Deep Learning for Computer Vision

Image in the Frequency Domain

Vineeth N Balasubramanian

Department of Computer Science and Engineering
Indian Institute of Technology, Hyderabad

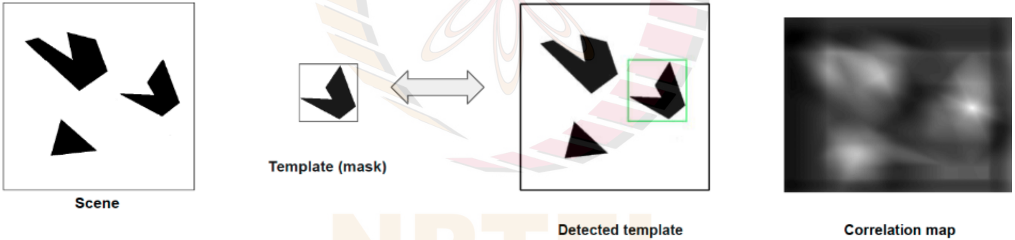
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Review: Questions to Think About

- Do we then need (cross)-correlation at all?
- Are all filters always linear?

Is Correlation Still Useful?

- Can be used for Template Matching
- Filters look like objects they are intended to find $\implies$ use Normalized Cross-correlation (to control relative brightness) score to find a given pattern in an image



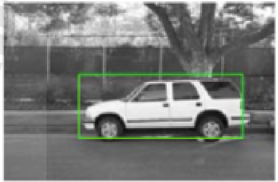
Credit: K Grauman, Univ of Texas Austin

Is Correlation Still Useful?

- Even if the template is not identical to some subimage in the scene, match can be meaningful, if scale, orientation and general appearance is right.

Scene

Template

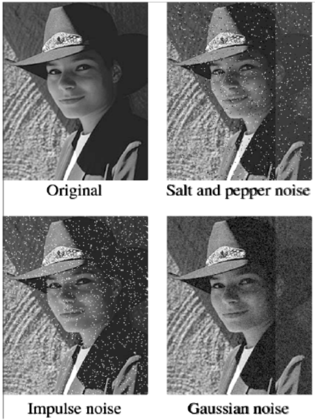


Detected template

Credit: K Grauman, Univ of Texas Austin

Non-Linear Filters

Different types of noise in images



Reducing Salt-and-Pepper noise using Gaussian filters

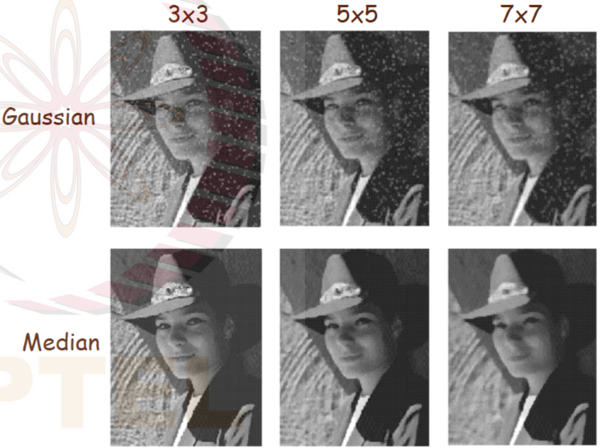
See the problem? What do we do?

Credit: S Seitz, Univ of Washington & J Košecká, George Mason University

Non-Linear Filters: Median Filter

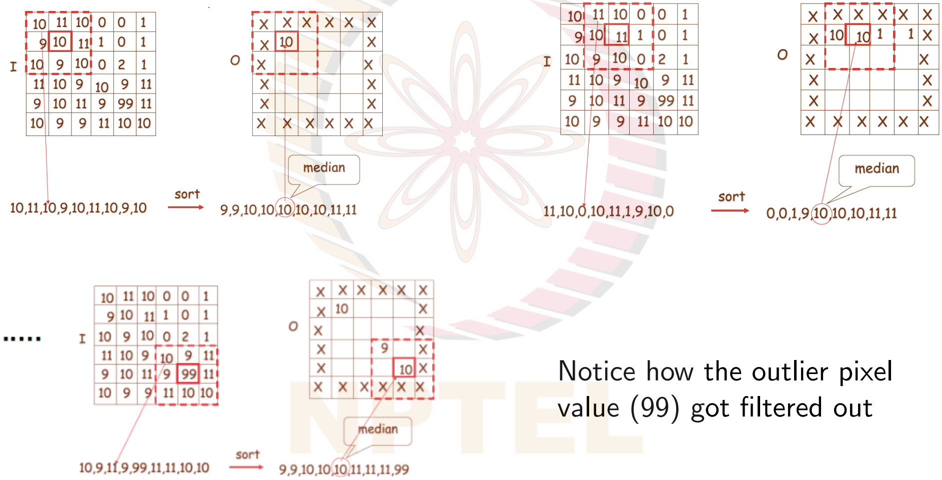
- Replace each pixel with MEDIAN value of all pixels in neighbourhood

- Properties:
  - Non-linear
  - Does not spread noise
  - Can remove spike noise
  - Robust to outliers, but not good for Gaussian noise



Credit: J Košecká, George Mason University

Median Filter: Example



Notice how the outlier pixel value (99) got filtered out

Credit: J Košecká, George Mason University

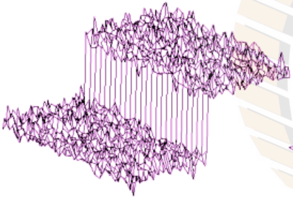
Non-Linear Filters: Bilateral Filtering

- Noise removal comes at expense of image blurring at edges
- **Bilateral filtering:** Simple, non-linear edge-preserving smoothing
- Reject (in a soft manner) pixels whose values differ too much from the central pixel value.
- Output pixel value is weighted combination of neighboring pixel values: $g(i,j) = \frac{\sum_{k,l} I(k,l)w(i,j,k,l)}{\sum_{k,l} w(i,j,k,l)}$
- Data-dependent bilateral weight function composed of domain and range kernel:

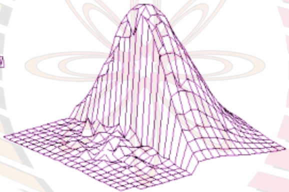
$$w(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{||I(i,j) - I(k,l)||^2}{2\sigma_r^2}\right)$$

Credit: Wikipedia; CVOnline

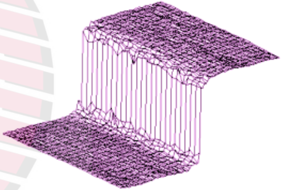
Bilateral Filters: Example



Given region in an image



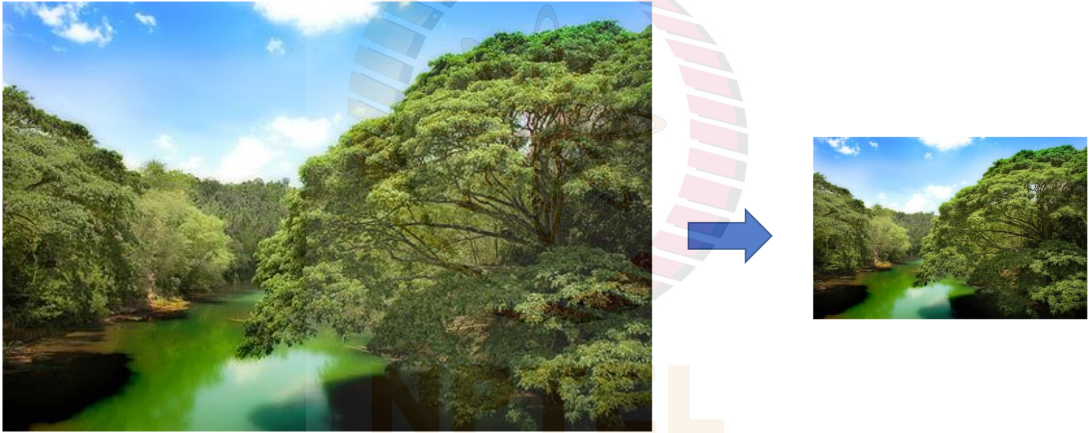
23 x 23 filter centered
two pixels to right of step
in previous figure



Filtered output

Credit: *CVOnline*

What do we lose in a low-resolution image?



Credit: Derek Hoeim, UIUC; James Hays, Gatech

Fourier

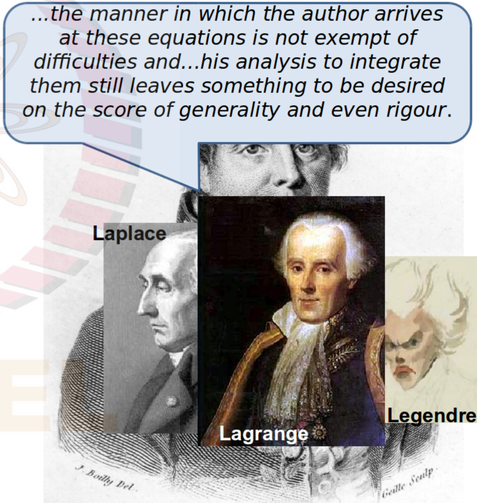
Jean Baptist Joseph Fourier (1768-1830) had an idea in (1807).

- **Idea:** Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.

Fourier

Jean Baptist Joseph Fourier (1768-1830) had an idea in (1807).

- **Idea:** Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.
- **Of course, what's new?**
  - Many including Lagrange, Laplace, Poisson and other big wigs did not believe him
  - Not translated into English until 1878!
- **(Mostly) true!**
  - Called Fourier Series
  - Some subtle restrictions



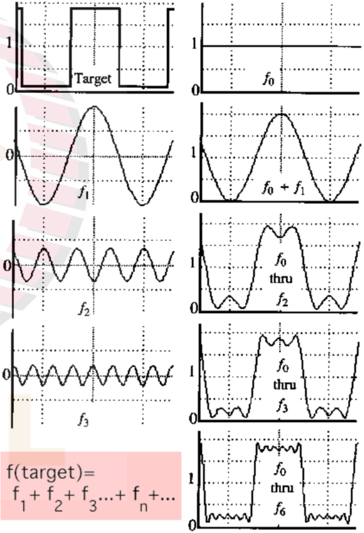
Credit: James Hays

A sum of sines

- Building block:

$$A \sin(\omega x + \phi)$$

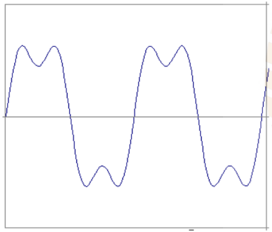
- Add enough of them to get any signal f(x) you want!



Credit: James Hays, Gatech

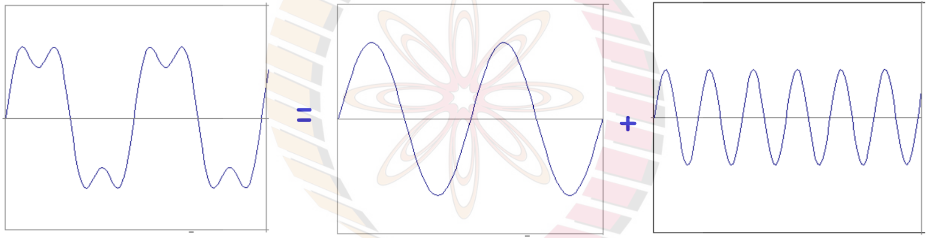
The Fourier Spectrum

Example: $g(t) = \sin(2pft) + (1/3)\sin(2p(3f)t)$



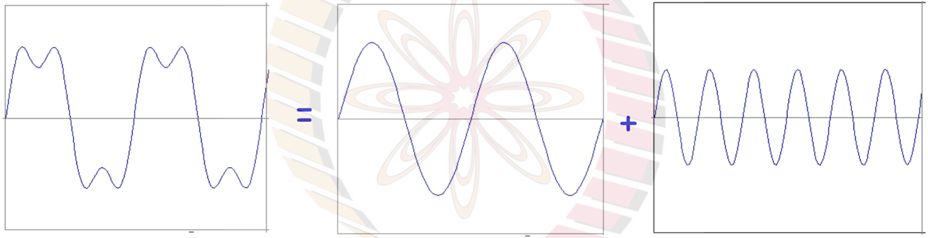
The Fourier Spectrum

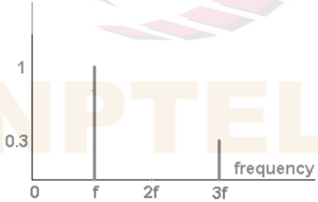
Example: $g(t) = \sin(2pft) + (1/3)\sin(2p(3f)t)$



The Fourier Spectrum

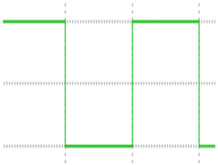
Example: $g(t) = \sin(2pft) + (1/3)\sin(2p(3f)t)$



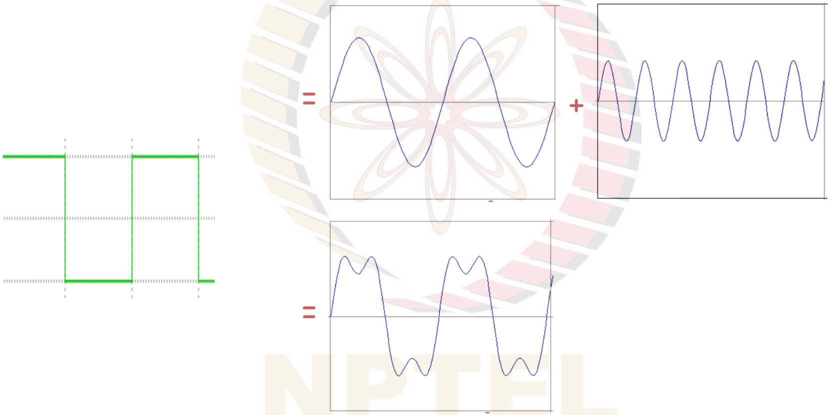


Credit: Alexei Efros, UCBerkeley; James Hays, Gatech

The Fourier Spectrum

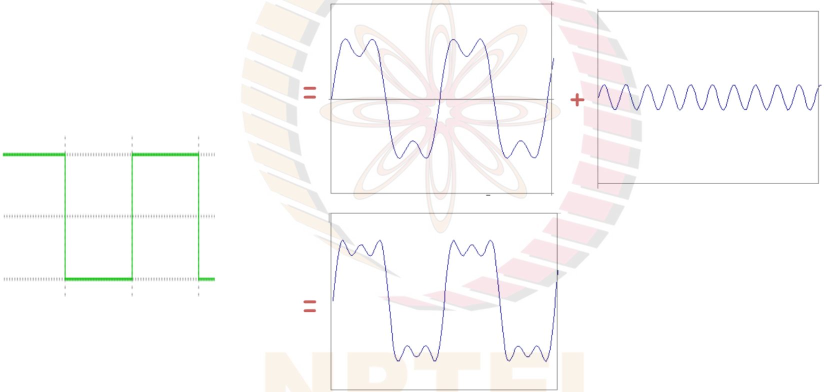


The Fourier Spectrum



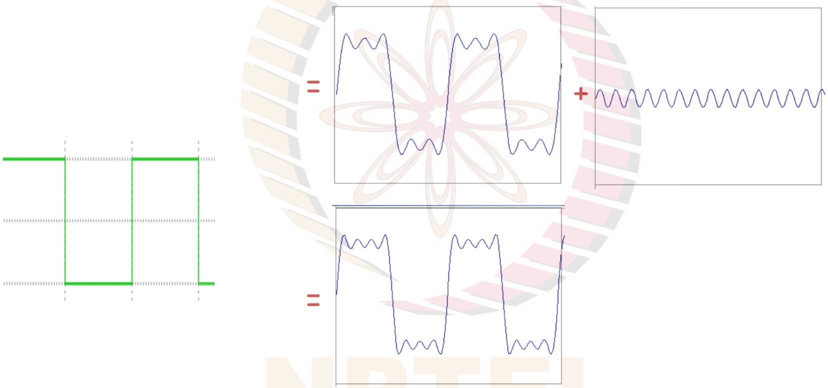
Credit: James Hays, Gatech

The Fourier Spectrum



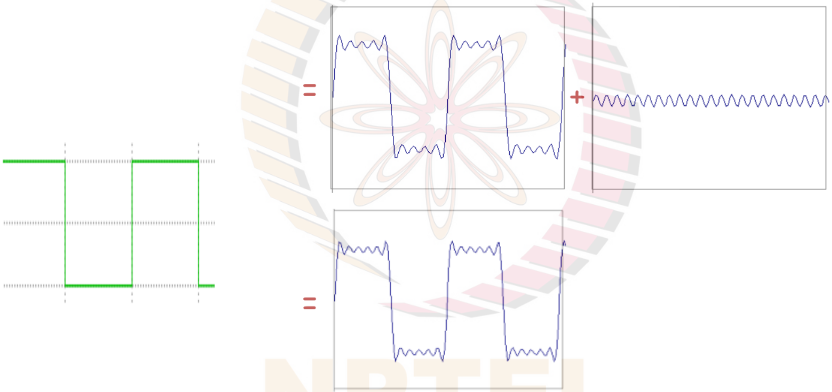
Credit: James Hays, Gatech

The Fourier Spectrum



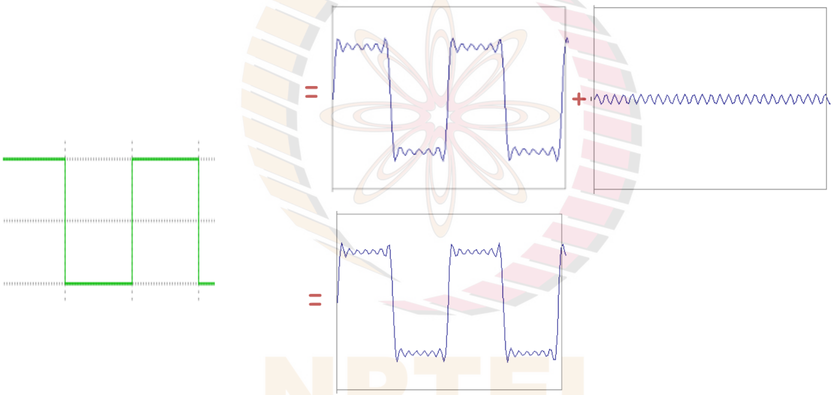
Credit: James Hays, Gatech

The Fourier Spectrum



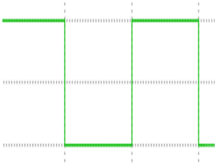
Credit: James Hays, Gatech

The Fourier Spectrum



Credit: James Hays, Gatech

The Fourier Spectrum



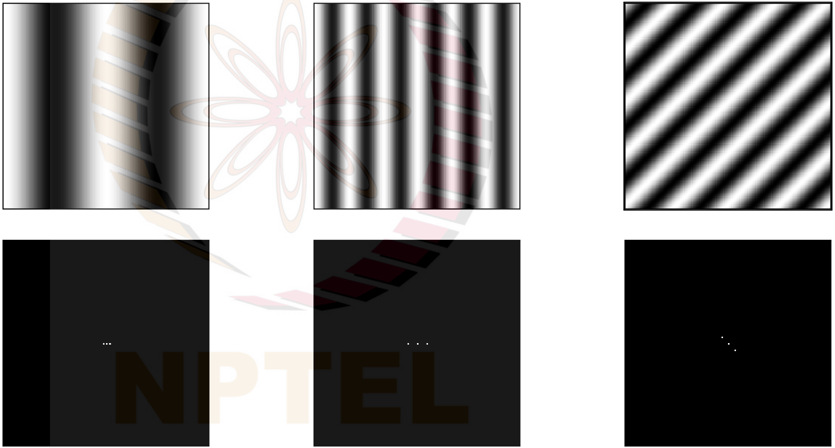
$$= A \sum_{1}^{\infty} \frac{1}{k} \sin(2\pi k t)$$

Credit: James Hays, Gatech

Fourier Analysis of Images

Intensity Image

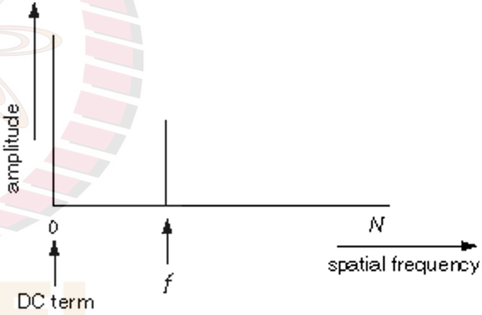
Fourier Image



Credit: Derek Hoeim, UIUC

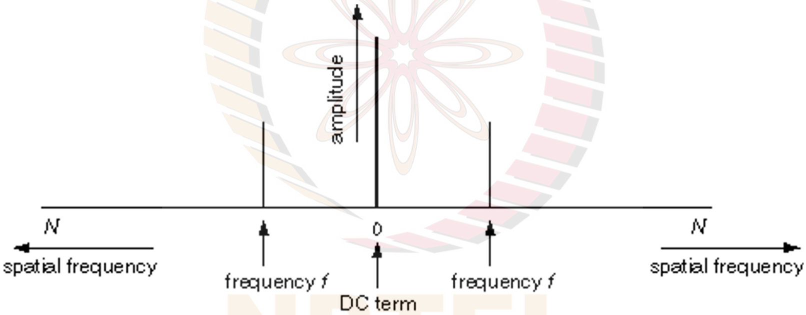
Fourier Analysis of Images

- Encodes a whole series of sinusoids through a range of spatial frequencies from zero all the way up to '*Nyquist frequency*' (more on this later)
- Signal containing only a single spatial frequency of frequency f is plotted as:
  - a single peak at point f along spatial frequency axis
  - height of that peak corresponding to the amplitude, or contrast of that sinusoidal signal



Fourier Analysis of Images

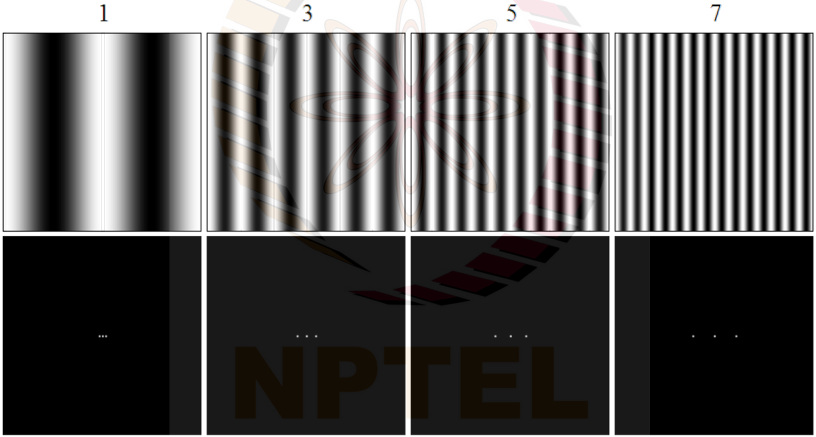
Mirror-image reflections along the axes



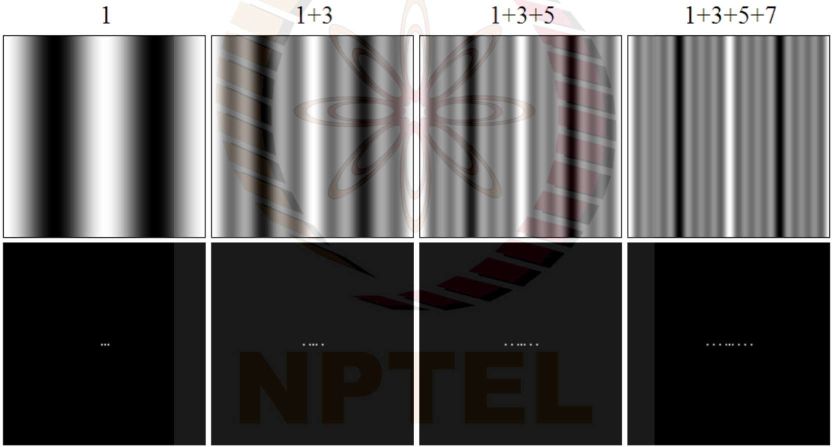
Why? See

http://dsp.stackexchange.com/questions/4825/why-is-the-fft-mirrored

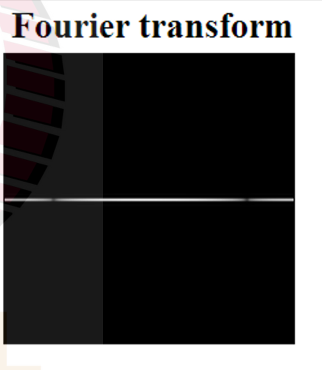
Fourier Analysis of Images: Examples



Fourier Analysis of Images: Examples



Fourier Analysis of Images: Examples



Fourier Analysis of Images: Examples

Credit: Forsyth and Ponce, Computer Vision: A Modern Approach, 2003

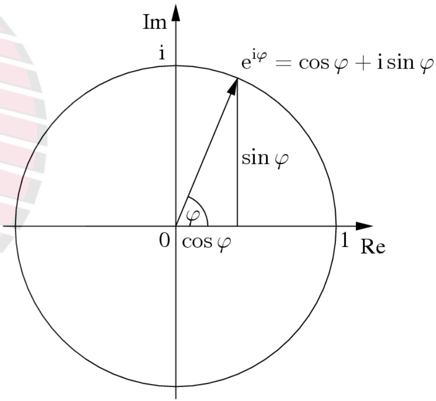
Fourier Transform: Magnitude and Phase

- Fourier transform stores the **magnitude** and **phase** at each frequency
  - For mathematical convenience, this is often denoted in terms of real and complex numbers
  - **Magnitude** encodes how much signal is there at a particular frequency

$$A = \pm\sqrt{Re(\varphi)^2 + Im(\varphi)^2}$$

  - **Phase** encodes spatial information (indirectly)

$$\phi = \tan^{-1}\frac{Im(\varphi)}{Re(\varphi)}$$



Credit: Wikimedia Commons

Credit: Derek Hoeim, UIUC

Fourier Transform: Magnitude and Phase

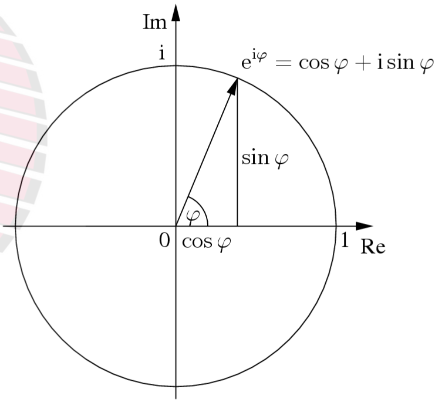
- Fourier transform stores the **magnitude** and **phase** at each frequency
  - For mathematical convenience, this is often denoted in terms of real and complex numbers
  - **Magnitude** encodes how much signal is there at a particular frequency

$$A = \pm\sqrt{Re(\varphi)^2 + Im(\varphi)^2}$$

  - **Phase** encodes spatial information (indirectly)

$$\phi = \tan^{-1}\frac{Im(\varphi)}{Re(\varphi)}$$

Credit: *Derek Hoeim, UIUC*



Credit: Wikimedia Commons

Fourier Transform: Magnitude and Phase

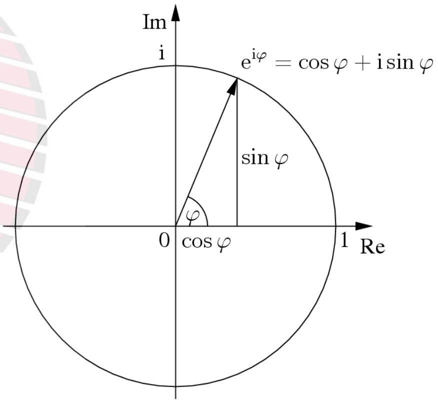
- Fourier transform stores the **magnitude** and **phase** at each frequency
  - For mathematical convenience, this is often denoted in terms of real and complex numbers
  - **Magnitude** encodes how much signal is there at a particular frequency

$$A = \pm\sqrt{Re(\varphi)^2 + Im(\varphi)^2}$$

  - **Phase** encodes spatial information (indirectly)

$$\phi = \tan^{-1}\frac{Im(\varphi)}{Re(\varphi)}$$

Credit: Derek Hoeim, UIUC



Credit: Wikimedia Commons

Continuous vs Discrete Fourier Transform

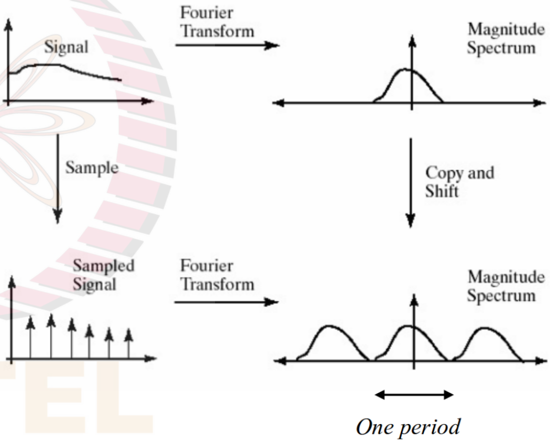
- **Continuous Fourier transform (FT):**

$$H(\omega) = \int_{-\infty}^{\infty} h(x) e^{-j\omega x} dx$$

- **Discrete Fourier Transform (DFT):**

$$H(\omega) = \sum_{0}^{N-1} h(x) e^{-j\frac{2\pi kx}{N}} dx$$

where N is the length of the sampled signal.



Credit: Elgammal, Rutgers University

More on the Fourier Transform

If you want to learn more on the Fourier transform

- An intuitive explanation (highly recommended if you don't have a background in signal processing): An Interactive Guide to the Fourier Transform
- Other good tutorial-styled references:
  - Lecture by Lennart Lindegren, Lund University
  - An Introduction to the DFT
  - Wikipedia: Discrete Fourier Transform
- Any Digital Signal Processing course on NPTEL

Convolution Theorem

- Fourier transform of convolution of two functions is product of their Fourier transforms:

$$F[g * h] = F[g]F[h]$$

- **Convolution** in spatial domain can be obtained through **multiplication** in frequency domain!

$$g * h = F^{-1}[F[g]F[h]]$$

Credit: James Hays, Gatech

Properties of Fourier Transform

Property	Signal		Transform
superposition	$f_1(x) + f_2(x)$		$F_1(\omega) + F_2(\omega)$
shift	$f(x - x_0)$		$F(\omega)e^{-j\omega x_0}$
reversal	$f(-x)$		$F^*(\omega)$
convolution	$f(x) * h(x)$		$F(\omega)H(\omega)$
correlation	$f(x) \otimes h(x)$		$F(\omega)H^*(\omega)$
multiplication	$f(x)h(x)$		$F(\omega) * H(\omega)$
differentiation	$f'(x)$		$j\omega F(\omega)$
domain scaling	$f(ax)$		$1/aF(\omega/a)$
real images	$f(x) = f^*(x)$	$\Leftrightarrow$	$F(\omega) = F(-\omega)$
Parseval's Theorem	$\sum_x [f(x)]^2$	$=$	$\sum_\omega [F(\omega)]^2$

Credit: Szeliski, Computer Vision: Algorithms and Applications, 2010

Fast Fourier Transform

- Number of arithmetic operations to compute Fourier transform of N numbers (i.e., function defined at N points) is....

Fast Fourier Transform

- Number of arithmetic operations to compute Fourier transform of N numbers (i.e., function defined at N points) is.... proportional to N^2
- Possible to reduce this to $N \log N$ using **Fast Fourier Transform (FFT)**

For more, see `https://www.karlsims.com/fft.html`

Fast Fourier Transform

- Number of arithmetic operations to compute Fourier transform of N numbers (i.e., function defined at N points) is.... proportional to N^2
- Possible to reduce this to $N \log N$ using **Fast Fourier Transform (FFT)**
- FFT is a **recursive divide-and-conquer algorithm** for computing DFT

For more, see https://www.karlsims.com/fft.html

Fast Fourier Transform

- Number of arithmetic operations to compute Fourier transform of N numbers (i.e., function defined at N points) is.... proportional to N^2
- Possible to reduce this to $N \log N$ using **Fast Fourier Transform (FFT)**
- FFT is a **recursive divide-and-conquer algorithm** for computing DFT
- Applications of FFT? Examples: Convolution, correlation

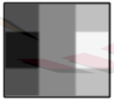
For more, see https://www.karlsims.com/fft.html

Filtering in Spatial Domain

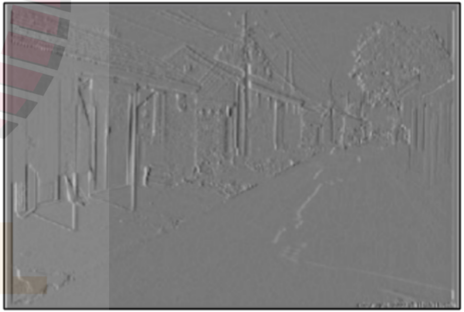
1	0	-1
2	0	-2
1	0	-1

Intensity Image

$*$

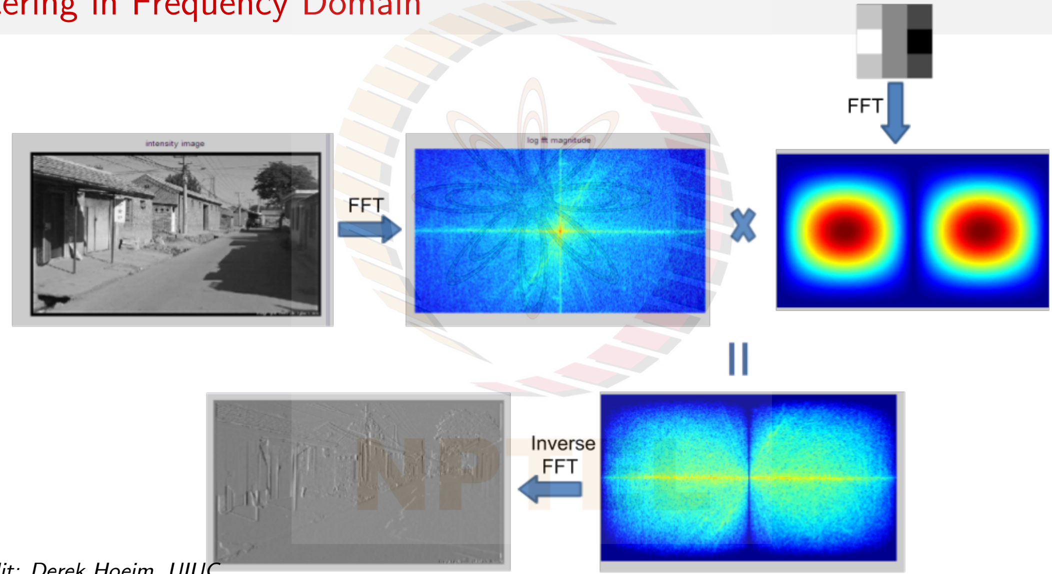


$=$



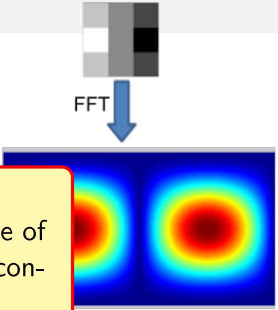
Credit: Derek Hoeim

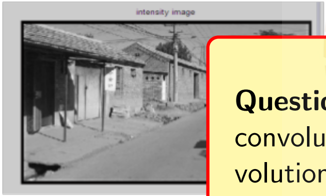
Filtering in Frequency Domain

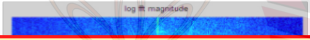


Credit: Derek Hoeim, UIUC

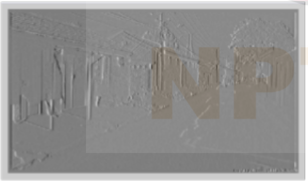
Filtering in Frequency Domain

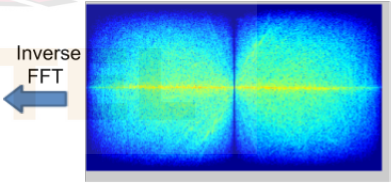






Question: What cost improvement does use of convolution theorem (or FFT over vanilla convolution) give?





Low-Pass and High-Pass Filters

- **Low-Pass Filters:** Filters that allow low frequencies to pass through (block high frequencies). Example?

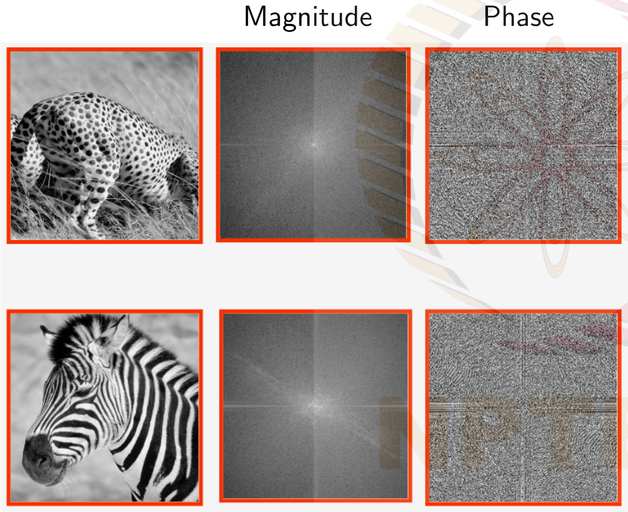
Low-Pass and High-Pass Filters

- **Low-Pass Filters:** Filters that allow low frequencies to pass through (block high frequencies). Example?
  - Gaussian filter
- **High-Pass Filters:** Filters that allow high frequencies to pass through (block low frequencies). Example?

Low-Pass and High-Pass Filters

- **Low-Pass Filters:** Filters that allow low frequencies to pass through (block high frequencies). Example?
  - Gaussian filter
- **High-Pass Filters:** Filters that allow high frequencies to pass through (block low frequencies). Example?
  - Edge filter

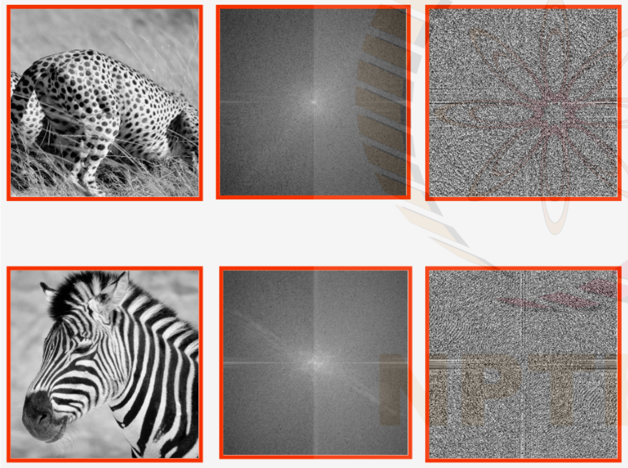
Which has more information: Magnitude or Phase?



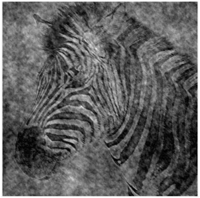
Swap phase and reconstruct?

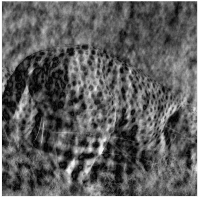
Which has more information: Magnitude or Phase?

Magnitude Phase



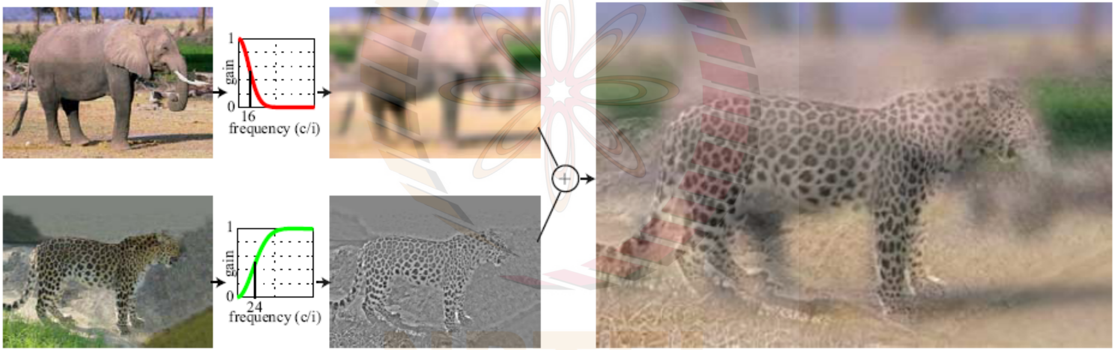
Swap phase and reconstruct?





Credit: Forsyth and Ponce, Computer Vision: A Modern Approach, 2003

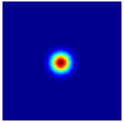
Hybrid Images



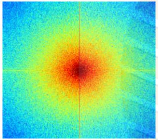
Credit: A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Exercise: Match spatial domain image to Fourier magnitude image

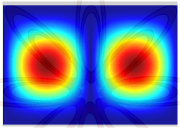
1



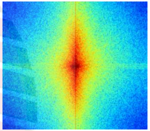
2



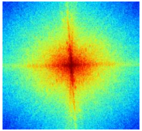
3



4



5

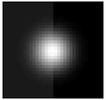


A

B

C

D



E

Homework

Readings

- Chapter 3.4, Szeliski, *Computer Vision: Algorithms and Applications*
- For more information on fourier transforms:
  - http://www.thefouriertransform.com/
  - http://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/
  - http://wwwpub.zih.tu-dresden.de/~ds24/lehre/bvme_ss_2013/ip_05_fourier.pdf
- Other links provided on respective slides

Questions/Exercises

- What cost improvement does convolution theorem give?
- Complete the matching exercise

References

- Richard Szeliski. *Computer Vision: Algorithms and Applications*. Texts in Computer Science. London: Springer-Verlag, 2011.
- David Forsyth and Jean Ponce. *Computer Vision: A Modern Approach*. 2 edition. Boston: Pearson Education India, 2015.
- *Hays, James, CS 6476 - Computer Vision (Fall 2018)*. URL: https://www.cc.gatech.edu/~hays/compvision/ (visited on 04/28/2020).
- *Hoiem, Derek, CS 543 - Computer Vision (Spring 2011)*. URL: https://courses.engr.illinois.edu/cs543/sp2017/ (visited on 04/25/2020).
- *Oliva, Aude, 6.819/6.869 - Advances in Computer Vision (Fall 2015)*. URL: http://6.869.csail.mit.edu/fa15/ (visited on 04/28/2020).