NEUTRINO SCALE WITHIN A UNIVERSAL TEXTURE AND MAJORANA MASSES MODEL

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Abstract

It is shown how using the concept of Universal Texture Constraint and corrections from the Majorana mass contribution to neutrino masses, a relationship between parameters from other sectors within the standard model and the neutrino sector can be established in order to predict the absolute mass scale of the heaviest neutrino under normal ordering, giving as a result:

$$m_3 \in [2.488\times10^{-3}, 2.544\times10^{-3}]\ ,\ (\delta_1', \delta_2') \in [0.4766\times10^{-1}, 0.5157\times10^{-1}]$$

Introduction

The neutrino sector allows two contributions for the masses given by a Dirac and a Majorana description. For quarks and charged leptons it is possible to find invariant parameters that depend on the ratios of the masses. Such invariants may represent low energy limits for flavor symmetries on higher energy scales.

Universal Texture Constraint (UTC)

We call Universal Texture to the mechanism that allows to use invariant parameters from one fermionic sector in any other fermionic sector in order to find universal values which can generate mass matrices in the latter sector.

Matrices; Gell-Mann, fermions

The Gell-Mann matrices are used to describe the three-generation mixing of neutrino and strong interactions. Although being eight matrices, only those with a diagonal not composed out of zeros only are used. Namely; matrices 3 and 8, plus the identity matrix $\underline{I}$.

$$\ell_3 = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix} \quad , \quad \ell_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix} \quad , \quad \underline{I} = \sqrt{\tfrac{2}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} = \ell_0$$

Mass matrices for fermions are modified by multiplying them for the inverse of the heaviest fermion within each sector. These are written as:

$$\underline{M_i} = \frac{1}{m_{3,i}}\underline{m_i} = \frac{1}{m_{3,i}}\begin{pmatrix}m_{1,i}&0&0\\0&m_{2,i}&0\\0&0&m_{3,i}\end{pmatrix} = C_0\cdot\underline{I} + C_3\cdot\ell_3 + C_8\cdot\ell_8 \qquad ; i = u,d,l$$

It's important that any result obtained reproduce the experimental data for the squared masses difference for neutrinos given by:

$$(\Delta m^2_{21})_{\rm exp} = 7.53\pm0.18\times10^{-5}eV^2,\ \ (\Delta m^2_{21})_{\rm exp} = 2.453\pm0.034\times10^{-3}eV^2$$

Procedure

The universal texture permits taking $C_{k,i}$ as constant parameters in all sectors. A comparative between error sectors is given in Figure 1:

$$\Delta C_{k,i} = \sum_{j=1}^{3}\frac{dC_{k,i}}{dm_j}\cdot\delta_{\rm err}m_j; \quad C_{k,i} = \frac{1}{2}Tr(\ell_0\cdot M_i)$$

Coefficients	Value	Error
$C_{0,l}$	0.4326	4.0365×10^{-6}
$C_{0,u}$	0.4112	1.0585×10^{-4}
$C_{0,d}$	0.4178	2.0038×10^{-3}

Figure 1 : Values for C_0 for; Quarks Up, Quarks Down and Charged Leptons families.

Dirac Masses

Matrices $C_0,\ C_3,\ C_8$ are computed for the charged leptons case then this matrices are changed for those in the neutrino case, giving the expressions:

$$C_{0,\nu} = \frac{m_1+m_2}{m_3}+1 \quad , \quad C_{3,\nu} = \frac{m_1+m_2}{m_3} \quad , \quad C_{8,\nu} = \frac{1}{\sqrt{3}}\left(\frac{m_1+m_2}{m_3}-2\right)$$

Solving for m_1 and m_2 :

$$m_1 = \frac{m_3(C_{0,l}+C_{3,l})}{2} - \frac{m_3}{2} \quad , \quad m_2 = \frac{m_3(C_{0,l}-C_{3,l})}{2} - \frac{m_3}{2} \qquad (Ec.1)$$

Majorana Masses

Charged fermions don't allow Majorana masses for they are Dirac fermions. But neutrinos having a lack of charge, and with the introduction of sterile right-handed neutrinos, make possible the addition of a Majorana mass term. By including a correction coming from the Majorana mass term the following can be obtained:

Case 1

With the condition:

$$m_1 \to m_1+\delta_1\,, \quad m_2 \to m_2+\delta_2 \quad | \quad \delta_1-\delta_2 = \delta\,, \quad \delta_1+\delta_2 = 0$$

$$\Longrightarrow \left.\begin{aligned} m_{1\mathscr{M}} &= \frac{m_3(C_{0,l}+C_{3,l})}{2} - \frac{m_3+\delta}{2} \\ m_{2\mathscr{M}} &= \frac{m_3(C_{0,l}-C_{3,l})}{2} - \frac{m_3+\delta}{2}\end{aligned}\right\} (Ec.2)$$

Case 2

$$m_1 \to \frac{m_1}{\delta_1}\,, \quad m_2 \to \frac{m_2}{\delta_2}$$

$$\Longrightarrow \left.\begin{aligned} m_{1\mathscr{M}} &= \frac{m_3\cdot\delta_1'\cdot(C_{0,l}+C_{3,l})}{2} - \frac{m_3\cdot\delta_1'}{2} \\ m_{2\mathscr{M}} &= \frac{m_3\cdot\delta_2'\cdot(C_{0,l}-C_{3,l})}{2} - \frac{m_3\cdot\delta_2'}{2}\end{aligned}\right\} (Ec.3)$$

Contour plots are made to observe the behaviour of the functions: $f_1 = \Delta m^2_{21} - (\Delta m^2_{21})_{\rm exp}$ $f_2 = \Delta m^2_{32} - (\Delta m^2_{32})_{\rm exp}$ in order to find the level zero contour.

Results

Dirac Masses

Replacing the numerical values for a fixed value of m_3 and comparing with the experimental data:

$$\begin{matrix} m_1 = 1.437\times10^{-5}\ \text{eV} & , & \Delta m^2_{21} = 8.839\times10^{-6}\text{eV}^2 & \Longrightarrow & \Delta m^2_{21} \neq (\Delta m^2_{21})_{\rm exp} \\ m_2 = 2.973\times10^{-3}\ \text{eV} & , & \Delta m^2_{32} = 2.491\times10^{-3}\text{eV}^2 & & \Delta m^2_{32} \approx (\Delta m^2_{32})_{\rm exp}\end{matrix}$$

Majorana Masses

Case 1:

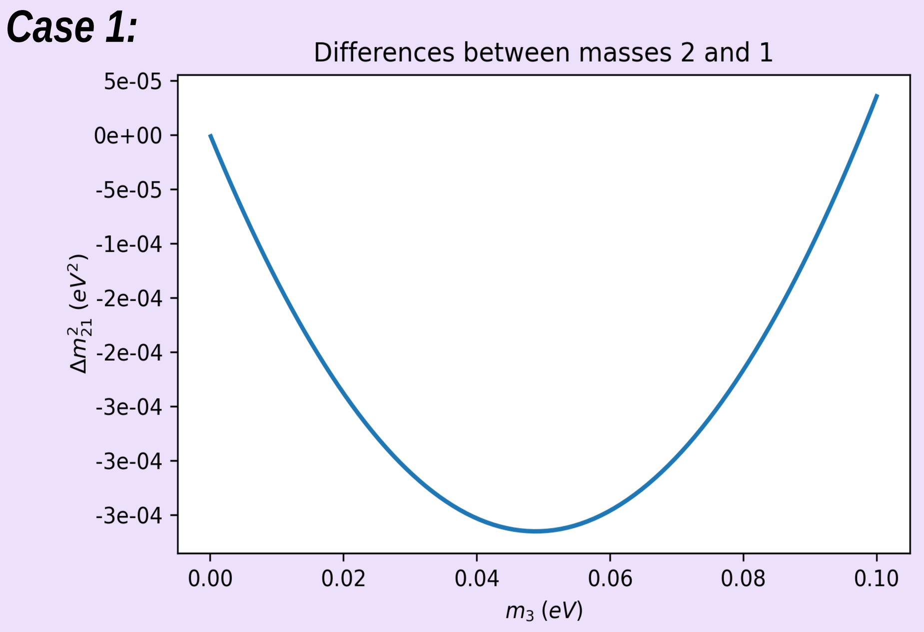

Figure 2: Curve for Δm^2_{21}, when; $m_3 \in (0, 1\times10^{-1})$.

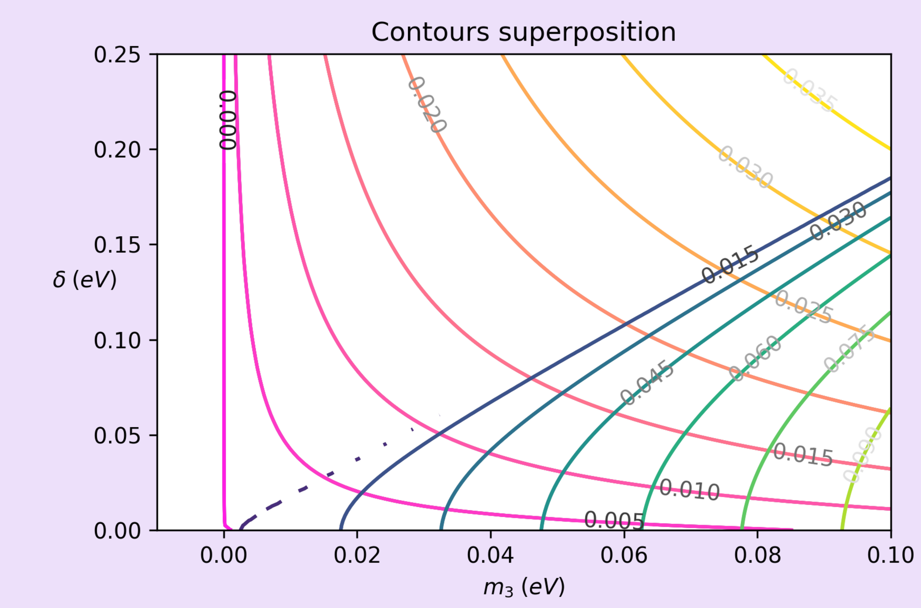

Figure 3: Curves; f_1, f_2 using the Ec. 1 masses. Level zero curves do not intersect.

Case 2:

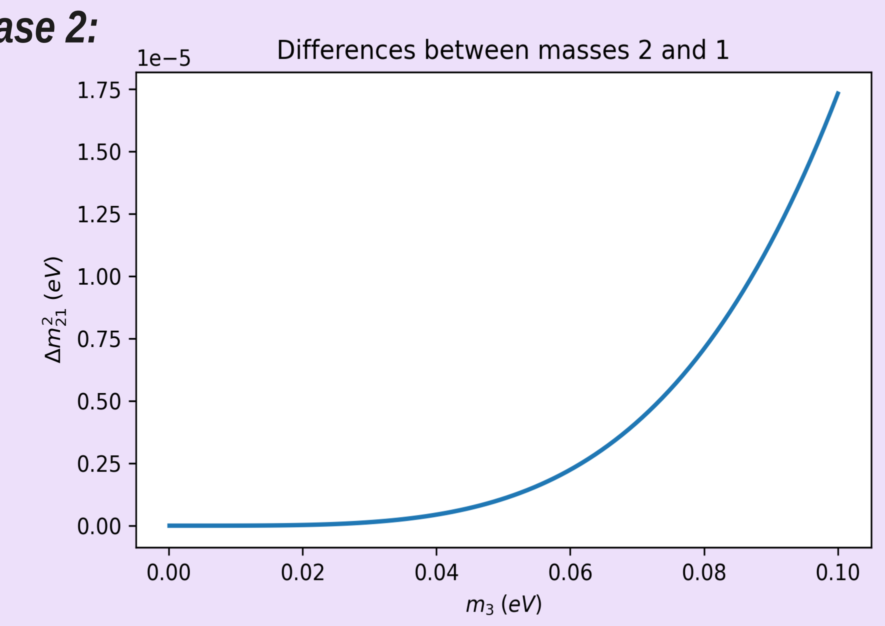

Figure 4: Curve for Δm^2_{21}, when; $m_3 \in (0, 1\times10^{-1})$.

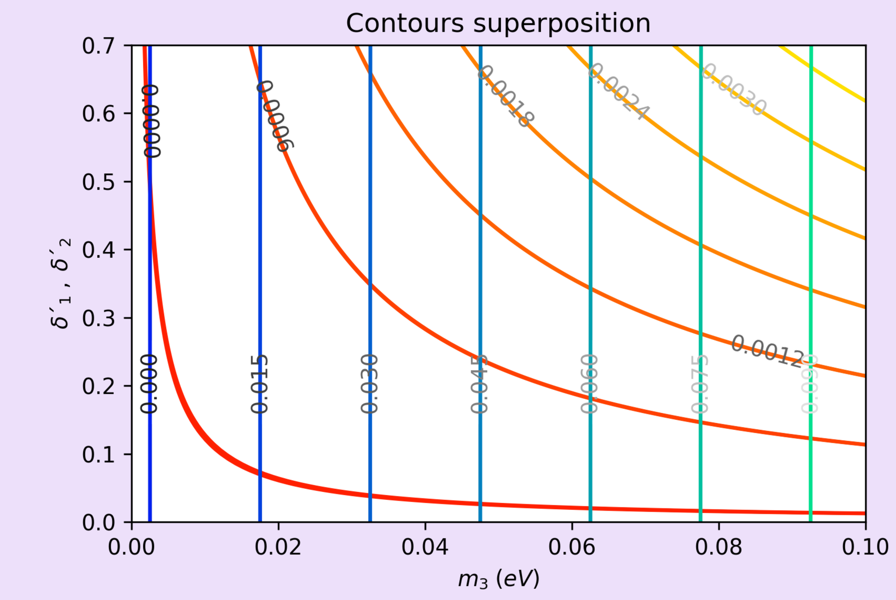

Figure 5: Curves; f_1, f_2 using the masses on Ec. 2. Level zero curves do intersect.

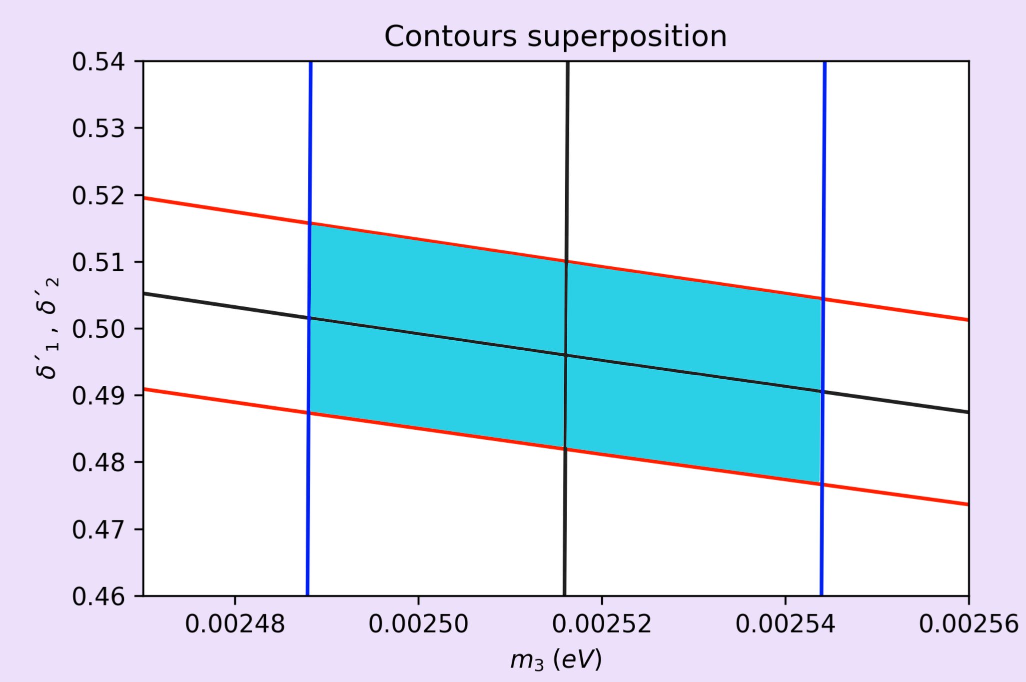

Figure 6: Prediction for the values $\delta_1',\ \delta_2',\ m_3$;

$$m_3 \in [2.488\times10^{-3}, 2.544\times10^{-3}]\ \text{eV}$$

$$(\delta_1', \delta_2') \in [0.4766\times10^{-1}, 0.5157\times10^{-1}]$$

so that;

$$\Delta m^2_{21} = (\Delta m^2_{21})_{\rm exp}\ \ ,\ \ \Delta m^2_{32} = (\Delta m^2_{32})_{\rm exp}.$$

Conclusion & Perspectives

- **There is a set of values for $\delta_1', \delta_2', m_3$ such that the theoretical Δm^2_{ij} agree with the experimental measurements and they are given by:**

$$m_3 \in [2.488\times10^{-3}, 2.544\times10^{-3}]\ ,\ (\delta_1', \delta_2') \in [0.4766\times10^{-1}, 0.5157\times10^{-1}]$$

- **It is required to study the relationship between the Majorana correction and the electro-weak Lagrangian as well as it's effect on the See-Saw mechanism to complement the given prediction. It is also required to study the congruence of the three-generation mixing mechanism for neutrinos and the Majorana correction.**

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You can find more here!

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Acknowledgements

J. A. Hernández-Acosta deeply acknowledges his supervisors; S. Gómez-Ávila and L. López-Lozano for the academic support provided in the making of this work as well as the patience and comprehension in the development and continuation of the same. Acknowledgements to M. B. Palacios-Hernández, J. F. Landaverde-Cañedo and A. Castelan-Hernández.

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