chapter 1

SETS

SETS :-

A set is a well defined collection of distinct objects

sets are denoted by capital Letters A, B, C..etc

Elements

The members of a set are called its elements

if 'x' is an element of a set A, we denote it as $x \in A$ [read as x belongs to A] [$\in$ - Epsilon]

if 'x' is not an element of A, we denote. $x \notin A$ [read as x doesnot belongs to A]

Commonly used sets in Mathematics

N: The set of all natural numbers

Z: The set of all Integers

Q: The set of all rational numbers

R: The set of all real numbers

Z^+ : The set of all +ve integers

Q^+ : The set of all +ve rationals

R^+ : Set of all +ve real numbers.

Representation of sets

A set can be represented by two forms namely

1) Roster form / tabular form
11) Set builder form

In Roster form we just arrange the distinct objects in arbitary manner by introducing comma b/w them and enclose them with braces ({ }).

Eg:- set of all +ve even integers less than or equal to 10

$= \{2, 4, 6, 8, 10\}$

In setbuilder form the memers of the sets are represented by stating ther commen property. The members are described by the letter x (or y or z, etc) which is followed by the colon (:) (~~for~~ read as such that) ~~After~~ After the colon we write the characteristic property of the members of the set. finally we enclose these within the braces.

Eg:- Consider the set of all +ve even integers less than or equal to 10.

In set builder form it is written as

$\{x : x \text{ is a positive even integer and } x \leq 10\}$

1. Consider the set $\{x : x = 2^n + 1, n \in N, n \leq 5\}$ write this set in Roster form

→ $\{2^n + 1$ where $n \leq 5, n \in N$

$\therefore \{2^1 + 1, 2^2 + 1, 2^3 + 1, 2^4 + 1, 2^5 + 1\}$

$= \{3, 5, 9, 17, 33\}$

0 write the solution of the eqn $x^2 - 5x + 6$ in Roster form.

a) $x^2 - 5x + 6 \Rightarrow (x-2)(x-3) = 0$

$x = 2, 3 \quad \therefore$ set $A = \{2, 3\}$

Write the following sets in the ~~set builder~~ Roster form

i) $A = \{x :$ x is an integer and $-3 < x < 7\}$

a) $A = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

ii) $B = \{x :$ x is ~~an~~ a prime number which is a divisor of $60\}$

a) $B = \{2, 3, 5,\}$

Write the following sets in the set builder form

i) $\{3, 6, 9, 12\}$

a) Set builder form $= \{x : x = 3n$ n is a natural number, $n < 5\}$

ii) $\{2, 4, 6\}$ set builder form $= \{x : x = 2n$ and n is a natural number$\}$

Write the following sets in roster form

i) $A = \{x : x$ is a natural numbers less than $6\}$

→ $A = \{1, 2, 3, 4, 5\}$

ii) Consider the sets $A = \{x : x$ is an integer, $0 \leq x < 4\}$

→ $A = \{0, 1, 2, 3\}$

iii) $B = \{x : x$ is a prime number less than $5\}$

→ $B = \{2, 3,\}$

Write the following sets in set builder form

i) $\{1, 3, 5, 7, 9\}$

→ Set builder form - $\{x : x$ is an odd natural number less than $10\}$

or

$\{x : x = 2n + 1$; where $n \in W, n \leq 4\}$

ii) $\{2, 3\}$

Set builder form = $\{x : x$ is a natural number, $1 < x < 4\}$

Q write solution of the eqn $x^2 + x - 2 = 0$ in roster form?

ans
$$x^2 + x - 2 = 0$$
$$(x+2)(x-1) = 0$$
$$x = -2, 1$$
$$x = \{-2, 1\}$$

Null set or Empty Set

A set which is not containing any element is called a null set or an empty set or the void set. It is denoted by 'ϕ' (Phi) or $\{\}$.

Eg:-

1. Let $A = \{x : 1 < x < 2, x \in N\}$

Then A is an empty set, because there is no natural number b/w 1 and 2.

2) $B = \{x : x^2 = 4, x \text{ is an odd}\}$.

Then B is ~~t~~ an empty set because the eqn $x^2 = 4$ is not satisfied by any odd values of 'x'.

Finite and infinite Set

A set which ~~is~~ contains only finite number (definite number) of elements is called a finite set. Otherwise, the set is infinite.

Eg:- $\{2, 4, 6, 8, 10\}$ is a finite set

If a is the set of all points on a line. Then a is an ~~if~~ infinite set.

1. $\{x : x \in N \text{ and } (x-1)(x-2)=0\}$

Given Set $= \{1, 2\}$. Hence it is a finite set.

2. $\{x : x \in N \text{ and } x \text{ is odd}\}$

Since there are infinitely many odd numbers the given set is infinite.

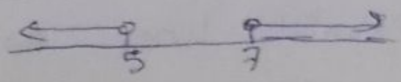
Q which of the following are ~~an~~ example of null set

i) Set of odd natural numbers divisible by 2 ✓

ii) Set of even prime numbers.

iii) $\{x : x$ is a natural number, $x < 5$ & $x > 7\}$ ✓

emptyset bcoz x



State which of the following sets are finite or infinite:-

i) $\{x : x \in N \text{ and } x^2 - 3x + 2 = 0\}$

a) $$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$x = 2, 1$ set $= \{1, 2\}$

Hence it is finite

ii) $\{x : x \in N \text{ and } x \text{ is even}\}$

ans Since there are infinitly many even natural numbers, the given set is infinite

iii) $\{x : x \in N \text{ and } x^2 = -2\}$

a) Square of a number will never become -ve for any real number so, the given set is an empty set it is also finite.

iv) The set of lines which are parallel to x axis

ans Infinite set. Because there are many no; of parallel lines as Real numbers which is also infinite.

The set of letters in the English Alphabet

no: Letters in English Alphabet = 26

So, the given set is finite

Equal Sets

The two sets A and B are said to be equal if every element of A is an element of B and every element of B is an element A. In such a case we write $A = B$

Eg:- Set $A = \{x : x \in N \ 3 < x < 8\}$ and
$B = \{4, 6, 7, 5\}$ Then $A = B$

[Since $3 < x < 8 = \{4, 5, 6, 7\} = \{4, 6, 7, 5\}$]

Eg 2:-
Let $A = \{x : x$ is a letter in word Follow$\}$
$B = \{y : y$ is a letter in the word WOLF$\}$

$A = \{F, O, L, W\}$
$B = \{W, O, L, F\}$ $\therefore A = B$//

Q. Are the following pair of sets equal? Give reason.

i) $A = \{2, 3\}$

$B = \{x : x$ is a solution of $x^2 + 5x + 6 = 0\}$

a) B :- $x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0 \Rightarrow x = -2, -3$

ie, $B = \{-2, -3\} \neq \{2, 3\}$

so set $A \neq$ set B

ii) $A = \{2, 4, 6, 8, 10\}$

$B = \{x : x$ is a +ve even Integer $\leq 10\}$

a) $B = \{2, 4, 6, 8, 10\}$

so set $A =$ Set B //

iii) $A = \{x : x$ is multiple of $10\}$

$B = \{10, 15, 20, 25, 30 \ldots\}$

a) set $A \neq$ Set B

[Since set B contains multiples of 5]

Subset

A set 'A' is said to be a subset of a set B if every element of A is an element of B. we can write in symbol $A \subset B$ [read as A is a subset of B or A is contained in B].

Note :- * If $A \subset B$ and $B \subset A$, then $A = B$

* For any set A, $A \subset A$ and $\phi \subset A$

* Let A and B be two sets. If $A \subset B$ and $A \neq B$ then A is called a proper subset of B and B is called the superset of A.

eg:- $A = \{1, 2, 3\}$ is a proper subset of
$B = \{1, 2, 3, 4\}$.

Singleton set:-

A set containing only one element is called a singleton set.

* If A is not contained in B then we write $A \not\subset B$. ~~(A⊇B)~~

eg:- $A = \{7, 8, 9\}$ $D = \{1, 2, 3\}$

$B = \{2, 3, 4, 5, 1\}$

Then $A \not\subset B$

$D \subset B$

Q Examine whether the following statements are 'T' or F

i) $\{a, b\} \not\subset \{b, c, a\}$ - F

ii) $\{a, e\} \subset \{x : x$ is a vowel in the English Alphabet$\}$

iii)

Q write down all the substets of the following sets

i) $\{a\}$

Subsets - ϕ, $\{a\}$

ii) $\{1, 2\}$ -

Subsets - are - ϕ, $\{1\}$, $\{2\}$, $\{1, 2\}$

iii) $\{a, b, c\}$

Subsets - ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$
$\{b, c\}$, $\{a, b, c\}$

iv) $\{1, 2, 3\}$ Subsets are -

$= \phi, \{1\}, \{2\}, \{3\}, \{1,2\}\ \{1,3\}\ \{3,2\}$
$\{1,2,3\}$.

v) ϕ Subsets ~~are~~ is only ϕ ~~bcoz~~ itself (bcoz $2^0 = 1$).

Four types of intervals

If $a, b, \in R$ so that $a<b$, the we may define four types of intervals as follows

Closed interval : $[a, b]$

$= \{x : x \in R \text{ and } a \leq x \leq b\}$

eg:- $[7, 13] = \{x : x \in R \text{ and } 7 \leq x \leq 13\}$

Open interval : $(a, b) = \{x : x \in R \text{ and } a < x < b\}$

eg:- $(-3, 0) = \{x : x \in R \text{ and } -3 < x < 0\}$

Left Open right closed interval :- $(a, b]$

$= \{x : x \in R \text{ and } a < x \leq b\}$

eg:- $(6, 11] = \{x : x \in R \text{ and } 6 < x \leq 11\}$

Left Closed Right open Interval : $[a, b)$

$= \{x : x \in R \text{ and } a \leq x < b\}$

Eg:- $[-22, 5) = \{x : x \in R \text{ and } -22 \leq x < 5\}$

Write following as intervals

i) $\{x : x \in R, -4 < x \leq 6\}$

ans) $(-4, 6]$

ii) $\{x : x \in R, -12 < x < -10\}$

ans $(-12, -10)$

iii) Write ~~to~~ the following Intervals in set builder form.

i) $[6, 12] - \{x : x \in R \ \& \ 6 \leq x \leq 12\}$

ii) $(-5, 0) - \{x : x \in R \ \& \ -5 < x < 0\}$

iii) $(8, 12] - \{x : x \in R \ \& \ 8 < x \leq 12\}$

iv) $[-23, 5) - \{x : x \in R \ \& \ -23 \leq x < 5\}$

Power Set of a Set

If 'A' is any set then the set all possible subset of 'A' is named as the powerset of 'A' and it is denoted by $P(A)$.

Note :-

If 'A' contains 'n' elements, then A will have 2^n subsets.

Eg:- Consider $A = \{1, 2, 3\}$

Then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$n(A) = 3$ $\quad$ $n(P(A)) = 2^3 = 8$ //

Universal Set

In any discussion of sets, it is possible to identify a super set containing all other sets in the discussion. Such ~~as~~ a superset is called the Universal set and it is denoted by 'U'

Eg:- In the study of ~~sets of~~ all sets containing real numbers, R is the Universal Set.

Q. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$ which of the following may be considered as universal sets for all the three A, B and C.

i) $\{0, 1, 2, 3, 4, 5, 6\}$

ii) ϕ

iii) $\{0, 1, 2, 3, 4, 5, \ldots 10\}$

iv) $\{1, 2, 3, 4 \ldots 8\}$

ans) iii) $\{0, 1, 2, 3 \ldots 10\}$

Q write down ~~all the~~ powersets of the following sets

1) $A = \{a, b, c\}$

a) $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{ab\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

2) $A = \{1, 2\}$

a) $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

3. How many elements has $P(A)$ if $A = \phi$

If $A = \emptyset$ A contains no element.
$\therefore$ A has only one subset which is $\emptyset$ itself.

$\therefore$ $P(A) = \{\emptyset\}$ and there for $P(A)$ has only one element.

$$\boxed{\text{If } n(A) = 0, \quad n(P(A)) = 2^0 = 1}$$

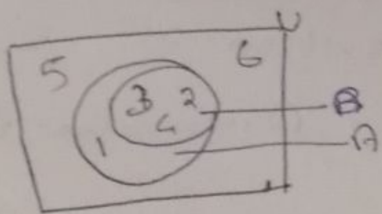
Venn diagram

Pictorical representation of sets.

eg. Suppose $U = \{1,2,3,4,5,6\}$

$A = \{1,2,3,4\}$ and $B = \{2,3,4\}$

Then we may represent these sets as shown below



1) Introduced by John Venn (English Logician)
2) Invented by Leonhard Euler (mathematician)

Operations of sets

(1) Union of sets

Combining two sets.

two sets say A and B

The Union of A & B ($A \cup B$) [read as A Union B] is the set of all elements which are ~~written~~ contained in A and B.

In set builder form

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Eg:- Suppose $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 4\}$ and $B = \{3, 4, 5\}$

Then $A \cup B = \{1, 2, 3, 4, 5\}$

Intersection of A and B

The Intersection of two sets say A and B ($A \cap B$ - read as A intersection B) is the set of elements common in A and B.

*) $A \cap B$, $A \cup B$ are subsets of U for any two sets A and B

In set builder form

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

eg:- $U = \{1, 2, \ldots 6\}$

$A = \{1, 2, 4\}$

$B = \{3, 4, 5\}$

Then $A \cap B = \{3, 4\}$

Disjoint Sets

If A and B are two sets such that $A \cap B = \emptyset$ then A and B are called disjoint sets

eg:- $A = \{a, b, c\}$ and $B = \{e, f, g\}$ are disjoint sets bcoz $A \cap B = \emptyset$

Properties

1. $A \cup B = B \cup A$; ~~$B \cap A$~~ $A \cap B = B \cap A$ (commutative laws)
2. $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$ (assossiative laws)
3. $A \cup \emptyset = A$; $A \cap U = A$ (Identity laws)
4. $A \cup A = A$; $A \cap A = A$ (Idempotent laws)
5. $A \cup U = U$; $A \cap \emptyset = \emptyset$
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributive law)

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(6) If $A \subset B$, then $A \cup B = B$ and $A \cap B = A$

Q. If $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$

$C = \{5, 6, 7, 8\}$

Find?

i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

ii) $B \cup A$ = "

iii) $(A \cup B) \cup C = \{1, 2, 3 \ldots 8\}$

iv) $A \cup (B \cup C) = \{1, 2, 3, \ldots 8\}$

v) $A \cap B = \{3, 4\}$

vi) $(A \cap B) \cap C = \emptyset$

vii) $A \cap (B \cap C) = \emptyset$

viii) $A \cup A = A = \{1, 2, 3, 4\}$

ix) $A \cap A = A$ = "

x $A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}$

xi) $(A \cup B) \cap (A \cup C)$ = "

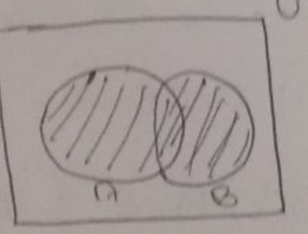
xii) $A \cap (B \cup C) = \{3, 4\}$

$(A \cap B) \cup (A \cap C) = \{3, 4\}$

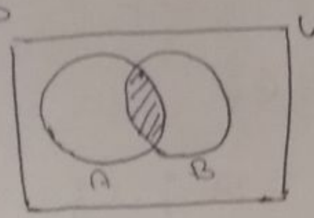
Venn diagrams for operations on sets

Let A, B, C are ~~two~~ three sets

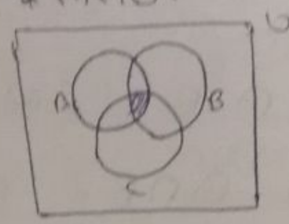
1) $A \cup B$



2) $A \cap B$



$A \cap B \cap C$



Difference of sets

If A and B are two sets, then the difference of A and B is defined as the set of all elements in A which doesnot belong to B ~~and its~~. It is written ~~as~~ symbolically as $A-B$

~~A-B~~ in set builder form $A-B = \{x : x \in A \ \& \ x \notin B\}$

Q. If $X = \{a, b, c, d\}$ & $Y = \{f, b, d, g\}$

find i) $X-Y$ ii) $Y-X$

as) i) $x-y = \{a, c\}$ ii) $y-x = \{f, g\}$

) $X \cap Y = \{b, d\}$

. If $A = \{x : x$ is a letter in the word 'MATHEMATICS'$\}$

$B = \{y : y$ is a letter in the word 'STATISTICS'$\}$

then find

i) $A - B$ ii) $A \cap B$

) $A = \{M, \underline{A}, \underline{T}, H, E, \underline{I}, \underline{C}, \underline{S}\}$

$B = \{\underline{S}, \underline{T}, \underline{A}, \underline{I}, \underline{C}\}$

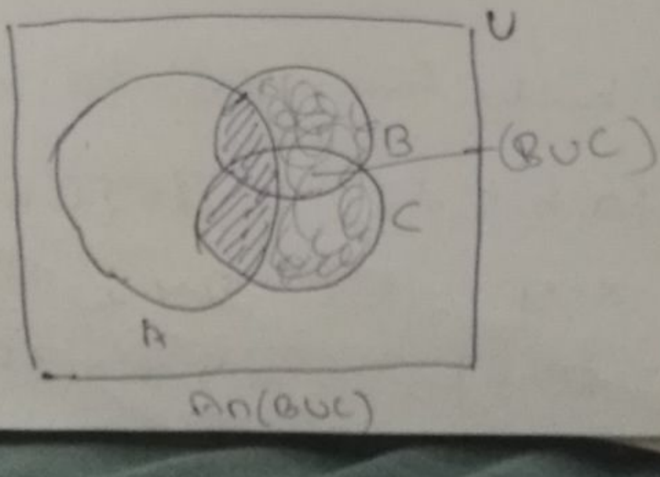
i) $A - B = \{M, H, E\}$

ii) $A \cap B = \{S, T, A, I, C\}$

Draw Venn diagrams.

i) $A \cap (B \cup C)$

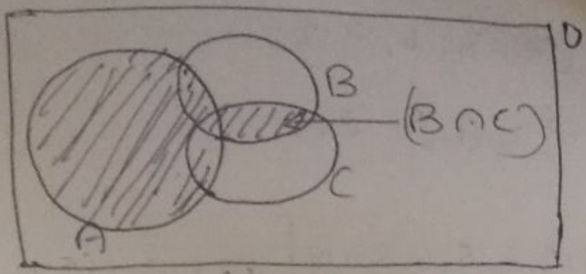
Assuming that set A and set B, set C are not disjoint. And s



$A \cap (B \cup C)$

2. $A \cup (B \cap C)$

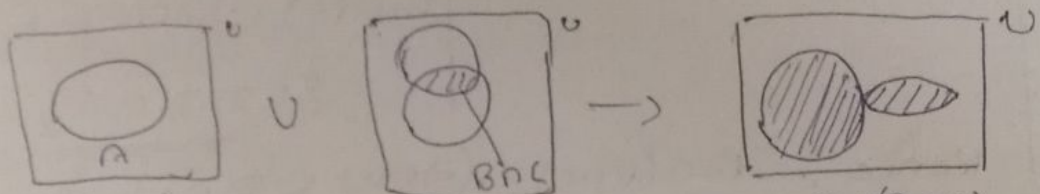
If set A, B, C, are not disjoint.



$A \cup (B \cap C)$

Case :2

If Set A and set $(B \cap C)$ are disjoint



Set A $\quad$ $B \cap C$ $\quad$ $A \cup (B \cap C)$

Q Let $A = \{x : x \text{ is a prime number less than } 11\}$

and $B = \{x : x \text{ is an integer such that } 2 \leq x \leq 8\}$

write $C = A \cap B$

Ans) $A = \{2, 3, 5, 7\}$

$B = \{2, 3, 4, 5, 6, 7, 8\}$

$A \cap B = \{2, 3, 5, 7\}$

Q Let $A = \{x : x \in N, 1 \leq x \leq 5\}$

$B = \{2, 3, 6, 9\}$ & $C = \{1, 4, 5, 8, 9, 10\}$

Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 6, 9\}$

$C = \{1, 4, 5, 8, 9, 10\}$

$B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$

$A \cap (B \cup C) = \{1, 2, 3, 4, 5\}$

$A \cap B = \{2, 3\}$

$A \cap C = \{1, 4, 5\}$

$(A \cap B) \cup (A \cap C) =$
$= \{1, 2, 3, 4, 5\}$

Hence, $A \cap (B \cap C) = (A \cap B) \cup (A \cap C) = \{1, 2, 3, 4, 5\}$

Verified

Q If U is the Universal set and A is any set. then $U \cap A =$ ____

a) A

Q If A is a subset of the set B

then $A \cap B = \underline{A}$ $A \cup B = \underline{B}$

Q) Let $A = \{x : x \in W ; x < 5\}$ and

$B = \{x : x$ is a prime number less than $5\}$

$U = \{x : x$ is an Integer, $0 \le x \le 8\}$

write A, B in roster form

A) $A = \{0, 1, 2, 3, 4\}$

$B = \{2, 3, \}$

Q find $(A-B) \cup (B-A)$

An $A - B = \{0, 1, 4\}$; $B - A = \{\}$ or ϕ

so $(A-B) \cup (B-A) = \{0, 1, 4\}$

Consider the set $A = \{2, 3, 5, 7\}$ and

$B = \{1, 2, 3, 4, 6, 12\}$

a) Find $A \cap B$

b) Find $A-B$, $B-A$ and hence show that

$(A \cap B) \cup (A-B) \cup (B-A) = A \cup B$

c) write the power set of $A \cap B$

a) a) $A \cap B = \{2, 3, \}$

5) $A - B = \{\cancel{1, 4, 6, 12}\}$ $\{5, 7\}$

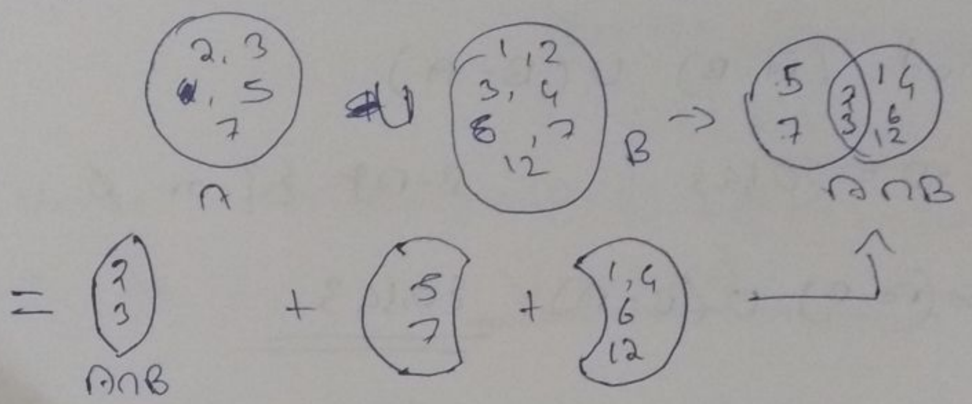
$B - A = \{\cancel{5, 7}\}$ $\{1, 4, 6, 12\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 12\}$

$(A \cap B) \cup (A - B) \cup (B - A) = \{1, 2, 3, 4, 5, 6, 7, 12\}$

~~So, Hence verified //~~

By venn diagrams ~~A = O B = O~~



Hence verified. //

Q c) $A \cap B = \{2, 3\}$

$P(A \cap B) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$

Q 16 $A = \{3, 6, 9, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$D = \{5, 10, 15, 20\}$

Find i) $A - B$ ii) $A - C$

iii) $A - D$ iv) $B - A$ v) $C - A$ vi) $D - A$

vii) $B - C$ ~~$B - D$~~ viii) $B - D$ ix) ~~$D - B$~~ $C - B$

x) $D - B$ xi) $C - D$ xii) $D - C$

A)

i) $A - B = \{3, 6, 9, 15, 18, 21\}$ (12 is gone)

ii) $A - C = \{3, 9, 15, 18, 21\}$ (6, 12 is gone)

iii) $A - D = \{3, 6, 9, 12, 18, 21\}$ (15 is gone)

iv) $B - A = \{4, 8, 16, 20\}$ (12 is gone)

v) $C - A = \{2, 4, 8, 10, 14, 16\}$ (6, 12 gone)

vi) $D - A = \{5, 10, 20\}$ (15 is gone)

vii) $B - C = \{20\}$ (4, 12, 16, 8 gone)

viii) $B - D = \{4, 8, 12, 16\}$ (20 gone)

ix) $C - B = \{2, 10, 6, 14\}$

x) $D - B = \{5, 10, 15\}$ (20 gone)

xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$ (10 gen)

xii) $D - C = \{5, 15, 20\}$ (10 gen)

Complement of a set :-

If A is any set, then the complement of A is defined as the set of all element in U which ~~doesnot~~ ~~donot to~~ donot belong to A.

The complement of A is denoted by A^c or $\bar{A}$ or A'.

Eg:- Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4, 5\}$ and $B = \{5, 6, 7, 8, 9\}$

Then $A' = \{6, 7, 8, 9\}$ and $B' = \{1, 2, 3, 4\}$

$\therefore A' \cap B' = \phi$ & $A' \cup B' = \{1, 2, 3, 4, 6, 7, 8, 9\}$

2) $U - A = \{6, 7, 8, 9\} = A'$

$A - B = \{1, 2, 3, 4\}$

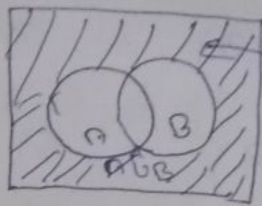
$A \cap B' = \{1, 2, 4, 3\}$

$A - B = A \cap B'$

$(A')' = \{1, 2, 3, 4, 5\} = A$ i.e., $\underline{\underline{(A')' = A}}$

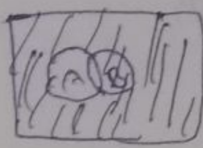
Q Draw appropriate Venn diagram for each of the following

i) $(A \cup B)' \rightarrow$



U $(A \cup B)'$

ii) $A' \cap B' \rightarrow$

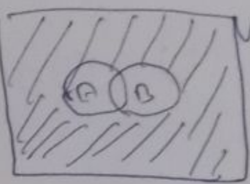


A'

$\cap$

B'

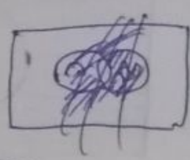
$=$



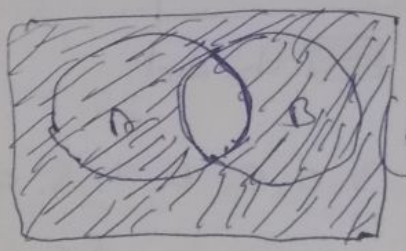
U $A' \cap B'$

So, $(A \cup B)' = A' \cap B'$

iii) $(A \cap B)' \longrightarrow$

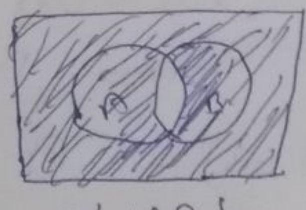


$\rightarrow$



U $(A \cap B)'$

iv) $A' \cup B' \longrightarrow$



U

$A' \cup B'$

So, $(A \cap B)' \Rightarrow A' \cup B'$

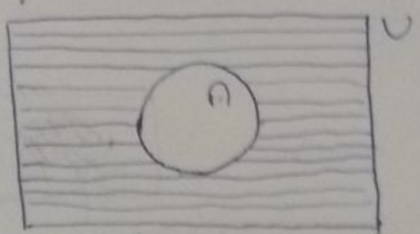
Properties ~~of Complement~~

1. $A \cup A' = U$ & $A \cap A' = \emptyset$ Complement laws

2. $(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$ } De morgan's laws

3. $\emptyset' = U$ and $U' = \emptyset$.

Venn diagram of A'



A' ▤ shaded.

Q If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$ verify
the ~~laws~~ De morgan's laws

i) $(A \cup B)' = A' \cap B'$

LHS $(A \cup B)' = \{2, 4, 6, 8, 5, 7\}' = \{1, 9\}$ //

$A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$ //

Hence $(A \cup B)' = A' \cap B'$ //

ii) $A \cap B = \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

Hence $(A \cap B)' = A' \cup B'$ //

Q Consider the set $U = \{a, b, c, d, e, f, g\}$ $A = \{b, c, d, e\}$ and $B = \{a, c, g\}$ find A' and B' and verify that $(A \cup B)' = A' \cap B'$

A) $A' = \{a, f, g\}$, $B' = \{b, d, e, f\}$

$A' \cap B' = \{f\}$

$(A \cup B)' = \{a, b, c, d, e, g\}' = \{f\}$

Hence $(A \cup B)' = A' \cap B'$ verified

Theory

1) If A is any set then the number of elements in A is denoted by $n(A)$

2) If A and B are any two sets then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If A and B are disjoint sets then

$$n(A \cup B) = n(A) + n(B)$$

iii) If A, B and C are any three sets, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

If A, B and C are pair wise disjoint then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$

eg: If X and Y are two sets such that $X \cup Y$ has 50 elements. X has 28 elements and Y has 32 elements. How many elements does $X \cap Y$ have?

a) Given that

$n(X \cup Y) = 50$ $\left[n(A \cup B) = nA + nB - n(A \cap B) \right]$

$n(X) = 28$

$n(Y) = 32$

$n(X \cap Y) = ?$ $(n(X \cup Y) = n(X) + n(Y) - n(X \cap Y))$

$\therefore n(X \cap Y) = nX + nY - nX \cup Y$

$= 28 + 32 - 50 = 10$ //

Q. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$, and $n(X \cup Y) = 38$. Find $n(X \cap Y)$

A) Given that $n(X) = 17$

$n(Y) = 23$

$n(X \cup Y) = 38$

we know $n(X \cup Y) = nX + nY - nX \cap Y$

so $n X \cap Y = 17 + 23 - 38 = 2$ //

Q. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English

A) Given data - Hindi - 250 English - 200

Total people - 400

$n(A \cap B) = nA + nB - n(A \cup B)$

$= 250 + 200 - 400 = 50$ //

Q In a Committee, 50 people speak French, 20 speak Spanish, and 10 speak both Spanish and French. How many speak atleast one of these languages?

ans Given data – $N_{French} - 50$, $N_{spanish} - 20$

$$N(\text{French} \cap \text{Spanish}) = 10$$

& $n(\text{French} \cup \text{spanish}) = ?$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\therefore n(\text{French} \cup \text{Spanish}) = 50 + 20 - 10$$
$$= 60$$

Q In a school there are 20 teachers who teach Mathematics or physics of these 12 teachers teach maths and 12 teach physics. How many teach both of the subject

ans $n(m) = 12$, $n(P) = 12$

$$n(m \cup P) = 20$$

$$n(m \cap P) = n(m) + n(P) - n(m \cup P)$$
$$= 12 + 12 - 20 = 4$$

Rationalize

$1+\sqrt{2} \longrightarrow$

Q ~~In a Committee, 50 people speak french 20 speak Spanish and 10 speak both~~

$\rightarrow n(A') = n(U) - n(A)$

$\rightarrow n(A-B) = n(A) - n(A \cap B)$

$\rightarrow$ If $A \subset B$ then $n(B-A) = n(B) - n(A)$

$\rightarrow n(A' \cap B') = n(U) - n(A \cup B)$

$\rightarrow n(A' \cup B') = n(U) - n(A \cap B)$

Q In a group of 65 people, 40 like cricket 10 like both cricket and tennis. How many like tennis only and not cricket?

How many like tennis?

A) Given data $n(C) = 40$

$n(C \cap T) = 10$

$n(U) = 65$

$\cancel{n(U) =}\ n(T) = n(U) + n(C \cap T) - n(C)$

$= 65 + 10 - 40 = 35$ //

like Tennis only $= 35 - 10 = 25$ //

Q In a group of people, 50 people speak Hindi, 20 speak English and 10 speak English and 10 speak both Hindi and English.

i) Find the number of people speaking only Hindi?

ii) Find the number of people speaking only English

iii) Find the number of people speaking at least one of these two languages.

as i) $n(H) = 50$
$n(E) = 20$
$n(H \cap E) = 10$

We have $n(H \cap E') = n(H) - n(H \cap E)$
$= 50 - 10 = 40$ //

ii) $n(E \cap H') = n(E) - n(E \cap H)$
$= 20 - 10 = 10$ //

iii) $n(E \cup H) = n(H) + n(E) - n(H \cap E)$
$= 50 + 20 - 10 = 60$ //

Q In a survey of 600 students in a school, 150 students were found to be taking tea. 225 taking Coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor Coffee.

as Given dat $U = 600$
$n(T) = 150$, $n(C) = 225$

$n(C \cap T) = 100$

Neither Tea nor coffee means

$n(U) - n(C \cup T)$

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$= 150 + 225 - 100$

$= 275$ //

$n(U) - n(C \cup T) = 60-$

$n(U) - n(C \cup T) = 600 - 275 = \underline{\underline{325}}$