot \cos\frac{\pi}{4\sqrt{2}} + \sin^2\left(\frac{\pi}{4\sqrt{2}}\right)\right)\right)\\ \left(\sin\frac{\pi}{4\sqrt{2}}\cos\frac{\pi}{4\sqrt{2}} - i\sin^2\left(\frac{\pi}{4\sqrt{2}}\right)\right) & \left(\cos^2\left(\frac{\pi}{4\sqrt{2}}\right) + i\cos\frac{\pi}{4\sqrt{2}} \cdot \sin\frac{\pi}{4\sqrt{2}}\right)\end{pmatrix}$

3.

1) $|\psi_f\rangle = \hat{U}\cdot\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \hat{U}\cdot\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{i}{\sqrt{2}}\end{pmatrix} =$

$$= \begin{pmatrix}\cos^2\left(\frac{\pi}{4\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}} - \sin^2\left(\frac{\pi}{4\sqrt{2}}\right)\cdot\frac{1}{\sqrt{2}}\\ \sqrt{2}\sin\left(\frac{\pi}{4\sqrt{2}}\right)\cdot\cos\frac{\pi}{4\sqrt{2}}\cdot\frac{i}{\sqrt{2}}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos\frac{\pi}{2\sqrt{2}}\\ \sin\frac{\pi}{2\sqrt{2}} - i\end{pmatrix}$$

2) $|\psi_f\rangle = \frac{1}{\sqrt{2}}\left(\left(\sin\left(\frac{\pi}{2\sqrt{2}}\right) - i\right)|1\rangle + \cos\frac{\pi}{2\sqrt{2}}|0\rangle\right)$

$|\psi_f\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\cdot\sin\left(\frac{\theta}{2}\right)|1\rangle)$

$$\begin{cases}\frac{1}{\sqrt{2}}\cdot\cos\frac{\pi}{2\sqrt{2}} = \cos\frac{\theta}{2}\\ \frac{1}{\sqrt{2}}\cdot\sin\left(\frac{\pi}{2\sqrt{2}}\right) - \frac{i}{\sqrt{2}} = e^{i\varphi}\cdot\sin\left(\frac{\theta}{2}\right)\end{cases}$$

$$\begin{cases}\theta = 2\cdot\arccos\left(\frac{1}{\sqrt{2}}\cdot\cos\left(\frac{\pi}{2\sqrt{2}}\right)\right)\\ e^{i\varphi}\cdot\sqrt{1-\frac{1}{2}\cdot\cos^2\left(\frac{\pi}{2\sqrt{2}}\right)} = \frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{2\sqrt{2}}\right) - \frac{i}{\sqrt{2}} \Rightarrow\end{cases}$$

$$\Rightarrow e^{i\varphi}\cdot\sqrt{\frac{2-\cos^2\left(\frac{\pi}{2\sqrt{2}}\right)}{2}} = \frac{1}{\sqrt{2}}\cdot\sin\left(\frac{\pi}{2\sqrt{2}}\right) - \frac{i}{\sqrt{2}}$$

$$e^{i\varphi}\sqrt{2-\cos^2\left(\frac{\pi}{2\sqrt{2}}\right)} = \sin\left(\frac{\pi}{2\sqrt{2}}\right) - i$$

$$\cos\varphi + i\sin\varphi = \frac{\sin\frac{\pi}{2\sqrt{2}}}{\sqrt{2-\cos^2\left(\frac{\pi}{2\sqrt{2}}\right)}} - i\cdot\frac{1}{\sqrt{2-\cos^2\left(\frac{\pi}{2\sqrt{2}}\right)}}$$

$$\begin{cases}\cos\varphi = \sqrt{\frac{\sin^2\left(\frac{\pi}{2\sqrt{2}}\right)}{1+\sin^2\left(\frac{\pi}{2\sqrt{2}}\right)}}\\ \sin\varphi = -\frac{1}{\sqrt{1+\sin^2\left(\frac{\pi}{2\sqrt{2}}\right)}}\end{cases}$$

$$\left[\begin{array}{l}\varphi = \arcsin\left(-\frac{1}{\sqrt{1+\sin^2\left(\frac{\pi}{2\sqrt{2}}\right)}}\right) + 2\pi\\ \theta = 2\cdot\arccos\left(\frac{1}{\sqrt{2}}\cdot\cos\left(\frac{\pi}{2\sqrt{2}}\right)\right)\end{array}\right] \text{- ответ}$$