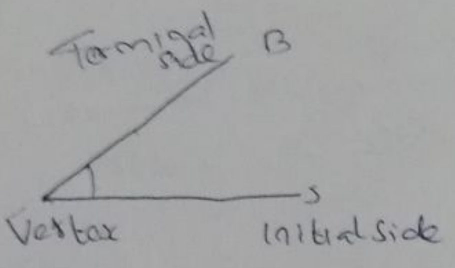
CHAPTER - 3

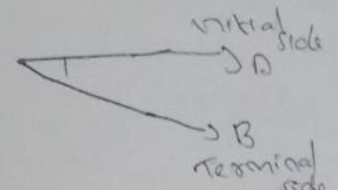
TRIGNOMETRIC FUNCTION

Angles

An Angle is a measure of rotation of a given ray about its initial point. The original ray is called initial side and final position of the ray after rotation is called terminal side of the angle. The point of rotation is called Vertex. If the direction of rotation is anticlockwise the, angle is said to be +ve and if the direction of rotation is clockwise, the angle is -ve.



+ve angle



-ve ~~st~~ angle.

There are two types of measures of an angle namely

1. Degree measure
2. Radian measure.

Degree measure

If a rotation from the initial side to the terminal side is $\left(\frac{1}{360^\circ}\right)^{th}$ of a revolution the angle is said to have a measure of one degree

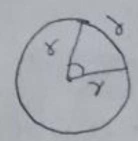
1° = 60 minutes, written as $60'$

1 minute $(1')$ = 60 seconds, written as $60''$

Radian measure

One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle

One radian is written as 1^c



$$180^\circ = \pi \text{ radian}$$

$$1 \text{ radian} = \frac{180}{\pi}$$

$$= 56^\circ 16' \text{ approx}$$

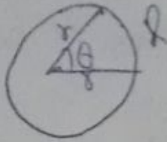
$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c = 0.01746 \text{ radians approx.}$$

$$1^\wedge = \pi/3 = 1.047 \text{ app}$$

$$1'' = \quad 2015 = 62.8.$$

If 'θ' radians is the angle subtended by an arc of length l at the centre of a circle of radius r, then

$$l = r\theta$$



$\theta = l/r$

$\Rightarrow l = r\theta$

→ If θ radians is the angle subtended by an ~~angle~~ arc $\overset{\frown}{AB}$ of a circle of radius r. at its centre C. then the area of the sector ACB. then area of the sector $ACB = \frac{1}{2} r^2\theta$. $(r\theta = l)$

$\Rightarrow \frac{1}{2} l r$

Express the following in radians

1) $30^\circ = \frac{\cancel{30}^{1} \times \pi}{\cancel{180}_{6}} = \frac{\pi}{6}^c$ //

2) $135^\circ = \frac{\cancel{135} \times \pi}{\cancel{180}} = \frac{3\pi}{4}^c$ //

$$\left[\text{Radian measures} = \frac{\pi}{180} \times \text{D. measure}\right.$$
$$\left.\text{Degree m} = \frac{180}{\pi} \times \text{Radian m.}\right]$$

Express the following in degrees

1) $\left(\frac{\pi}{4}\right)^c = \frac{\cancel{\pi}}{\cancel{4}} \times \frac{\overset{45}{\cancel{180}}}{\cancel{\pi}} = 45°$ //

2) $\left(\frac{2\pi}{3}\right)^c = \frac{2\cancel{\pi}}{\underset{1}{\cancel{3}}} \times \frac{\overset{60}{\cancel{180}}}{\cancel{\pi}} = 120°$ //

3) $\left(\frac{7\pi}{6}\right)^c = \frac{7\cancel{\pi}}{\cancel{6}} \times \frac{\overset{30}{\cancel{180}}}{\cancel{\pi}} = 210°$ //

Q Convert $40°, 20'$ into radian measure

Ans $40° 20' = 40° + \frac{\overset{1}{\cancel{20}}}{\underset{3}{\cancel{60}}}°$

$$\left[\begin{array}{c} 1° = 60' \\ \frac{1°}{60} = 1' \end{array}\right]$$

$= \left(40 + 1/3\right)°$

$= \frac{121}{3} \times \frac{\pi}{180} = \frac{121\pi}{540}$ radians

Q. Express the following in radians

ans $33^\circ 45' = 33 + \frac{\cancel{45}^{3}}{\cancel{60}_{4}} = 33 + 3/4$

$\Rightarrow \frac{135}{4} \to$ radians

$\Rightarrow \frac{\cancel{135}^{\cancel{27}\,3}}{4} \times \frac{\pi}{\cancel{180}_{4}}$

$= \left(\frac{3\pi}{16}\right)^c$

Q Express the following in degrees.

1 ① $\frac{11}{16}$ rad $\to$ degrees

$\boxed{\pi = 22/7}$

~~$\Rightarrow \frac{\pi}{16} \times \frac{180}{\pi} = \frac{45}{4}$~~

$\frac{11}{16} \times \frac{180}{\pi} \Rightarrow \frac{\cancel{11}^{1}}{\cancel{16}_{4}} \times \frac{\cancel{180}^{45}}{\cancel{22}_{2}/7}$

$\Rightarrow \frac{45}{8} \times 7 = \left(\frac{315}{8}\right)^\circ$

$$\frac{315}{8} = 39^\circ + \left(\frac{3}{8}\right)^\circ$$

$$\Rightarrow 39^\circ + \left(\frac{180}{8}\right)'$$

$$\Rightarrow 39^\circ + 22' + \frac{1}{2}''$$

$$\Rightarrow \underline{\underline{39^\circ + 22' + 30''}}$$

$\frac{3}{8} \to$ minute

$\Rightarrow \frac{3}{8} \times 60$

$= \frac{180}{8}$

$$\begin{array}{r} 39 \\ 8\overline{)315} \\ 24 \\ \hline 75 \\ 72 \\ \hline 3 \end{array}$$

$\Rightarrow$ $22 + \frac{4}{8}$

$$\begin{array}{r} 22 \\ 8\overline{)180} \\ 16 \\ \hline 20 \\ 16 \\ \hline 4 \end{array}$$

$\frac{1}{2} \times 60^\circ = 30$

-6 radians $= -6 \times \frac{180}{\pi}$

$$= -6 \times \frac{180}{\frac{22}{7}} \Rightarrow \frac{-6 \times 180 \times 7}{22}$$

$$= \left(\frac{-2520}{11}\right)^\circ \,//$$

$$\Rightarrow \left(-229 + \frac{1}{11}\right)^\circ$$

$$\Rightarrow 229^\circ + \left(\frac{60}{11}\right)'$$

$$\Rightarrow 229^\circ + 5' + \frac{5'}{11} \quad \left(\frac{5 \times 60}{11} = \frac{300}{11} \approx 27''\right)$$

$$\Rightarrow 229^\circ + 5' + 27'' \text{ approximately}$$

Q The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes (choose $\pi = 3.14$)

Ans In 60 minutes the minute hand of a watch complete one revolution.

$\therefore$ in 40 minute $\Rightarrow \frac{\cancel{40}^{2}}{\cancel{60}_{3}} = \frac{2}{3}$ of a revolution

$$\theta = \frac{2}{\cancel{3}} \times \cancel{360}^{120} = 240^\circ$$

$$240^\circ = \left(\cancel{240}^{4} \times \frac{\pi}{\cancel{180}_{3}}\right)^c = \frac{4\pi}{3}$$

Hence the required distance travelled is given by

$$l = r\theta = \cancel{1.5}^{1} \times \frac{\cancel{4}^{2}\pi}{\cancel{3}_{2}} \text{ cm} = 2\pi \text{ cm}$$

$2 \times 3.14 = 6.28$ cm

Q Find the angle in radian through which a pendulum swings if its length is 75 cm and the describes arc of length

1) 10 cm 2) 15 cm 3) 21 cm

ii) Required angle $= \frac{l}{r} = \frac{10}{75} = \frac{2}{15}$ rad.

iii) $\theta = \frac{l}{r} = \frac{\cancel{15}^{1}}{\cancel{75}_{5}} = \frac{1}{5}$ rad

iv) $\theta = l/r = \frac{21}{75_{25}} = 7/25$ rad.

Q A wheel makes 360 revolutions in one minute through How ~~any~~ many ~~dian~~ radians, does it turn in one second?

A The angle described by the wheel in one revolution $=$ 2π ~~rd~~ rad.

$\therefore$ The angle through which the wheel turn in one minute.

$= 2\pi \times 360$ rad

$\therefore$ The angle through which the wheel turns in one second

$$= \frac{2\pi \times \cancel{360}^{6}}{\cancel{60}_{1}} = 12\pi \text{ rad}$$

Q The relation b/w the degree and radian measures of certain commonly used angles are given below

Degree	30°	45°	60°	90°	135°	180°	270°	360°
Radian	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/4$	π	$\frac{3\pi}{2}$	2π

Convert -

① $-47° 30' \Rightarrow$ rads

$$\Rightarrow 47° + \frac{\cancel{30}}{\cancel{60}_2} = 47° + \frac{1}{2}°$$

$$= \frac{95°}{2}$$

$$\Rightarrow \frac{\cancel{95}^{19}}{2} \times \frac{\pi}{\cancel{180}_{36}} = \frac{-19\pi}{72} \text{ rad}$$

② $520° = \cancel{520} \times \frac{\pi}{\cancel{180}}$

$$\Rightarrow \frac{26\pi}{9} \text{ rad}$$

3) $\frac{5\pi}{3} \rightarrow$ degree

$$\frac{5\pi}{\cancel{3}} \times \frac{\cancel{180}^{60}}{\cancel{\pi}} = 300° //$$

4) $\frac{7\pi}{6} \Rightarrow$ degree

$$\frac{7\cancel{\pi}}{\cancel{6}} \times \frac{\cancel{180}^{30}}{\cancel{\pi}} = 210° //$$

$$\begin{array}{r} 480 \\ 41 \\ \hline 521 \end{array} \qquad \begin{array}{r} 26 \\ 15 \\ \hline 41 \end{array}$$

5) $16° \; 82' \; 30'' \rightarrow$ radians

$$\cancel{\left(16 + \frac{82}{60} + \frac{30}{60 \cdot 2}\right)^{\circ}} \Rightarrow \cancel{16° + \frac{13}{15} + \frac{1}{2}}$$

$$= \cancel{16 + \frac{41}{30}}$$

$$= \cancel{\frac{521}{30} \text{ rad.}}$$

$16° \; 52' \; 30''$

$\Rightarrow 16° \; 52' + \frac{1}{2}'$

$\Rightarrow 16° + \frac{105}{2 \times 60} = 16° + 7/8$

$$\Rightarrow \frac{128+7}{8} = \frac{135}{8}$$

$$\Rightarrow \frac{\cancel{135}}{8} \times \frac{\pi}{\cancel{180^\circ}} = \frac{3}{8} \times \frac{\pi}{4}$$

$$= \frac{3\pi}{32}$$

Q. Find the degree measure of the angle subtended at the centre of a circle of radius 100cm by an arc of length 22cm (use $\pi = 22/7$)

A. Here, use formulae $l = r\theta$

$$\Rightarrow \theta = l/r$$

$$\Rightarrow \theta = \frac{22}{100} = \frac{11}{50} \text{ radians}$$

$$\Rightarrow \left(\frac{11}{50} \times \frac{180}{\pi}\right)^\circ \Rightarrow \frac{11}{50} \times \frac{180}{22/7}$$

$$\Rightarrow \frac{1}{5}\times\frac{\overset{9}{\cancel{18}}\times 7}{\cancel{2}} = \left(\frac{63}{5}\right)^\circ$$

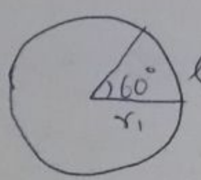
$$5\overline{)63}\,(12 \quad 60 \quad 3$$

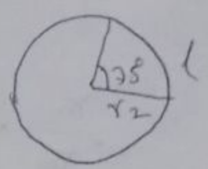
$$\Rightarrow 12^\circ + \left(\frac{3}{5}\right)^\circ$$

$$\Rightarrow 12^\circ + \left(\frac{3}{5}\times 60\right)'$$

$$\Rightarrow 12 + \frac{\overset{36}{\cancel{180}}}{\cancel{5}} \Rightarrow \underline{\underline{12^\circ\, 36'}}$$

If in two circles, arc of the same length subtend angles 60° and 75° at the centre. Find the ratio of their radii.





Let l be the common length of two arcs and let r_1 and r_2 be the radii of the circle. Then using the formula $l = r\theta$, we have

$$l = r_1 \frac{\pi}{3} \quad \left(60^\circ = \pi/3 \text{ rad}\right)$$

$$l = r_2 \frac{15\pi}{36} \quad \left(75^\circ = \overset{15}{\cancel{75}}\times\frac{\pi}{\underset{36}{\cancel{180}}} = \frac{15\pi}{36}\text{ rad}\right),$$

we have :-

$l_1 = r_1 \frac{\pi}{3}$

$l_2 = r_2 = \frac{\pi}{2}$

$\frac{l_1}{l_2} = \frac{r_1 \frac{\pi}{3}}{r_2 \frac{\pi}{2}}$

on dividing these eqns we have

$$\frac{l_1}{l_2} = \frac{r_1 \cdot \frac{\pi}{3}}{r_2 \cdot \frac{15\pi}{36}}$$

$$1 \Rightarrow \frac{r_1}{r_2} \cdot \frac{\pi}{\cancel{3}} \cdot \frac{\cancel{36}^{\cancel{12}\,4}}{\cancel{15}_{5}\cancel{\pi}}$$

$$1 = \frac{r_1}{r_2} \cdot 4/5 \Rightarrow \frac{r_1}{r_2} = 5/4$$

Thus required ratio is

$$r_1 : r_2 = 5 : 4$$

If the arcs of the same length in two circle's subtend angles 60° and 90° at their ~~radii~~ centres, find the ~~radi~~ ratio of their ~~radii~~

Let the common length of arcs be l and the radii of the two circles be r_1 and r_2

Expressing 60° and 90° in radians we have

$$60^\circ = \frac{\pi}{3} \text{ radians}$$

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

Using formula - $l = r\theta$

we have

$$l = r_1 \frac{\pi}{3}$$

$$l = r_2 \times \frac{\pi}{2}$$

$$\Rightarrow \frac{l}{l} = \frac{r_1 \frac{\pi}{3}}{r_2 \frac{\pi}{2}}$$

$$1 = \frac{r_1}{r_2} \frac{\pi}{3} \frac{2}{\pi} \Rightarrow 1 = \frac{r_1}{r_2} \cdot \frac{2}{3}$$

$$\frac{r_1}{r_2} = \frac{3}{2}$$

Hence ratio of the radii of the two circles $r_1 : r_2 = 3 : 2$

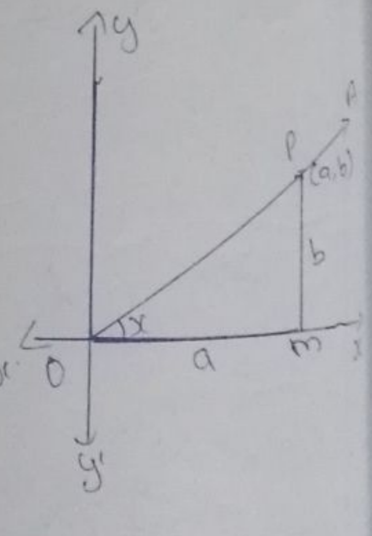
Trignometric functions or circular functions

Let x radians be the angle xOA and $P(a,b)$ be any point other than origin O on the terminal side.

Let $OP = r\ (>0)$. Then we define the six trignometric functions of x as

$$\cos x = \frac{OM}{OP} = \frac{a}{r}$$

$$\sin x = \frac{PM}{OP} = \frac{b}{r}$$



$$\tan x = \frac{PM}{OM} = \frac{b}{a} \left(\frac{\sin x}{\cos x}\right)$$

$$\sec x = \frac{OP}{OM} = \frac{r}{a} \left(\frac{1}{\cos}\right)$$

$$\operatorname{cosec} x = \frac{OP}{PM} = \frac{r}{b} \qquad \cot x = \frac{OM}{PM} = \frac{a}{b}$$

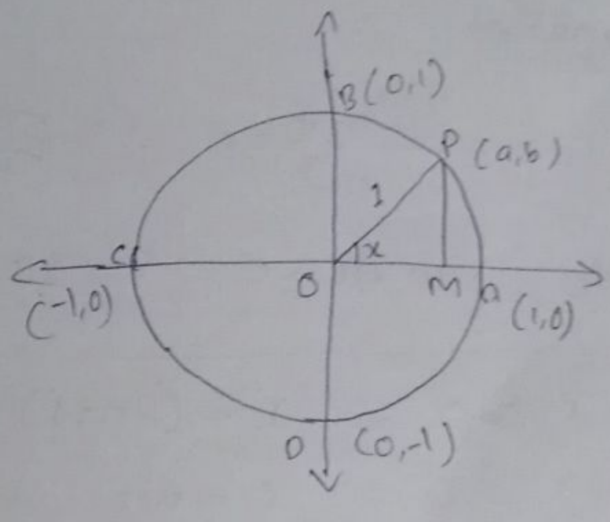
$\sec x = \dfrac{1}{\cos x}$ $\qquad$ $\operatorname{cosec} x = \dfrac{1}{\sin x}$

$\cot x = \dfrac{1}{\tan x}$ $\qquad$ $\tan x = \dfrac{\sin x}{\cos x}$

Using the unit circle we define the six trignometric functions as follows

Consider the circle with centre origin and radius 1cm. Let P (a,b) be any point on the circle. $\angle AOP = x$ radians

Then



(unit circle)

$\sin x = \dfrac{PM}{OP} = \dfrac{b}{1} = b$ $\qquad$ $\operatorname{cosec} x = \dfrac{1}{\sin x} = \dfrac{1}{b}$

$\sec x = \dfrac{1}{\cos x} = \dfrac{1}{a}$

$\cos x = \dfrac{OM}{OP} = \dfrac{a}{1} = a$

$\cot x = \dfrac{1}{\tan x} = \dfrac{a}{b}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{b}{a}$

$(\cos 0, \sin 0) = (1, 0)$

$\cos 0 = 1, \quad \sin 0 = 0$

* $(\cos 90, \sin 90) = (0, 1)$; $\cos 90 = 0, \sin 90 = 1$

* $(\cos 180, \sin 180) = (-1, 0)$; $\cos 180 = -1, \sin 180 = 0$

* $(\cos 270, \sin 270) = (0, -1)$, $\cos 270 = 0$, $\sin 270 = -1$

* $(\cos 360, \sin 360) = (1, 0)$; $\cos 360 = 1, \sin 360 = 0$

in general

$\cos \frac{\pi}{2} = 0, \cos \frac{3\pi}{2} = 0, \cos \frac{5\pi}{2} = 0, \ldots$

$\sin \pi = 0, \quad \sin 2\pi = 0, \quad \sin 3\pi = 0, \ldots$

$$\boxed{\begin{aligned} \cos x = 0 &\Rightarrow x = (2n+1)\pi/2 \\ \sin x = 0 &\Rightarrow x = n\pi \end{aligned}}$$

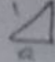
$$\left.\begin{aligned} \sin(2n\pi + x) &= \sin x \\ \cos(2n\pi + x) &= \cos x \end{aligned}\right\} n \in \mathbb{Z}$$

Key Results.

1) $\sin^2 x + \cos^2 x = 1$

2) $1 + \tan^2 x = \sec^2 x$

3) $1 + \cot^2 x = \operatorname{cosec}^2 x$



$a^2 + b^2 = 1$

$\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

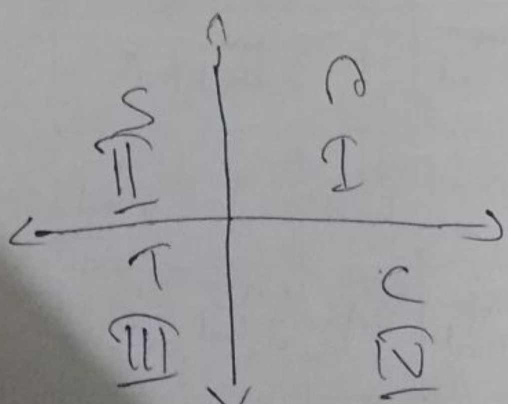
The trigonometric ratios of commonly used angles

x	$\sin x$	$\cos x$	$\tan x$	$\operatorname{cosec} x$	$\sec x$	$\cot x$
0	0	1	0	Not defined	1	Not defined
$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
$\pi/2$	1	0	Not defined	1	Not defined	0
π	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	-1	0	Not defined	-1	Not defined	0
2π	0	1	0	Not defined	1	Not defined

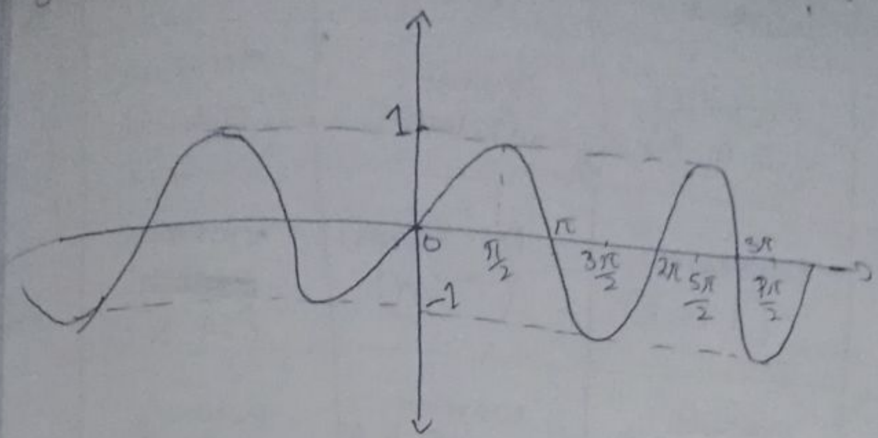
The sign of trignometric functions of angles in a different quadrants are given

Trignometric functions	I	II	III	IV
$\sin x$	+	+	−	−
$\cos x$	+	−	−	+
$\tan x$	+	−	+	−
$\cot x$	+	−	+	−
$\sec x$	+	−	−	+
$\mathrm{cosec}\, x$	+	+	−	−

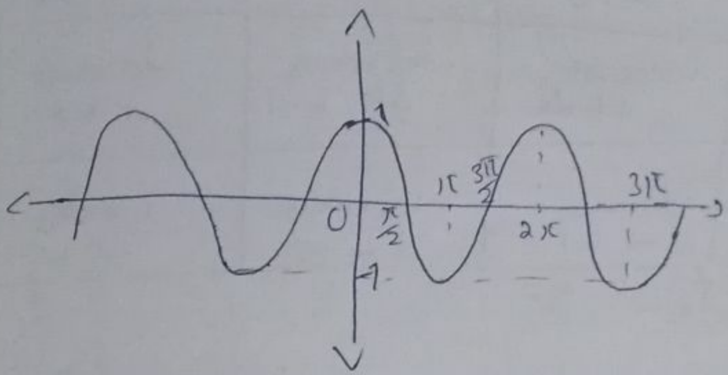
{ASTC Rule . A - All
S - Sine and cosec (reciprocal)
T - Tan
C - cos



$y = \sin x$ (graph)



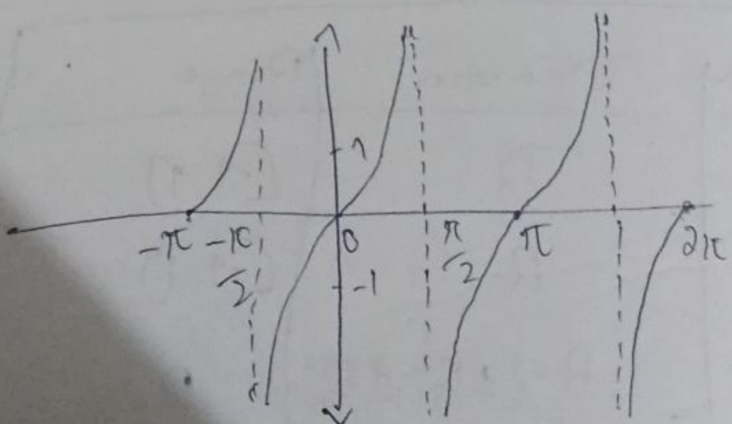
$y = \cos x$



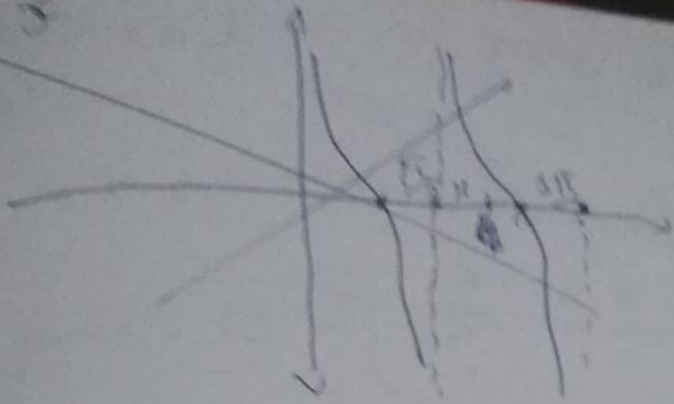
function	Domain	Range
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$(-1, 1)$
$\tan x$	$R - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \ldots\}$	R
$\cot x$	$R - \{0, \pm\pi, \pm 2\pi \ldots\}$	R
$\sec x$	$R - \{(2n+1)\frac{\pi}{2}, n \in Z\}$	$R - (-1, 1)$
$\operatorname{cosec} x$	$R - \{n\pi, n \in Z\}$	$R - (-1, 1)$

	Ist quadrant	IInd quadrant	IIIrd quadrant	IVth quadrant
Sin x	increases (0 to 1)	decreases (1 to 0)	decreases (0 to -1)	increases (-1 to 0)
Cos x	Decreases (1 to 0)	Decreases (0 to -1)	increases (-1 to 0)	increases (0 to 1)
tan x	increases 0 to ∞	increases -∞ to 0	increases 0 to ∞	increases -∞ to 0
Cot x	decreases ∞ to 0	decreases 0 to -∞	decreases ∞ to 0	decreases 0 to -∞
Sec x	increases 1 to ∞	increases -∞ to -1	decreases -1 to ∞	decreases ∞ to 1
Cosec x	decreasy ∞ to 1	increase 1 to ∞.	increas -∞ to -1	decrease -1 to ∞

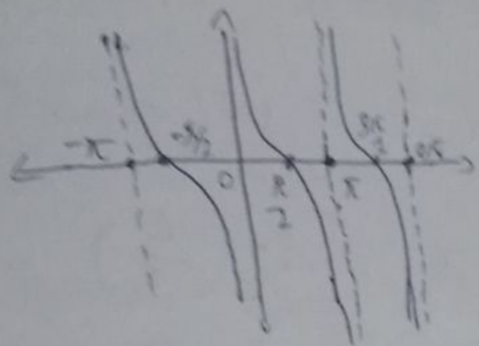
Tan x



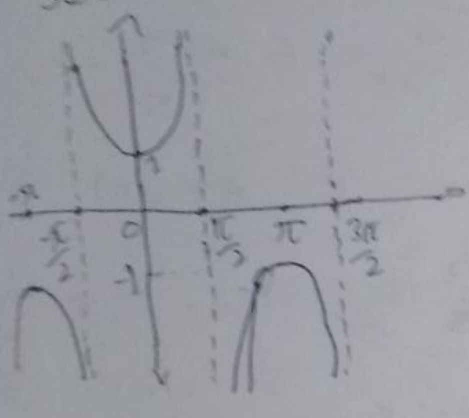
tan x



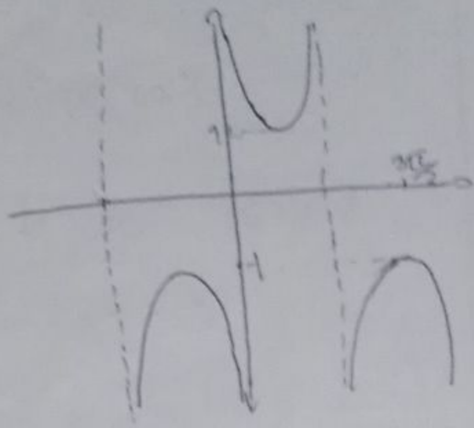
cot x



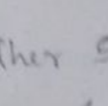
sec x



cosec x



If $\sin x = -4/5$ and x lies in the III^{rd} quadrant find the values of other 5 Trigonometric function



So, Third side = 3

$\sin = \frac{opp}{hyp} = \frac{-4}{5}$

so, $\cos x = \frac{-3}{5}$ //

$\sec x = \frac{-5}{3}$ //

$\operatorname{cosec} x = \frac{-5}{4}$ //

$\tan x = (+ve) \; \frac{4}{3}$ //

$(+4/\cancel{5} / +3/\cancel{5})$

$\cot x = \frac{3}{4}$ //

Or alternate method.

Result $\sin^2 x + \cos^2 x = 1$

$$\cos^2 x = 1 - (\sin x)^2$$

$$\cos^2 x = 1 - \left(\frac{-4}{5}\right)^2$$

$$\Rightarrow 1 - \frac{16}{25} = \frac{9}{25}$$

$$\cos x = \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

becoz x is in 3rd quad

$\cos x = -\frac{3}{5}$ // $\quad \sec x = -5/3$ //

Then $\tan x = \frac{-4/\cancel{5}}{+3/\cancel{5}} = 4/3$ //

$\cot x = 3/4$ //

~~$\sec x =$~~

$\operatorname{cosec} x = -5/4$ //

ind the value of the other five trigonometric function if $\cos x = -\frac{1}{2}$, x lies in the third quadrant

$$\sin^2 x + \cos^2 x = 1$$

$$\sin x^2 = 1 - \cos^2 x \Rightarrow 1 - \left(-\tfrac{1}{2}\right)^2$$

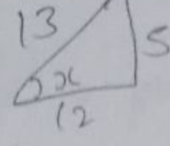
$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$\sin x = -\sqrt{3}/2$ $\qquad \sec x = -2$

$\operatorname{cosec} x = -2/\sqrt{3}$ $\qquad \tan x = \dfrac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

$\cot x = \dfrac{1}{\sqrt{3}}$

Q. If $\tan x = -5/12$, x lies in the second quadrant then find values of other 5 Trig functions



$\sqrt{12^2 + 5^2}$
$= \sqrt{169}$
$= 13$

$\sin x = \dfrac{5}{13}$

$\cos x = \dfrac{-12}{13}$ $\qquad \sec x = \dfrac{-13}{12}$

$\cot x = \dfrac{-12}{5}$ $\qquad \operatorname{cosec} x = \dfrac{13}{5}$

$$1 + \tan^2 x = \sec^2 x$$

$$\cancel{\tan^2 x = \sec^2 x - 1}$$

$$\sec^2 x = 1 + \left(\frac{-5}{12}\right)^2$$

$$= 1 + \left(\frac{25}{144}\right)$$

$$= \frac{169}{144}$$

$$\sec x = \pm\sqrt{\frac{169}{144}} = \frac{-13}{12}$$ (because x is in II quad)

So, $\cos x = \frac{-12}{13}$ //

$\cot x = \frac{-12}{5}$ //

$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$\Rightarrow 1 + \left(\frac{-12}{5}\right)^2 \Rightarrow 1 + \frac{144}{25}$$

$$= \frac{169}{25}$$

$$\operatorname{cosec} x = \sqrt{\frac{169}{25}} = \frac{13}{5}$$ //

$\sin x = \frac{5}{13}$ //

$\sin x = 3/5$ x lies in the II^{nd} quad

find the values of other five trig functions

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2 \Rightarrow 1 - \frac{9}{25} = \frac{16}{25}$

$\cos x = \pm\sqrt{\frac{16}{25}} = \underline{\underline{-\frac{4}{5}}}$ (−ve becoz x is in the 2nd quad)

$\tan x = \frac{\sin x}{\cos x} = \frac{3/\cancel{5}}{-4/\cancel{5}} = -3/4$

$\Rightarrow$ $\operatorname{cosec} x = 5/3$

$\cos x = -4/5$

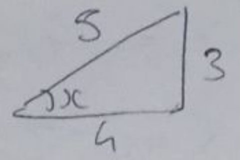
$\sec x = -5/4$

$\tan x = -3/4$

$\cot x = -4/3$

OR alternate way

$\sin x = 3/5 = \frac{\text{opp}}{\text{hyp}}$



$\sqrt{5^2 - 3^2}$
$= 4$

$\cos x = -4/5$

$\sec x = -5/4$

$\tan x = -3/4$

$\cot x = -4/3$

$\operatorname{cosec} x = 5/3$

Q. If $\cot x = 3/4$ and x lies in the 3rd quad, find the values of the other five trig functions

ans $1 + \cot^2 x = \operatorname{cosec}^2 x$

$$\operatorname{cosec}^2 x = 1 + \left(3/4\right)^2 = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\operatorname{cosec} x = \pm\sqrt{\frac{25}{16}} = -\frac{5}{4} \left(\begin{matrix}\text{-ve because} \\ x \text{ is in the} \\ \text{3rd quad}\end{matrix}\right)$$

$\therefore \sin x = -4/5$

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(-\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25} = \frac{9}{25}$$

$$\cos x = \pm\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$\tan x = 4/3$ | $\sin x = -4/5$

$\cos x = -3/5$ | $\operatorname{cosec} x = -5/4$

$\sec x = -5/3$

$\cancel{\sin(n2\pi)\ n \in Z + \sin}$

* $\sin(n2\pi + x) = \sin x \quad n \in Z$

* $\cos(2n\pi + x) = \cos x$

* $\tan(n\pi + x) = \tan x$

* $\cot(n\pi + x) = \cot$

Q find the value of $\sin(765^\circ)$

$\sin(765^\circ) = \sin(720^\circ + 45^\circ)$

$= \sin(2\pi + \pi/4) = 1/\sqrt{2}$

OR -

$765 \times \frac{\pi}{180} = \frac{17\pi}{4}$

$\sin\left(\frac{17}{4}\right)\pi = \sin\left(4 + \frac{1}{4}\right)\pi$

$= \sin\left(2 \times 2\pi + \frac{\pi}{4}\right) = \sin(\pi/4)$

$= 1/\sqrt{2}$ //

Q find the value of $\tan\left(\frac{19\pi}{3}\right)$

ans $\tan\left(\frac{19\pi}{3}\right) = \cancel{\tan 3}$ $\left(Q + \frac{R}{D}\right)$ $3\overline{)19}$ 6 (Q) $\frac{18}{1}$ (R) (D)

$\tan\left(6 + \frac{1}{3}\right)\pi$

$= \tan\left(6\pi + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ ans

$\{\tan(n\pi + x) = \tan x\}$

Q Find the value $\sin\left(-\frac{11\pi}{3}\right)$

ans $\cancel{\sin\left(\frac{-11\pi}{3}\right) = \sin\left(-3 - \frac{2}{3}\right)\pi}$ $3\overline{)11}$ -3 $\frac{-9}{2}$

$= \cancel{\sin\left(-3\pi - \frac{2\pi}{3}\right)} \times 2$

$\cancel{= 6\pi - \frac{4\pi}{3}}$

ans $\sin\left(-11\frac{\pi}{3}\right) = \sin\left(-4\pi + \frac{\pi}{3}\right)$

$\frac{11}{3}$ can be written as $-4 + \frac{1}{3}$

$= -\frac{11}{3}$

$\sin(2n\pi + x) = \sin x$

$\rightarrow \sin\left(-4\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3}$

$\sin \pi/3 = \sqrt{3}/2$

find $\sin\left(\frac{31\pi}{3}\right)$?

$\sin\left(\frac{31\pi}{3}\right) = \sin\left(10\pi + \frac{\pi}{3}\right)$

$\sin\left(10\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cot\left(-\frac{15\pi}{4}\right)$?

~~$\cot\left(-4\pi + \frac{\pi}{4}\right)$.~~

$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-4\pi + \frac{\pi}{4}\right)$

$= \cot\left(-4\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4}$

$\left[\frac{\sqrt{a}}{a} = \frac{1}{\sqrt{a}}\right]$

$\left[\cot(n\pi + x) = \cot x\right]$

$\cot \pi/4 = 1$ //

Q ~~Find the value of cos (−1710°)~~

Ans −1710 → radians

$\cancel{-1710} \times \frac{\pi}{\cancel{180}} = -\frac{\cancel{57}\pi}{\cancel{6}\,2}$

$\cancel{\cos\left(6\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2}}$

$\cancel{\cos(-1710) =}$

$\cos(-1710) = \cos(-1440 - 270)$

$= \cancel{\cos(-8\pi} - \frac{3\pi}{2}$

$6\overline{)57}$ 9, 54, 3

$6\overline{)19}$ 3, 18, 1

$5\overline{)170}$ 15, 21, 20, 10

$3\overline{)57}$ 19, 3, 27

$3\overline{)171}$ 15, 21

Q Find value of $\cos(-1710)^\circ$?

Ans $-1710^\circ \to \text{rads} = -\cancel{1710^\circ} \times \frac{\pi}{\cancel{180}} = \frac{\cancel{57}\pi}{\cancel{6}}$ (19, 36, 2)

$= \frac{-19\pi}{2} = -10\pi + \frac{\pi}{2}$

So, $\cos(-1710) = \cos\left(\frac{-19\pi}{2}\right)$

$= \cos\left(-10\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2} = \underline{\underline{0}}$

$\left[\cos(2n\pi + x) = \cos x\right]$

Find the value of $\operatorname{cosec}(-1410)^\circ$?

$$\operatorname{cosec}(-1410^\circ) = \operatorname{cosec}(-1440^\circ + 30^\circ)$$

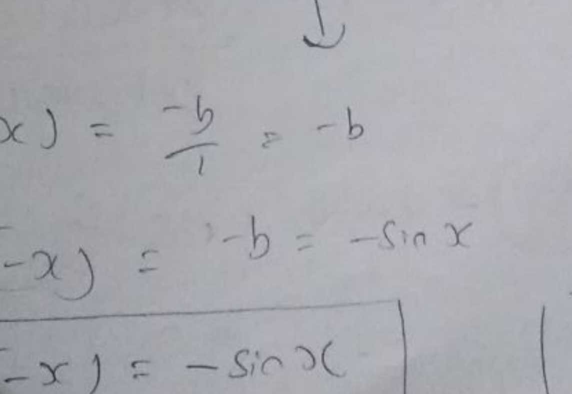
$$\left[\operatorname{cosec}(2n\pi + x) = \operatorname{cosec} x\right]$$

so, $\operatorname{cosec}(-1440^\circ + 30^\circ) = \cancel{\operatorname{cosec} 30^\circ}$

$$= \operatorname{cosec}\left(-8\pi + \frac{\pi}{6}\right) = \operatorname{cosec}\frac{\pi}{6}$$

$$\operatorname{cosec}\left(\frac{\pi}{6}\right) = \underline{\underline{2}}$$

Trignometric Identities



$$\sin(-x) = \frac{-b}{1} = -b \qquad \cos(-x) = \frac{a}{1} = a$$

$$\sin(-x) = -b = -\sin x \qquad \cos(-x) = \cancel{\cos x}\ a$$

$$\boxed{\sin(-x) = -\sin x} \qquad \boxed{\cos(-x) = \cos x}$$

$\tan(-x) = -\tan x$ | $\operatorname{cosec}(-x) = -\operatorname{cosec}(x)$

$\sec(-x) = \sec x$ | $\cot(-x) = -\cot(x)$

* $\sin(x+y) = \sin x \cos y + \cos x \sin y$
* $\sin(x-y) = \sin x \cos y - \cos x \sin y$
* $\cos(x+y) = \cos x \cos y - \sin x \sin y$
* $\cos(x-y) = \cos x \cos y + \sin x \sin y$
* $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
* ~~$\tan(x-y) = \dfrac{\tan x - \tan y}{\cot x \cot y}$~~
* $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$
* $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$
* $\cot(x+y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

* $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ | * $\sin\left(\frac{\pi}{2} + x\right) = \cos x$
* $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ | * $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\sec\left(\pi/2 - x\right) = \operatorname{cosec} x$

$\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\operatorname{cosec}\left(\frac{\pi}{2} - x\right) = \sec x$

$\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

$\sec\left(\frac{\pi}{2} + x\right) = -\operatorname{cosec} x$

$\cot\left(\frac{\pi}{2} + x\right) = -\tan x$

$\operatorname{cosec}\left(\frac{\pi}{2} + x\right) = \sec x$

1. identify the quadrant.
2. Check the sign
3. Check angles
4. , 0, π, 2π .. $n\pi$ no change in form
5. $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$ .. Change in form

Sin → Cos, cos → Sin

tan → cot Cot → tan

cosec → sec Sec → cosec

$\left(\frac{\pi}{2} + x\right)$ $90 + x$

$(\pi - x)$ $180 - x$

$90 - x$ $360 + x$ $(2\pi + x)$

$\pi/2 - x$

$(\pi + x)$ $180 + x$

$\left(\frac{3\pi}{2} - x\right)$ $270 - x$

$360 - x$ $(2\pi - x)$

$270 + x$ $\left(\frac{3\pi}{2} + x\right)$

$\sin(\pi - x) = \sin x$

$\cos(\pi - x) = -\cos x$

$\tan(\pi - x) = -\tan x$

$\sec(\pi - x) = -\sec x$

$\operatorname{cosec}(\pi - x) = \operatorname{cosec} x$

$\cot(\pi - x) = -\cot x$

$\sin(\pi + x) = -\sin x$

$\cos(\pi + x) = -\cos x$

$\tan(\pi + x) = \tan x$

$\sec(\pi + x) = -\sec x$

$\operatorname{cosec}(\pi + x) = -\operatorname{cosec} x$

$\cot(\pi + x) = \cot x$

$\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

$\operatorname{cosec}\left(\frac{3\pi}{2} - x\right) = -\sec x$

$\tan\left(\frac{3\pi}{2} - x\right) = \cot x$

$\sec\left(\frac{3\pi}{2} - x\right) = -\operatorname{cosec} x$

$\cot\left(\frac{3\pi}{2} - x\right) = \tan x$

$\cos\left(\frac{3\pi}{2} + x\right) = \sin x$

$\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$

$\tan\left(3\pi/2 + x\right) = -\cot x$

$\sec\left(\frac{3\pi}{2} + x\right) = \operatorname{cosec} x$

$\operatorname{cosec}\left(\frac{3\pi}{2} + x\right) = -\sec x$

$\cot\left(\frac{3\pi}{2} + x\right) = -\tan x$

$\cos(2\pi - x) = \cos x$

$\sin(2\pi - x) = -\sin x$

$\tan(2\pi - x) = -\tan x$

$\sec(2\pi - x) = \sec x$

$\operatorname{cosec}(2\pi - x) = -\operatorname{cosec} x$

$\cot(2\pi - x) = -\cot x$

Trigonometric ratios and co-ratios

$\sin x \to \cos x$	$\tan x \to \cot x$	$\sec x \to \operatorname{cosec} x$
$\cos x \to \sin x$	$\cot x \to \tan x$	$\operatorname{cosec} x \to \sec x$

Q $\cos(-300)$?

$\cos(-360° + 60) = \cos(60°) = 1/2$

or —

$\cos(-x) = \cos x$

$\cos(-300) = \cos(300) = \cos(270° + 30°)$

$= \cos\left(\frac{3\pi}{2} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \underline{\underline{1/2}}$

Q $\sin(-330°)$?

~~$\sin(-330°) = -\sin(330°)$~~

~~$-\sin(330°)$~~ $= \sin(-360° + 30°)$ ~~$\cos(30)$~~

$=$ ~~[illegible]~~ $= \sin(30°) = 1/2$ //

or $\sin(270° + 60°) = \sin(330°)$

$\sin(-330°) = -\sin(330°)$

$\sin(270° + 60°) = -(-\cos(60°))$

$= -(-1/2) = 1/2$ //

Q $\tan(75^\circ)$?

an $\tan(75^\circ) = \tan(30+45)$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\Rightarrow \tan(30+45) = \frac{\tan(30) + \tan(45)}{1 - \tan(30)\tan(45)}$$

$$\Rightarrow \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\cancel{\sqrt{3}}} \cdot \frac{\cancel{\sqrt{3}}}{\sqrt{3}-1} = \underline{\underline{\frac{\sqrt{3}+1}{\sqrt{3}-1}}}$$

(multiply and divide by Conjugate of Denominator)

$$\rightarrow \cancel{\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{\sqrt{3}^2 - 1^2}$$

$\Rightarrow \dfrac{3+1+2\sqrt{3}}{3-1} = \dfrac{\cancel{2}^{2}+\cancel{2}\sqrt{3}}{\cancel{2}}$

$\Rightarrow 2+\sqrt{3}$

Q. $\tan(15^\circ)$?

$\tan(45^\circ - 15^\circ) = \tan(15^\circ)$

$\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x \tan y}$ so, $\tan(45-15)^\circ$

$\Rightarrow \dfrac{\tan(45) - \tan(30)}{1+\tan(45)\tan(30)} = \dfrac{1-1/\sqrt{3}}{1+1/\sqrt{3}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1} = \dfrac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2} = \dfrac{3+1-2\sqrt{3}}{3-1}$

$\Rightarrow \dfrac{\cancel{2}^{2}-\cancel{2}\sqrt{3}}{\cancel{2}} = 2-\sqrt{3}$

Q. Prove that $\tan\left(\frac{\pi}{4}+x\right) = \dfrac{1+\tan x}{1-\tan x}$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x \tan y}$

So, $\tan\left(\frac{\pi}{4}+x\right) = \dfrac{\tan(\pi/4) + \tan(x)}{1-\tan(\frac{\pi}{4})\tan y} = \dfrac{1+\tan x}{1-\tan x}$ AND

TRIGNOMETRIC FUNCTIONS

(Continued)

Trignometric Identities

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

Replacing y by x

$\cos(x+x) = \cos x \cos x - \sin x \sin x$

$$\boxed{\cos(2x) = \cos^2 x - \sin^2 x}$$

$\cos 2x = \cos^2 x - (1 - \cos^2 x)$ $\left[\begin{array}{l} \sin^2 x + \cos^2 x = 1 \\ \sin^2 x = 1 - \cos^2 x \end{array}\right]$

$= \cos^2 x - 1 + \cos^2 x$

$$\boxed{\cos(2x) = 2\cos^2 x - 1}$$ $\left[\cos^2 x = 1 - \sin^2 x\right]$

$\cos 2x = 1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x$

$$\boxed{\cos(2x) = 1 - 2\sin^2 x}$$

$$\cos(2x) = \frac{\cos^2 x - \sin^2 x}{1} \quad \left[\cos^2 x + \sin^2 x = 1\right]$$

$$\Rightarrow \cos(2x) = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

dividing throughout by $\cos^2 x$

$$\Rightarrow \frac{\dfrac{\cos^2 x}{\cos^2 x} - \dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}}$$

$$\boxed{\cos(2x) = \frac{1 - \tan^2 x}{1 + \tan^2 x}}$$

$$\boxed{\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}\end{aligned}}$$

k for Class XI

$\sin(2x) = 2\sin x \cos x$

derivation

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x+x) = \sin x \cos x + \cos x \sin x$

$\Rightarrow \sin 2x = 2\sin x \cos x$

$\sin(2x) = \dfrac{2\tan x}{1+\tan^2 x} = \dfrac{2\tan x}{\sec^2 x}$

$\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$

$1-\tan^2 x = \sec^2 x$

so, ~~$\tan(2x) = \dfrac{2\tan x}{\sec^2 x}$~~

$\sin 2x = \dfrac{2\sin x \cos x}{1}$

$= \dfrac{2\sin x\cos x}{\sin^2 x + \cos^2 x}$

$\dfrac{2\sin x \cos x / \cos^2 x}{\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x}}$

$= \dfrac{2\tan x}{\tan^2 x + 1}$

$1+\tan^2 x = \sec^2 x$

$= \dfrac{2\tan x}{\sec^2 x}$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

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$$\tan(x+x) = \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$\boxed{\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}}$$

$\underline{\sin(3x)}$:-

$$\sin(x+y) = \sin x \cos y + \sin y$$

$$\sin(2x + x) = \sin 2x \cos x + \cos 2x +$$

$\Rightarrow$

$$\sin(3x) = 3\sin x - 4\sin^3 x$$

$$\cos(3x) = 4\cos^3 x - 3\cos x$$

$\rightarrow \tan(3x) = \dfrac{3\tan x - \tan^3 x}{1 - \tan^2 x}$

NOTE -

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = \cos 2x + 1$

$$\boxed{\cos^2 x = \frac{\cos 2x + 1}{2}}$$

Similarly $\cos 2x = 1 - 2\sin^2 x$

$\rightarrow \sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\rightarrow \sin^3 x = \dfrac{3\sin x - \sin 3x}{4}$

$\rightarrow \cos^3 x = \dfrac{3\cos x + \cos 3x}{4}$

$\rightarrow \dfrac{\sin 2x}{1 + \cos 2x} = \tan x$

$$\frac{\sin 2x}{1+\cos 2x} = \frac{2\sin x \cos x}{2\cos^2 x} = \tan x$$

$\tan x$

Sum formulae

→ $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

→ $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

→ $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

→ $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Product formulae

→ $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

→ $2\cos x \sin y = \sin(x+y) - \sin(x-y)$

→ $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

→ $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

* $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$
* $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$

Q Prove that $\sin\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right) = -\frac{1}{2}$.

A) $\sin \pi/6 = 1/2 = \cos(\pi/3)$

$\tan(\pi/4) = 1$

$\sin(\pi/6) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$

$= 1/2 + \left(\frac{1}{2}\right)^2 - (1)^2$

$= \frac{1}{2} + \frac{1}{4} - 1$ $\qquad \frac{1}{2} + \frac{1}{4}$

$= \frac{1}{2} - 1 = \frac{-1}{2}$

Q Prove that $2\sin^2\left(\frac{\pi}{6}\right) + \text{Cosec}^2\left(\frac{7\pi}{6}\right) \cdot \cos^2\left(\frac{\pi}{3}\right) = 3/2$

a) $\sin\left(\frac{\pi}{6}\right) = \sqrt{3}/2$

$\text{cosec}\left(\frac{7\pi}{6}\right) = \text{cosec}\left(\pi + \frac{\pi}{6}\right)$

$= -\text{cosec}\left(\frac{\pi}{6}\right) = \left(-\frac{2}{\sqrt{3}}\right)$

$\cos\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$2\sin^2\left(\pi/6\right) + \text{cosec}^2\left(\frac{7\pi}{6}\right) \cdot \cos^2\left(\pi/3\right)$

$\Rightarrow 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2\Big) \cdot \left(\frac{2}{\sqrt{3}}\right)^2$

$\cancel{2} \cdot \frac{3}{\cancel{4}2} + \left(-\frac{4}{3}\right) * \frac{4}{3}$

$= \cancel{\frac{17}{63} \cdot \frac{4^2}{3}} = \frac{1}{\cancel{6}} \cdot \frac{4}{3}^2 = 2/3$

$$\sin\left(\frac{\pi}{6}\right) = 1/2$$

$$\operatorname{cosec}\left(\frac{7\pi}{6}\right) = \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) =$$

$$-\operatorname{cosec}\left(\frac{\pi}{6}\right) = -2$$

$$\cos\left(\frac{\pi}{3}\right) = 1/2$$

$$2\sin^2\left(\frac{\pi}{6}\right) + \operatorname{cosec}^2\left(\frac{7\pi}{6}\right) \cdot \cos^2\left(\frac{\pi}{3}\right)$$

$$= 2\left(\frac{1}{2}\right)^2 + (-2)^2 \cdot (1/2)^2$$

$$\cancel{= \cancel{2} \cdot \frac{\cancel{2}\,1}{\cancel{4}2} + 4 \cdot \frac{1}{4}}$$

$$=$$

$$\cancel{\Rightarrow 2 \; \frac{1}{2})}$$

$$\Rightarrow \left(\cancel{2} \cdot \frac{1}{\cancel{4}2} + 4\right) \cdot \frac{1}{4}$$

$$\frac{9}{4} \cdot \frac{1}{4} = \frac{9}{16}$$

$$= \cancel{\;} \frac{1}{2} + 1 = \underline{\underline{3/2}}$$

Q prove that $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)$

$-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$ $\sin\left(\frac{\pi}{4}-y\right)$

$= \sin(x+y)$

a) ~~$\sin(x+y) = \sin x \cos y$~~

$\left(\cos(x+y) = \cos x \cos y - \sin x \sin y\right)$

Here the given expression is in a same form

$\rightarrow \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

So, $x = \left(\frac{\pi}{4}-x\right)$

$y = \left(\frac{\pi}{4}-y\right)$

So, $\cos\left(\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right)$

$= \cos\left(\frac{\pi}{2} - (x+y)\right)$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

~~Here~~ $\cancel{(-x) = (-x-y)}$

Here $x = x + y$

~~So,~~ So, $\cos\left(\frac{\pi}{2} - (x+y)\right)$

$= \sin(x+y)$

Q Prove that $\dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Textbook for

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan x}{1 + \tan x}$$

$$\frac{\tan(\pi/4 + x)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\frac{1 + \tan x}{1 - \tan x}}{\frac{1 - \tan x}{1 + \tan x}}$$

$$\Rightarrow \frac{1 + \tan x}{1 - \tan x} \times \frac{1 + \tan x}{1 - \tan x}$$

$$\Rightarrow \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$ Hence proved

Q $\sin(75^\circ) = ?$

a) $\sin(75)^\circ = \sin(\cancel{60}^{30} + 45)$

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

ook for Class XI

$\Rightarrow \sin(30^\circ)\cos(45^\circ) + \cos(30)\sin(45)$

$= \frac{1}{2}\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}+1}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} = \cancel{\frac{2\sqrt{3}+2}{4\sqrt{3}}}$

$= \frac{1}{2} + \frac{1}{2\sqrt{3}}$

Q. P.T $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$= -\sqrt{2}\sin x$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\Rightarrow$

$\cancel{\cos\left(\frac{3\pi}{4}+x\right) - \cos}$

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*Smooth Writing
*'A' Grade Paper
*Non Transparent

$\Rightarrow \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$= -2\sin\left[\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right] \sin\left[\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right]$

$\Rightarrow -2\sin\left(\frac{3\pi}{4}\right)\sin(x)$

$\Rightarrow -2\cdot\frac{1}{\sqrt{2}}\sin(x)$

$= -\sqrt{2}\sin(x)$

Hence proved //

$\frac{2}{\sqrt{2}} = \sqrt{2}$

$\sin\left(\frac{3\pi}{4}\right)$
$= \sin\left(\pi - \frac{\pi}{4}\right)$
$= \sin\left(\frac{\pi}{4}\right)$
$= \frac{1}{\sqrt{2}}$

Q P.T. $\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}\sin x$

A) $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) =$

$= -2\sin\left[\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right]$

$\sin\left[\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right]$

$\Rightarrow -2\sin\left(\frac{3\pi}{4}\right)\sin(x)$

$\Rightarrow -2 \times \frac{1}{\sqrt{2}}\sin(x)$ $\boxed{\frac{a}{\sqrt{a}} = \sqrt{a}}$

$\Rightarrow -\sqrt{2}\sin x$

$\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = 1/\sqrt{2}$

Hence proved.

Q PT $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ Q

$\sin 5x + \sin 3x = 2\sin\left(\frac{5x+3x}{2}\right)$
$\cos\left(\frac{5x-3x}{2}\right)$

$= 2\sin(4x)\cos(x)$

$\cos 5x + \cos 3x = 2\cos\left(\frac{5x+3x}{2}\right)$
$\cos\left(\frac{5x-3x}{2}\right)$
$= 2\cos(4x)\cos(x)$

$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \frac{\cancel{2}\sin(4x)\cancel{\cos(x)}}{\cancel{2}\cos(4x)\cancel{\cos(x)}}$ Q.

$\Rightarrow \tan(4x)$

Hence proved.

Q P.T $\dfrac{\sin 3x + \sin x}{\cos 3x + \cos x} = \tan 2x$

$\sin x + y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x + y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\dfrac{\sin 3x + \sin x}{\cos 3x + \cos x} = \dfrac{2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}$

$= \tan 2x$ Hence proved

P.T. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

~~$\sin^2 x = (1 - \cos^2 x)$~~

$$\frac{-(\sin 3x - \sin x)}{\cancel{-(\sin^2 x + \cos^2 x)}\ (\cos^2 x - \sin^2 x)}$$

$$= 2\cos\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)$$

$$= 2\cos(2x) \sin(x)$$

$$(\cos^2 x - \sin^2 x) = \cos(2x)$$

$$\Rightarrow \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)} = \frac{\cancel{-}(2\cancel{\cos(2x)} \sin x)}{\cancel{-}(\cancel{\cos 2x})}$$

$$= 2\sin(x)$$ Hence proved

Q) P.T $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

a) $\cos(2x) = 1 - 2\sin^2 x$

$$\cos\left(2(2x)\right) = 1 - 2\sin^2(2x)$$

$$\sin(2x) = 2\sin x \cos x$$

$$\sin^2 2x = 4\sin^2 x \cos^2 x$$

$$\Rightarrow 1 - 2\left(4\sin^2 x \cos^2 x\right)$$

$$\Rightarrow 1 - 8\sin^2 x \cos^2 x$$

Hence proved.

Q) P.T $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

$$\Rightarrow \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{\sin 4x}{\cos 4x}\frac{\cancel{\cos x}}{\cancel{\cos x}} = \tan 4x$$

Hence proved //

Q) PT $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

$$\Rightarrow \frac{2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)}$$

$$= \frac{2\sin(2x)\cos x}{2\cos(2x\ \cancel{\cos x}} = \tan x$$

Hence proved

Q) PT. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

$\Rightarrow$ $\left[\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]$

$\left[\cos^2 x - \sin^2 x = \cos 2x\right]$

$$\frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} = \frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x}$$

$$= \frac{2\cancel{\cos 2x}\ \sin x}{\cancel{\cos 2x}} = 2\sin x \text{ RHS}$$

Hence proved

Q) PT $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

$$\left[\begin{array}{l} \cos 2x = 1 - 2\sin^2 x \\ \sin 2x = 2\sin x\cos x \end{array}\right]$$

$\to$ $\cos 4x = \cos(2x \cdot 2x)$

$\Rightarrow 1 - 2\sin^2 2x$

$[\sin^2 2x = \sin 2x \sin 2x)$

$= 1 - 2(2\sin x\cos x)^2$

$\Rightarrow 1 - 2(4\sin^2 x\cos^2 x)$

$\Rightarrow 1 - 8\sin^2 x\cos^2 x$ RHS

Hence proved

Q PT $\dfrac{\sin 2x + 2\sin 4x + \sin 6x}{\sin 4x} = 4\sin^2 x$

$[\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$(1 + \cos 2x) = 2\cos^2 x]$

LHS. $\sin 2x + 2\sin 4x + \sin 6x$

$= \sin 6x + \sin 2x + 2\sin 4x$

$\Rightarrow 2\sin\left(\frac{6x+2x}{2}\right)\cos\left(\frac{6x-2x}{2}\right) + 2\sin 4x$

$\Rightarrow 2\sin 4x \cos 2x + 2\sin 4x$

$\Rightarrow 2\sin 4x\,[\cos 2x + 1]$

$\Downarrow$

$= 2\sin 4x \cdot 2\cos^2 x$

$= \cancel{4\sin\cos}\ 4\cos^2 x \sin 4x$ RHS

Hence proved

P.T. $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

$= \dfrac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$

$\Rightarrow \dfrac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$

$$\frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x}$$

$$= \frac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x+1)} = \frac{\cos 3x}{\sin 3x}$$

$$= \cot 3x$$ RHS Hence proved //

Theory (Trignometric equations

An eqn involving trignometric ratio of an unknown is called a trignometric eqn. A value of the ~~unkwon~~ unknown satisfying the eqn is called a solution of the eqn. The solutions b/w 0 and 2π are known as the principal solutions. Any solution generalised by means of periodicity is called a general solution.

① $\sin x = 0$ if and only if $x = n\pi$, $n \in$

② $\cos x = 0$ iff $x = (2n+1)\frac{\pi}{2}$, $n \in Z$

③ $\tan x = 0$ iff $x = n\pi$, $n \in Z$

④ The general solution of the eqn $\sin x = \sin\alpha$ (α is fixed) given by
$x = n\pi + (-1)^n\alpha$ $n \in Z$

⑤ The general solution of the eqn $\cos x = \cos\alpha$ (α is fixed) given by
$x = 2n\pi \pm \alpha$ $n \in Z$

⑥ The general solution of the eqn $\tan x = \tan\alpha$ (α is fixed) is given by
$x = n\pi + \alpha$, $n \in Z$

⑦ The general solution of $\sin^2 x = \sin^2\alpha$
or $\cos^2 x = \cos^2\alpha$
or $\tan^2 x = \tan^2\alpha$ is given by
$x = n\pi \pm \alpha$, $n \in Z$

Q) $\sin x + \sin 3x + \sin 5x = 0$
solve $\cos x$

Sol $\underbrace{\sin 5x + \sin x}_{\downarrow} + \sin 3x = 0$

$\Rightarrow 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x\,[2\cos 2x + 1] = 0$

1PP

$\sin 3x = 0$ or $2\cos 2x + 1 = 0$

$\sin 3x = 0$

$3x = n\pi$

$x = \frac{n\pi}{3}$

$2\cos 2x = -1$

$\Rightarrow \cos 2x = -\frac{1}{2}$

$\Rightarrow \cos 2x = \cos\left(\pi - \frac{\pi}{3}\right)$

$\cos 2x = \cos\frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{3}$ or $x = n\pi \pm \frac{\pi}{3}$,

$n \in Z$

Q) $\cos 3x + \cos x - \cos 2x = 0$

Solve $\cos x$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x$

$\Rightarrow 2\cos 2x \cos x - \cos 2x$

$\cos 2x\,[2\cos x - 1] = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\cos 2x = 0$	$2\cos x = 1$
$2x = (2n+1)\frac{\pi}{2}$	$\cos x = 1/2$
$x = \frac{(2n+1)\pi}{4}$	$\cos x = \cos\left(\frac{\pi}{3}\right)$
	$x = 2n\pi \pm \frac{\pi}{3}$ $n \in$

So, $x = \frac{(2n+1)\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$ $n \in$

Q) $\operatorname{cosec} x = -2$ Find general and principal solution

$\operatorname{cosec} x = -2 \Rightarrow \sin x = \frac{-1}{2}$

$\Rightarrow \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\frac{7\pi}{6}\right)$

The general solution is given by

$x = n\pi \pm (-1)^n \frac{7\pi}{6}$ $n \in Z$,

The principal solutions are obtained by putting $n=0$. and $n=3$

They are $\frac{7\pi}{6}$ and $3\pi - \frac{7\pi}{6} = \frac{11\pi}{6}$

Q Find the general solution for each of the following eqn -

1) $\cos 4x = \cos 2x$

$\cos 4x = \cos 2x \Rightarrow \cos 4x - \cos 2x = 0$

$\Rightarrow 2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$

$\Rightarrow \underbrace{2\sin 3x}\ \underbrace{\sin x} = 0$

$2\sin 3x = 0 \Rightarrow \sin 3x = 0$

$3x = n\pi$

$x = \frac{n\pi}{3}$

$\sin x = 0$

$x = n\pi$ $n \in Z$

or x $n\pi$, $n \in Z$

1) Find the general solution of eqn ①

$\sin x = \frac{1}{\sqrt{2}}$

ans) This eqn may be written as

$\sin x = \sin \pi/4$

General Solution of the eqn is

$x = n\pi + (-1)^n \frac{\pi}{4}, n \in Z$

2) ~~Cos~~

$\cos x = \frac{1}{2}$ find the general solution

ans) eqn may be written as

$\cos x = \cos \pi/3$

∴ General solution of the eqn

$x = 2n\pi \pm \frac{\pi}{3} \quad n \in Z$

Q) Find the principal and general solution of the following eqns?

$\tan x = \sqrt{3}$

$\tan x = \sqrt{3} \Rightarrow \tan x = \tan \frac{\pi}{3}$

The general is given by

$$x = n\pi + \frac{\pi}{3} \quad n \in Z$$

These solution lying in the interval $[0, 2\pi]$ are called principal solutions. They are obtained by putting

$n = 0$ and $n = 1$

$\therefore$ The principal solutions are $\frac{\pi}{3}$

and $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$