Chapter - 2

RELATIONS AND FUNCTIONS

Ordered pair

~~(a, b)~~ → in form (a, b)

Cartesian product of sets

If A and B are two sets then their cartesian product is defined as

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

eg:- Let $P = \{1, 2, 3\}$ $Q = \{a, b\}$

Then $P \times Q = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$Q \times P = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

$A \times B$ need not be equal to $B \times A$

Note → Two ordered pairs (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$.

→ If $n(A) = m$ and $n(B) = n$ then $n(A \times B) = mn$

→ If A is non empty and B is infinite then $A \times B$ is infinite

→ $A \times A \times A = \{(a, b, c)\ a, b, c \in A\}$

(a, b, c) is called an ordered triplet.

Q $P = \{m, n\}$ and $Q = \{n, m\}$ then find

$P \times Q = \{(m, n), (m, m), (n, m), (n, n)\}$

Q $A = \{1, 2\}$ and $B = \{3, 4\}$ find $A \times B$

ans) $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

$A \times B$ can be represented graphically

Take the elements of B along a vertical line, Then represent each pair of elements of A

$4b^2 = 7$

$b^2 = 7/4$ $\quad \frac{\sqrt{7}}{2}$

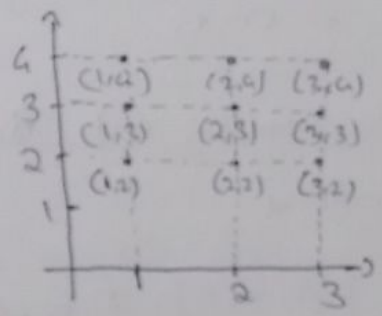
$b =$

$\frac{2P-1}{2(4P}$

and B by dot. The diagram so obtained gives the graphical representation:

eg: $A = \{1, 2, 3\}$ and $B\{2, 3, 4\}$ Then the graphical representation of $A \times B$

as



Q If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of x and y?

as $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$

$\Rightarrow \frac{x+1}{3} = \frac{5}{3}$ and $y - \frac{2}{3} = \frac{1}{3}$

$\frac{x+3}{\cancel{3}} = \frac{5}{\cancel{3}}$ | $\frac{3y-2}{\cancel{3}} = \frac{1}{\cancel{3}}$

$x = 2$ // | $y = 1$ //

Q. If $(x+1, y-2) = (3, 1)$ write value of x and y?

A) $x+1 = 3$ and $y-2=1$

$x = 2$ $y = 3$

Q. Let $P = \{1, 2\}$ find $P \times P \times P$

$P \times P \times P = (P \times P) \times P$

$= \{(1,1)\ (1,2), (2,2), (2,1)\} \times (1,2)$

$= \{(1,1,1), \cancel{(1,2,2)}, (1,1,2), (1,2,1), (1,2,2)$

$(2,2,1)\ (2,2,2), (2,1,1)\ (2,1,2)\}$

Q Let $A = \{-1, 1\}$ find $A \times A \times A$

A) $(A \times A) \times A = \{(1,1), (-1,1)\ (-1,-1)\ (1,-1)\} \times (-1,1)$

$= \{(1,1,-1)\ (1,1,1)\ (-1,1,-1)\ (-1,1,1), (-1,-1,-1),$

$(-1,-1,1), (1,-1,-1)\ (1,-1,1)\}$

$4b^2 = 7$

$b^2 = 7/4$ $\frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

Q Let $A = \{7, 8\}$, $B = \{5, 4, 2\}$ find $A \times B$ and $B \times A$

a) $A \times B = \underset{A}{\{7, 8\}} \times \underset{B}{\{5, 4, 2\}}$

$= \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

$B \times A = \{5, 4, 2\} \times \{7, 8\}$

$= \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

Q Let $A = \{1, 2\}$ & $B = \{3, 4\}$ write $A \times B$. How many subsets will $A \times B$ have? List them

a) $A \times B = \{1, 2\} \times \{3, 4\}$

$= \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

No: of subsets $= 2^4 = 16$ //

Subsets $= \{ \underset{1}{\phi}, \underset{2}{\{(1, 3)\}}, \underset{3}{\{(1, 4)\}}, \underset{4}{\{(2, 3)\}}, \underset{5}{\{(2, 4)\}},$
$\underset{6}{\{(1, 3), (1, 4)\}}, \cancel{\{(1, 3), (1, 4)\}}, \underset{7}{\{(1, 3), (2, 3)\}}$

$\{(1,3),(2,4)\}, \{(1,4),(2,3)\}, \{(1,4),(2,4)\},$
8 9
$\{(2,3),(2,4)\}, \{(1,3),(1,3)\}, \{(1,4),(1,3),(2,3)\}$ ~~$\{(1,4),(1,4)\}$~~
11 12 13
~~$\{(2,3),(2,3)\}, \{(2,4),(2,4)\}$~~, $\{(1,3),(1,4),$ $(2,3),(2,4)\}$
15 16
$\{(1,4),(2,3),(2,4)\}, \{(1,3),(2,3),(2,4)\}$

Q Let $A = \{1,2,3\}$, $B = \{3,4\}$ and $C = \{4,5,6\}$ then find.

i) $A \times (B \cap C)$

$(B \cap C) = \{3,4\} \cap \{4,5,6\} = \{4\}$

$A \times (B \cap C) = \{1,2,3\} \times \{4\}$

$= \{(1,4), (2,4), (3,4)\}$

ii) $(A \times B) \cap (A \times C)$

$A \times B = \{1,2,3\} \times \{3,4\}$

$= \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

$(A \times C) = \{1,2,3\} \times \{4,5,6\}$

$= \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6)$

$(3,4), (3,5), (3,6)\}$

$4b^2 = 7$

$b^2 = 7/4$ $\frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

$(A \times B) \cap (A \times C) = \{(1,4), (2,4), (3,4)\}$

Alternate way

$B \cap C = 4$ //

$A \cap B = 3$ //

$A \cap C = \emptyset$

1st element of ordered pair will be ~~set~~.
element of set A (ie, 1, 2, 3).
The common element in B and C is 4.

$\therefore$ common ordered pairs $= (1,4) (2,4), (3,4)$

iii) $A \times (B \cup C)$

$B \cup C = \{\cancel{1}2, 3, 4, 5, 6\}$ $A = \{1, 2, 3\}$

$A \times (B \cup C) = \{\cancel{(1,1)}, \cancel{(1,2)}, (1,3), (1,4), (1,5) (1,6)$
$\cancel{(2,1)}, \cancel{(2,2)} (2,3), (2,4) (2,5), (2,6)$
$\cancel{(3,1)}, \cancel{(3,2)}, (3,3), (3,4), (3,5) (3,6)\}$

iv) $(A \times B) \cup (A \times C)$

$(A \times B) = \{1,2,3\} \times \{3,4\}$

$= \{(1,3)\ (1,4)\ (2,3)\ (2,4)\ (3,3)(3,4)\}$

$(A \times C) = \{1,2,3\} \times \{4,5,6\}$

$= \{(1,4)\ (1,5), (1,6). (2,4)(2,5)(2,6)$

$(3,4), (3,5), (3,6)\}$

$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6)$

$(2,3), (2,4), (2,5)\ (2,6), (3,4)\ (3,3),$

$(3,5), (3,6)\}$ Total 12 elements

$(6+9-3 = 12)$

Relation

Let A and B be two non-empty sets. Then a relation from A to B is a subset of $A \times B$ and is denoted by R.

$4b^2 = 7$

$b^2 = 7/4$

$b = \frac{\sqrt{7}}{2}$

$\frac{2P-1}{2(4P}$

Eg:-

Let $P = \{a, b, c\}$ $Q = \{$ Anu, Bini, Binoy, Chinnu, Divya$\}$

$P \times Q = \{$(a, Anu), (a, Bini), (a, Binoy), (a, Chinnu), (a, Divya), (b, Anu), (b, Bini), (b, Binoy), (b, Chinnu), (b, Divya), (c, Anu), (c, Bini), (c, Binoy), (c, Chinnu), (c, Divya)$\}$

$R = \{(x, y)$; x is the first letter of the name y, $x \in P, y \in Q\}$

$\therefore R = \{$(a, Anu), (B, Binu), (b Binoy), (c, Chinnu)$\}$

Eg 2:- $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ Then

$R_1 = \{(1,3), (2,4), (3,5)\}$

$R_2 = \{(2,3), (2,5), (3,4), (1,5)\}$ etc.

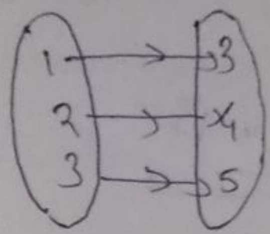
are relation from A to B because all of these are subset of $A \times B$.

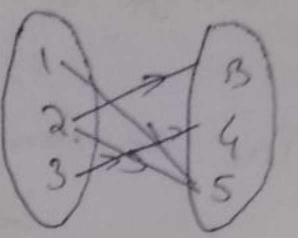
Arrow diagram

Arrow matic representation of relation.

Consider the above problem

$A \xrightarrow{R_1} B$ $\qquad\qquad$ $A \xrightarrow{R_2} B$





* If $(a,b) \in R$ we say 'a' is related to b. and we write in symbol aRb

Here b is called the image of a

* $\text{Domain}(R) = \{a ; a \in A \ (a,b) \in R\}$

$\text{Range}(R) = \{b ; b \in B \ (a,b) \in R\}$

* Here B may be named as Co-domain of R
* Range is always a subset of the Codomain

Eg:- Consider the $R = \{(1,1), (1,3) (2,1), (1,4), (2,2), (3,3)\}$

From $A = \{1,2,3\}$

$B = \{1,2,3\}$

$4b^2 = 7$

$b^2 = 7/4$

$b = \frac{\sqrt{7}}{2}$

$\frac{2P-1}{2(4P}$

Domain of $R = \{1, 2, 3\} = A$

The range of $R = \{1, 2, 3, 4\} = B$

The co-domain of $R = B$

Note: [If $n(A) = m$, $n(B) = n$, then the number of possible relations from A to B $= 2^{mn}$]

Q If R is a relation from a non-empty set itself ~~A to B~~, the we say R is a relation on A

eg:- if $A = \{1, 3, 5\}$ then $R = \{(1,1) (1,3), (3,5)$
$(3,3), (5,1)\}$

is a relation on A because $R \subset A \times A$

Q $A = \{1, 2, 3, \ldots 14\}$ A relation R from A to A
$R = \{(x, y) : 3x - y = 0$, where $x, y \in A\}$
write down it's domain and Range.

a $3x - y = 0 \Rightarrow 3x = y$

$\therefore R = \{(1,3), (2,6), (3,9), (4,12)\}$

Domain $= \{1, 2, 3, 4\}$

Range $= \{3, 6, 9, 12\}$

Co-domain = set A

3. Given that relations

i) $R_1 = \{(x, y) : x, y \in N \text{ and } x+y=6\}$

ii) $R_2 = \{(x, y) : x, y \in N \text{ and } x^2+y^2 \le 10\}$

Find the domain and Range of R_1 and R_2.

A) i) $R_1 = \{(x, y) ; x, y \in N \text{ and } x+y=6\}$

$= \{(1,5), (2,4)\ (3,3)\ (5,1)\ (4,2)\}$

$Dom\,(R_1) = \{1, 2, 3, 4, 5\}$

$Range\,(R_1) = \{1, 2, 3, 4, 5\}$

ii) $R_2 = \{(x, y) ; x, y \in N \text{ and } x^2+y^2 \le 10\}$

$R = \{(1,1), (1,2), (2,1), (1,3), (3,1)\ (2,2)\}$

$Dom\,(R_2) = \{1, 2, 3\}$

$Range\,(R_2) = \{1, 2, 3\}$

$4b^2 = 7$

$b^2 = 7/4 \quad \frac{\sqrt{7}}{2}$

$\frac{2P-1}{2(4P}$

Q If $A = \{a, b\}$ writ all relation on A?

ans) ϕ, $\{(a,a), (a,b), (b,a), (b,b)\}$

$\{(a,a)\}, \{(a,b)\}, \{(b,a)\}, \{(b,b)\}$

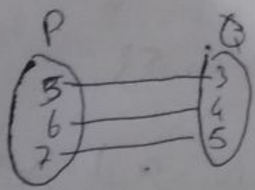
$\{(a,a)(a,b)\}$, $\{(a,a)(b,a)\}$, $\{(a,a), b,b\}$.

$\{(a,b), (b,a), (a,a)\}$: $\{(a,b), (b,a), (b,b)\}$

$\{(a,a), (a,b), (b,b)\}$, $\{(a,a), (b,a), (b,b)\}$

Q ~~In~~ a figure, shows a relationship b/w the sets P and Q. write this relation

i) Set builder form. ii) roster form



a) i) Set builder form = $\{(x,y) : y = x-2, x = \{5, 6, 7\}\}$

ii) Roster form - $\{(5,3), (6,4), (7,5)\}$.

Domain $= \{5, 6, 7\}$
Range $= \{3, 4, 5\}$

Q Find the domain and range of the relation

$R = \{(x,y) : y = x^3, x \text{ is a +ve prime no.}\}$

A) $x = \{2, 3, 5, 7\}$

$\therefore R = \{(2,8), (3,27), (5,125), (7,343)\}$

Domain $= \{2, 3, 5, 7\}$

Range $= \{8, 27, 125, 343\}$

Q Find the no: of relations which can be defined from $P = \{1, 2, 3\}$ to $Q = \{x, y\}$

A) $n(P) = 3$, $n(Q) = 2$

No: of relation $= 2^{3\times 2} = 2^6 = 64$ //

Q Determine the domain, and range relation R defined by

$R = \{(x, x+5) ; x \in \{0, 1, 2, 3, 4, 5\}\}$

A) Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{5, 6, 7, 8, 9, 10\}$ //

$4b^2 = 7$

$b^2 = 7/4$ $\quad \frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

Q) Let R be the relation on Z defined by

R. $R = \{(a,b) ; a, b \in Z,\ a-b \text{ is an integer}\}$

find the domain and Range of R.

a) Domain = Range = $\mathbb{Z}$

Q) Let $A = \{1,2\}$, $B = \{3,4\}$

① write $A \times B$

a) $A = \{1,2\}$ $B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

② write relation from A to B in roaster form.

Ans) Subsets of $\{(1,3), (1,4), (2,3), (2,4)\}$

$= \underset{1}{\phi}, \underset{2}{\{(1,3)\}}, \underset{3}{\{(1,4)\}}, \underset{4}{\{(2,3)\}}, \underset{5}{\{(2,4)\}}$

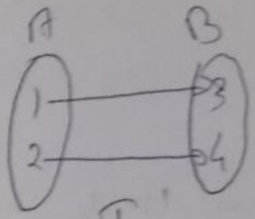
$\underset{6}{\{(1,3),(1,4)\}}, \underset{7}{\{(1,3),(2,3)\}}\ \underset{8}{\{(1,3),(2,4)\}}\ \underset{9}{\{(1,4),(2,3)\}}$

$\underset{10}{\{(1,4),(2,4)\}}, \underset{11}{\{(2,3),(2,4)\}}\ \underset{12}{\{(1,3),(1,4),(2,3)\}}$

$\underset{13}{\{(1,3),(1,4),(2,4)\}}\ \underset{14}{\{(1,4),(2,3),(2,4)\}}$

$\{(1,3), (2,3)\ (2,4)\}, \{(1,3),(1,4),(2,3),(2,4)\}$
15 16

(3) Represent all possible functions from A to B. (Arrow diagram may be used)



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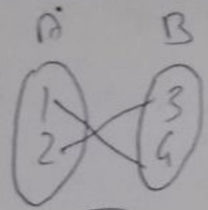
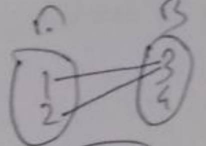


II



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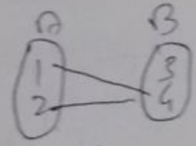


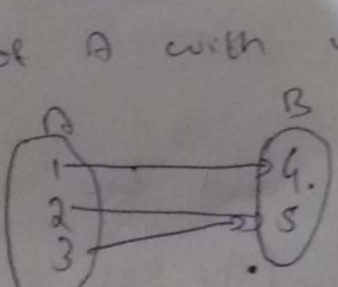
IV

Total 4 functions //

Function?

-> Special type of relation.

Let A and B which associates every element of A with unique element of B.



$4b^2 = 7$
$b^2 = 7/4$
$b = \frac{\sqrt{7}}{2}$

$\frac{2P-1}{2(4P}$

Here A is called the domain of f and
b is called the co-domain of 'f'

→ If 'f' associates the ~~ele~~ element $a \in A$ with the element $b \in B$ then b is called the image of a under f and we write in symbol $f(a) = b$.

→ In this case, we also say that 'a' is the pre-image of b and write -

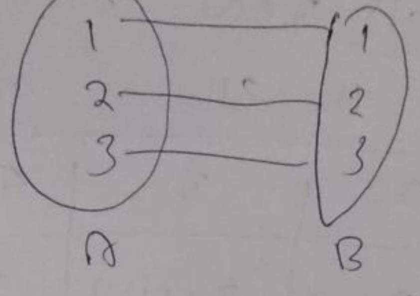
$a = f^{-1}(b)$

→ If ~~A~~ 'f' is a function from A to B, we write ~~A~~ $f : A \to B$

Note [If the range of a function is a subset of R, then 'f' is called a real valued. function]

Real function

If the domain and the range of a function lie on R, (The set of all real numbers) then the function is called a real function.



Eg:- Let N be the set of all natural No:s and R be a relation on N defined by $R = \{(x,y) : y = 2x,\ x, y \in N\}$

In Roster form $R = \{(1,2),(2,4),(3,6)\ldots\}$

Domain of $R = N$

Range of $R =$ Set of all even Natural No:s.

Co-domain $= N$

Representation of a function

A real function can be represented by a formulae where the formula gives the rule under the elements of domain and co-domain are associated.

$4b^2 = 7$
$b^2 = 7/4$ $\sqrt{7}$

$\frac{2P-1}{2(4P}$

eg:. Let $f: N \to N$ be a function defined by
$f(x) = 2x + 1$ for all $x \in N$

This function gives the following association

x	1	2	3	4	5...
$f(x)$	$2\times1+1=3$	$2\times2+1=5$	$2\times3+1=7$	9	11...

The range of this function $= \{3, 5, 11, 9, 11 \ldots\}$

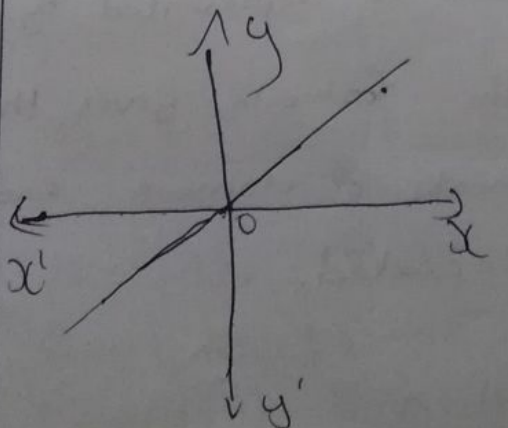
Graph of a function

A real function can be represented by its graph. To draw the graph of a function $f(x)$ we proceed as

Take the values of x along the x axes and the values of $f(x)$ along the y axis

Some standard functions and their graphs

Identity function - A function $I: R \to R$ is said to be Identity function. If $I(x) = x \ \forall \ x \in R$



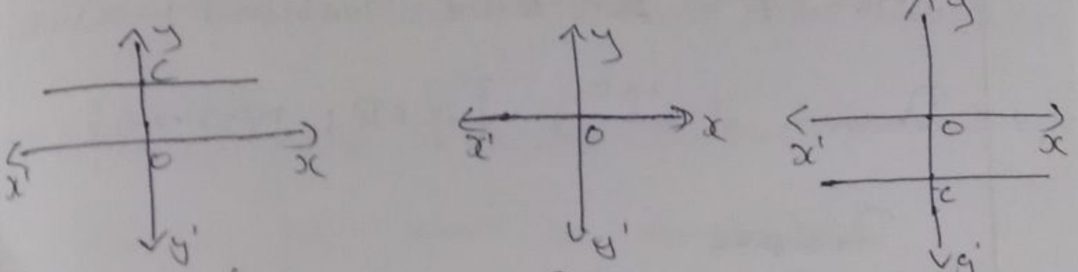
Domain$(I) = R$

Range of $I = R$.

Constant function

A function $f: R \to R$ is said to be a constant function if for every real value of x, $f(x) = c$, where 'c' is a fixed number.

Its graph is a straight line which is parallel to x axis



$f(x) = c \; (c>0)$ $\quad$ $c=0$ $\quad$ $c<0$

Polynomial function

A function of the form $f(x) = a_0x^n + a_1x^{n-1} + \dots a_n$ where $a_0, a_1, a_2, \dots a_n$ are real constants with $a_0 = 0$ and $n \in N$ is called a polynomial function of degree 'n'.

eg:- $f(x) = 3x^3 - 2x^2 + x + 1$ is a polynomial fun. of degree 3.

$P(x) = x^4 + 3$ is a degree 4 polynomial fun.

In particular a polynomial function of degree 1 is called linear function,

degree 2 - Quadratic function

degree 3 - Cubic function.

Rational function

A function of the form

$f(x) = \frac{g(x)}{h(x)}$ where both $g(x)$ and $h(x)$ are polynomial functions with $h(x) \neq 0$. is called Rational function.

Domain of 'f' = $\{x \in R : h(x) \neq 0\}$

~~Recipro~~

Eg:- $f(x) = \frac{x^2 - 3x + 4}{x+5}$ $x \neq (-5)$

$D(f) = \cancel{\{R\} - \{-5}$ $R - \{-5\}$

Reciprocal function

A function of the form $f(x) = 1/x$ where $x \neq 0$ is called the Reciprocal function.

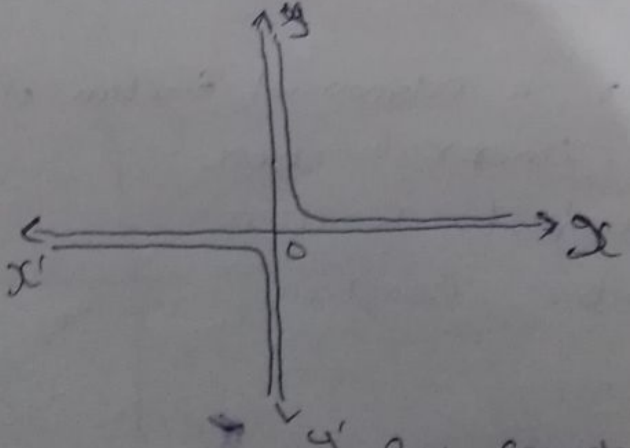


fig: Graph

Since $f(x) = 1/x$ is defined for all $(\forall)$ $x \neq 0$

Domain of 'f' $= R - \{0\}$

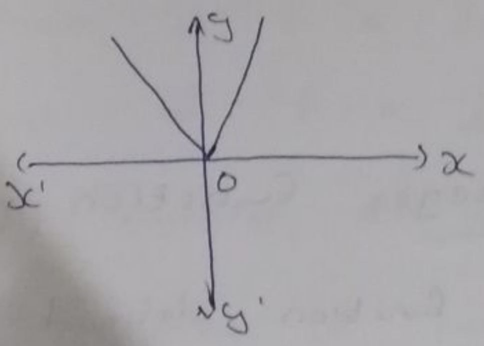
Range of 'f' $= R - \{0\}$

Modulus / Absolute Value function

~~Modulus / Ab~~ It is the function defined by.

$f(x) = |x|$ which is equal to -

$$|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$



Domain of 'f' $= R$

Range of 'f' $=$ All non -ve Real values

$R \geq 0$

Signum function

It is a function defined by $f x = \left\{ \frac{|x|}{x}, \; x \neq 0 \right.$

$$f(x) = \begin{cases} 0, & \text{when } x = 0 \\ 1, & \text{when } x > 0 \\ -1, & \text{when } x < 0 \end{cases}$$

$4b^2 = 7$

$b^2 = 7/4$ $\quad \frac{\sqrt{7}}{2}$

$\frac{2P-1}{2(4P}$

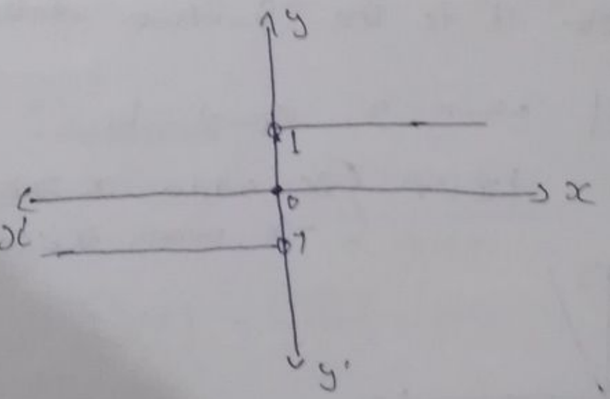
Domain of 'f' = R

Range = {1, 0, -1}

→ If $x > 0$ $f(x) = \frac{x}{x} = 1$

→ If $x < 0$ $f(x) = \frac{-x}{x} = -1$

→ If $x = 0$ $f(x) = 0$



Greatest Integer function (G.I.F)

which is the function defined by

$f(x) = [x] \ \forall \ x \in R$, where

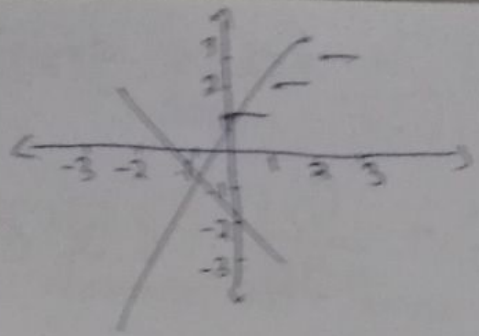
[ ]. denotes the Greater Integer less than or equal to x

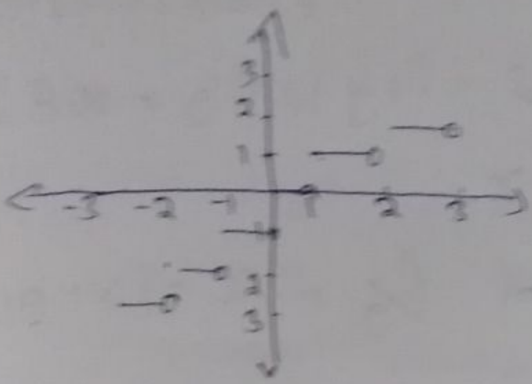
Eg:- $[2] = 2.$

$[0] = 0$

$[1.01] = 1$

$[-2.1] = -3$





Domain of 'f' = R

Range of 'f' = Z

This function also called step function

Algebra of real functions

If 'f' and 'g' are two real functions with the same domain x and α is a scalar we ~~dine~~ defined:-

$$(f+g)(x) = f(x) + g(x) \ \forall \ x \in R$$

$$(f-g)(x) = f(x) - g(x) \ \forall \ x \in R$$

$$(\alpha f)(x) = \alpha f(x) \ \forall \ x \in R$$

$4b^2 = 7$

$b^2 = 7/4$ $\frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

$(fg)(x) = f(x) * g(x) \ \forall x \in R$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \ \forall x \in R$, provided $g(x) \neq 0$.

The domain of $f+g$, $f \cdot g$, αf, and fg are all real x.

The domain of $f/g = x - \{x : g(x) = 0\}$

Eg:-

Consider the real functions defined by $f(x) = x^2$ and $g(x) = 3x + 1 \ \forall x \in R$

Then $(f+g)(x) = f(x) + g(x)$

$= x^2 + 3x + 1, \ \forall x \in R$

$(f-g)(x) = f(x) - g(x) = x^2 - (3x+1)$

$= x^2 - 3x - 1$

$(\alpha f)(x) = \alpha . f(x) = \alpha x^2$.

α is a scalar

$(fg)(x) = f(x) g(x) = x^2(3x+1)$

$= 3x^3 + x^2$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{3x+1} \quad \forall\, x \in R - \left\{\frac{-1}{3}\right\}$$

Q. Find the domain and Range of the following real functions.

i) $f(x) = -|x|$

A) $f(x) = -|x|$ is real for any real value of x.

$\therefore$ The domain of 'f' = R

Since $|x|$ Takes every non negative real value

$-|x|$ takes every non +ve real value

Thus the Range of 'f' = The set of all non +ve numbers.

ii) $f(x) = \sqrt{9-x^2}$

A) $f(x)$ is real iff $9 - x^2 \geq 0$

ie, iff $x^2 \leq 9$.

$\Rightarrow -3 \leq x \leq 3$

Thus, the domain of 'f' = $[-3, 3]$ //

$4b^2 = 7$

$b^2 = 7/4$ $\quad \frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

write $y = \sqrt{9-x^2}$

$\Rightarrow y^2 = 9 - x^2$ (squaring on both sides)

$\Rightarrow 9 - x^2 = y^2$

$\Rightarrow x^2 = 9 - y^2$

$x = \sqrt{9-y^2}$

x is real iff $9 - y^2 \geq 0$

$\Rightarrow 9 \geq y^2$

$\Rightarrow -3 \leq y \leq 3$

But y is non-ve [$\because \sqrt{9-x^2}$ is regarded as a +ve square root

Thus Range of 'f' = $[0, 3]$

Q Consider real valued fun $f(x) = \dfrac{x-3}{x^2-x-6}$

a) find the domain of $f(x)$

$$f(x) = \frac{x-3}{x^2-x-6}$$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \Rightarrow x = 3, -2$

$\therefore$ Domain = $\mathbb{R} - \{3, -2\}$

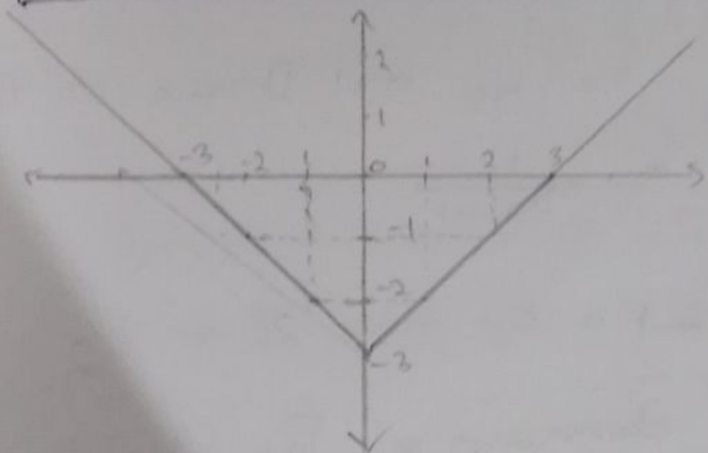
Consider the function $f(x) = |x| - 3$.

a) Draw the graph of $f(x)$

b) write the domain and Range of $f(x)$

a) $f(x) = |x| - 3$

x	0	1	2	3	-1	-2 ...
$y = f(x)$	-3	-2	-1	0	-2	-1 ...



b) Domain = $\mathbb{R}$

Range = $[-3, 2]$

Q The Domain of the function $f(x) \frac{1}{x-1}$ is

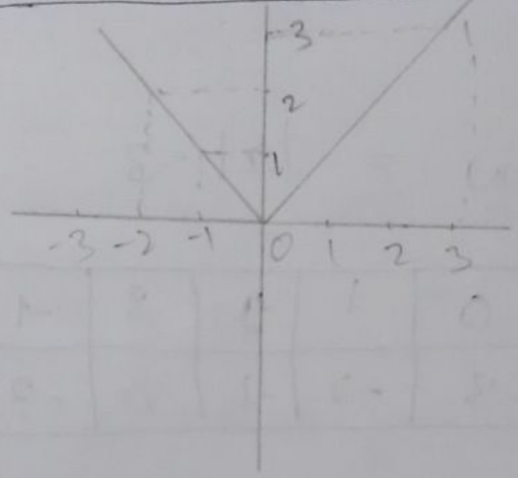
$f(x) = \frac{1}{x-1}$ | $x - 1 = 0$
$x = 1$
Domain = $\mathbb{R} - \{1\}$

$4b^2 = 7$
$b^2 = 7/4$ $\quad \frac{\sqrt{7}}{2}$
$b =$

$\frac{2P-1}{2(4P}$

Q Draw the graph of the function $f(x) = |x|$, $x \in R$

x	-3	-2	-1	0	1	2	3
$f(x)$	3	2	1	0	1	2	3



Q Find the range and Domain of the function

i) $f(x) = \frac{1}{2x-1}$

a) $2x - 1 = 0 \Rightarrow x = 1/2$

So, domain $= R - \{1/2\}$

$y = \frac{1}{2x-1}$

$y(2x-1) = 1 \Rightarrow 2xy - y = 1$

$\Rightarrow 2x = \frac{1+y}{y}$

$x = \frac{1+y}{2y}$ So, Range $= \underline{\underline{R - \{0\}}}$

$2y = 0$; $y = 0$

Q The Total no. of relations which can be defined from $P = \{1, 2, 3\}$ to $Q = \{x, y\}$

No of relation $= 2^{mn}$

So, $= 2^{3 \times 2} = 2^6 = 64$ //

Q Determine the domain and Range of the relation R defined by

$$R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$$

Ans In Roster form $R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{5, 6, 7, 8, 9, 10\}$

Q Let $A = \{1, 2\}$, $B = \{3, 4\}$

(1) write $A \times B$

(2) Write relation $A \times B$ in roster form

(3) Represent all possible functions from A to B (Arrow diagram may be used)

$4b^2 = 7$

$b^2 = 7/4$ $\frac{\sqrt{7}}{2}$

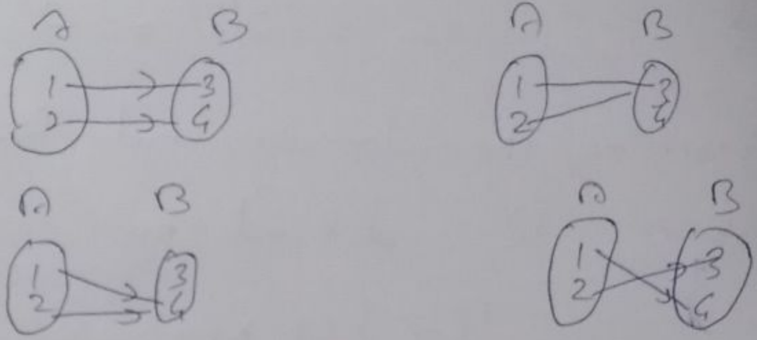
$b =$

$\frac{2P-1}{2(4P}$

ans i) $A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

2) Any subset of $A \times B$, $R = \{(1,3), (2,4)\}$

3)



Q Let R be the relation on Z defined by $R = \{(a,b) : a, b \in Z \; a-b \text{ is an integer}\}$ find the domain and Range

Sol. : For any two integers, a, b. a-b is an integer

$\therefore$ Domain of 'f' = Z

Range of 'f' = Z //

~~Domain of the function~~ $\cancel{f(x) = \frac{1}{x-1}}$

Q $g(x) = 6x^2 + 3x - 2$. The value of $g(-1)$ is

$\Rightarrow$ ~~$g(-1)$ — $g(x)$~~

i) $g(-1) = 6(-1)^2 + 3\times(-1) - 2$

$= 6 - 3 - 2 = 1$ //

ii)

Q $f(x) = x+1 \quad g(x) = 2x-1$

Then the value of $\frac{f(x)}{g(x)}$ is

ans) $\frac{f(x)}{g(x)} = \frac{x+1}{2x-1} \quad x \neq \frac{1}{2}$

Q $f(x) = x^2$ the value of $\frac{f(2) - f(1)}{2-1}$

$\Rightarrow$ $f(2) - f(1) = 4 - 1 = 3$

$\frac{f(2) - f(1)}{2-1} = \frac{3}{1} = 3$ //

Q If 'f' is a Signum function then $f(100)$

ans $f(x) = \frac{|x|}{x}$

$f(100) = \frac{|100|}{100} = 1$ //

$4b^2 = 7$

$b^2 = 7/4 \quad \frac{\sqrt{7}}{2}$

$b =$

$\frac{2P-1}{2(4P}$

Q Let $g(x) = 2-3x$, $x \in R$ $x>0$ and a

$h(x) = x^2-3x+2$, $x \in R$.

i) find the range of $g(x)$

ii) find the domain of $\frac{g(x)}{h(x)}$

a) $g(x) = 2-3x = y$

$\Rightarrow y+3x = 2 \Rightarrow x = \frac{2-y}{3}$ $(x>0)$

So, $2-y>0 \Rightarrow 2>y$

$y \in (-\infty, 2)$

ii

$$\frac{g(x)}{h(x)} = \frac{2-3x}{x^2-3x+2}$$

$x^2-3x+2=0$

$(x-2)(x-1)=0 \mid x = 2, 1$

Domain $\left(\frac{g(x)}{h(x)}\right) = R - \{2,1\}$

a $f(x) = 2x - 5$ write down the values of 32

$f(0)$, $f(7)$, $f(-3)$

$f(0) = -5$ // $\left(2(0) - 5\right) = -5$ //

$f(7) = 2(7) - 5 = 14 - 5 = 9$ //

$f(-3) = -6 - 5 = -11$ //