Sensor Positioning in Wireless Ad-hoc Sensor Networks Using Multidimensional Scaling

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Abstract—Sensor Positioning is a fundamental and crucial issue for sensor network operation and management. In the paper, we first study some situations where most existing sensor positioning methods tend to fail to perform well, an example being when the topology of a sensor network is anisotropic. Then, we explore the idea of using dimensionality reduction techniques to estimate sensors coordinates in two (or three) dimensional space, and we propose a distributed sensor positioning method based on multidimensional scaling technique to deal with these challenging conditions. Multidimensional scaling and coordinate alignment techniques are applied to recover positions of adjacent sensors. The estimated positions of the anchors are compared with their true physical positions and corrected, The positions of other sensors are corrected accordingly. With iterative adjustment, our method can overcome adverse network and terrain conditions, and generate accurate sensor position. We also propose an on demand sensor positioning method based on the above method.

I. INTRODUCTION

Wireless ad-hoc sensor networks has recently attracted much interest in the wireless research community as a fundamentally new tool for a wide range of monitoring and data-gathering applications. Many applications with sensor networks are proposed, such as habitat monitoring [7], [11], [19], [31], health caring [2], [27], battle-field surveillance and enemy tracking [8], [16], and environment observation and forecasting [1], [9], [24]. A general setup of a wireless sensor network consists of a large number of sensors randomly and densely deployed in a certain area. Each compact sensor usually is capable of sensing, processing data at a small scale, and communicating through omni-directional radio signal [2], [15]. Because omni-direction radio signal attenuates with distance, only sensors within certain range can communicate with each other. This range is called radio range r. Wireless sensor networks significantly differ from classical networks on their strict limitations on energy consumption, the simplicity of the processing power of nodes, and possibly high environmental dynamics.

Determining the physical positions of sensors is a fundamental and crucial problem in wireless ad-hoc sensor network operation for several important reasons. We briefly list two of them in the following: first, in order to use the data collected by sensors, it is often necessary to have their position information stamped. For example, in order to detect and track objects with sensor networks, the physical position of each sensor should be known in advance for identifying the positions of detected objects. In addition, many communication protocols of sensor networks are built on the knowledge of the geographic positions of sensors [4], [10], [29]. However, in most cases, sensors are deployed without their position information known in advance, and there is no supporting infrastructure available to locate them after deployment. It is necessary to find an alternative approach to identify the position of each sensor in wireless sensor networks after deployment. In the general model of wireless ad-hoc sensor network, there are usually some landmarks or nodes named *anchor* nodes, whose position information is known, within the area to facilitate locating all sensors in a sensor network.

The work presented in the paper is the extension of our previous study on sensor positioning methods and dimensionality reduction techniques [17], [18], [32], [33], [34]. In the paper, we first analyze some challenges of sensor positioning problem in real applications. The conditions that most existing sensor positioning methods fail to perform well are the anisotropic topology of the sensor networks and complex terrain where the sensor networks are deployed. Moreover, cumulative measurement error is a constant problem of some existing sensor positioning methods [10], [20], [25]. In order to accurately position sensors in anisotropic network and complex terrain and avoid the problem of cumulative errors, we propose a distributed method based on the multidimensional scaling technique. In detail, a series of local maps of adjacent sensors along the route from a starting anchor to an ending anchor are computed. We apply multidimensional scaling (MDS), a technique that has been successfully used to capture the inter-correlation of high dimensional data at low dimension in social science, to compute the local maps (or relative positions) of adjacent sensors with high error-tolerance. These local maps are then pieced together to get the approximation of the physical positions of the sensor nodes. Because the position information of the starting anchor is known, with the stitched maps, the position of the ending anchor can be estimated, which may be different from the true position of the ending anchor. When aligning the calculated position and the true position of ending anchor, the positions of sensors in the stitched maps will approximate their true positions effectively. With three or more anchors, our approach usually generate more accurate sensor position information in a network with

anisotropic topology and complex terrain than any other positioning methods. The method is also efficient in eliminating cumulative measurement errors. At last, we propose a method that is able to position sensors on demand. When a sensor or several adjacent sensors want to get location information, one of them initializes flooding and recover a series of local maps to three or more anchors. Then, the location of the sensor(s) can be estimated as well as location information of sensors along the route from the sensor(s) to anchors.

The focus of the paper is on the sensor position estimation algorithm instead of communication protocol details. Our methods are illustrated with 2-D sensor networks and they can be easily extended to 3-D cases. The rest of the paper is organized as follows: We review previous research on sensor positioning in Section II and list the challenges for positioning sensors in Section III. We present the multidimensional scaling algorithm in Section IV and position alignment method in Section V. In Section VI, we propose a distributed sensor positioning method with anchor sensors and an on demand positioning method based on the MDS and position alignment method. The details of our experiments is presented in Section VII. We conclude the paper in Section VIII.

II. PREVIOUS WORK

Global Positioning System (GPS) has been widely used for positioning service [30]. Although it is possible to find the position of each sensor in a wireless sensor network with the aid of Global Positioning System (GPS) installed in all sensors, it is not practical to use GPS due to its high power consumption, expensive price and line of sight conditions.

There has been many efforts on the sensor positioning problem. They mainly fall into one of the following four classes or the combinations of them. The first class of methods improve the accuracy of distance estimation with different signal techniques. The Received Signal Strength Indicator (RSSI) technique was employed to measure the power of the signal at the receiver. Relatively low accuracy is achieved in this way. However, because of its simplicity, it is widely used in previous research. Later, Time of Arrival (ToA) and Time Difference of Arrival (TDoA) are used by Savvides et al. [26] and Priyantha et al. [22] to reduce the errors of range estimation, but these methods require each sensor node being equipped CPU with powerful computation capability. Recently, Niculescu et al. use Angle of Arrival (AoA) to measure the positions of sensors [21]. The AoA sensing requires each sensor node installed with antenna array or ultrasound receivers.

The second class of positioning methods relies on a large amount of sensor nodes with positions known densely distributed in a sensor network [4], [5], [6]. These nodes with positions known, which are also named as *beacons* or *anchor* nodes, are arranged in grid across the network to estimate other nodes' positions nearby them.

The third class of methods employ distance vector exchange to find the distances from the non-anchor nodes to the anchor nodes. Based these distances, each node can estimate its

position by performing a trilateration or multilateration [20], [26]. The performance of the algorithms is deteriorated by range estimation errors and inaccurate distance measures, which are caused by complex terrain and anisotropic topology of the sensor network. Savarese [28] try to improve the above approach by iteratively computing. However, this method adds a large number of communication costs into the algorithm and still cannot generate good position estimation in some circumstances. Moreover, the accuracy of this class of algorithms relies on the average radio range estimation, and it tends to deteriorate when the topology of sensor network is anisotropic. For example, in Figure 1, in which sensors are
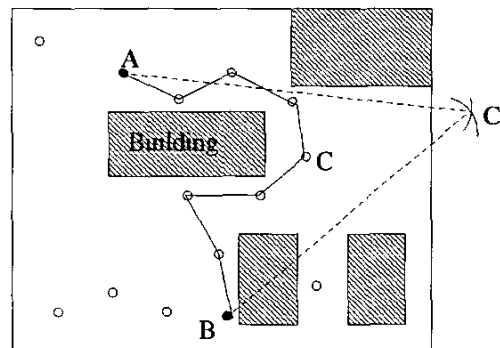

Fig. 1. Sensor Network deployed in a square area with obstacles

deployed in a square area. But there are some buildings that are marked by shadowed rectangle areas, and sensors cannot access them. Thus, the routes for between a pair of sensors detour severely by the buildings in the square area, and the estimated distances of AC and BC are increased significantly. Similar situation happens to the case in Figure 2, sensors are
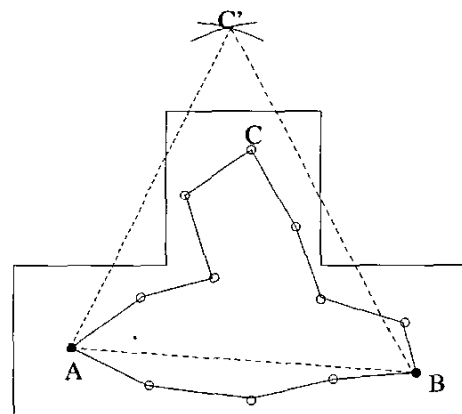

Fig. 2. Sensor Network in non-square area

deployed in a T-shape area, instead of a square area which is assumed and used as the fundamental condition by most existing research works. A and B are two anchors, A may estimate radio range with the distance of AB and hop count in the route from A to B. If A and B estimate their distances to C with the estimated radio range, the estimated distances

will be increased a lot by error. Another example is that the ideal radio range of a sensor is a circle centered in the sensor. However, a sensor usually has an irregular radio pattern, which is represented with the black curve Figure 3, in real world. This meas that the radio range of a sensor is different at different directions. In Figure 4, sensors are deployed on a square area
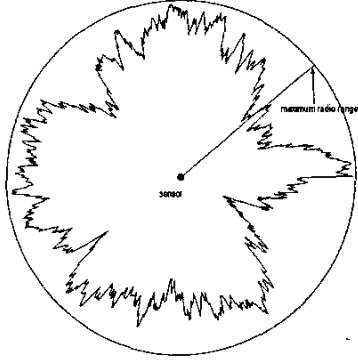

Fig. 3. Irregular radio pattern of a sensor

with deep grass or bush on the left part and clear ground on the right. The complexity of and terrain leads to different signal attenuation factors and radio ranges in the field.
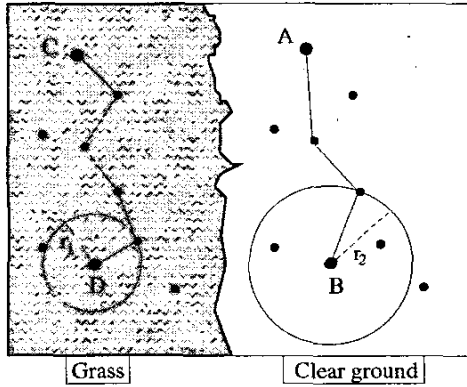

Fig. 4. Anisotropic terrain condition leading to different radio ranges

The last class of methods [10], [20], [25] locally calculate maps of adjacent nodes with trilateration or multilateration and piece them together to estimate nodes' physical or relative positions. The performance of these algorithms relies heavily on the average radio range estimation and suffers from the cumulative range error during the map stitching.

III. CHALLENGES

Considering a real sensor network application scenario, there are several challenges in designing positioning algorithm. Firstly, since a large number (up to thousands) of sensors are usually used when they are randomly deployed across an given area, we hope to achieve good position estimation as well as keep the hardware design of sensors simple and cheap. Secondly, in many circumstances it is impossible to

get a large number of anchor nodes deployed uniformly aross the area to assist position estimation of non-anchor nodes. Thus, it is desirable to design a sensor positioning method that is able to generate accurate position estimation with as few anchors as possible. Thirdly, sensors may be deployed in battle fields or urban areas with complex terrain and vegetation (Figure 1,Figure 2). The sensor network may have a high level of anisotropicity (Figure 4). However, most existing research explored sensor positioning algorithms based on isotropic network topology in a square area. Neither their algorithms nor their experimental environment dealt with sensor network with anisotropic topology like Figure 1, Figure 2, Figure 3, and Figure 4. Finally, most of previous methods estimate an average radio range and broadcast it to whole network. In many cases, sensors may be deployed on an area with anisotropic vegetation and terrain condition (Figure 4). Thus, sensors at different locations of the area can have different radio ranges, and a uniform radio range calculation will lead to serious errors (in [20], [26], [28]) and the errors may propagate throughout the sensors in the network [10], [25].

IV. CALCULATING RELATIVE POSITIONS WITH MULTIDIMENSIONAL SCALING

The multidimensional scaling (MDS), a technique widely used for the analysis of dissimilarity of data on a set of objects, can discover the spatial structures in the data [3], [13], [23]. We use it as a data-analytic approach to discover the dimensions that underlie the judgements of distance and model data in a geometric space. The main advantage in using the MDS for position estimation is that it can always generates relatively high accurate position estimation even based on limited and error-prone distance information. There are several varieties of MDS. We focus on classical MDS and the iterative optimization of MDS, the basic idea of which is to assume that the dissimilarity of data are distances and then deduce their coordinates. More details about comprehensive and intuitive explanation of MDS are available in [3], [13], [23].

Inspired by the above multidimensional scaling techniques, we present a multivariate optimization based iterative algorithm for sensor location calculation. $T = [t_{ij}]_{n \times 2}$ denotes the true locations of the set of n sensor nodes in 2-dimensional space. $d_{ij}(T)$ stands for the distance between sensor i and j based on their position in T and

$$d_{ij}(T) = (\sum_{a=1}^{m}(t_{ia} - t_{ja})^2)^{1/2}.$$

The collected distance between node i and j is δ_{ij}. If we ignore the errors in distance measurement, δ_{ij} is equal to $d_{ij}(T)$. We will discuss the error effects to location estimation caused by differences between δ_{ij} and $d_{ij}(T)$ later. If only a portion of pairwise distances are collected, some δ_{ij} are undefined for some i, j. In order to assist computation, we define weights w_{ij} with value 1 if δ_{ij} is known and 0 if δ_{ij} is unknown and assume

$$\delta_{ij} = d_{ij}(T)$$

in the following induction. $X = [x_{ij}]_{n \times 2}$ denotes the estimated locations of the set of n sensor nodes in 2-dimensional space. X is randomly initialized as $X^{[0]}$ and will be updated into $X^{[1]}$, $X^{[2]}$, $X^{[3]}$ ... to approximate T with our iterative algorithm. $d_{ij}(X)$ means the calculated distance between sensor i and j based on their estimated positions in X and

$$d_{ij}(X) = (\sum_{a=1}^{m}(x_{ia} - x_{ja})^2)^{1/2}.$$

We hope to find the a position matrix X to approximate T by minimizing

$$\sigma(X) = \sum_{i<j} w_{ij}(d_{ij}(X) - \delta_{ij})^2.$$

This is a quadratic function without containts. The minimum value of such functions is reached when its gradient is equal to 0. For our problem, we have the following observations:

$$\sigma(X) = \sum_{i<j} w_{ij}\delta_{ij}^2 + \sum_{i<j} w_{ij}d_{ij}^2(X) - 2\sum_{i<j} w_{ij}\delta_{ij}d_{ij}(X),$$

$$\sum_{i<j} w_{ij}d_{ij}^2(X) = \sum_{i<j} tr(X'(w_{ij}A_{ij})X)$$

$$= tr(X'(\sum_{i<j} w_{ij}A_{ij})X) = tr(X'VX)$$

where where A_{ij} is a matrix with $a_{ii} = a_{jj} = 1$, $a_{ij} = a_{ji} = -1$, and all other elements zeros, $V = \sum_{i<j} w_{ij}A_{ij}$, tr the $trace$ function and

$$-\sum w_{ij}\delta_{ij}d_{ij}(X) =$$

$$-\sum w_{ij}\delta_{ij}\frac{(\sum_{a=1}^{m}(x_{ia} - x_{ja})^2)^{1/2}(\sum_{a=1}^{m}(t_{ia} - t_{ja})^2)^{1/2}}{d_{ij}(T)}$$

$$\leq -\sum w_{ij}\delta_{ij}\frac{(\sum_{a=1}^{m}(x_{ia} - x_{ja})(t_{ia} - t_{ja}))}{d_{ij}(T)}$$

$$= -\sum w_{ij}\delta_{ij}\frac{tr(X'A_{ij}T)}{d_{ij}(T)} = tr(X'(\frac{w_{ij}\delta_{ij}}{d_{ij}(T)}A_{ij})T)$$

where the equality achieved when $X = T$. Thus, we get

$$\sigma(X) = \sum_{i<j} w_{ij}\delta_{ij}^2 + tr(X'VX) - 2tr(X'(\frac{w_{ij}\delta_{ij}}{d_{ij}(X)}A_{ij})X)$$

$$\leq \sum_{i<j} w_{ij}\delta_{ij}^2 + tr(X'VX) - 2tr(X'(\frac{w_{ij}\delta_{ij}}{d_{ij}(T)}A_{ij})T),$$

and the equality is achieved when $X = T$. This means that the derivative of the right side of the inequation is zero when the equality is achieved. Based on the above idea, we easily induce the update formular of the SMACOF algorithm

$$VX = (\frac{w_{ij}\delta_{ij}}{d_{ij}(T)}A_{ij})T,$$

or

$$X = V^{-1}(\frac{w_{ij}\delta_{ij}}{d_{ij}(T)}A_{ij})T.$$

If V^{-1} does not exist, we should replace it with Moore-Penrose inverse of V given by $V^{-} = (V + 11')^{-1} - n^{-2}11'$ [3].

In a summary, if all pairwise distances of sensors in T are collected, we can use the classical multidimensional scaling algorithm to estimate the positions of sensors:

1) Compute the matrix of squared distance D^2, where $D = [d_{ij}]_{n \times n}$;
2) Compute the matrix J with $J = I - e * e^T/n$, where $e = (1, 1, \ldots, 1)$;
3) Apply double centering to this matrix with $H = -\frac{1}{2}JD^2J$;
4) Compute the eigen-decomposition $H = UVU^T$;
5) Suppose we want to get the i dimensions of the solution ($i = 2$ in 2-D case), we denote the matrix of largest i eigenvalues by V_i and U_i the first i columns of U. The coordinate matrix of classical scaling is $X = U_iV_i^{\frac{1}{2}}$.

In many situation, the distances between some pairs of sensors in the local area are not available. When this happens, the iterative MDS is employed to compute the relative coordinates of adjacent sensors. We summarize the iteration steps as:

1) Initialize $X^{[0]}$ as random start configuration, set $T = X^{[0]}$ and $k = 0$, and compute $\sigma(X^{[0]})$;
2) Increase the k by one;
3) Compute $X^{[k]}$ with the above update formula and $\sigma(X^{[k]})$;
4) If $\sigma(X^{[k-1]}) - \sigma(X^{[k]}) < \epsilon$, which is a small positive constant, then stop; Otherwise set $T = X^{[k]}$ and go to step 2.

The ϵ is an empirical threshold based on accuracy requirement. We usually set it as 5% of the average radio range. This algorithm generates the relative positions of sensor nodes in $X^{[k]}$.

The above methods can estimate the relative locations of sensor nodes based on their pairwise distances. We also need position alignment techniques to map the relative coordinates to physical coordinates based on three or more anchor sensors.

V. ALIGNING RELATIVE POSITIONS

Since we hope to compute the physical positions of sensors eventually, it is necessary to align the relative positions to physical positions with the aid of sensors with positions known. For an adjacent group of sensors, at least three sensors' physical positions are needed in order to identify the physical positions of remaining nodes in the group in 2-D case. Thus, each group of adjacent sensors must contain at least three nodes with physical positions known, which can be anchors or nodes with physical positions calculated previously.

The alignment usually includes shift, rotation, and reflection of coordinates. $R = [r_{ij}]_{2 \times n} = (R_1, R_2, \ldots, R_n)$ denotes the relative positions of the set of n sensor nodes in 2-dimensional space. $T = [t_{ij}]_{2 \times n} = (T_1, T_2, \ldots, T_n)$ denotes the true positions of the set of n sensor nodes in 2-dimensional space. In following explanation, we assume the nodes 1,2,3 are anchors. A vector R_i may be shifted to $R_i^{(1)}$ by $R_i^{(1)} = R_i + X$,

where $X = R_i^{(1)} - R_i$. It may be rotated counterclockwise through an angle α to $R_i^{(2)} = Q_1 R_i$, where

$$Q_1 = \begin{bmatrix} cos(\alpha) & -sin(\alpha) \\ sin(\alpha) & cos(\alpha) \end{bmatrix}.$$

It may also be reflected across a line

$$S = \begin{bmatrix} cos(\beta/2) \\ sin(\beta/2) \end{bmatrix}$$

to $R_i^{(3)} = Q_2 R_i$, where

$$Q_2 = \begin{bmatrix} cos(\beta) & sin(\beta) \\ sin(\beta) & -cos(\beta) \end{bmatrix}.$$

Before alignment, we only know R and three or more anchor sensors' physical positions T_1, T_2, T_3. Based on them, we computer $T_4, T_5, \ldots, T_n$. Based on the above rules, we have

$$(T_1 - T_1, T_2 - T_1, T_3 - T_1) = Q_1 Q_2 (R_1 - R_1, R_2 - R_1,$$

$$R_3 - R_1). \tag{1}$$

With $R_1, R_2, R_3, T_1, T_2,$ and T_3 known, we can compute

$$Q = Q_1 Q_2 = (R_1 - R_1, R_2 - R_1, R_3 - R_1)/(T_1 - T_1,$$

$$T_2 - T_1, T_3 - T_1). \tag{2}$$

Then, $(T_4, T_5, \ldots, T_n)$ can be calculated with

$$(T_4 - T_1, T_5 - T_1, \ldots, T_n - T_1) = Q(R_4 - R_1, R_5 - R_1,$$

$$\ldots, R_n - R_1), \tag{3}$$

$$(T_4, T_5, \ldots, T_n) = Q(R_4 - R_1, R_5 - R_1, \ldots, R_n - R_1)$$

$$+(T_1, T_1, \ldots, T_1). \tag{4}$$

VI. DISTRIBUTED SENSOR POSITIONING METHODS

In our sensor positioning method, the above MDS techniques are used in a distributed manner to estimate a local map for each group of adjacent sensors, and then these maps are aligned together based on the alignment method. In this section, the details of distributed sensor positioning method are presented.

Here we employ the widely used distance measurement model of Received Signal Strength Indication (RSSI). A circle centered in a sensor node bounds the maximal range for the direct communication of the sensor's radio signal, which is called the *radio range*. The power of the radio signal attenuates exponentially with distance, and this property enables the receiver to estimate the distance to the sender by measuring the attenuation of radio signal strength from the sender to the receiver. It is necessary to point out that some other distance measure approaches, such as TOA, TDOA, AoA, and Ultrasound, can also be applied here. They even generate more accurate distance measure than RSSI, but they usually need complex hardware equipped in each sensor. In the paper, we intend to use RSSI and simple hardware configuration to achieve competitive performance.

A. Distributed Position Estimation with Anchor Sensors

An anchor node named as *starting anchor* initializes flooding to the whole network. When other anchor nodes, named *ending anchors*, get the flooding message, they pass their positions back to the starting anchor along the reverse routes from starting anchor to each of them. Then, the starting anchor knows the positions of ending anchors and routes to each of them. The average radio ranges in different directions from the starting anchor to different ending anchors can be estimated with the hop counts and physical distances between the starting sensor to these anchor sensors. Figure 5 shows a flooding initialized by the starting anchor in up-left corner of the square area. Black lines are the routes that the flooding passed, and blue circles represent the adjacent areas where sensors position will be estimated with MDS. After the
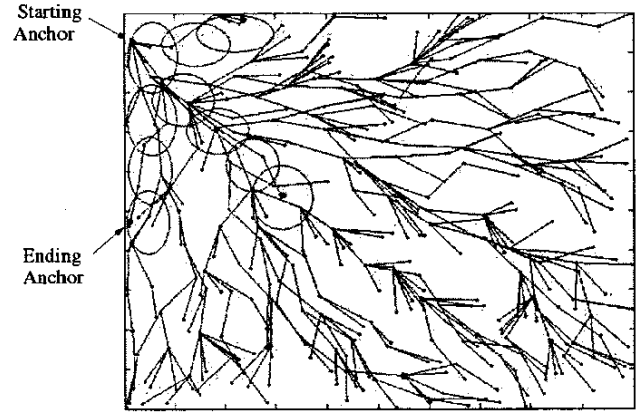

Fig. 5. Flooding from a starting anchor to the whole network. Red nodes are anchor nodes.

flooding, the starting anchor will initialize sensor positioning for sensors along the routes from the starting anchor to each ending anchor.
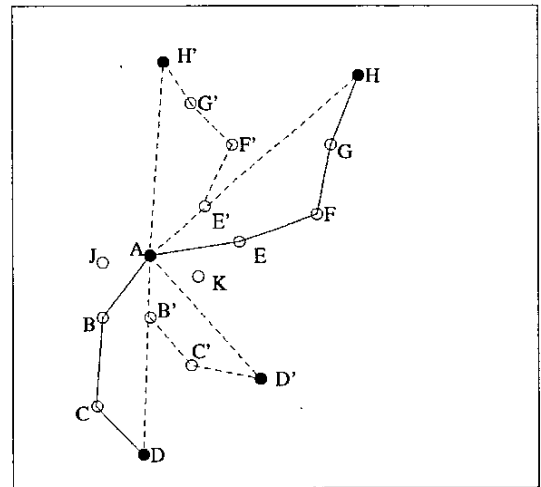

Fig. 6. Position estimation in the adjacent area of a starting anchor sensor.

Starting anchor first estimates the positions of those sensors

that are on these routes and one hop away from it. Figure 6 illustrates the procedure: A is the starting anchor, D and H are the ending anchors. A knows the positions of D and H as well as the routes to them, which are (A, B, C, D) and (A, E, F, G, H), respectively. A estimates that the position of B is B' on dashed line AD and the position of E is E' on dashed line AH. A also estimates the average radio ranges in the direction of AD and AH, respectively.

With the collection of pairwise distances among neighboring nodes by RSSI sensing, MDS can be performed to calculate the local map (or the relative positions) for neighboring sensor nodes. In Figure 6, the relative positions of neighboring nodes A, B, E, J, K are calculated by A. Through aligning the relative positions of A, B, E with their physical positions, the physical positions of J, K can be calculated as well. In the same way, localized mapping and alignment are performed for sensor nodes along a route from the starting anchor to an ending anchor. Figure 7 illustrates the procedure of propagated position estimation from starting anchor to ending anchor.
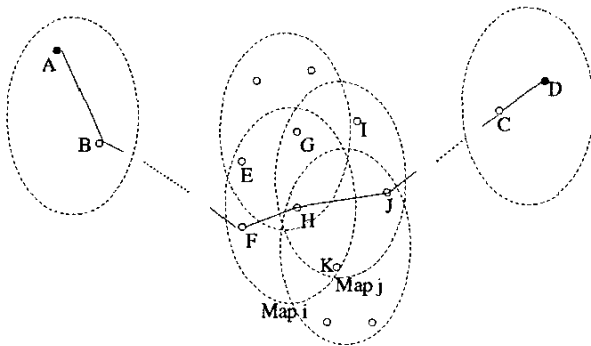

Fig. 7.   The propagation of position estimation

In Figure 7, A is the starting anchor and D is the ending anchor. The remaining nodes are along the route of flooding from A to D and each local map is represented with a dash ellipse. Map i contains adjacent sensors E, F, G, H, K. Since the physical positions of E, F, G are calculated previously, the physical positions of H, K can be computed with the above MDS and alignment techniques. Then $H, K, I, J,$ and G are adjacent sensors and build map j to further estimate I and J's positions. Figure 8 illustrates four adjacent sensors $A, B, C,$ and D. r is the radio range. $A, B,$ and C are nodes with positions known. D collects the position of $A, B,$ and C, and then calculate their pairwise distances. D also has its distances to A, B, C, respectively. Thus, D can perform a classical MDS to compute the local map (or relative positions of the four sensors).

Figure 9 illustrates an example of six adjacent sensors $A, B, C, D, E,$ and F. r is the radio range. Sensors $A, B, C,$ and D know their positions, and sensors E and F don't know their positions. E collects the position of A, B and its distances to them. Then E relays this information to F. F collects the positions of C, D, and its distances to them. Thus, F can compute the pairwise distances of the six sensors except the

distances of AF, BF, CE, DE. F can perform an iterative MDS to compute the local map (or the relative positions of the six sensors).
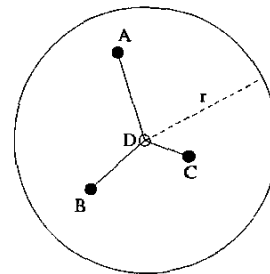

Fig. 8.   Classical MDS

Then, positions of all nodes around a route from a starting anchor to an ending anchor and the ending anchor itself can be estimated. For example, in Figure 6, the estimated position of nodes E, F, G are E', F', G', respectively. With the physical position of G known in advance, we can compare G' and G and align them if they are not equal (rotate $\angle G'AG$ with A as center and then scale AG' to AG). We can also apply the same alignment to the coordinates of all sensors along the route, such as E' and F'. In general, the positions of E' and F' are effectively corrected and approximated to their true positions, respectively. The above position estimation procedures are executed iteratively on a route from a starting anchor to an ending anchor until estimated positions converge. Our experimental results indicate that this procedure usually generate accurate position estimation for sensors along a route. Then, those nodes with positions accurately estimated are viewed as anchor nodes, and they initialize other position estimation for sensors along different routes. The estimation method can be performed on different portion of sensors in an ad hoc sensor network simultaneously until all sensors know their positions.
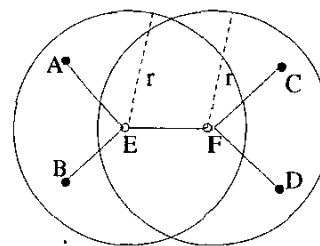

Fig. 9.   Iterative MDS

B. On Demand Distributed Position Estimation

In many application of sensor networks, there is no need to estimate all sensors location information in the sensor network. For example, sensors in a small area collaboratively detect intruders into the area. Position information of sensors within the small area should be estimated only. Since positioning all sensors usually consumes a large amount of time and energy, it is desirable to enable on demand sensor positioning.

We propose a distributed on demand positioning method based on the above position estimation method with anchor sensors. Without loss of generality, we study the case that one sensor's position is needed to be estimated. The sensor (starting sensor) first initializes flooding to pass its message to three or more anchor sensors, which are called ending anchors. The ending anchors send their locations and the flooding routes
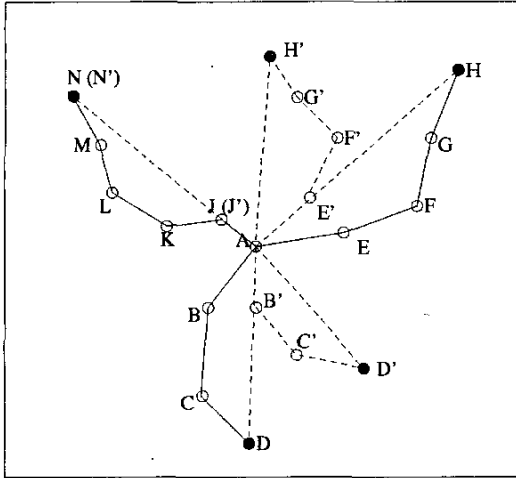

Fig. 10. Position estimation in the adjacent area of a sensor without position known.

back to the starting sensor. Then, the starting sensor knows the positions of ending anchors and routes to each of them. The starting sensor first simply estimates its physical position with a trilateration based on its hop distances to ending anchors, which is similar to the distance vector exchange based method [20]. Then, it estimates the positions of those sensors that are on these routes and on hop away from it. Figure 10 illustrates the procedure: A is the starting sensor, D, H, and N are the ending anchors. A knows the positions of D, H, and N as well as the routes to them, which are (A, B, C, D), (A, E, F, G, H), (A, J, K, L, M, N), respectively. A estimates that the position of B is B' on dashed line AD, the position of E is E' on dashed line AH, and the position of J is J' on dashed line AN.

MDS is used to calculate the local map (or the relative positions) for neighboring sensor nodes. In Figure 10, the relative positions of neighboring nodes A, B, E, J are calculated by A. As shown in Figure 7, localized mapping and alignment are performed for sensor nodes along a route from the starting sensor to each ending anchor. Eventually, ending anchors' physical location are calculated and sent back to the staring sensor. Then, the starting sensor aligns the estimated three or more ending anchors' locations to their physical locations based on the alignment technique presented in Section V. With the calculated alignment parameters during the alignment, the starting sensor maps itself from the estimated position to its physical position.

The above position estimation procedures can be executed for several times from a starting sensor to the ending anchors
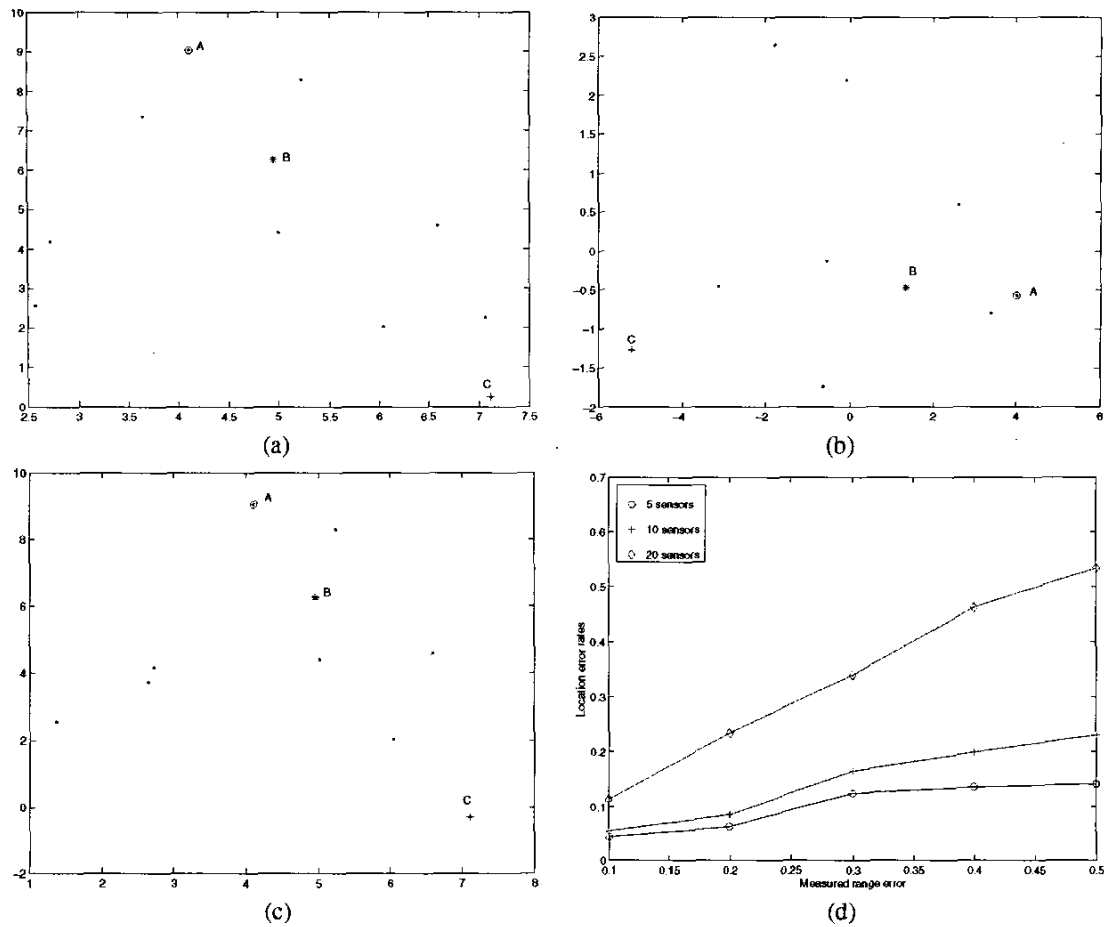
until estimated positions of starting sensor converge. At the same time, location information of sensors along the routes from the starting sensor to all ending anchors are also estimated as bonus. Our experimental results indicate that the procedure usually generate accurate position estimation for sensors along the routes and the starting sensor.

VII. EXPERIMENTAL RESULTS

A. Simulations Model

We simulated our proposed distributed positioning methods with Matlab. In order to exam the performance of our distributed positioning method, different sensor deployment strategies are considered to model anisotropic network topology and complex terrain. The first strategy is that 400 nodes are randomly placed in a 100-by-100 square region, and the average radio range is 10. The second strategy is that 400 nodes are randomly placed in a 100 by 100 square region, and the region is equally divided into four non-overlapped square regions. Sensors have different radio ranges. The average radio ranges in different small square regions are 7, 8.5, 10, and 11.5.

We also consider the errors of neighboring sensor distance estimation with RSSI. The measurement error is in the range 0% $-$ 50% of the average radio range, uniformly distributed.

B. Evaluation Criteria

We measure the performance of the algorithm with mean error, which is widely used in previous research works:

$$error = \frac{\sum_{i=m+1}^{n} \|x_{est}^i - x_{real}^i\|_2}{(n - m) \times (radio - range)}, \quad (5)$$

where n and m are the total number of sensors and the number of anchors, respectively. A low error means good performance of the evaluated method.

C. Results

In order to understand how the classic MDS and iterative MDS work, we first study the performance of classic MDS and iterative MDS in recovering adjacent sensors' position. Figure 11(a), (b), and (c) show the procedure of recovering sensors positions within a small area with classical MDS. Sensors A, B, and C are the three anchor senors. The Figure 11(c) shows the estimated physical position for all sensors within the area. Figure 11(d) indicates that when the error of measured distances for pairwise adjacent sensors increase, the error rates of sensor positioning increases. We vary the density of sensor deployment so that different number of sensors are enclosed in the area. When the number of sensors in the area is small, the error rates barely increase even the measured distance error increases. When there are more sensors in the area, the error rates of sensor positioning increase faster. The increase of error rates different conditions is always slower than the increase of distance measurement error. This indicates the classical MDS is robust in tolerating measurement errors of sensor distance. Based on the experiments, we get the observation that it is preferred to estimate positions for less number of sensors

Fig. 11. (a)The physical positions of sensors in a adjacent area.(b) The recovered relative positions of sensors in the adjacent area based on classical MDS. (c) These sensors' physical positions after alignment. (d)When the error of measured distances for pairwise adjacent sensors increases, the error rates of estimated sensor positions increases.
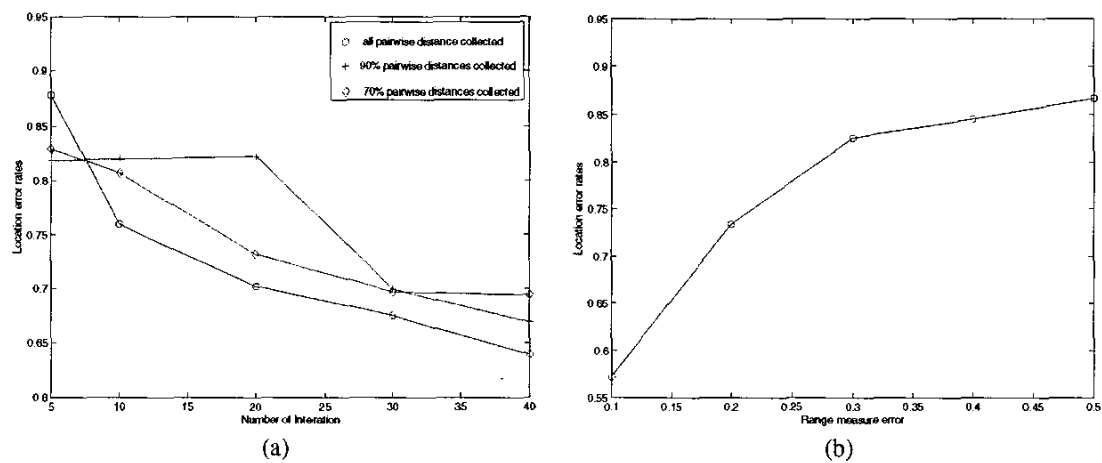

Fig. 12. (a)Error rates of sensor positioning increase when the percentage of sensor pairwise distances collected and the number of iteration increase. (b)When the collected pairwise distance and the number of iteration are fixed, the error rates of sensor positioning increase with the increase of distance measurement error.

within a small area, which tends to generate more accurate sensor positioning.

Figure 12 is the experimental results about recovering sensors' location with iterative MDS. When the number of iteration increases, the error rates of sensor positioning decreases. But, a large number of iteration steps mean high computation costs and computation time. In Figure 12(a), the three curves correspond to error rates with different percentage of pairwise distances collected during sensor positioning. When more pairwise distances collected for sensor positioning based on iterative MDS, the error rates decrease as well. The error rates of sensor positioning with iterative MDS is larger than that with classical MDS. In Figure 12(b), when the collected pairwise distance and the number of iteration are fixed, the error rates of sensor positioning increase with the increase of distance measurement error. The increase of sensor positioning error rates are slower than the increase of distance measurement. This indicates the iterative MDS is also robust in tolerating errors of pairwise sensor distance measurement.
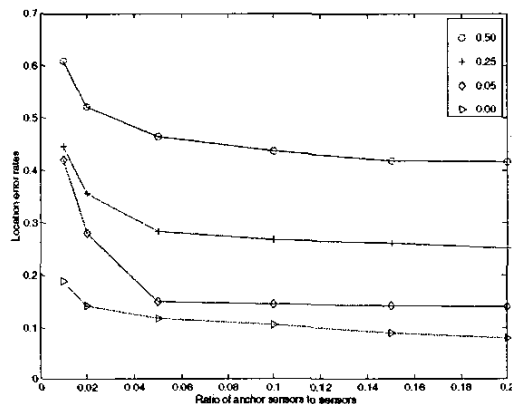

Fig. 13. Errors when applying the distributed positioning method with anchor sensors to all sensors in a square region with an uniform radio range and different distance measurement errors

Figure 13 and 14 are the experimental results with our proposed distributed method with anchor sensors for sensor positioning. When 400 sensors are deployed randomly and uniformly in a square area and their radio ranges are same, the error rates of sensor positioning decrease with the increase of number of anchor sensors in Figure 13. We find that when the total number of anchor sensor in the square area is small, a litter increase of the total number of anchor sensors will improve the error rates a lot. But, when there are about 10% sensors in the square area are anchor sensors, the error rates almost reach its minimum. Pure increase of the anchor sensors will not bring in much improvement of error rates any more. The distance measurement error rates vary: 0.0, 0.05, 0.25, and 0.50 in Figure 13. Small distance measurement error definitely generate accurate sensor positioning.

Similar observations are obtained from Figure 14. When 400 sensors are randomly and uniformly deployed in a square area with different terrain at different portions of the area. The different terrain generates different radio attenuate ratios

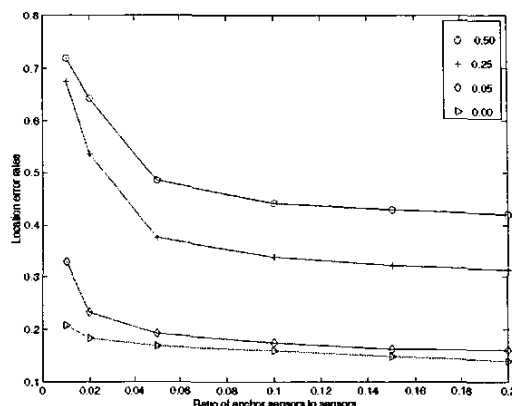and sensor radio ranges. When we compare the minimum error

Fig. 14. Errors when applying the distributed positioning method with anchor sensors to all sensors in a square region with different signal attenuation factors (radio ranges)

rates in Figure 14 and that in Figure 13, we find they are close. This indicates our proposed distributed sensor positioning method is robust in deal with complex terrain and anisotropic network topology.

In order to study the performance of our proposed on demand sensor positioning method, we do experiments on positioning one sensor in the square area with uniform radio attenuation ratio as well that with four different radio attenuation ratio. The number of anchor sensors are eight, which is 5% of the total number of sensors deployed in a square area. We also vary the distance measurement error rates to see its effect on the sensor positioning. The error rates with sensors deployed in the square area with an uniform terrain are slightly lower than that in the square area with different terrain condition. The overall error rates of on demand sensor positioning is not as good as its error rates estimated with distributed sensor positioning method.
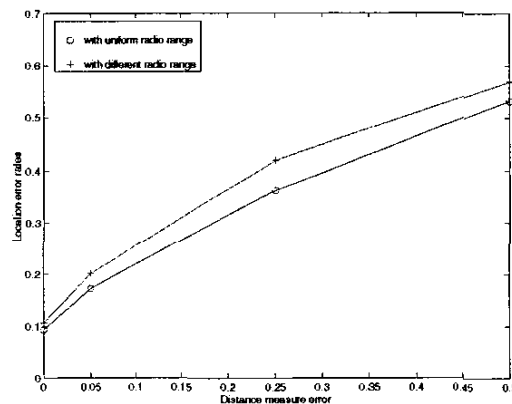

Fig. 15. Errors when applying the distributed on demand positioning method to one sensor in two square regions with uniform and different signal attenuation factors, respectively.

VIII. CONCLUSION AND FUTURE WORK

In this paper we address shortcomings, which are caused by anisotropic network topology and complex terrain, of existing sensor positioning methods. Then, we explore the idea of using multidimensional scaling technique to compute relative positions of sensors in a wireless sensor network. A distributed sensor positioning method based on multidimensional scaling is proposed to get the accurate position estimation and reduce error cumulation. Comparing with other positioning methods, with very few anchors, our approach can accurately estimate the sensors' positions in network with anisotropic topology and complex terrain as well as eliminate measurement error cumulation. We also propose an on demand position estimation method based on multidimensional scaling for one or several adjacent sensors positioning. Experimental results indicate that our distributed method for sensor position estimation is very effective and efficient.

For future work, we plan to carry out analysis of the communication costs during the operation of our methods. We will do experiments related to message complexity and power consumption. We also plan to investigate localization problems for *mobile* sensors in wireless ad-hoc sensor networks.

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