

# Deep Learning for Data Science

## DS 542

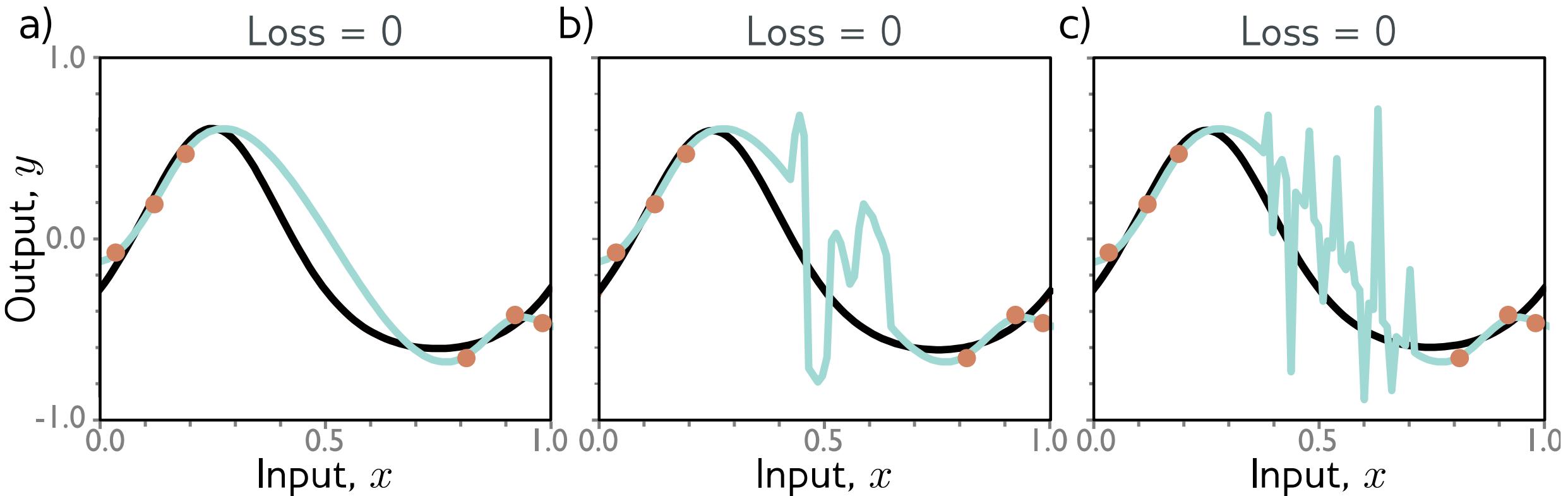
<https://dl4ds.github.io/sp2026/>

Regularization

# Regularization

- Why is there a generalization gap between training and test data?
  - Overfitting (model describes statistical peculiarities)
  - Model unconstrained in areas where there are no training examples
- **Regularization** = methods to reduce the generalization gap
- Technically means adding terms to loss function
- But colloquially means any method (hack) to reduce gap between training and test data

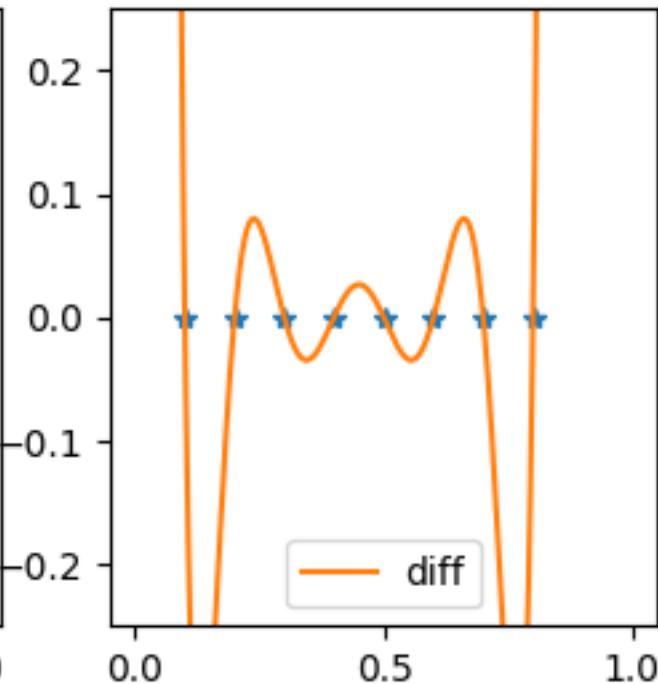
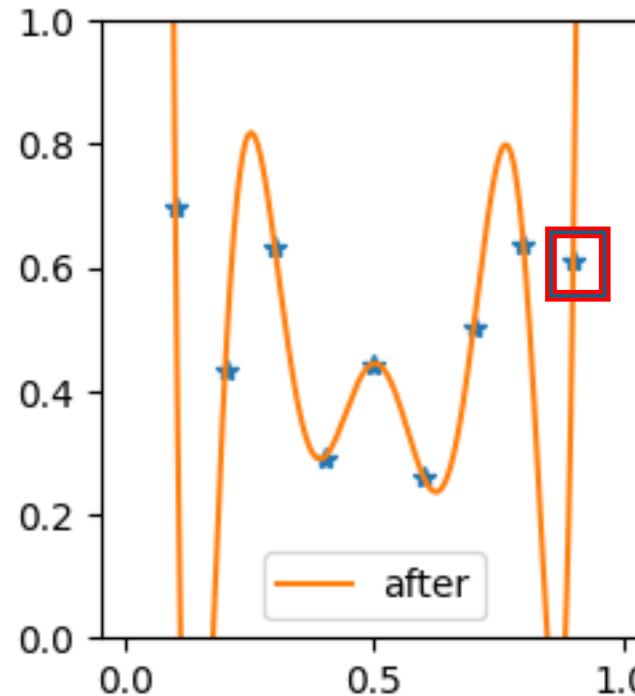
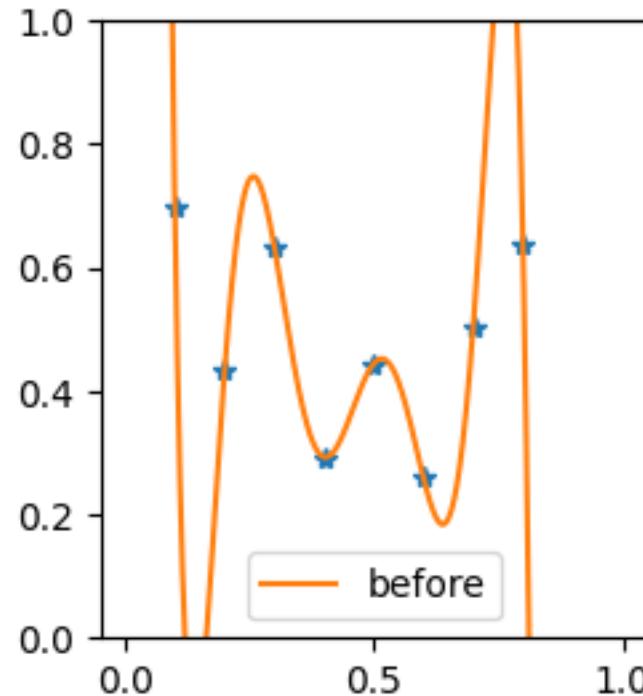
# How to bias for smoothness?



- All of these solutions are equivalent in terms of loss.
- Why should the model choose the smooth solution?
- Tendency of model to choose one solution over another is **inductive bias**

# Interpolating Polynomials Overfit A Lot

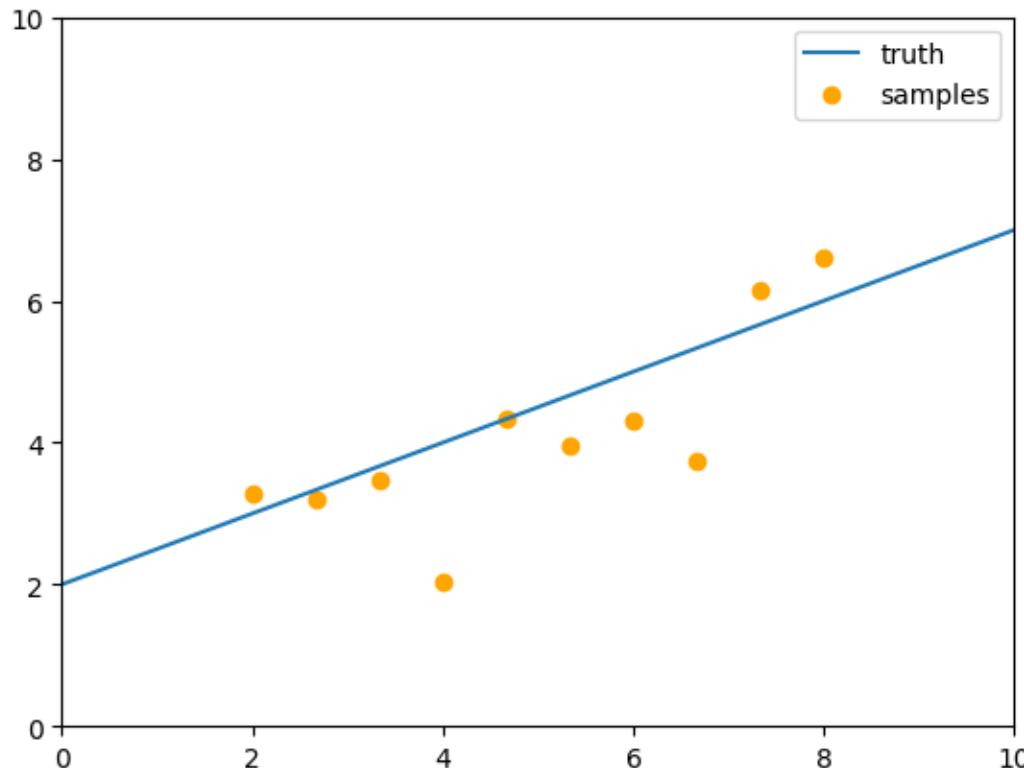
If our “before” model was good, why do we change it so much?



# Plan for Today

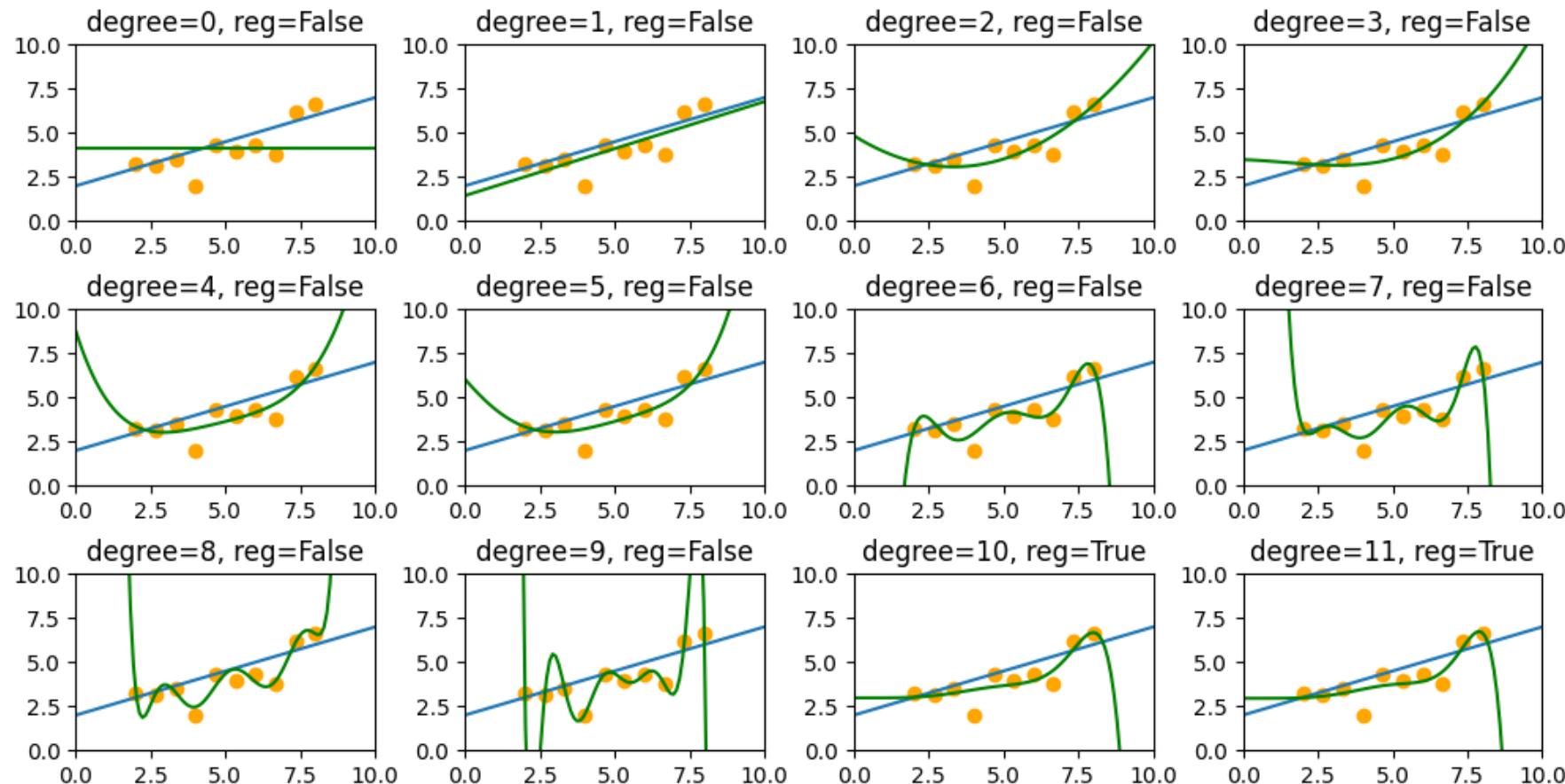
- Reproducing Double Descent Demystified
- Explicit regularization
- Implicit regularization
- Early stopping
- Ensembling
- Dropout
- Adding noise
- Transfer learning, multi-task learning, self-supervised learning
- Data augmentation

# Toy Setup

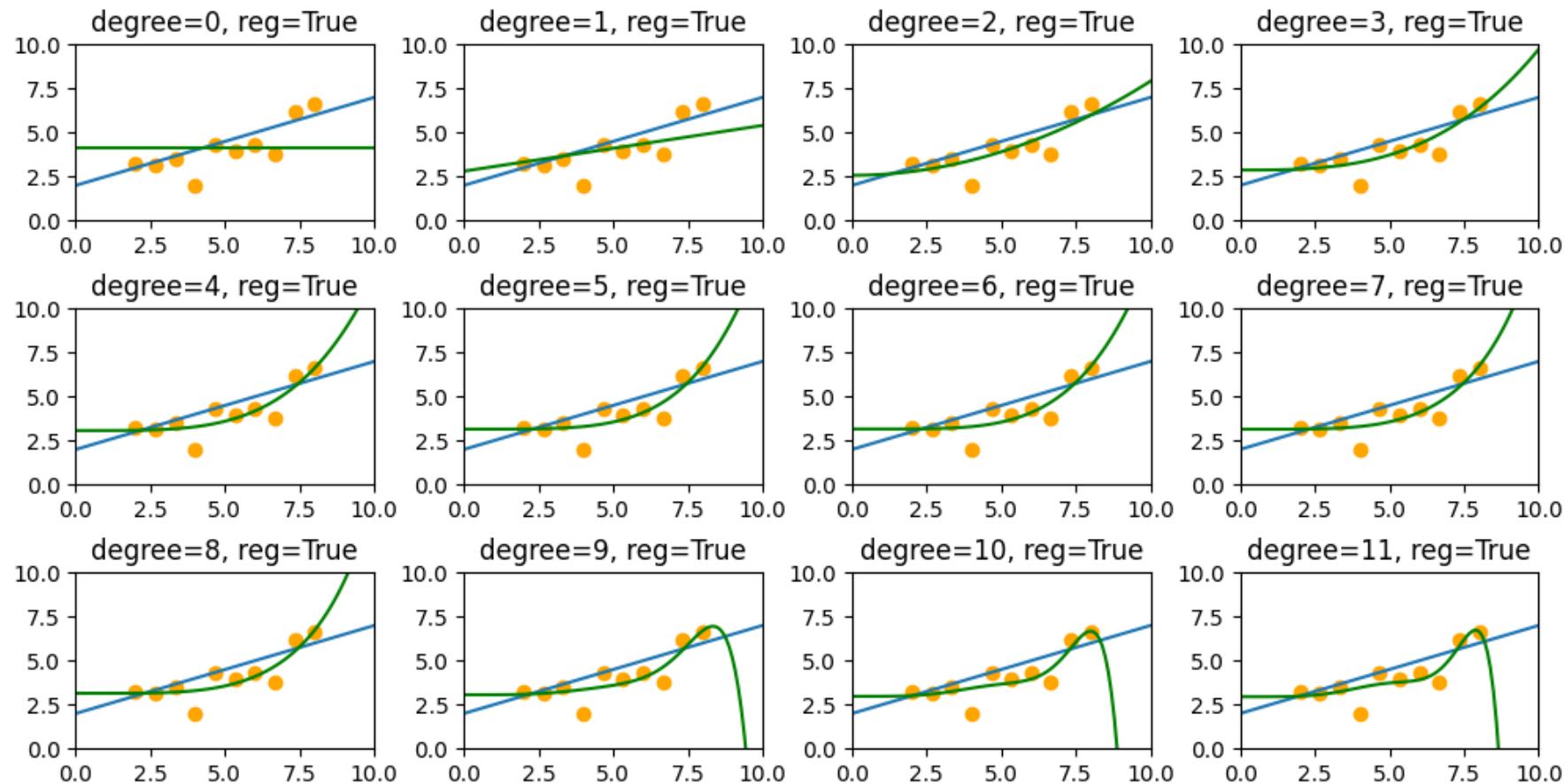


<https://colab.research.google.com/drive/1p6DYrIO6p1zGePiD3NJ9JPfkgsP70ojM>

# Reproducing Double Descent Demystified...



# Regularizing from the Beginning



# Any Questions?

???

## Moving on

- Reproducing Double Descent Demystified
- Explicit regularization
- Implicit regularization
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# Explicit regularization

- Standard loss function:

$$\begin{aligned}\hat{\phi} &= \operatorname{argmin}_{\phi} [L[\phi]] \\ &= \operatorname{argmin}_{\phi} \left[ \sum_{i=1}^I \ell_i[\mathbf{x}_i, \mathbf{y}_i] \right]\end{aligned}$$

# Explicit regularization

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- Regularization adds an extra term

$$\hat{\phi} = \operatorname{argmin}_{\phi} \left[ \sum_{i=1}^I \ell_i[\mathbf{x}_i, \mathbf{y}_i] + \lambda \cdot g[\phi] \right]$$

# Explicit regularization

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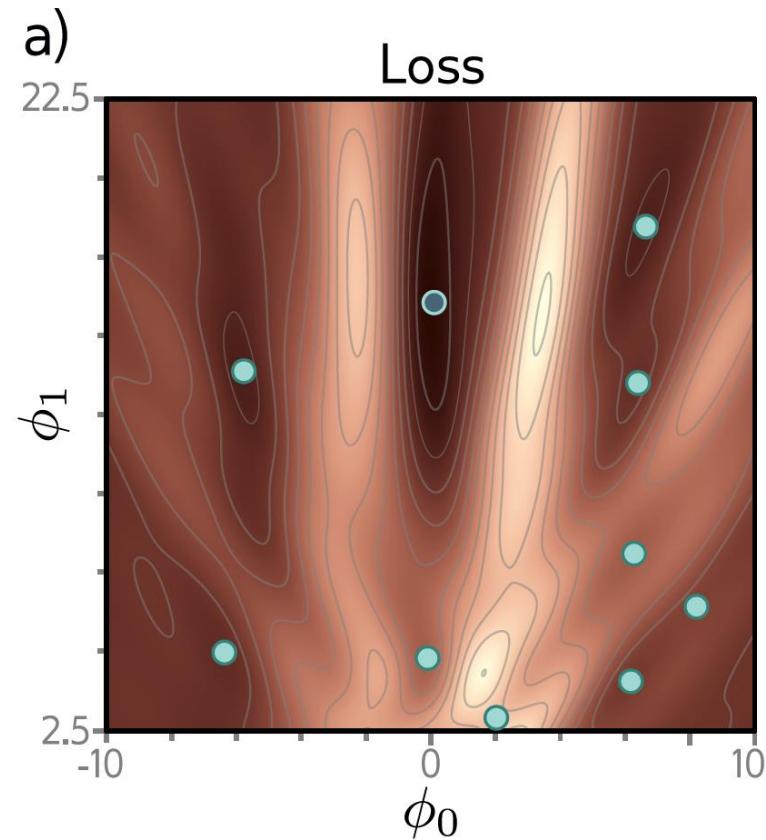
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- Where  $g[\phi]$  is smaller for preferred parameters
- $\lambda > 0$  controls the strength of influence

# Explicit regularization

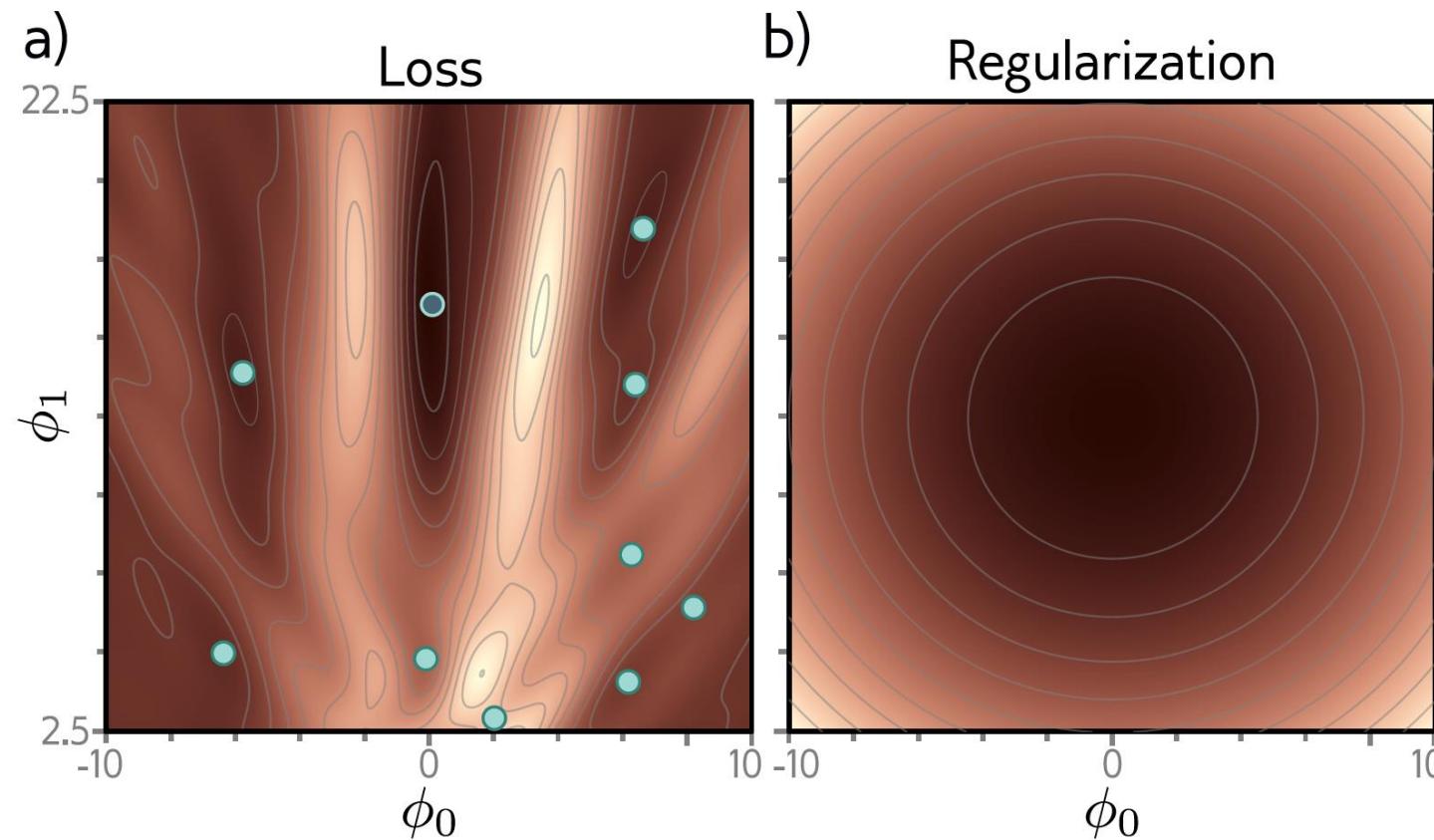


Loss function for Gabor  
model of Lecture 6 and  
Chapter 6.



denotes local minima

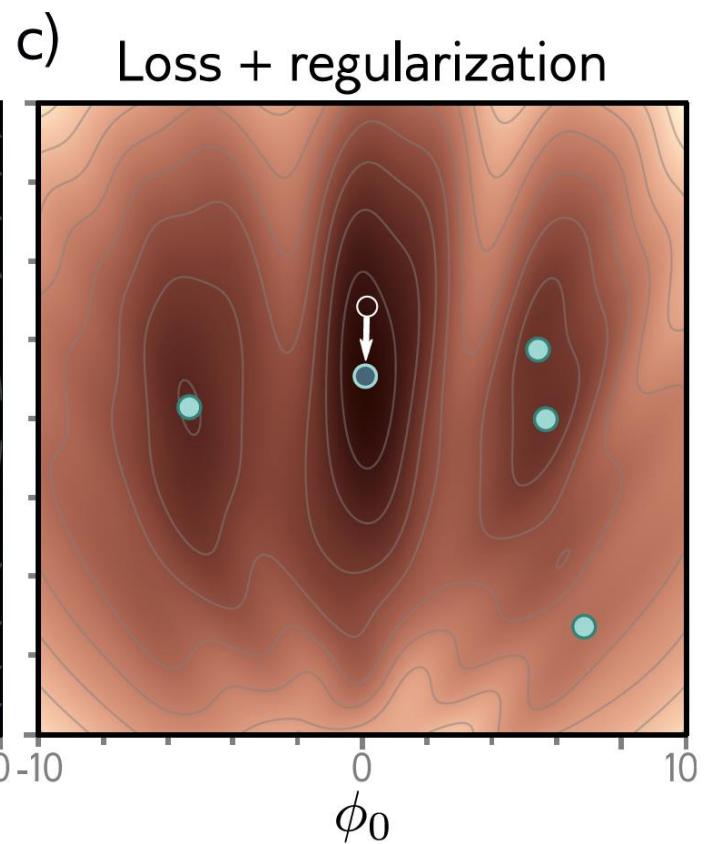
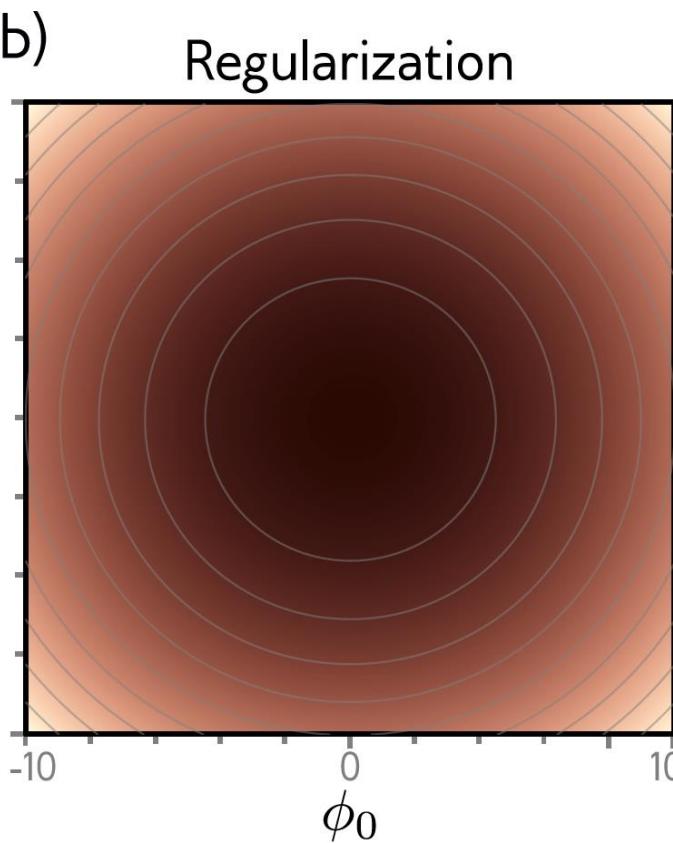
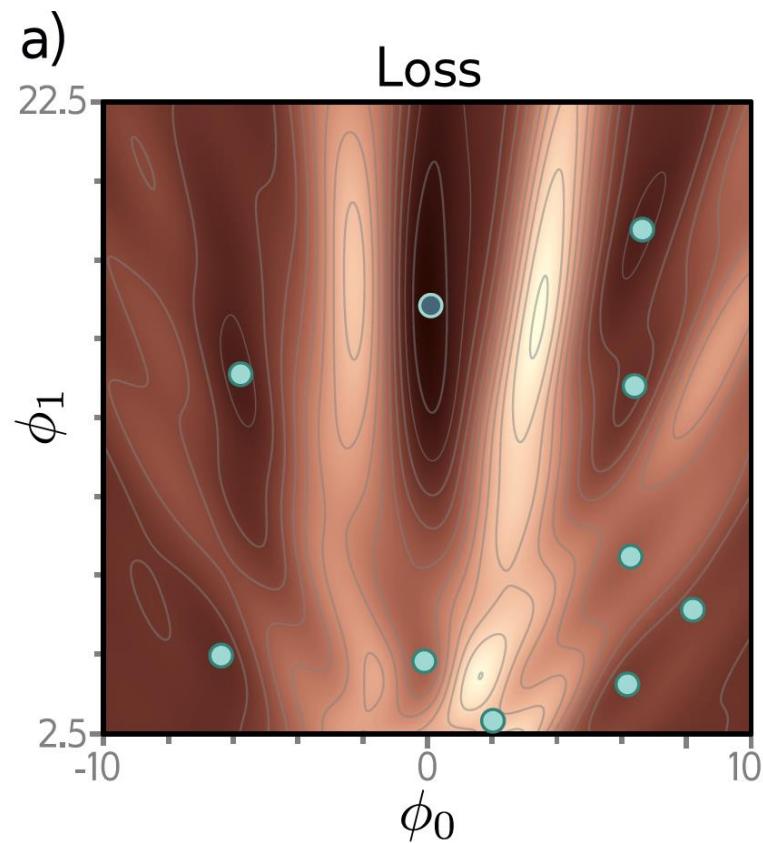
# Explicit regularization



Example of a regularization function that prefers parameters close to 0.

# Explicit regularization

Fewer local minima and the absolute minimum has moved.



● denotes local minima

# Probabilistic interpretation

- Maximum likelihood:

$$\hat{\phi} = \operatorname{argmax}_{\phi} \left[ \prod_{i=1}^I Pr(\mathbf{y}_i | \mathbf{x}_i, \phi) \right]$$

- Regularization is equivalent to adding a **prior** over parameters

$$\hat{\phi} = \operatorname{argmax}_{\phi} \left[ \prod_{i=1}^I Pr(\mathbf{y}_i | \mathbf{x}_i, \phi) Pr(\phi) \right]$$

*Maximum a posteriori or  
MAP criterion*

... what you know about parameters *before* seeing the data

# Equivalence

- Explicit regularization:

$$\hat{\phi} = \operatorname{argmin}_{\phi} \left[ \sum_{i=1}^I \ell_i[\mathbf{x}_i, \mathbf{y}_i] + \lambda \cdot g[\phi] \right]$$

- Probabilistic interpretation:

$$\hat{\phi} = \operatorname{argmax}_{\phi} \left[ \prod_{i=1}^I Pr(\mathbf{y}_i | \mathbf{x}_i, \phi) Pr(\phi) \right]$$

- Converting to Negative Log Likelihood (e.g.  $-\log(\cdot)$ ):

$$\lambda \cdot g[\phi] = -\log[Pr(\phi)]$$

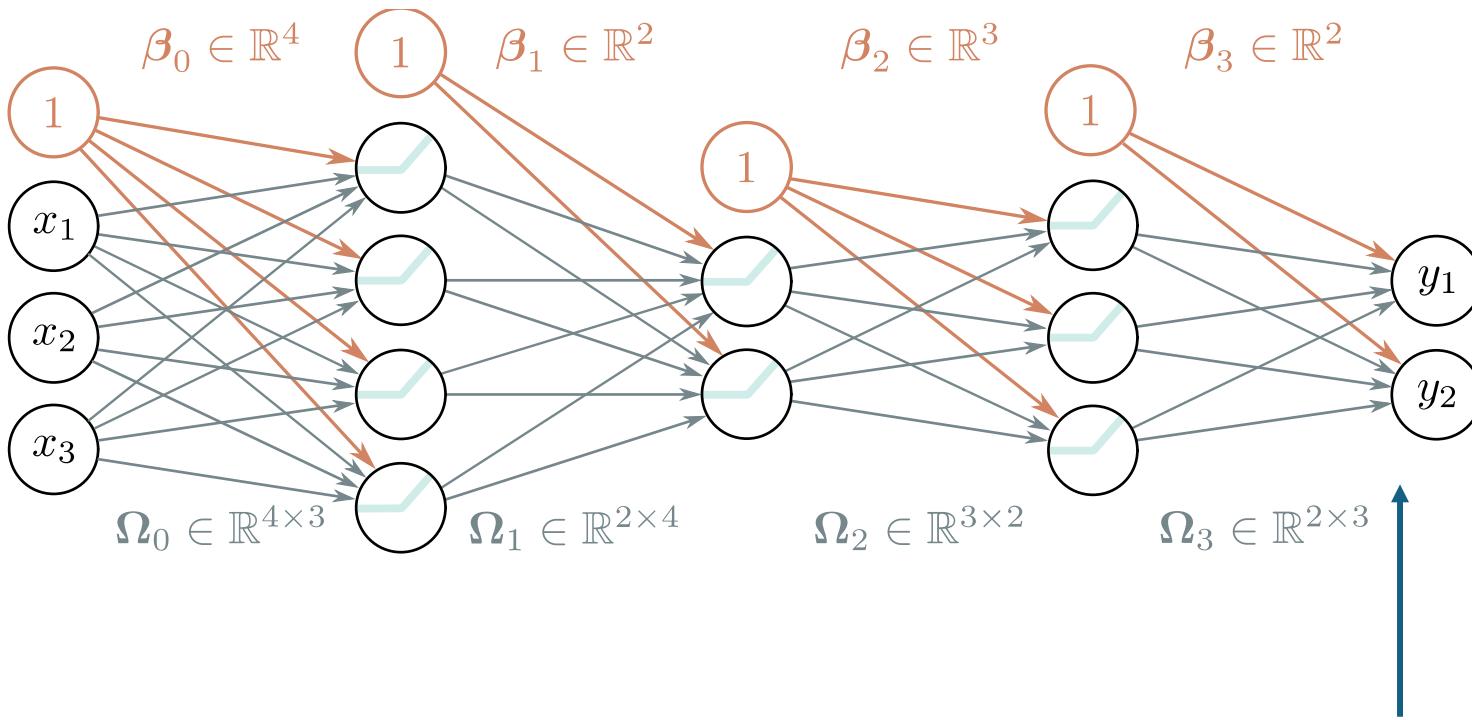
# L2 Regularization

- Most common regularization is L2 regularization
- Favors smaller parameters (like in previous example)

$$\hat{\phi} = \operatorname{argmin}_{\phi} \left[ L[\phi, \{\mathbf{x}_i, \mathbf{y}_i\}] + \lambda \sum_j \phi_j^2 \right]$$

- Also called Tikhonov regularization, ridge regression
- In neural networks, usually just for weights, and called weight decay

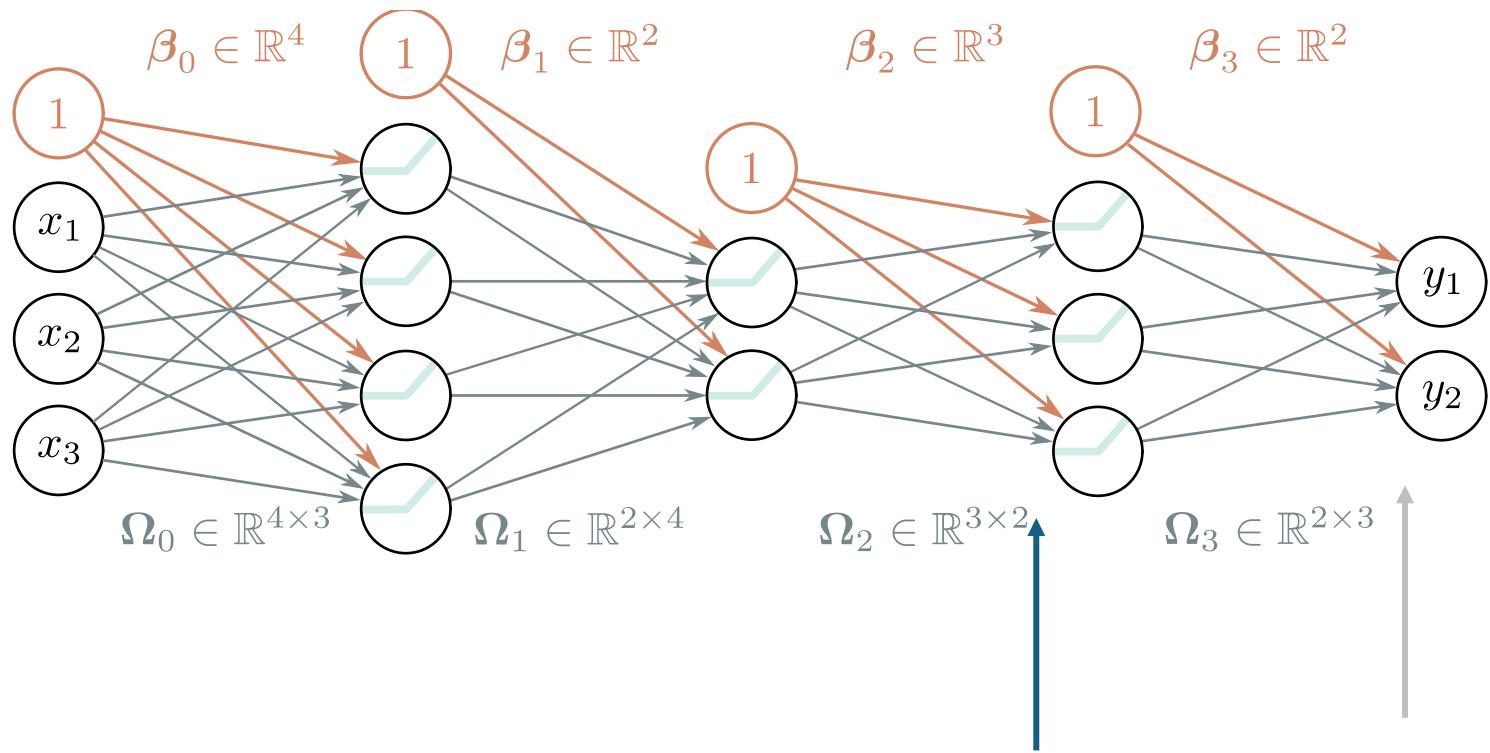
# Why does L2 regularization help?



Outputs are weighted linear combination of last layer activations.

Smaller weights attenuate changes.

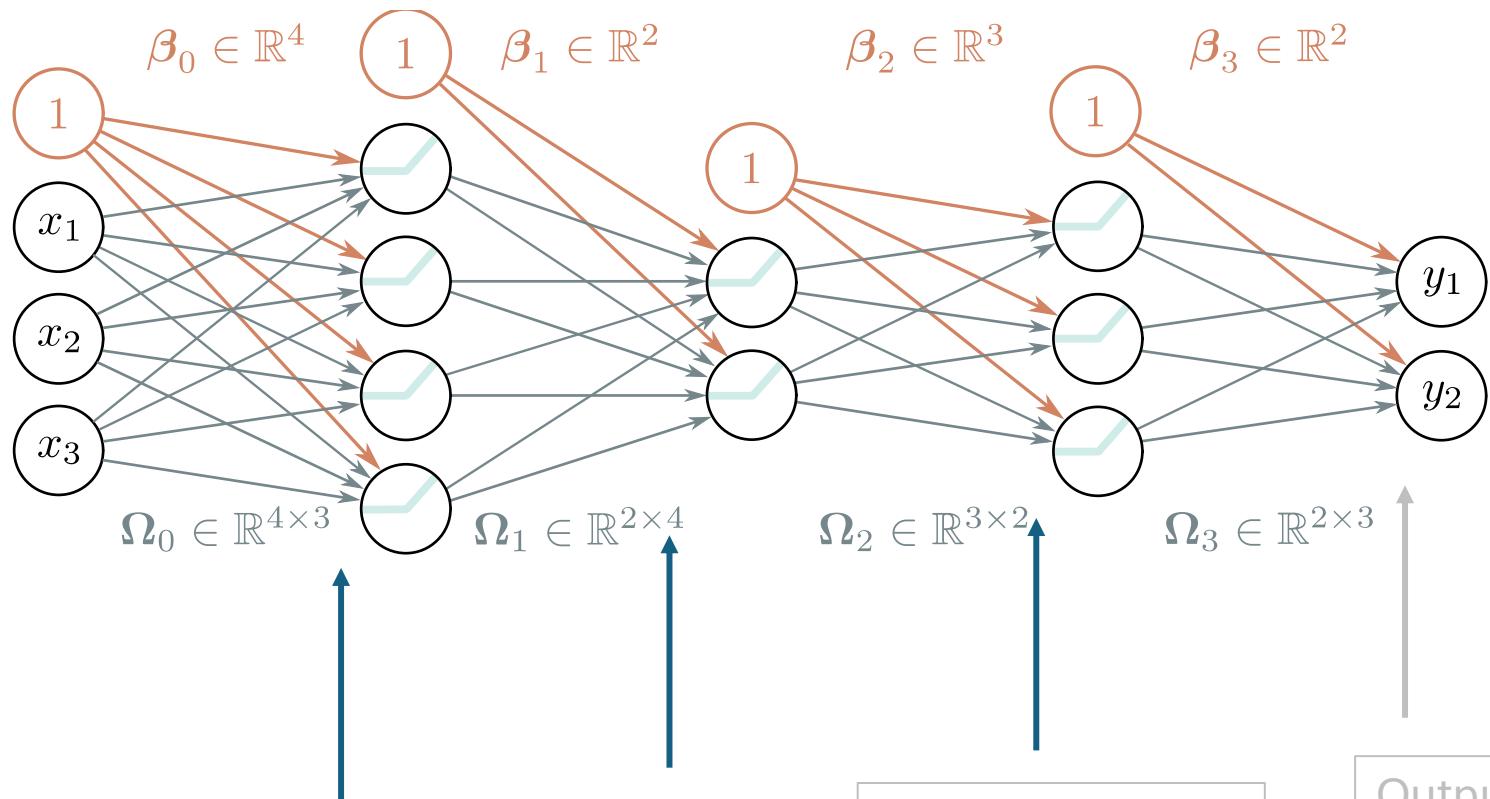
# Why does L2 regularization help?



Same for the  
pre-activations  
into the last layer

Outputs are weighted  
linear combination of  
last layer activations.  
Smaller weights  
attenuate changes.

# Why does L2 regularization help?



And so on...

All the way back.

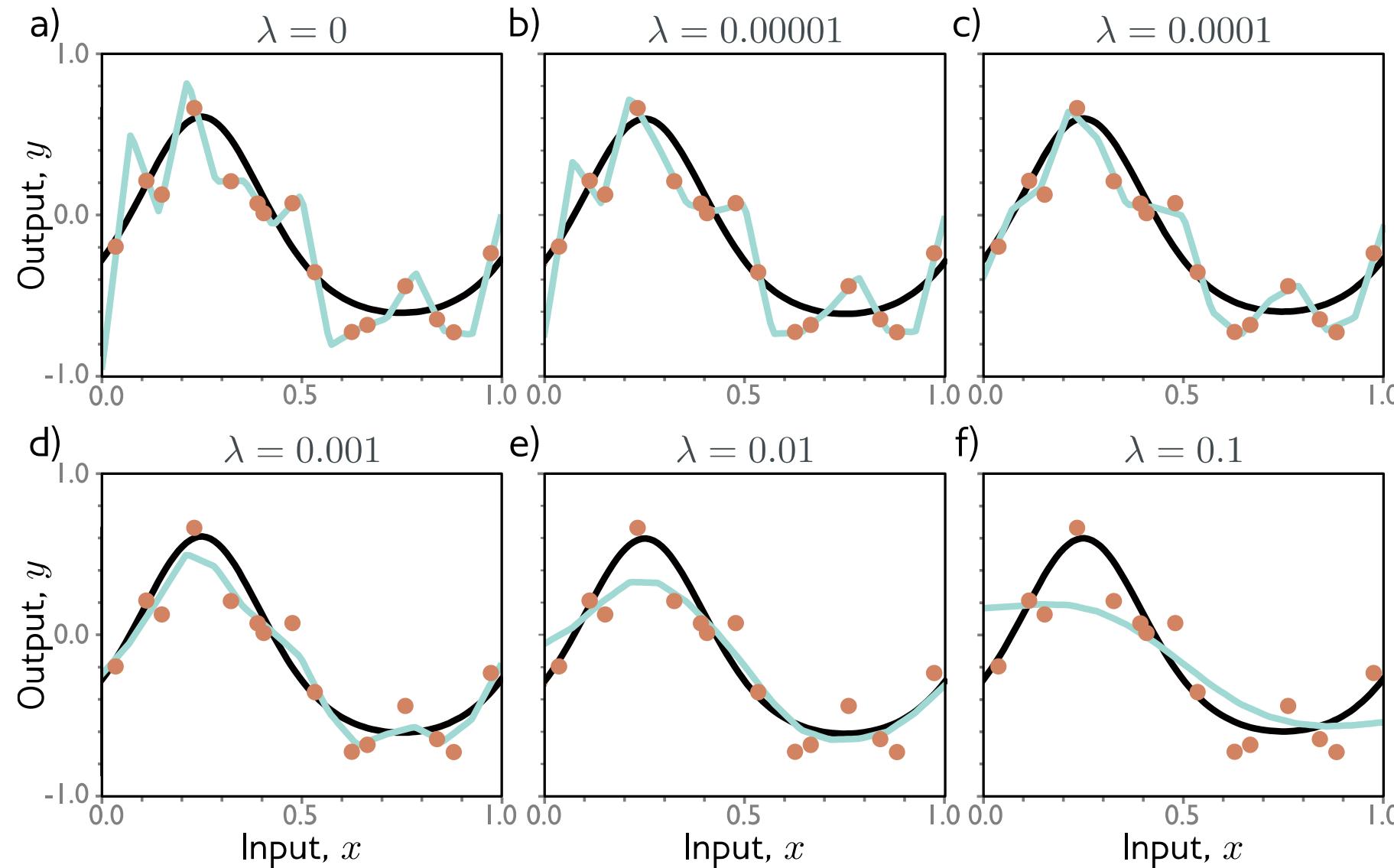
Same for the  
pre-activations  
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Outputs are weighted  
linear combination of  
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Smaller weights  
attenuate changes.

# Why does L2 regularization help?

- Discourages fitting excessively to the training data (overfitting)
- Encourages smoothness between datapoints
  - Specifically by making coefficients smaller, so small input changes have smaller output changes.

# L2 regularization (simple net from last lecture)



# PyTorch Explicit L2 Regularizer

## SGD

```
CLASS torch.optim.SGD(params, lr=0.001, momentum=0, dampening=0, weight_decay=0,  
nesterov=False, *, maximize=False, foreach=None, differentiable=False) [SOURCE]
```

Implements stochastic gradient descent (optionally with momentum).

### Parameters

- **params** (*iterable*) – iterable of parameters to optimize or dicts defining parameter groups
- **lr** (*float, optional*) – learning rate (default: 1e-3)
- **momentum** (*float, optional*) – momentum factor (default: 0)
- **weight\_decay** (*float, optional*) – weight decay (L2 penalty) (default: 0)

<https://pytorch.org/docs/stable/generated/torch.optim.SGD.html>

## ADAM

```
CLASS torch.optim.Adam(params, lr=0.001, betas=(0.9, 0.999), eps=1e-08,  
weight_decay=0, amsgrad=False, *, foreach=None, maximize=False,  
capturable=False, differentiable=False, fused=None) [SOURCE]
```

Implements Adam algorithm.

### Parameters

- **params** (*iterable*) – iterable of parameters to optimize or dicts defining parameter groups
- **lr** (*float, Tensor, optional*) – learning rate (default: 1e-3). A tensor LR is not yet supported for all our implementations. Please use a float LR if you are not also specifying fused=True or capturable=True.
- **betas** (*Tuple[float, float], optional*) – coefficients used for computing running averages of gradient and its square (default: (0.9, 0.999))
- **eps** (*float, optional*) – term added to the denominator to improve numerical stability (default: 1e-8)
- **weight\_decay** (*float, optional*) – weight decay (L2 penalty) (default: 0)

<https://pytorch.org/docs/stable/generated/torch.optim.Adam.html>

# Any Questions?

???

## Moving on

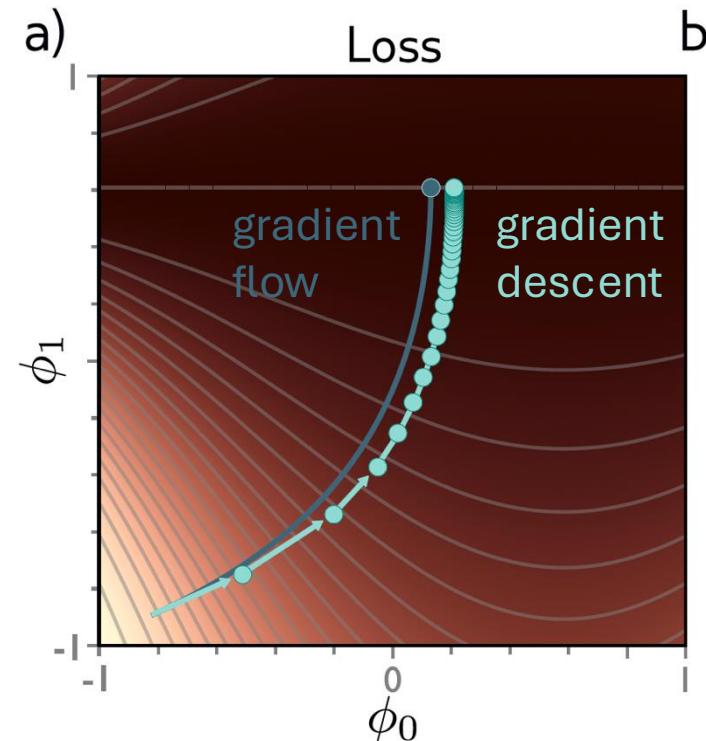
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- **Implicit regularization**
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# Implicit regularization

$$\phi_{t+1} = \phi_t - \alpha \frac{\partial L[\phi_t]}{\partial \phi} \quad \lim_{\alpha \rightarrow 0}$$

$$\frac{d\phi}{dt} = - \frac{\partial L}{\partial \phi}$$

- In the limit, as  $\alpha \rightarrow 0$ , the gradient descent equation becomes the gradient flow differential equation.
- Doesn't converge to the same place



# Implicit regularization

$$\phi_{t+1} = \phi_t - \alpha \frac{\partial L[\phi_t]}{\partial \phi}$$

$$\lim_{\alpha \rightarrow 0}$$

$$\frac{d\phi}{dt} = - \frac{\partial L}{\partial \phi}$$

- The implicit regularization can be derived:

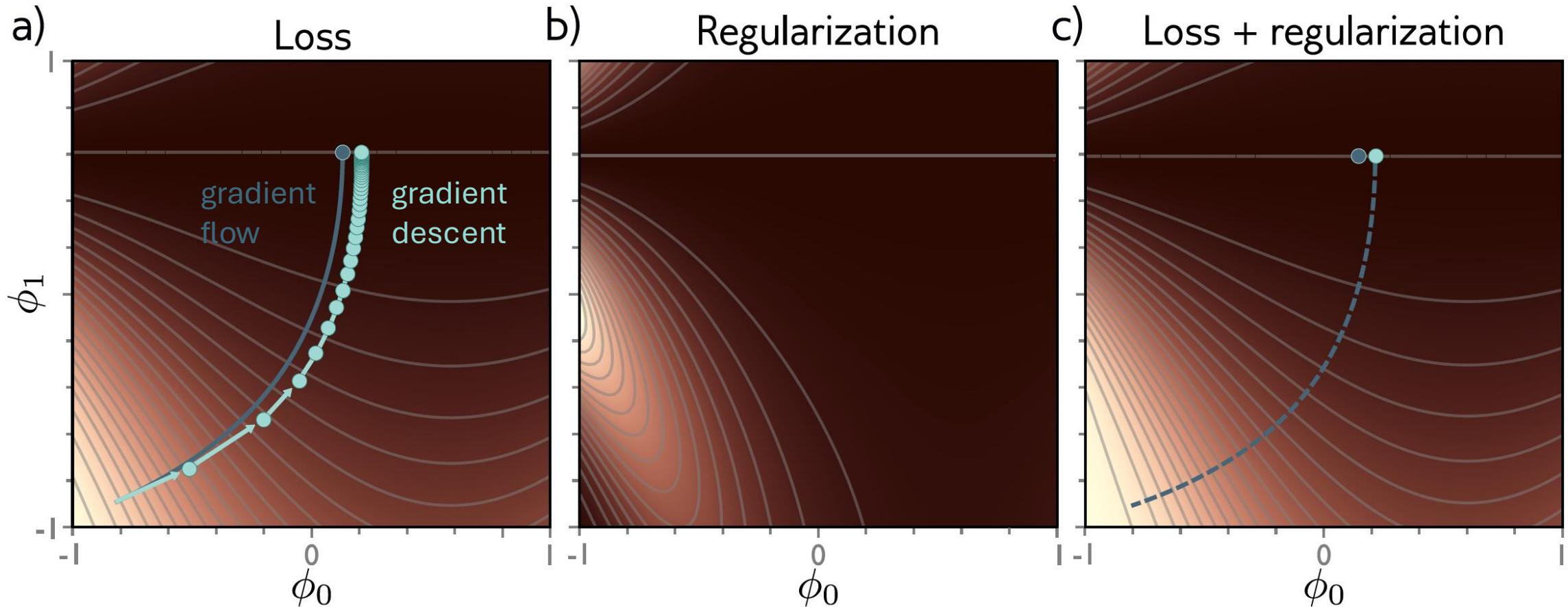
$$\tilde{L}_{GD}[\phi] = L[\phi] + \frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2$$



Penalty for large gradients.

See derivation at  
end of UDL Ch. 9

# Implicit regularization



Gradient descent doesn't converge to same location as (continuous) gradient flow.

$\sim \|\partial L / \partial \phi\|^2$

With regularization, continuous descent converges to same place

# Implicit regularization of SGD

- Gradient descent disfavors areas where gradients are steep

$$\tilde{L}_{GD}[\phi] = L[\phi] + \frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2$$

- SGD likes all batches to have similar gradients

$$\tilde{L}_{SGD}[\phi] = \tilde{L}_{GD}[\phi] + \frac{\alpha}{4B} \sum_{b=1}^B \left\| \frac{\partial L_b}{\partial \phi} - \frac{\partial L}{\partial \phi} \right\|^2$$


Want the batch variance to be small, rather than some batches fitting well and others not well...

Where  $L = \frac{1}{I} \sum_{i=1}^I \ell_i[\mathbf{x}_i, y_i]$  and  $L_b = \frac{1}{|\mathcal{B}|} \sum_{i \in \mathcal{B}_b} \ell_i[\mathbf{x}_i, y_i]$ .

# Implicit regularization of SGD

- Gradient descent disfavors areas where gradients are steep

$$\tilde{L}_{GD}[\phi] = L[\phi] + \frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2$$

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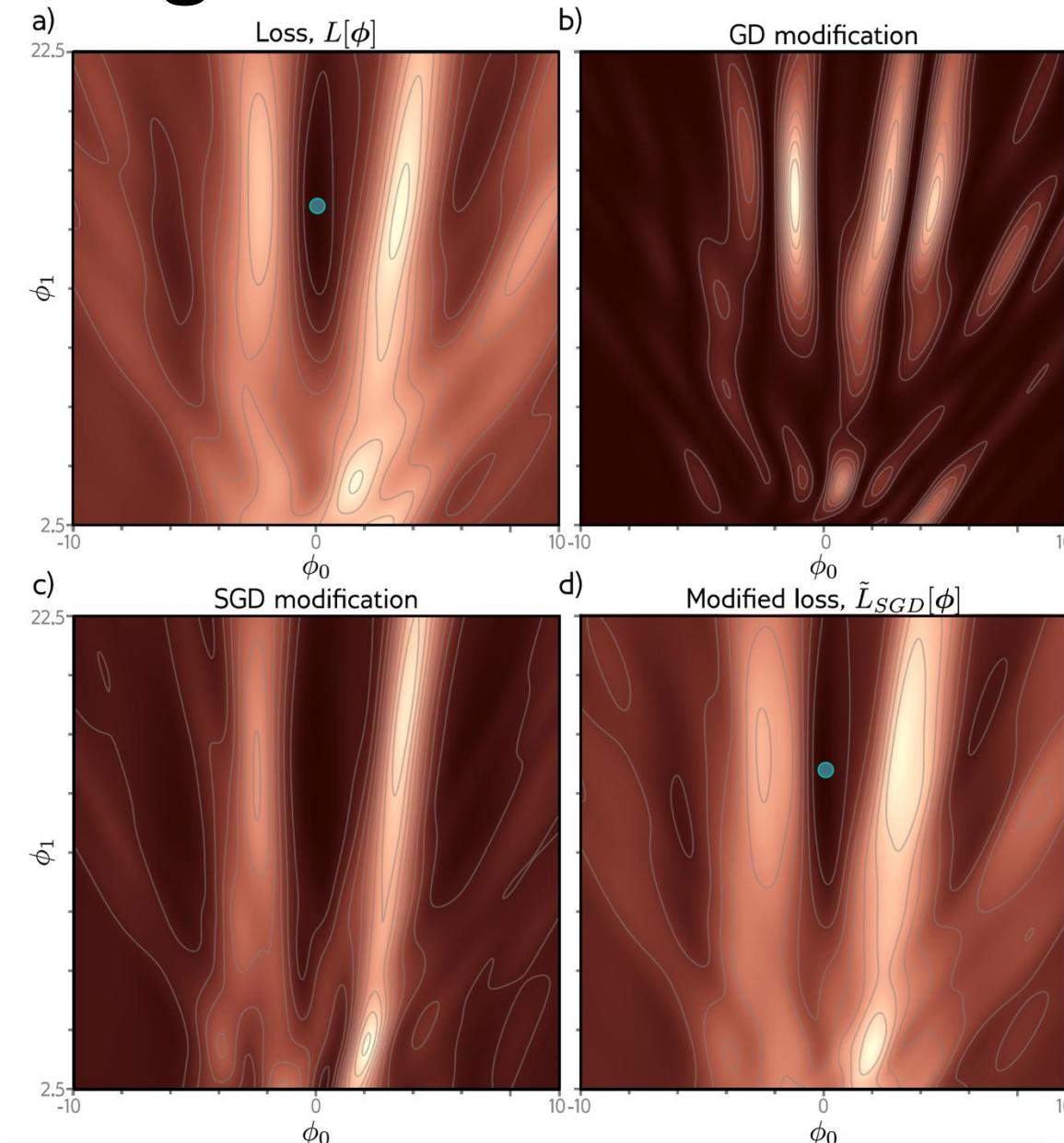
$$\begin{aligned}\tilde{L}_{SGD}[\phi] &= \tilde{L}_{GD}[\phi] + \frac{\alpha}{4B} \sum_{b=1}^B \left\| \frac{\partial L_b}{\partial \phi} - \frac{\partial L}{\partial \phi} \right\|^2 \\ &= L[\phi] + \frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2 + \frac{\alpha}{4B} \sum_{b=1}^B \left\| \frac{\partial L_b}{\partial \phi} - \frac{\partial L}{\partial \phi} \right\|^2\end{aligned}$$

- Depends on learning rate – perhaps why larger learning rates generalize better.

# Loss and Regularization Surfaces

Original Gabor Model  
Loss

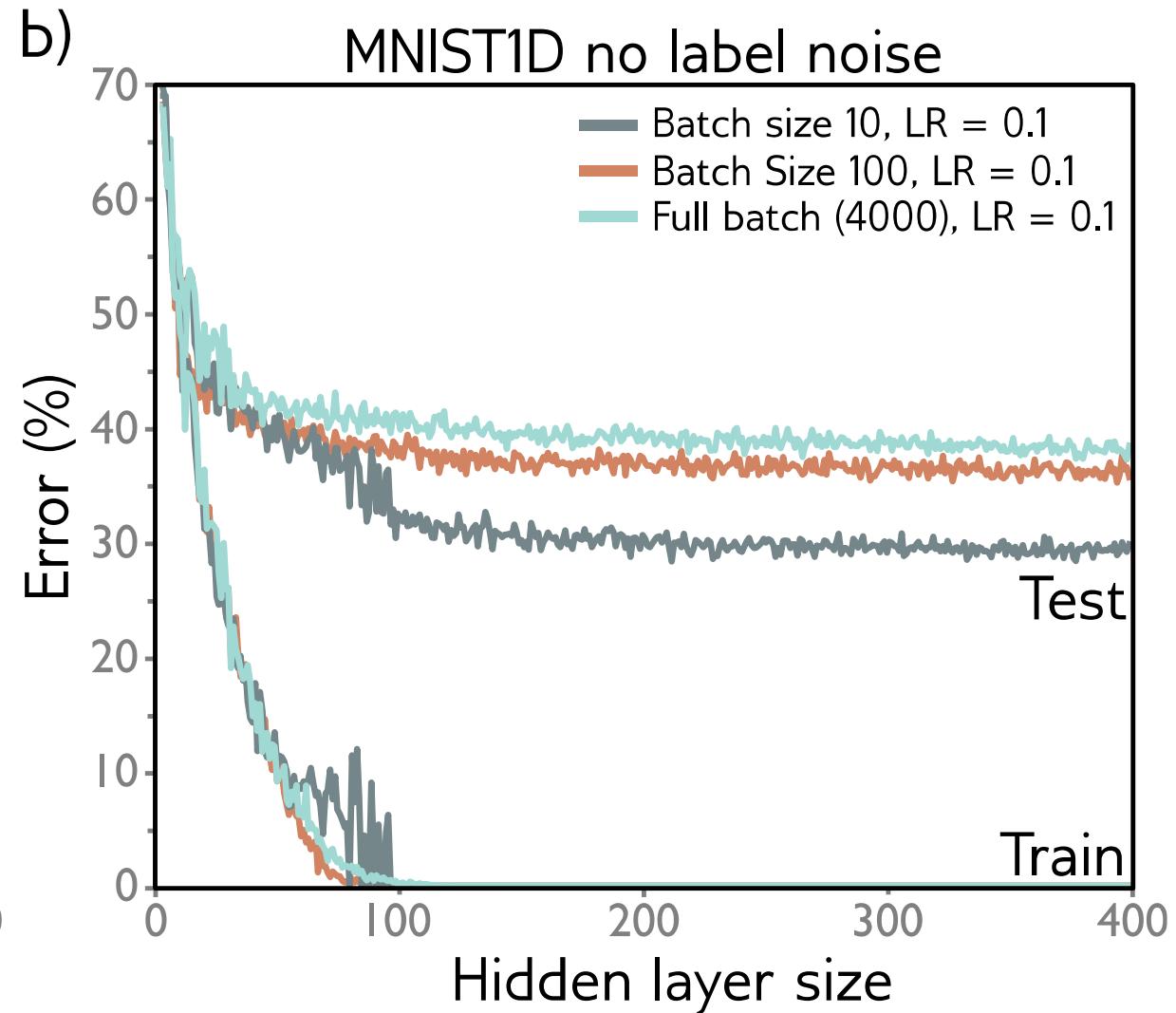
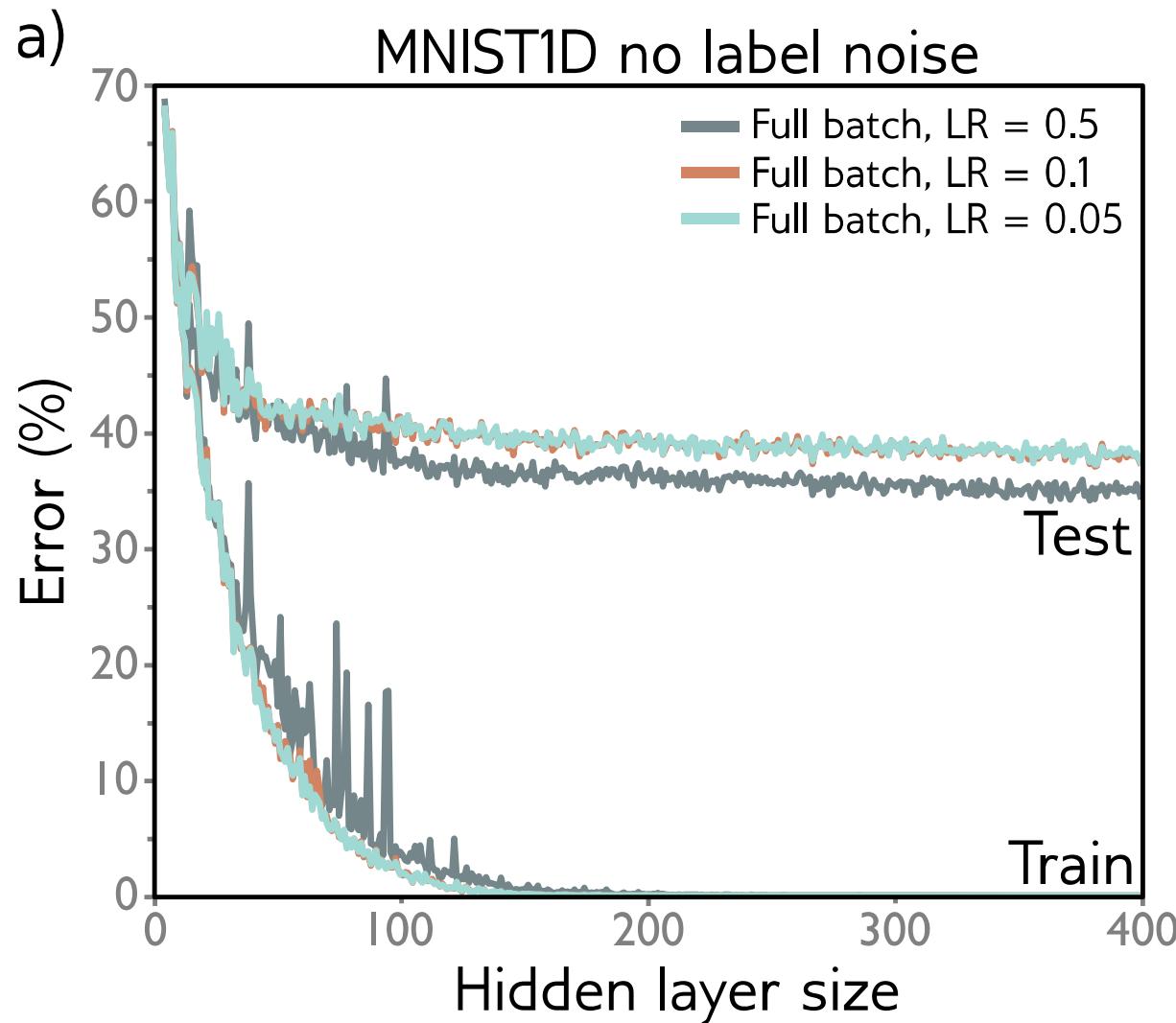
$$\frac{\alpha}{4B} \sum_{b=1}^B \left\| \frac{\partial L_b}{\partial \phi} - \frac{\partial L}{\partial \phi} \right\|^2$$



$$\frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2$$

$$\tilde{L}_{SGD}[\phi]$$

$$= L[\phi] + \frac{\alpha}{4} \left\| \frac{\partial L}{\partial \phi} \right\|^2 + \frac{\alpha}{4B} \sum_{b=1}^B \left\| \frac{\partial L_b}{\partial \phi} - \frac{\partial L}{\partial \phi} \right\|^2$$



Generally, performance is

- best for larger learning rates
- best with smaller batches

# Recap: Implicit regularization of GD and SGD

- Larger learning rates may lead to better generalization
- SGD seems to favor places where gradients are stable (all batches agreed on slope)
- SGD generalizes better than GD
- Smaller batches in SGD generally perform better than larger ones

# Any Questions?

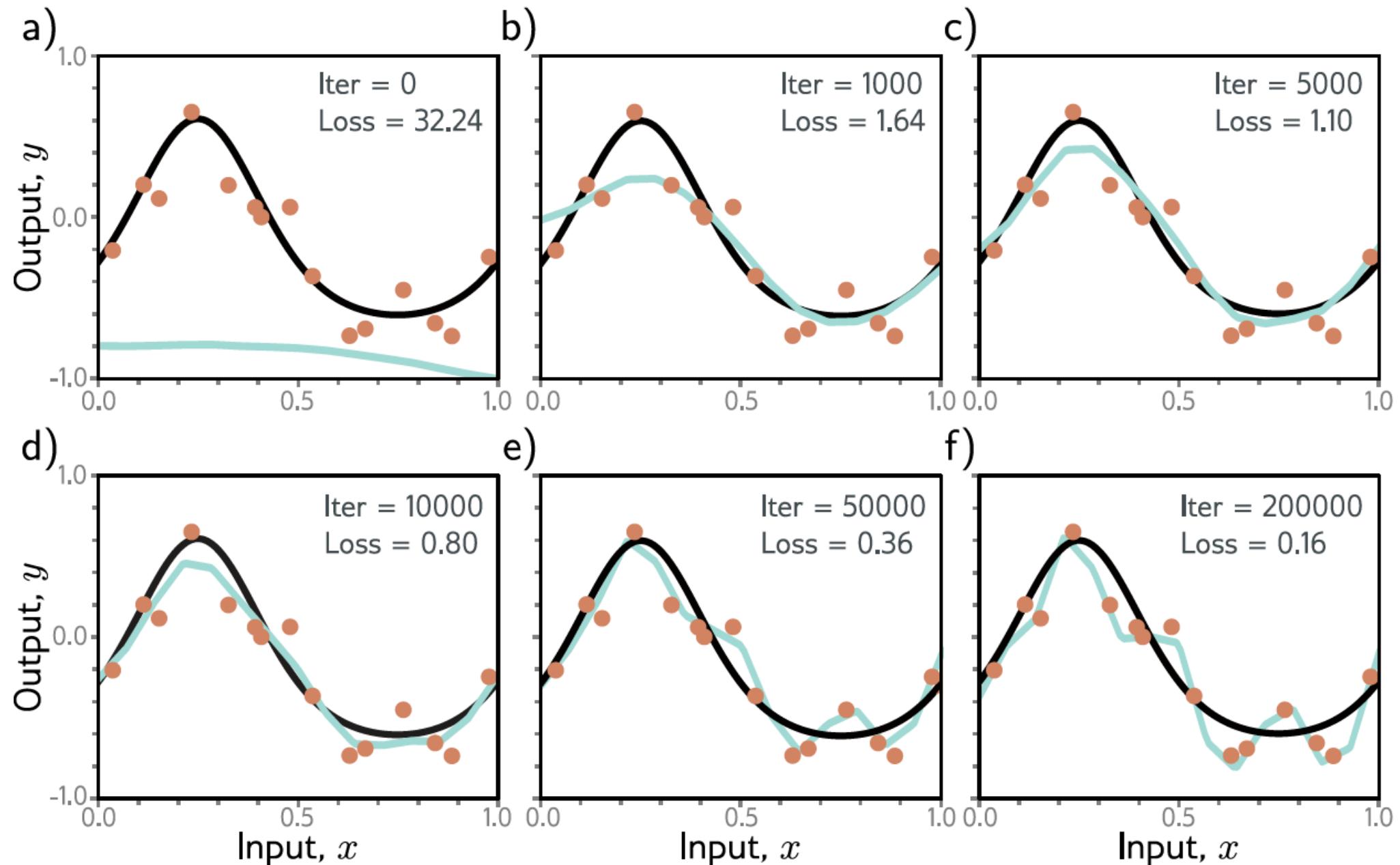
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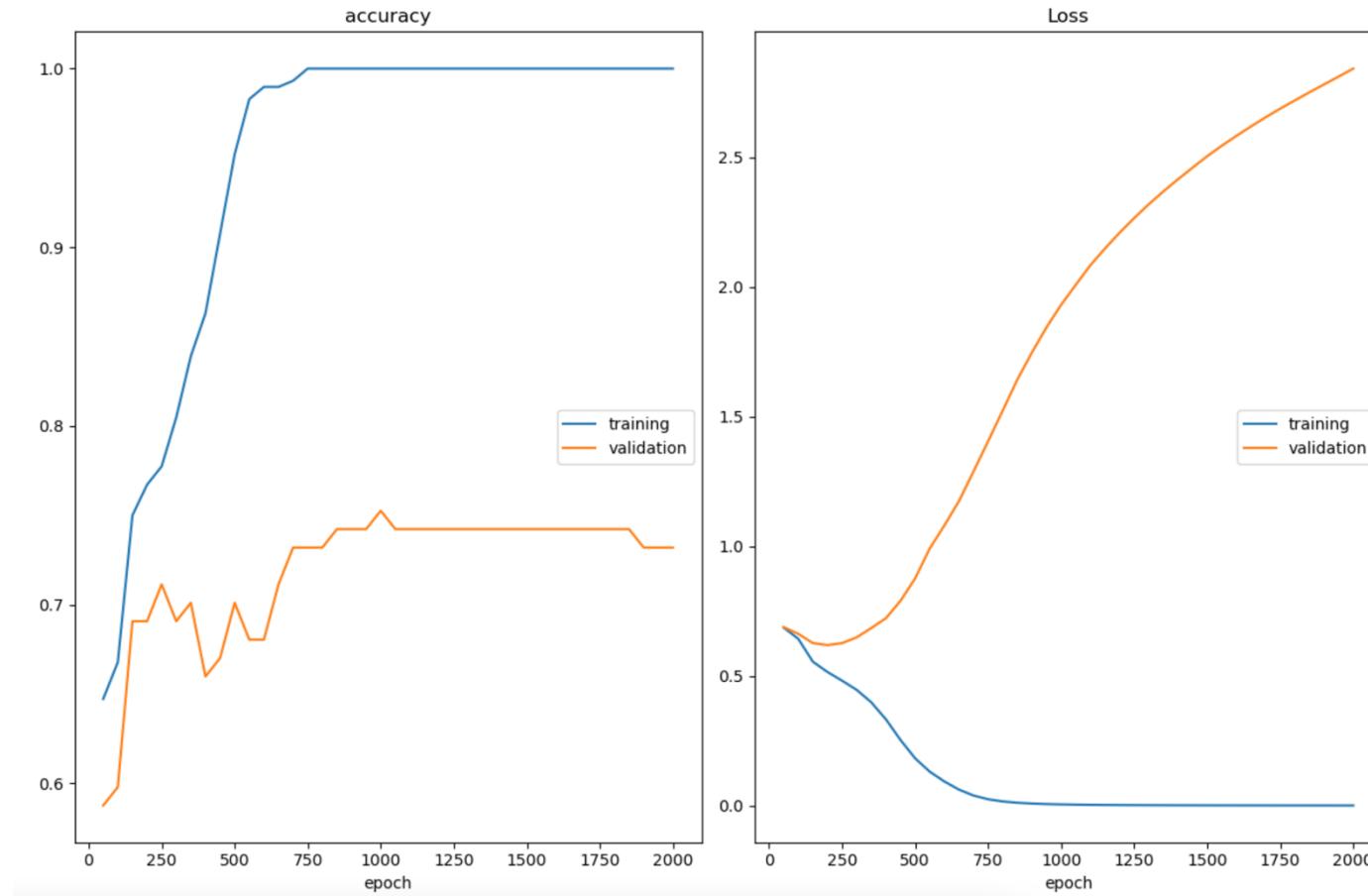
# Early stopping

- If we stop training early, weights don't have time to overfit to noise
- Weights start small, don't have time to get large
- Reduces effective model complexity
- Known as **early stopping**
- Don't have to re-train with different hyper-parameters – just “checkpoint” regularly and pick the model with lowest validation loss

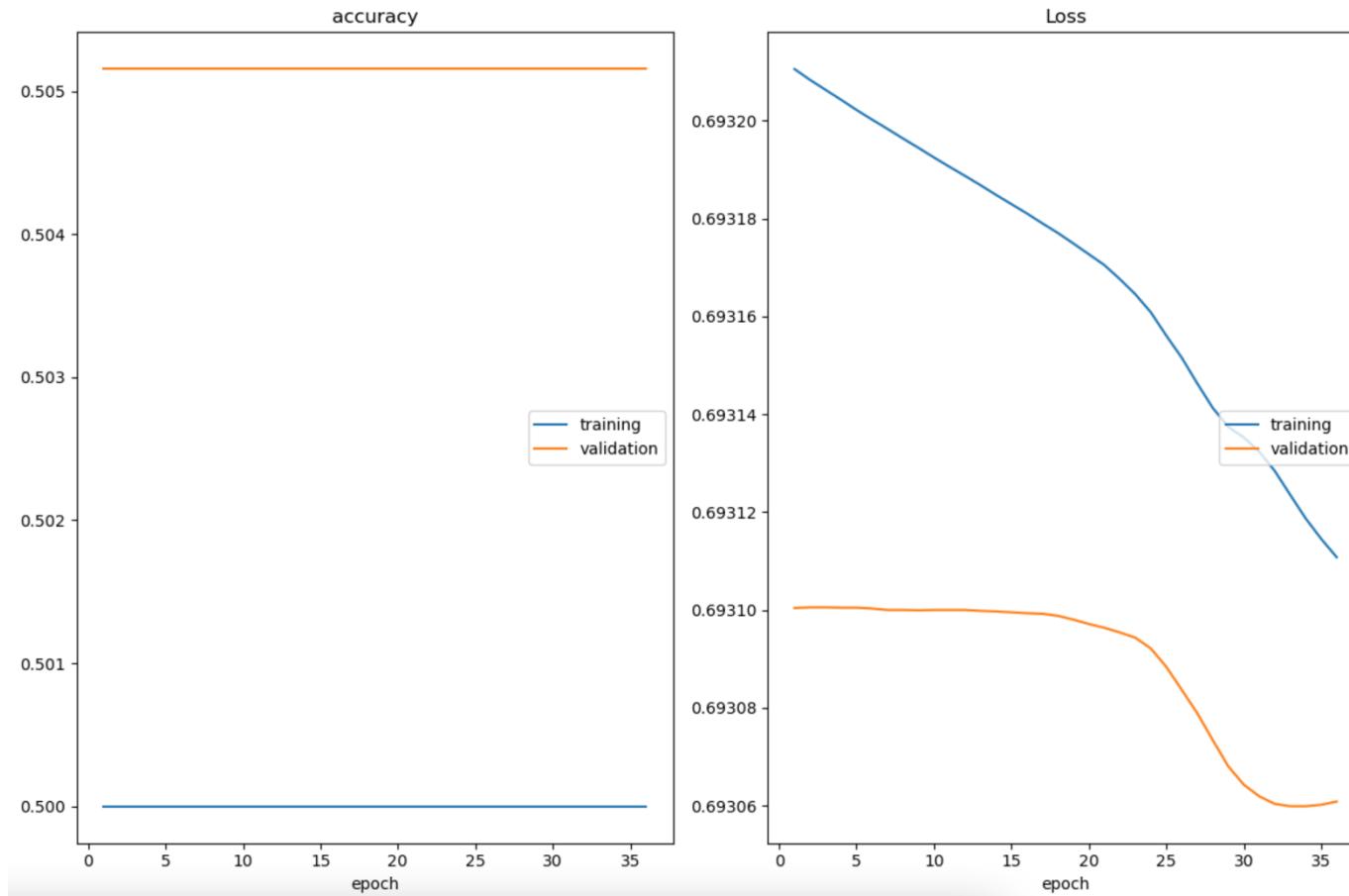


Simplified shallow network model with 14 linear regions initialized randomly (cyan curve in (a) ) and trained with SGD using a batch size of five and a learning rate of 0.05.

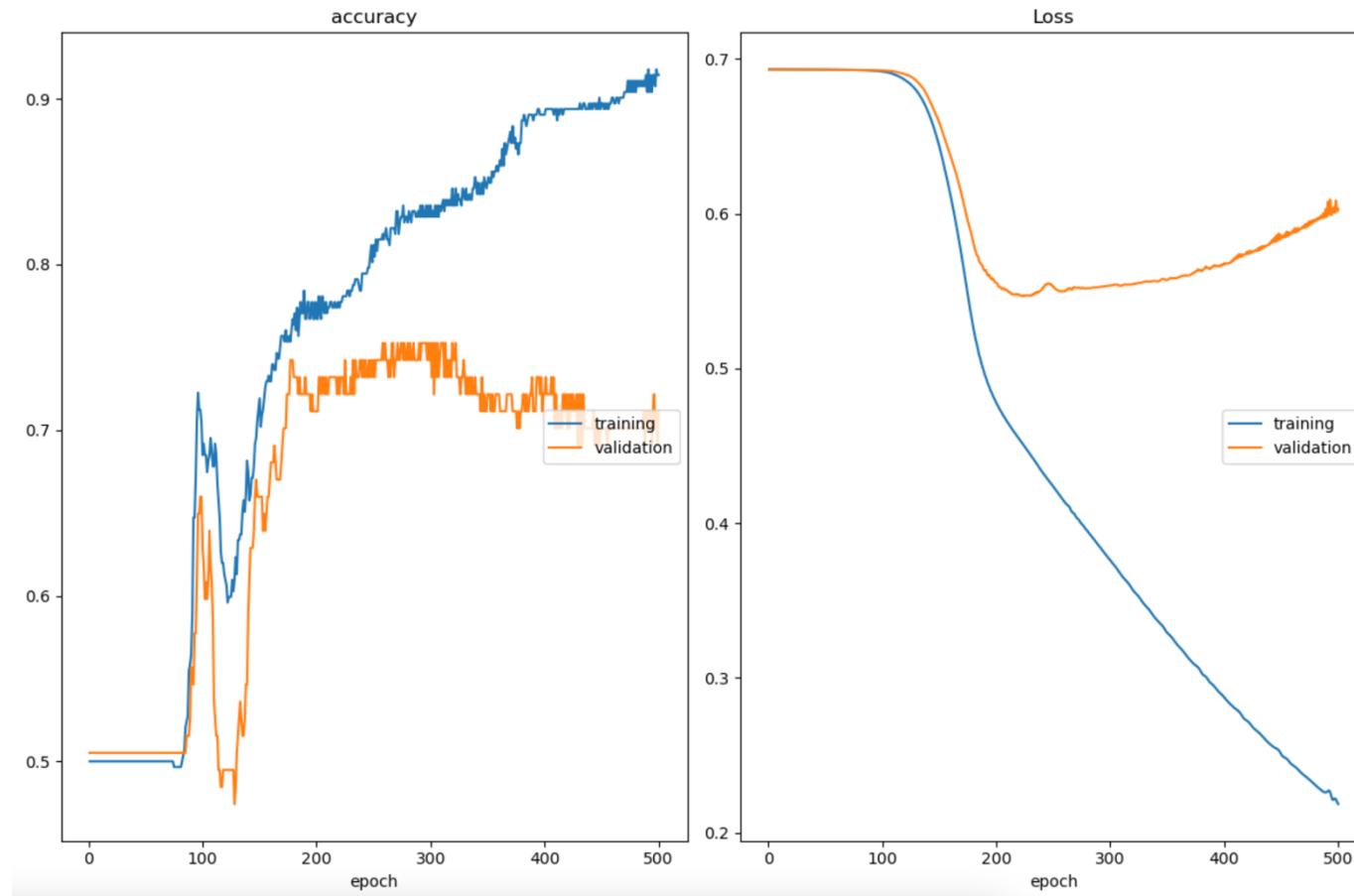
# Project 1 Provided Code



# Project 1 36 Epochs



# Same Seed (1) 500 Epochs



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???

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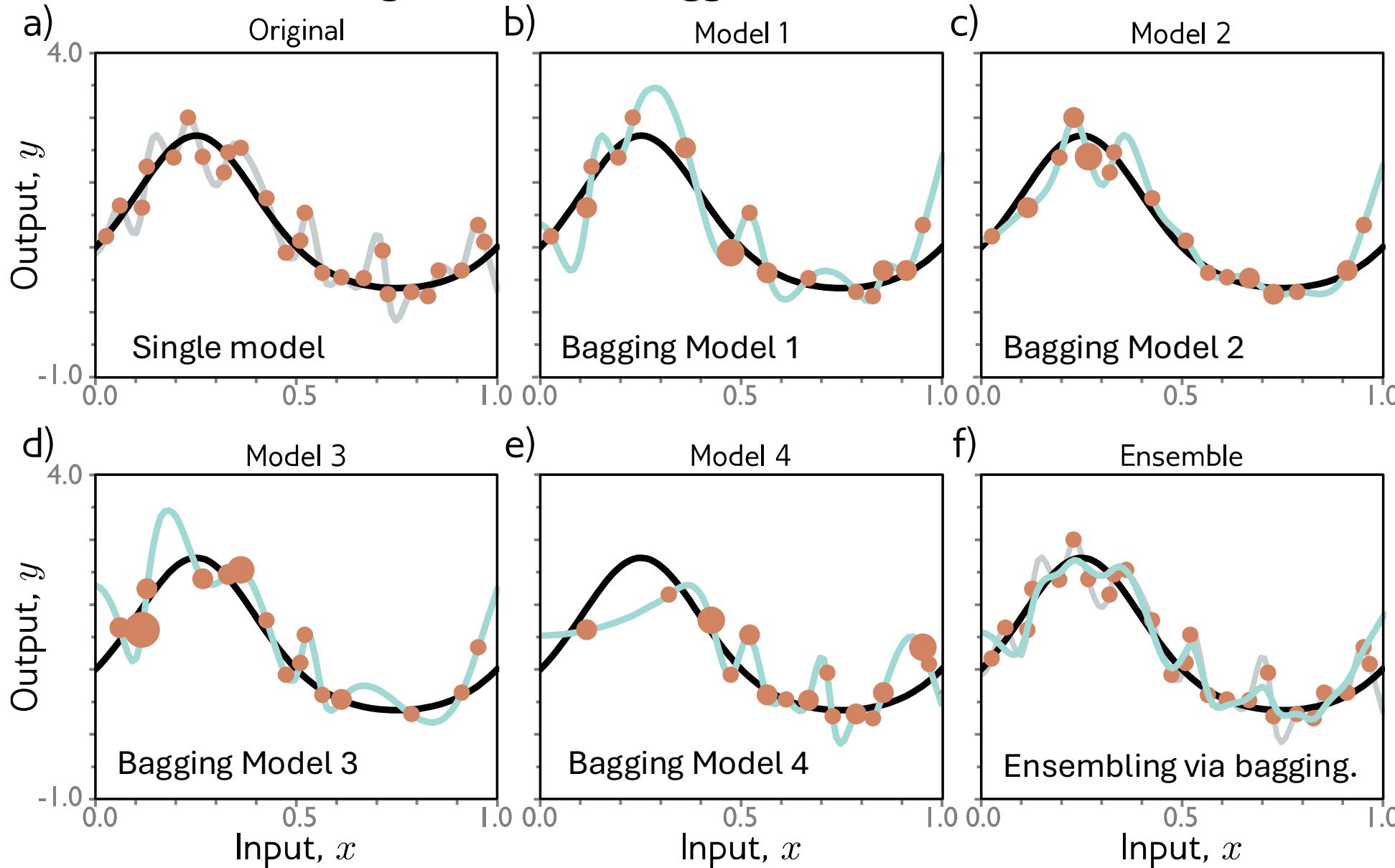
# Ensembling

- Combine several models – an **ensemble**
- Combining outputs

	Mean	Median/Frequent (Robust)
Regression	Mean of outputs	Median of outputs
Classification	Mean before softmax	Most frequent predicted class

- Can be simply different initializations or even different models
- Or train with different subsets of the data resampled with replacements – **bootstrap aggregating (bagging)**

# Single Model vs Bagged Ensemble



Size of orange point indicates number of times the data point was re-sampled.

# Any Questions?

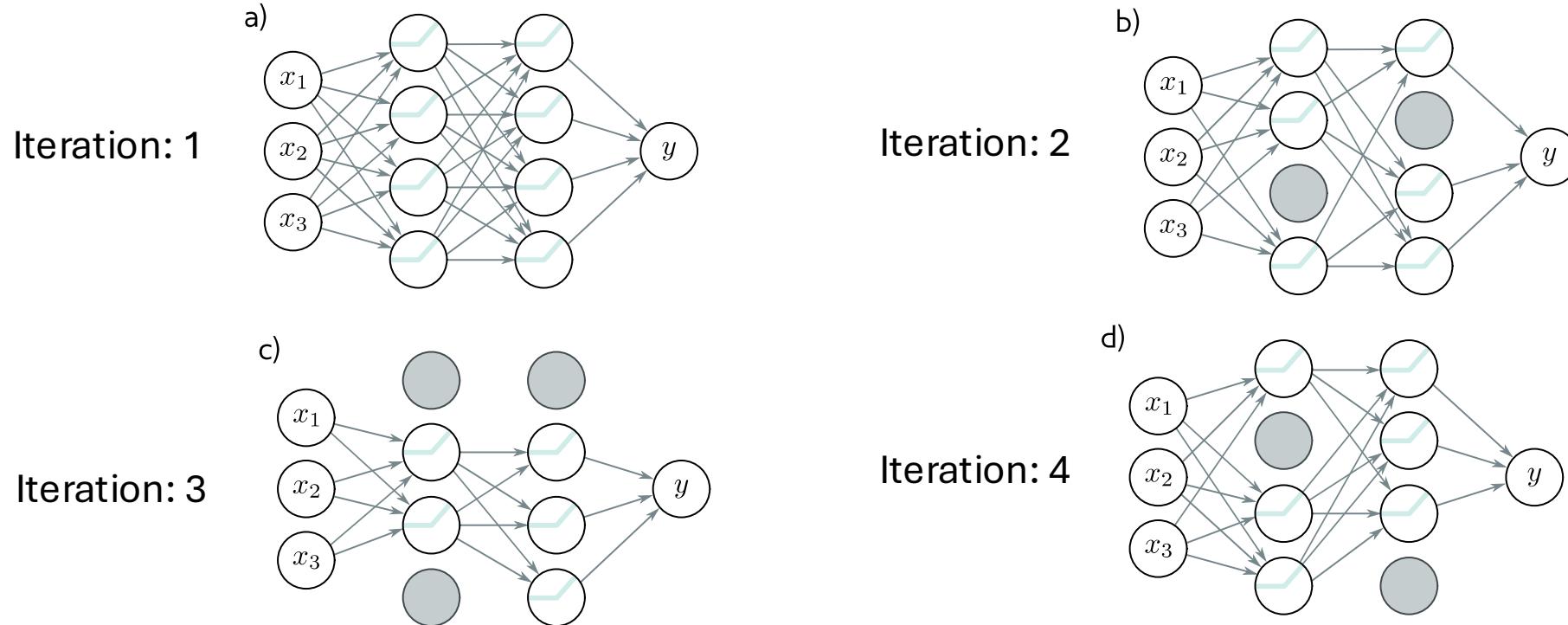
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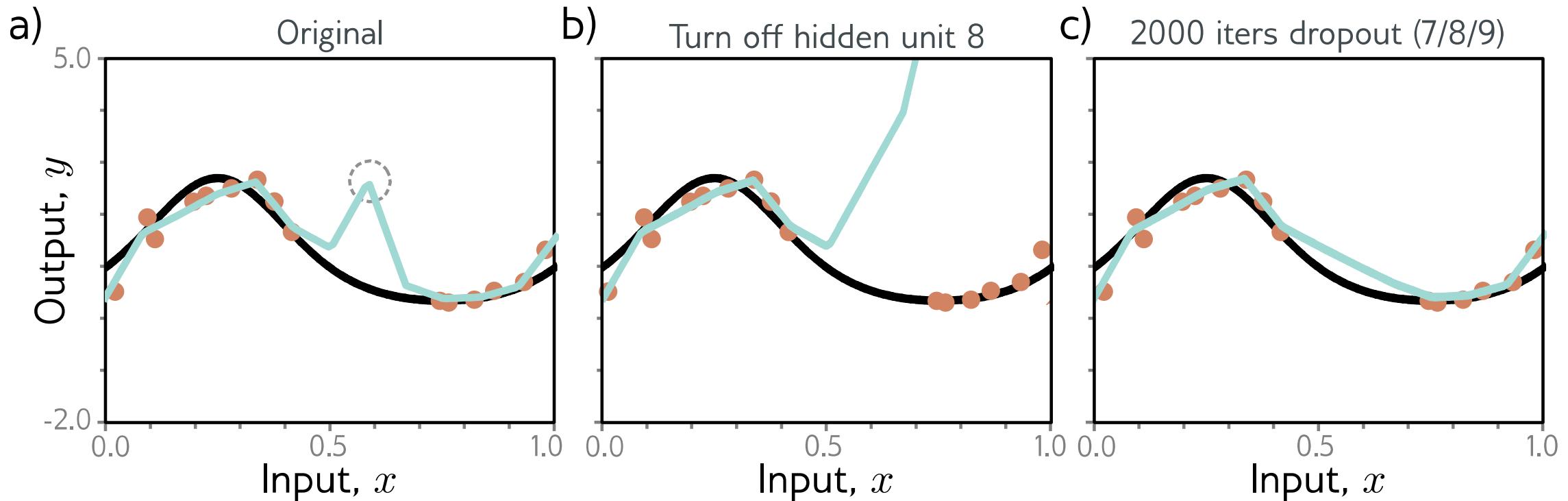
# Dropout

Randomly clamp ~50% of hidden units to 0 on each iteration.



- Makes the network less dependent on any given hidden unit.
- At test time, all hidden units are active, which was not the case during training
  - Must rescale using *weight scaling inference rule*
    - Multiply weights by  $(1 - \text{dropout probability})$  so average contribution is the same.

# Dropout



- Prevents situations where subsequent hidden units correct for excessive swings from earlier hidden units
- Can eliminate kinks in function that are far from data and don't contribute to training loss

# Monte Carlo Dropout for Inference (optional)

- Run the network multiple times with different random subsets of units clamped to zero (as in training).
- Combine the results using an ensembling method,
- This is closely related to ensembling in that every random version of the network is a different model; however, we do not have to train or store multiple networks here.

# Any Questions?

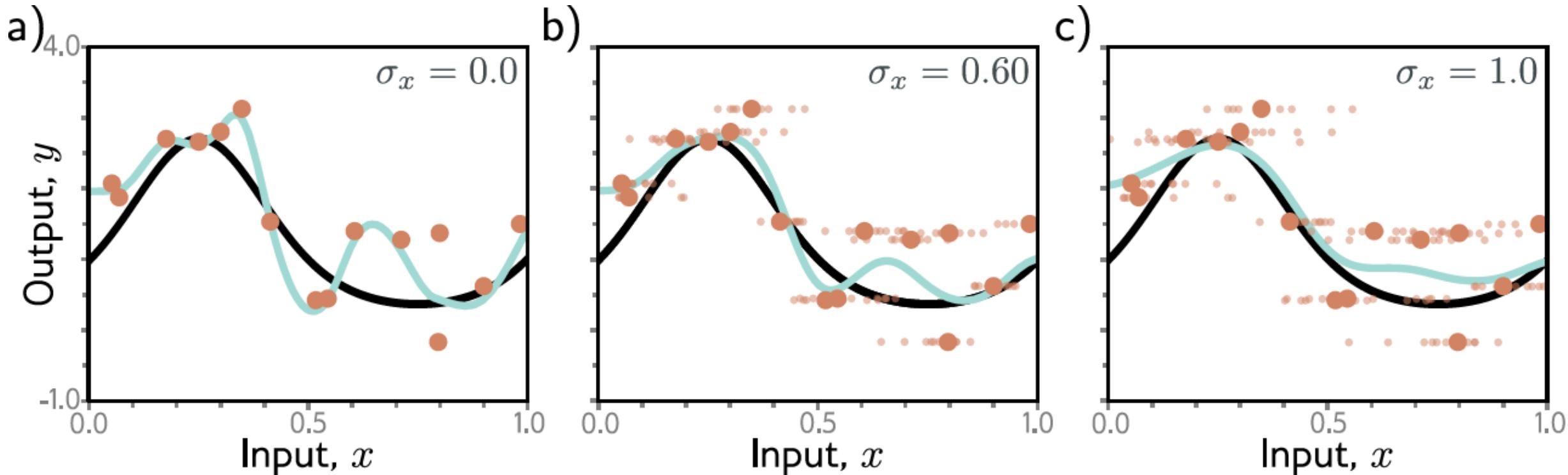
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# Adding noise

Adding noise to input with different variances.



- to inputs – induces weight regularization (see Exercise 9.3 in UDL)
- to weights – makes robust to small weight perturbations
- to outputs (labels) – reduces “overconfident” probability for target class

# Any Questions?

???

## Moving on

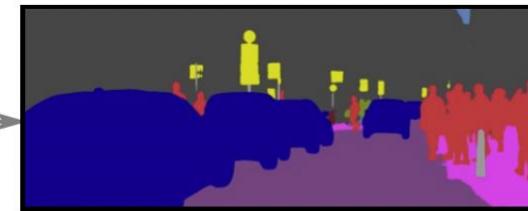
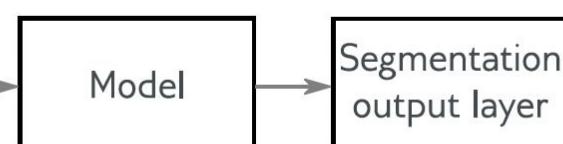
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# Transfer & Multitask Learning, Augmentation

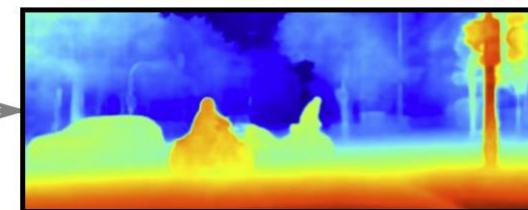
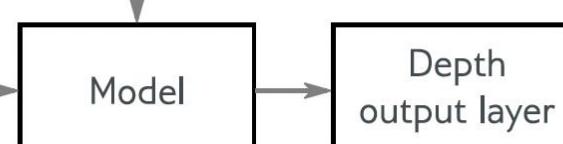
- Strictly speaking not regularization, but can help improve generalization when dataset sizes are limited

# Transfer Learning

(1) Train the model for segmentation



Assume we have lots of segmentation training data



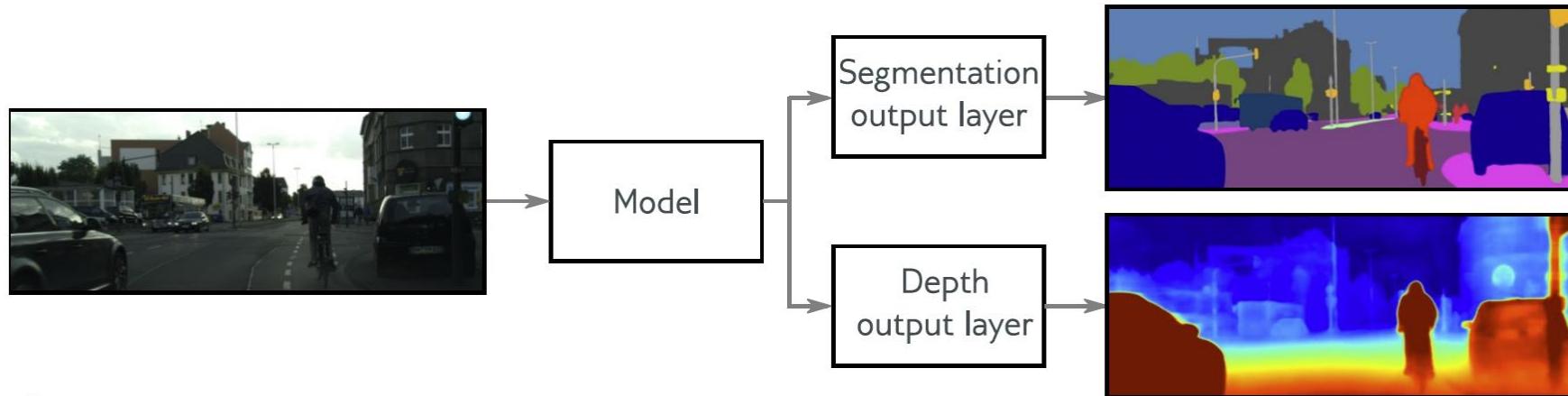
Assume we have limited depth training data

(2) Replace the final layers to match the new task and

(3) Either:

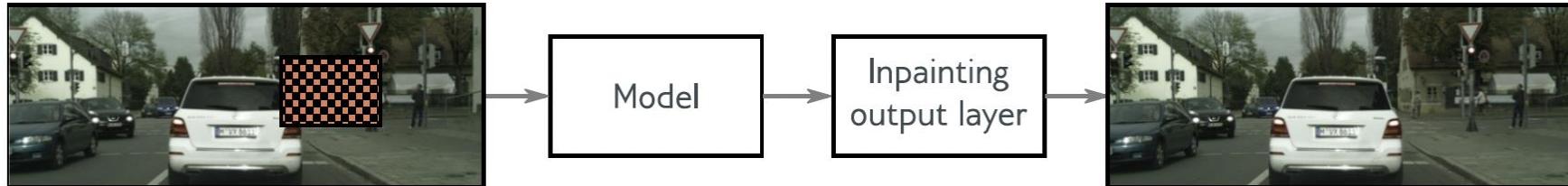
- a) Freeze the rest of the layers and train the final layers
- b) Fine tune the entire model

# Multi-Task Learning



- Train the model for 2 or more tasks simultaneously
  - Weighted combo of loss functions
$$L_{total} = \alpha \cdot L_{segmentation} + \beta \cdot L_{depth}$$
  - Less likely to overfit to training data of one task
  - Can be harder to get training to converge. Might have to vary the individual task loss weightings,  $\alpha$  and  $\beta$ .

# Self-Supervised Learning



*The animal didn't cross the  street because it was too tired.*

- Mask out part of the training data
- Train model to try to infer missing data
  - masked data is the target
- → Model learns characteristics of the data
- Then apply transfer learning

# Any Questions?

???

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# Data augmentation

a) Original



b) Flip



c) Rotate and crop



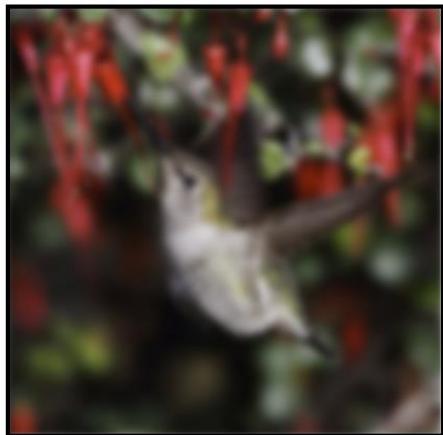
d) Vertical stretch



e) Color balance



f) Blur



g) Vignette



h) Pincushion



# Image augmentation in PyTorch

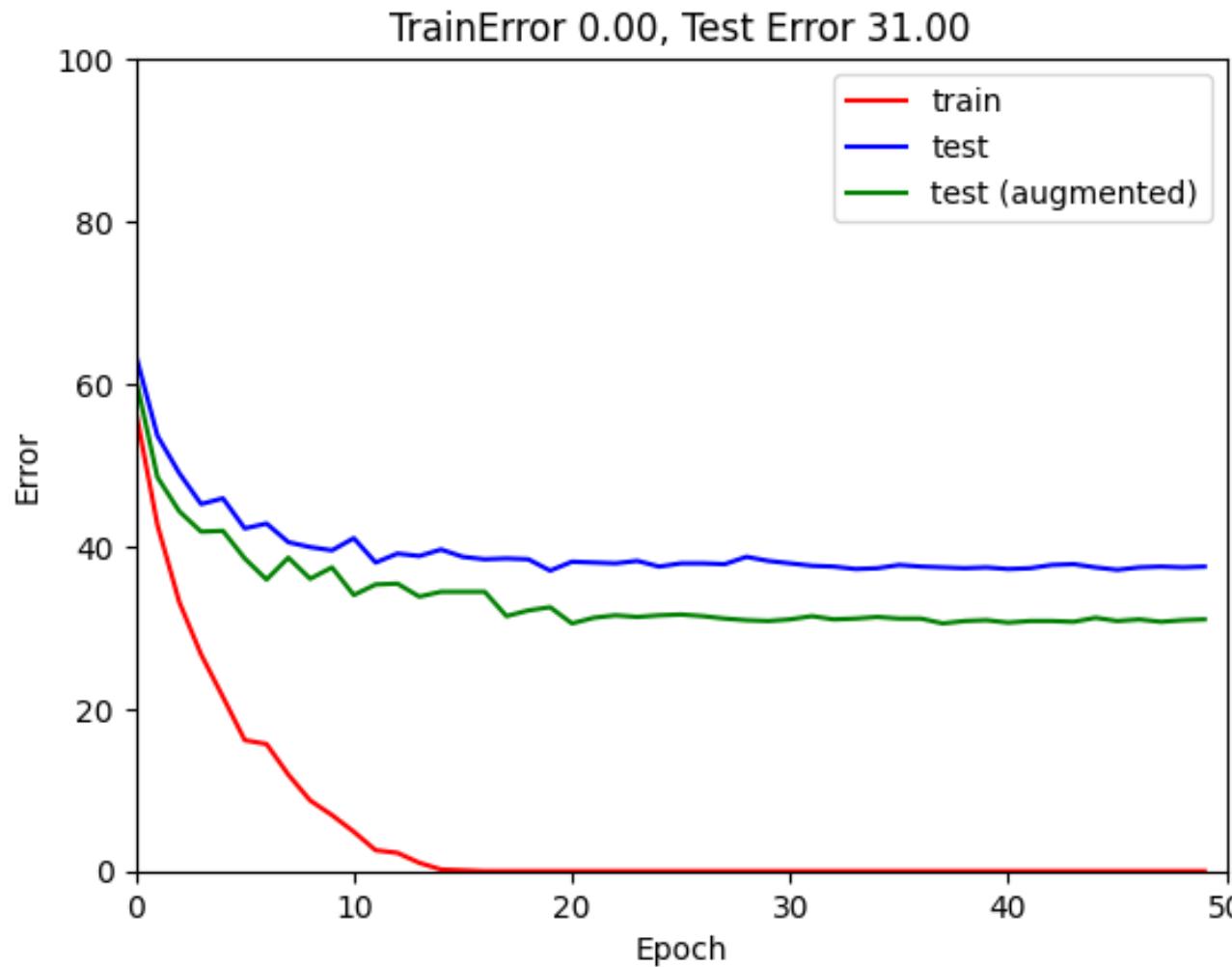
```
import torch
import torchvision.transforms as transforms

# Define augmentation pipeline
transform = transforms.Compose([
    transforms.RandomHorizontalFlip(p=0.5),
    transforms.RandomVerticalFlip(p=0.3),
    transforms.RandomRotation(degrees=30),
    transforms.ColorJitter(brightness=0.5, contrast=0.5, saturation=0.5),
    transforms.RandomAffine(degrees=20, translate=(0.2, 0.2), shear=10),
    transforms.RandomPerspective(distortion_scale=0.5, p=0.5),
    transforms.ToTensor(), # Convert image to tensor
])

# Apply transformations multiple times to visualize augmentation
augmented_image = transform(image)
```

<https://pytorch.org/vision/main/transforms.html>

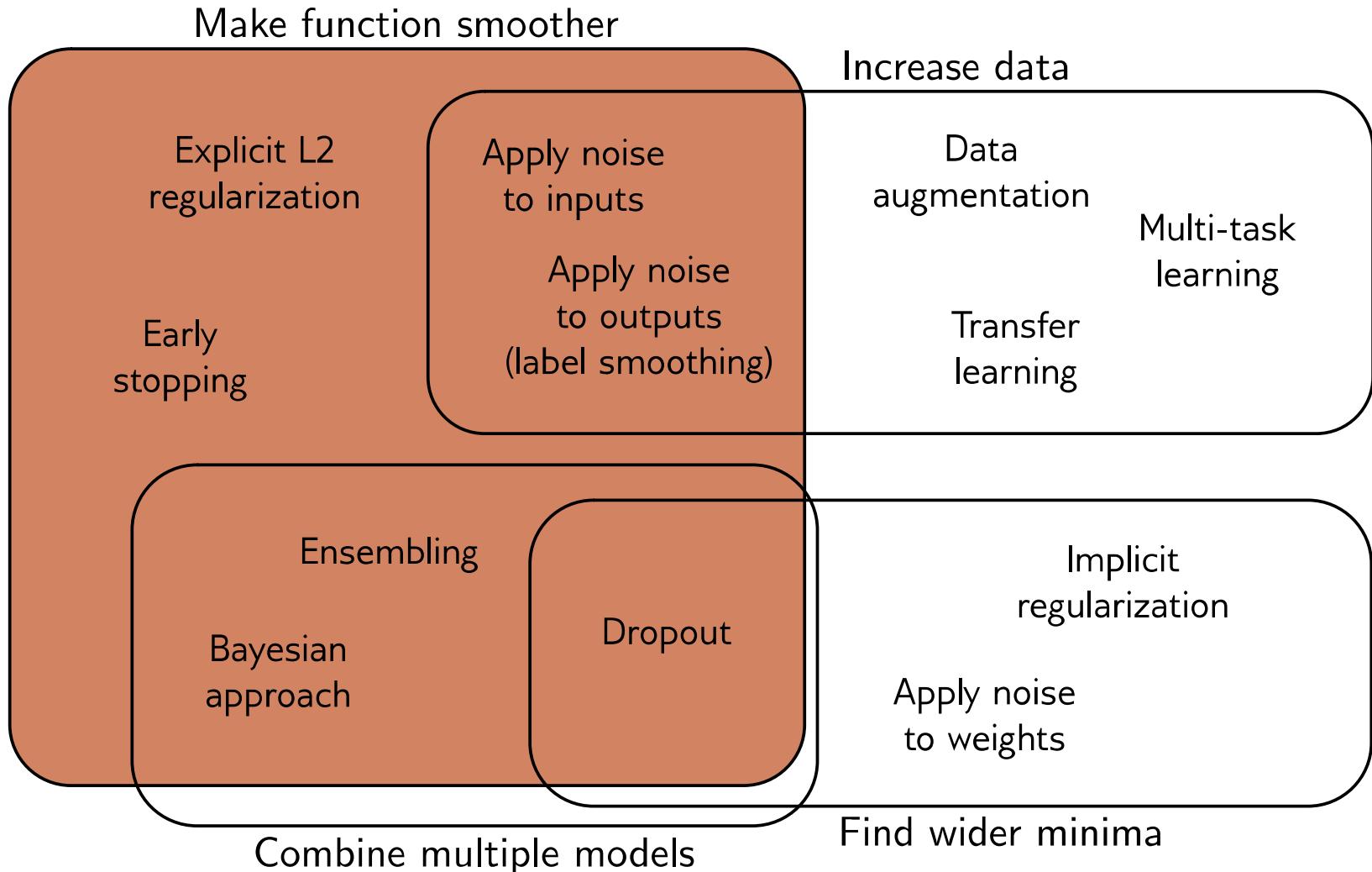
# Data Augmentation: MNIST1D



Examples in training set: 4000  
Examples in test set: 1000  
Length of each example: 40

- Randomly circularly rotate
- randomly scale between 0.8 and 1.2

# Regularization overview



# Any Questions?

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- Reproducing Double Descent Demystified
- Explicit regularization
- Implicit regularization
- Early stopping
- Ensembling
- Dropout
- Adding noise
- Transfer learning, multi-task learning, self-supervised learning
- Data augmentation