Convolutional Neural Networks: An Introduction

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Homework Exercises

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$; Parameters to be learned: γ , β

Output: $\{y_i = BN_{\gamma,\beta}(x_i)\}$

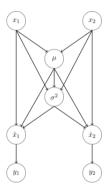
$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^{m} x_i \hspace{1cm} \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2$$
 // mini-batch variance

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$$
 // normalize

$$y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i)$$
 // scale and shift

Forward propagation is straight-forward:



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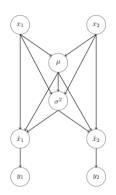
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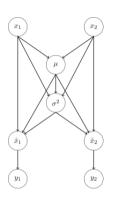
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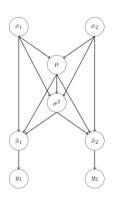
Backprop?

Image Credit: Aditya Agrawal



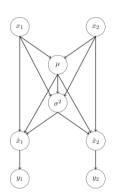
$$\frac{\partial L}{\partial \beta} = \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial \beta} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial \beta}$$
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$$\begin{split} \frac{\partial L}{\partial \gamma} &= \frac{\partial L}{\partial y_1} \frac{\partial y_1}{\partial \gamma} + \frac{\partial L}{\partial y_2} \frac{\partial y_2}{\partial \gamma} \\ &= \frac{\partial L}{\partial y_1} \hat{x}_1 + \frac{\partial L}{\partial y_2} \hat{x}_2 = \sum_{i=1}^2 \frac{\partial L}{\partial y_i} \hat{x}_i \end{split}$$

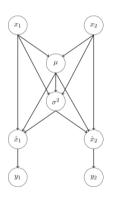


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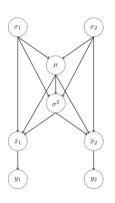
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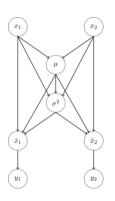


$$\frac{\partial L}{\partial \sigma^2} = \frac{\partial L}{\partial \hat{x}_1} \frac{\partial \hat{x}_1}{\partial \sigma^2} + \frac{\partial L}{\partial \hat{x}_2} \frac{\partial \hat{x}_2}{\partial \sigma^2} = \sum_{i=1}^2 \frac{\partial L}{\partial \hat{x}_i} \frac{\partial \hat{x}_i}{\partial \sigma^2}$$
$$= \sum_{i=1}^2 \frac{\partial L}{\partial \hat{x}_i} (x_i - \mu) \frac{-1}{2} (\sigma^2 + \epsilon)^{-3/2}$$

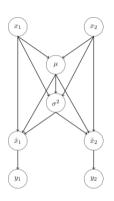
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$$\begin{split} \frac{\partial L}{\partial \mu} &= \frac{\partial L}{\partial \hat{x}_1} \frac{\partial \hat{x}_1}{\partial \mu} + \frac{\partial L}{\partial \hat{x}_2} \frac{\partial \hat{x}_2}{\partial \mu} + \frac{\partial L}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \mu} \\ &= \sum_{i=1}^2 \frac{\partial L}{\partial \hat{x}_i} \frac{\partial \hat{x}_i}{\partial \mu} + \frac{\partial L}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial \mu} \\ &= \sum_{i=1}^2 \frac{\partial L}{\partial \hat{x}_i} \frac{-1}{\sqrt{\sigma^2 + \epsilon}} + \frac{\partial L}{\partial \sigma^2} \frac{-2(x_1 - \mu) - 2(x_2 - \mu)}{2} \\ &= \sum_{i=1}^2 \frac{\partial L}{\partial \hat{x}_i} \frac{-1}{\sqrt{\sigma^2 + \epsilon}} + \frac{\partial L}{\partial \sigma^2} \frac{\sum_{i=1}^2 -2(x_i - \mu)}{2} \end{split}$$



$$\frac{\partial L}{\partial x_1} = \frac{\partial L}{\partial \hat{x}_1} \frac{\partial \hat{x}_1}{\partial x_1} + \frac{\partial L}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial x_1} + \frac{\partial L}{\partial \mu} \frac{\partial \mu}{\partial x_1}$$
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$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial \hat{x}_i} \frac{1}{\sqrt{\sigma^2 + \epsilon}} + \frac{\partial L}{\partial \sigma^2} \frac{2(x_i - \mu)}{m} + \frac{\partial L}{\partial \mu} \frac{1}{m}$$

Acknowledgements

• This lecture's content is largely based on **Lecture 11** of **CS7015** course taught by Mitesh Khapra at IIT Madras

Review: Convolution Operation

• Convolution is a mathematical way of combining two signals to form a third signal



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- As we saw in Part 5 of Week 1, it is one of the most important techniques in signal processing

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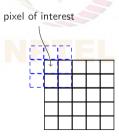
- Convolution is a mathematical way of combining two signals to form a third signal
- As we saw in Part 5 of Week 1, it is one of the most important techniques in signal processing
- In case of 2D data (grayscale images), the convolution operation between a filter $W^{k \times k}$ and an image $X^{N_1 \times N_2}$ can be expressed as:

$$Y(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} W(u,v)X(i-u,j-v)$$

• More generally, given a $K_1 \times K_2$ filter W, we can write it as:

$$Y(i,j) = \sum_{a = \lfloor -\frac{K_1}{2} \rfloor}^{\lfloor \frac{K_1}{2} \rfloor} \sum_{b = \lfloor -\frac{K_2}{2} \rfloor}^{\lfloor -\frac{K_2}{2} \rfloor} X(i-a,j-b) W\left(\frac{K_1}{2} + a, \frac{K_2}{2} + b\right)$$

This allows kernel to be centered on pixel of interest

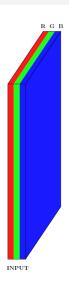


Pause and Ponder

- In the 1D case, we slide a one-dimensional filter over a one-dimensional input
- In the 2D case, we slide a two-dimensional filter over a two-dimensional input
- What would happen in the 3D case where your images are in color (RGB)?

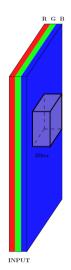
• What would a 3D filter look like?





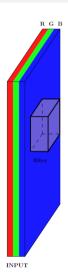
• What would a 3D filter look like?

 It will be in 3D too and we will refer to it as a volume



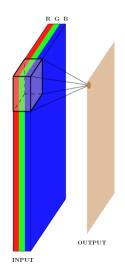
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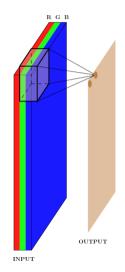


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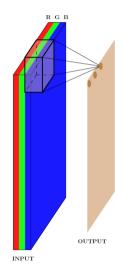


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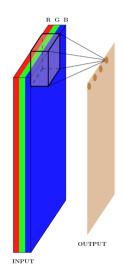
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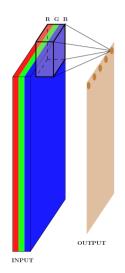
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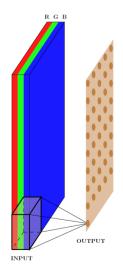


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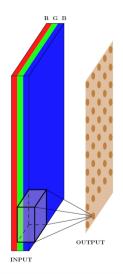
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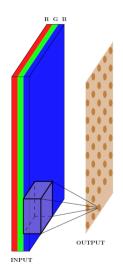
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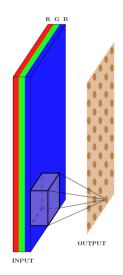
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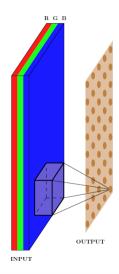
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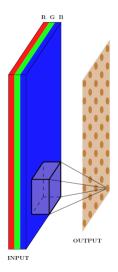
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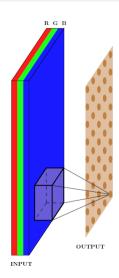


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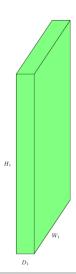


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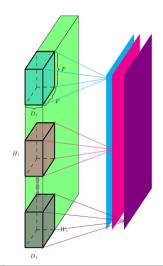
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- As a result the output will be 2D (only width and height, no depth)
- We can apply multiple filters to get multiple feature maps



• Input dimensions: Width $(W_1) imes \mathsf{Height}\ (H_1) imes \mathsf{Depth}\ (D_1)$

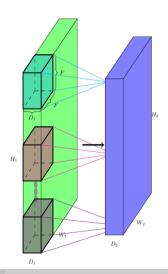


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- Spatial extent (F) of each filter (the depth of each filter is same as the depth of input)
- Output dimensions is $W_2 \times H_2 \times D_2$ (we will soon see a formula for computing W_2, H_2 and D_2)



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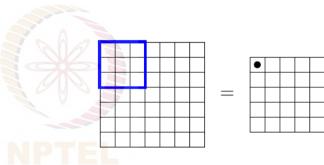
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- Stride (S) (explained in following slides)
- Number of filters K

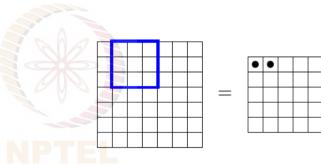


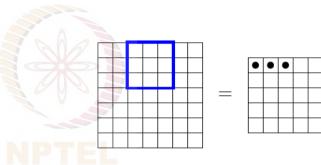
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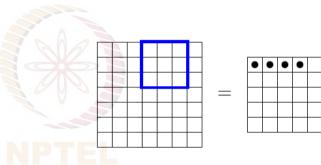


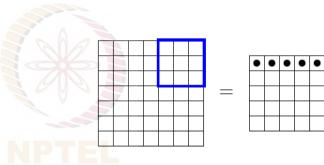
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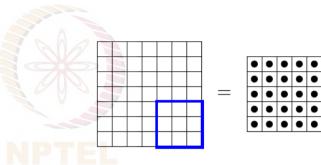




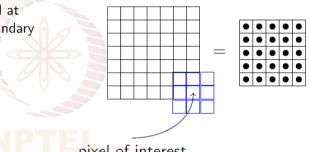






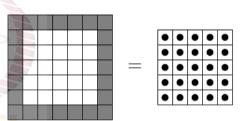


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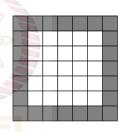


pixel of interest

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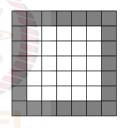


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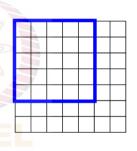


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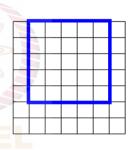


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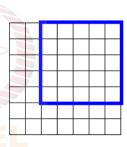


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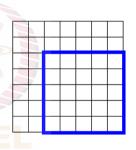


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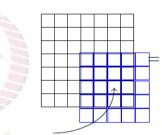


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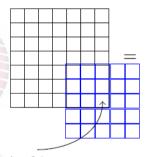
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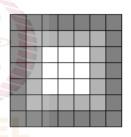




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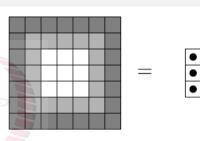
pixel of interest

- Let us compute dimensions (W_2, H_2) of output
- Recall that we can't place the kernel at corners as it will cross the input boundary
- This is true for all shaded points (the kernel crosses the input boundary)
- This results in an output which is of smaller dimensions than input
- As size of kernel increases, this becomes true for even more pixels
- ullet For example, let's consider a 5 imes 5 kernel
- We have an even smaller output now





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In general,

$$W_2 = W_1 - F + 1$$
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We will refine this formula further

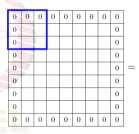
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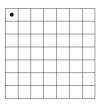


- What if we want output to be of same size as input?
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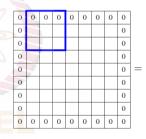


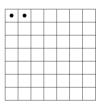
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- This means we will add one row and one column of 0 inputs at the top, bottom, left and right; recall there are other ways of padding, see Week 1 Part 5 lecture



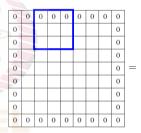


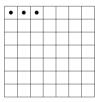
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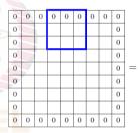


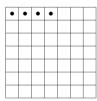
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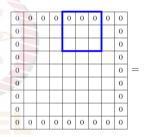


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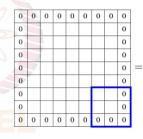


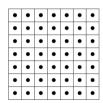
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We now have:

$$W_2 = W_1 - F + 2P + 1$$

$$H_2 = H_1 - F + 2P + 1$$

We will refine this formula further

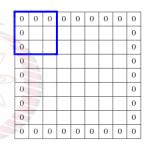
• What does **stride** S do?



- What does stride S do?
- It defines the intervals at which the filter is applied (here S=2)

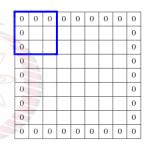


- What does **stride** S do?
- It defines the intervals at which the filter is applied (here S=2)
- Skip every 2nd pixel (S=2) which will result in an output of smaller dimensions



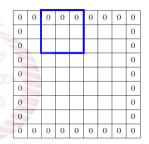


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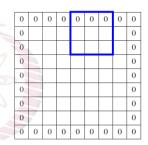


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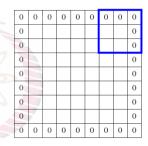


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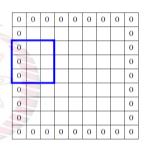


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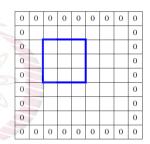


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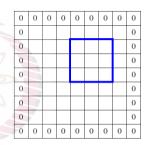


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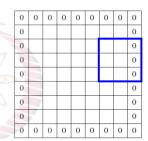


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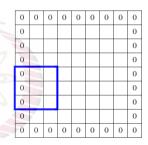


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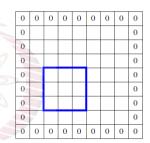


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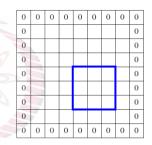


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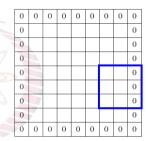


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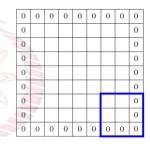


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So our final formula should mostly look like,

$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$
$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

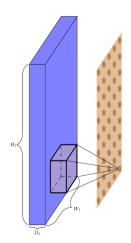
Not done yet, we will refine this formula further!

Finally, coming to depth of output

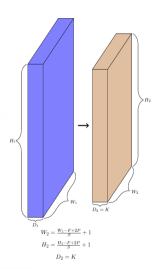


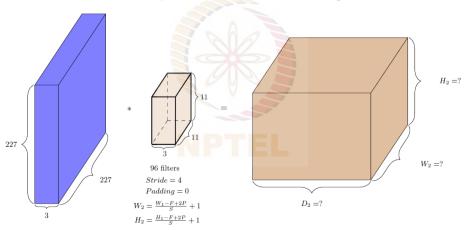
- Finally, coming to depth of output
- Each filter gives us one 2D output

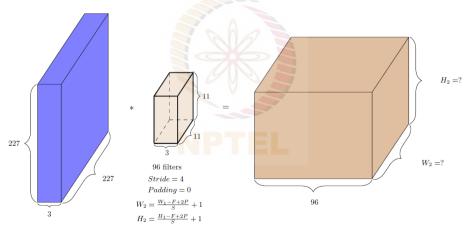


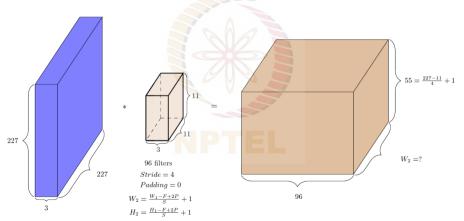


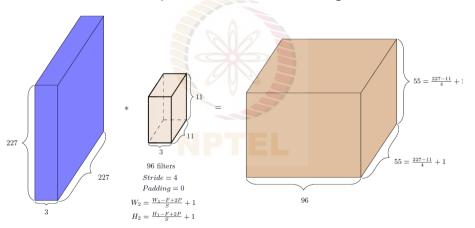
- Finally, coming to depth of output
- Each filter gives us one 2D output
- ullet K filters will give us K such 2D outputs
- We can think of resulting output as $K \times W_2 \times H_2$ volume
- Thus $D_2 = K$









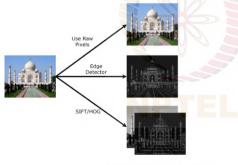


Pause and Ponder

- What is the connection between convolution and neural networks? Won't feedforward neural networks do?
- We will try to understand this by considering the task of "image classification"

Traditional Machine Learning for Vision

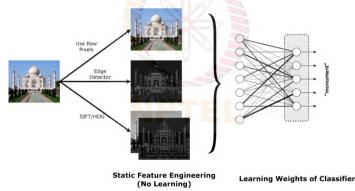
- Traditional ML-based computer vision solutions involve static feature engineering from images (e.g. recall SIFT, LBP, HoG, etc)
- Though effective, static feature engineering was a bottleneck of pre-DL vision solutions



Static Feature Engineering (No Learning)

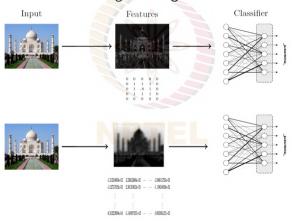
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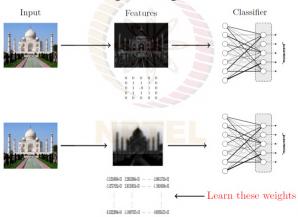


Vineeth N B (IIT-H) §5.1 Introduction to CNNs 19/37

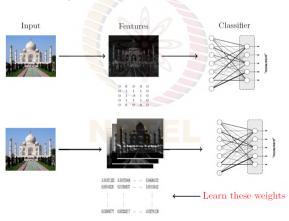
• Instead of using handcrafted kernels such as edge detectors can we **learn meaningful kernels/filters** in addition to learning the weights of the classifier?



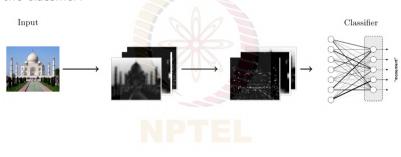
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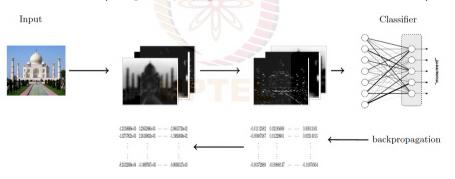
• Even better: Instead of using handcrafted kernels such as edge detectors can we learn multiple meaningful kernels/filters in addition to learning the weights of the classifier?



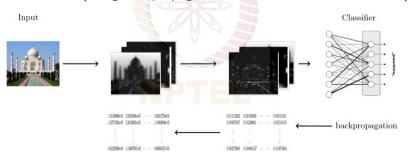
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- Can we learn multiple layers of meaningful kernels/filters in addition to learning the weights of the classifier? **Yes, we can!**
- Simply by treating these kernels as parameters and learning them in addition to the weights of the classifier (using backpropagation, discussed in the next lecture)



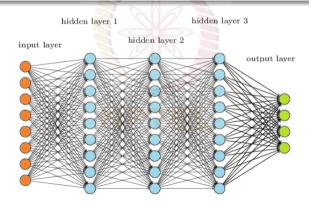
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Such a network is called a Convolutional Neural Network!

Pause and Ponder

- Learning kernels/filters by treating them as parameters definitely is interesting
- But why not directly use flattened images with fully connected neural networks (or feedforward neural networks, FNNs) instead?



Challenges of Applying FNNs to Images

MNIST Dataset

On a reasonably *simple* dataset like MNIST, we can get about 2% error (or even better) using FNNs, but

- Ignores spatial (2-D) structure of input images unroll each 28×28 image into a 784-D vector
 - Pixels that are spatially separate are treated the same way as pixels that are adjacent
- No obvious way for networks to learn same features (e.g. edges) at different places in the input image
- Can get computationally expensive for large images
 - For a 1MP color image with 20 neurons in the first hidden layer, how many weights in the first layer?

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 60 million!

- Local receptive fields, in which hidden units are connected to local patches of the layer below, serve two purposes:
 - Capture local spatial relationships in pixels (which would not be captured by FNNs)
 - Greatly reduces number of parameters in the model
 - For a 1MP color image a filter size of $K_1 \times K_2$ in the first hidden layer, how many weights in a convolutional layer?



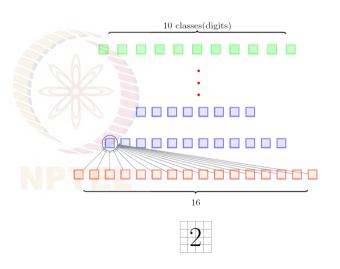
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- Weight sharing, which also serves two purposes:
 - Enables translation-invariance of neural network to objects in images
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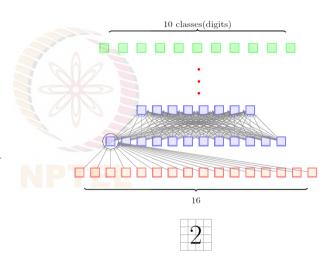
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- Pooling which condenses information from previous layer, serves two purposes:
 - Aggregates information, especially minor variations
 - Reduces size of output of a previous layer, which reduces number of computations in later layers

Credit: Steve Renals

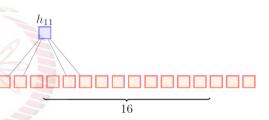
- This is what a regular feedforward neural network will look like
- There are many dense connections here

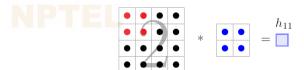


- This is what a regular feedforward neural network will look like
- There are many dense connections here
- All 16 input neurons are contributing to computation of h_{11}
- Let us contrast this to what happens in case of convolution

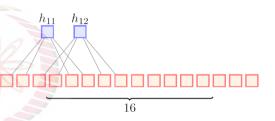


- Only a few local neurons participate in computation of h_{11}
- E.g. only pixels 1, 2, 5, 6 contribute to h_{11}



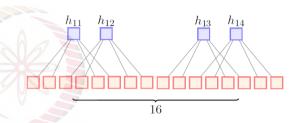


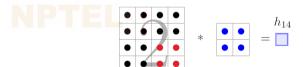
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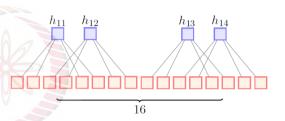


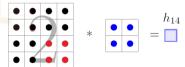
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- Similar for other pixels



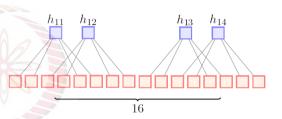


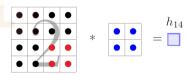
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- Similar for other pixels
- The connections are much sparser
- This sparse connectivity reduces the number of parameters in the model
- We are taking advantage of the structure of the image (interactions between neighboring pixels are interesting in images)





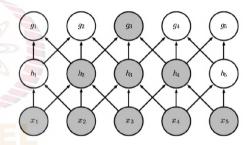
But is sparse connectivity really a good thing?



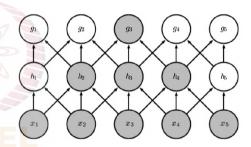
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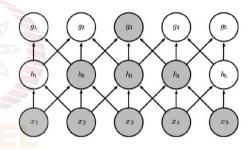
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- Well, not really
- The two highlighted neurons (x_1x_5) do not interact in layer 1

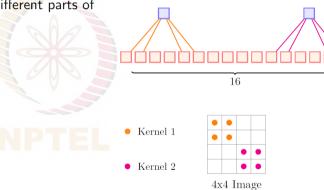


- But is sparse connectivity really a good thing?
- Aren't we losing information (by losing interactions between some input pixels)
- Well, not really
- The two highlighted neurons (x_1x_5) do not interact in layer 1
- But they indirectly contribute to the computation of g_3 and hence interact indirectly



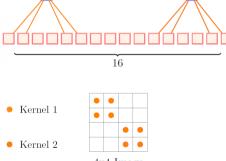
Weight Sharing

 Consider the following network; do we want the kernel weights to be different for different parts of the image?



Weight Sharing

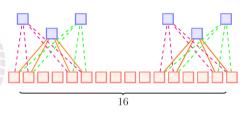
- Consider the following network; do we want the kernel weights to be different for different parts of the image?
- Not really. We would want the filter to respond to an object or an artefact in an image in the same way irrespective of where it is located in the image
 - ⇒ translation-invariance



4x4 Image

Weight Sharing

- Consider the following network; do we want the kernel weights to be different for different parts of the image?
- Not really. We would want the filter to respond to an object or an artefact in an image in the same way irrespective of where it is located in the image
 - **⇒** translation-invariance
- We can have as many different kernels to capture different kinds of artifacts, but each one is intended to give the same response on all parts of the image
- This is called weight sharing



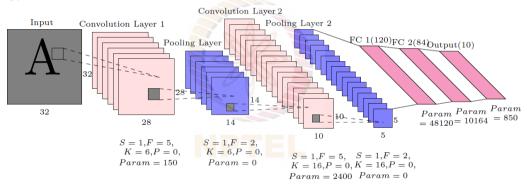
Convolutional Neural Network

• A typical CNN looks as follows:



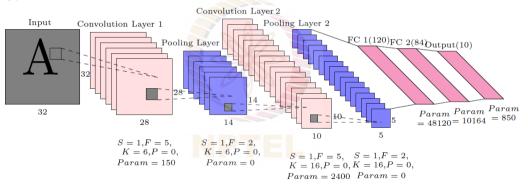
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Convolutional Neural Network

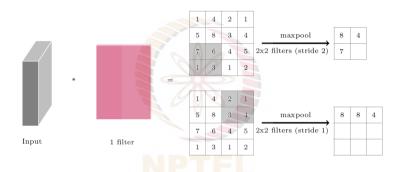
A typical CNN looks as follows:



- It has alternate convolution and pooling layers
- What do pooling layers do?











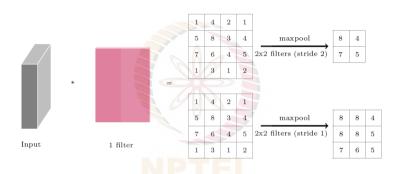












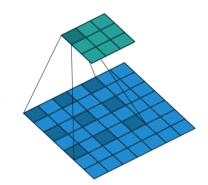
- Pooling is a parameter-free down sampling operation
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- ullet Instead of Max Pooling, we can also do Average Pooling, L_2 Pooling, etc
- Other notable mentions: Mixed Pooling (combines max and average pooling), Spatial Pyramid Pooling, Spectral Pooling - we'll see some of these in later lectures

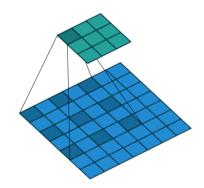
 Introduces another parameter to convolutional layer called dilation rate





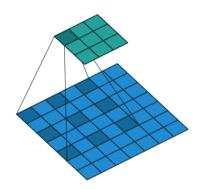
- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel



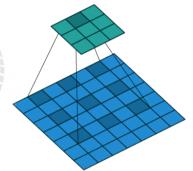


- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel
- Figure shows 3 × 3 kernel with dilation rate 2



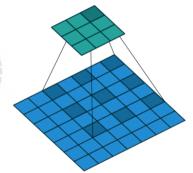


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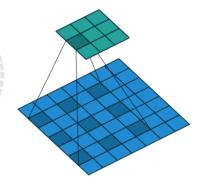
NPTEL

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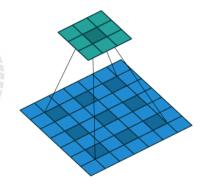
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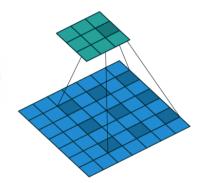
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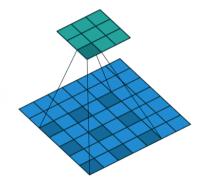
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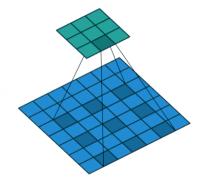
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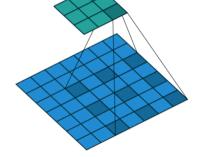
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NPTEL

Other Variants of Convolution: Dilated Convolution

- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel
- Figure shows 3×3 kernel with dilation rate 2
- Notice that dilated rate 1 is standard convolution



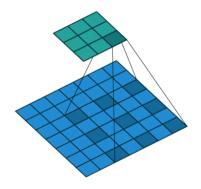


Image Credit: Vincent Dumoulin

Other Variants of Convolution: Dilated Convolution

- Introduces another parameter to convolutional layer called dilation rate
- Controls spacing between values in a kernel
- Figure shows 3×3 kernel with dilation rate 2
- Notice that dilated rate 1 is standard convolution
- A subtle difference between dilated convolution and standard convolution with stride > 1, what is it?

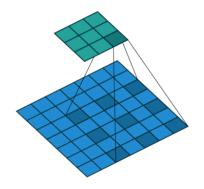


Image Credit: Vincent Dumoulin

Allows for learnable upsampling

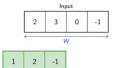
 Also known as Deconvolution (bad) or Upconvolution

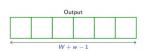
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- Let us see a 1D example

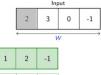
Transposed convolution layer



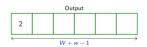


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Transposed convolution laver Input 2 -2 -3 Output

W + w - 1

Credit: François Fleuret

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Transposed convolution laver Input -1 2 -2 -3 0 0 Output W + w - 1

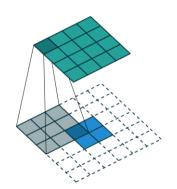
Credit: François Fleuret

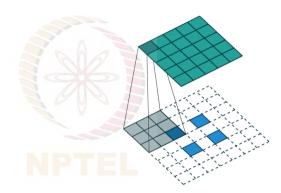
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Input -1 2 -2 -3 0 -1 Output -2 W + w - 1

Transposed convolution laver

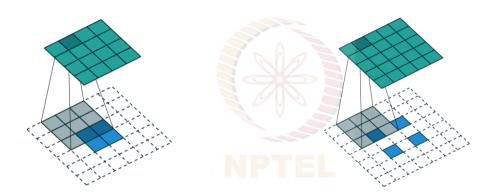
Credit: Francois Fleuret





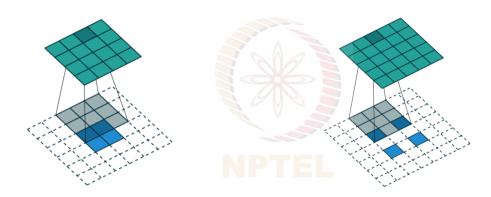
Upsampling 2×2 input to a 4×4 output

Upsampling 2×2 input to a 5×5 output



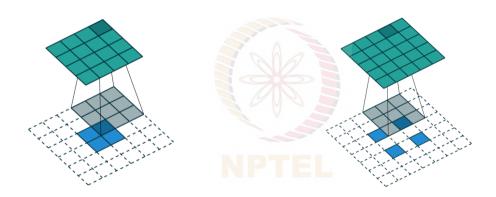
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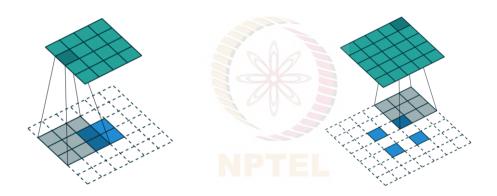
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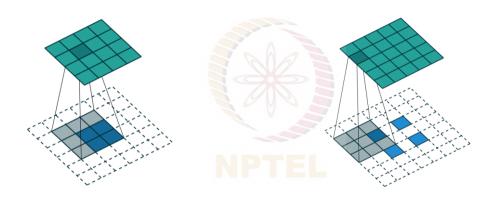
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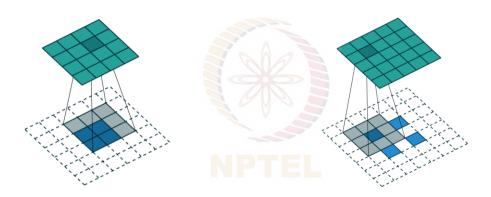
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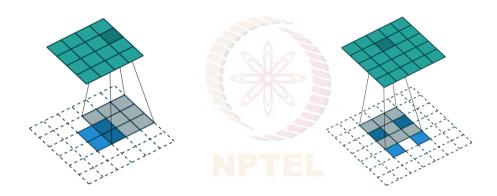
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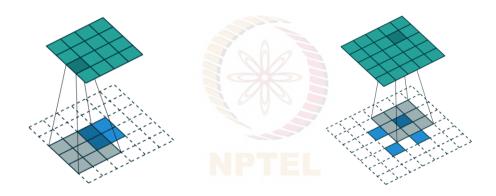
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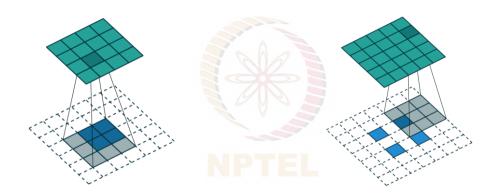
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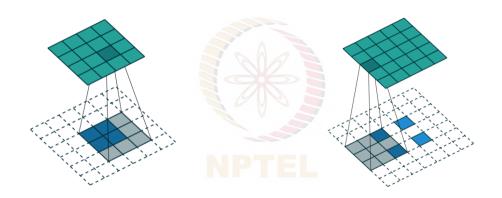
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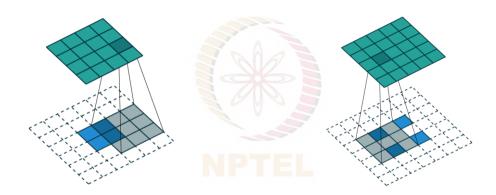
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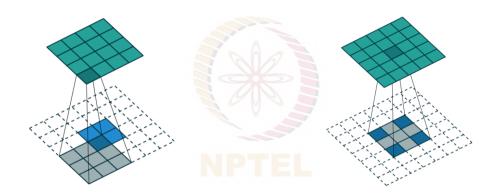
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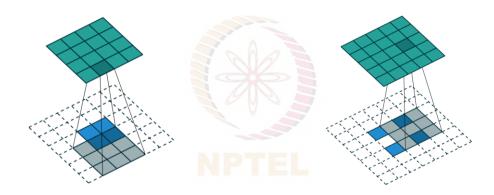
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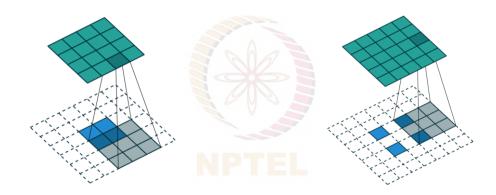


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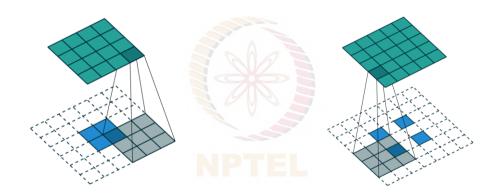
GIF Credit: Vincent Dumoulin

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Upsampling 2×2 input to a 4×4 output

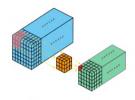
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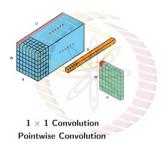
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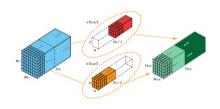
Upsampling 2×2 input to a 5×5 output

Other Variants of Convolution



3D Convolution





Grouped Convolution

Credit: Illarion Khlestov, Chi-Feng Wang

Other Variants of Convolution

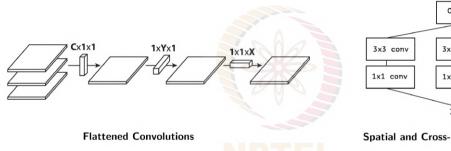


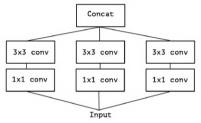
Spatial Separable Convolution



Credit: Chi-Feng Wang

Other Variants of Convolutions





Spatial and Cross-Channel Convolutions

Credit: Illarion Khlestov

Homework

Readings

- For an interactive illustration of the convolution operation, visit https://setosa.io/ev/image-kernels/
- Read more about deconvolution operation at Distill
- Other good resources:
 - Deep Learning Book: Chapter 9 Convolutional Networks
 - Stanford CS231n Notes

Questions

- Given a $32 \times 32 \times 3$ image and 6 filters of size $5 \times 5 \times 3$, what will be the dimension of the output volume when a stride of 1 and a padding of 0 is considered?
- Is the max-pooling layer differentiable? How to backpropagate across it?

References



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