UNIVERSITAT DE BARCELONA

Mediation and Moderation Models

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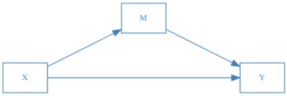
Mediation models

Introduction

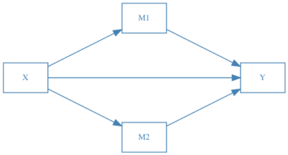
- In most occasions we are interested in establishing causal relationships in a set of variables. Therefore, we set functional relations in which some variables are antecedents (X) of some others (Y).
- Let's think of the mechanism by means of which a message on media (X) makes us to change our behavior (Y). For instance, how a negative message (i.e. photos showing ill people) influence on the decision of smoking cessation. Graphically we can represent that as: pre-main prep time: 1 ms

- In this regard, when aiming to establish causal relations we need to guarantee the following conditions:
  - Covariance
  - Temporal order
  - Control for alternative explanatory variables
- Frequently, researchers use observational data to study causal relations. In this studies is difficult to guarantee that the true relationship between X and Y is the specified causal mechanism.
- Using an experimental setting might solve some of these problems but presents some issues as ecological validity problems.

- Beyond the previous considerations, it is interesting to know under which conditions a specific effect from an exogenous variable is produced over an endogenous variable. That is the mechanisms by means of which X influences Y.
- A mediation model is used to account for the influence of certain predictor on a response through the influence of other variables that are causally between the previous ones. For example, using negative or positive advertisements may have an effect on the intention to quit smoking mediated by anxiety.
- We'll see that the total effect of X on Y through a (or several) mediator(s) M can be decomposed in direct and indirect (the mediated ones) effects. pre-main prep time: 0 ms pre-main prep time: 0 ms



Simple mediation

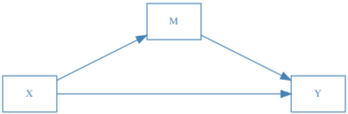


Multiple mediation

Mediation models

Simple mediation

- It is the simplest way of explaining biological, social, and emotional mechanisms by means of which X influences Y through the effect of other variables.
- M is usually called the mediator, intermediary, surrogate or intermediate variable, depending on the field.
- This model includes to consequent variables (Y and M) and two antecedent variables (X and M)



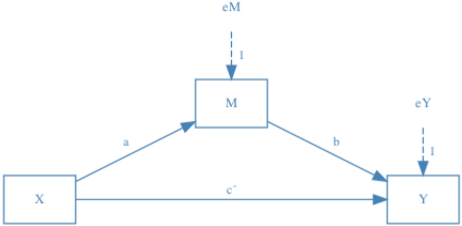
- There are two possible pathways by which X exerts an effect on Y in this model: direct and indirect effect.
- Association between X and Y was a pre-condition for a mediation model (see Baron and Kenny, 1986*). Nevertheless, this approach has been widely criticized and, thus, is not being used in the present course.

* Baron, R.M., Kenny, D.A. (1986). The moderator-mediator variable distinction in social psychology research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*, 1173-1182.

Mediation models

Simple mediation

- In order to estimate the coefficients and the effects let's represent the statistical model:
  pre-main prep time: 0 ms



- Therefore, the specified statistical model in this case is:

$$\begin{aligned} M &= i_M + aX + e_M \\ Y &= i_Y + c'X + bM + e_Y \end{aligned}$$

- The coefficients are estimates of the putative causal effects of each variable in the system. So need to estimate and combine them to provide with an estimate for the effects.

- The coefficients can be estimated using OLS by using `PROCESS` macro for SPSS. Or using the `medmod` module in jamovi.
- The direct effect (c') of X on Y is interpreted as the difference on Y between two cases that differ by one unit on X but present the same score on M. Thas is:

$$c' = [\hat{Y}|X = x, M = m] - [\hat{Y}|X = x - 1, M = m]$$

- Thus we can interpret the other effects (a and b) as follows:

$$\begin{aligned} a &= [\hat{M}|X = x] - [\hat{M}|X = x - 1] \\ b &= [\hat{Y}|M = m, X = x] - [\hat{M}|M = m - 1, X = x] \end{aligned}$$

- The indirect effect is estimated by means of the product $a \times b$. It is a composite of the effect of X on M (estimated in a simple regression) and the partial effect of M on Y (estimated in a multiple linear regression).
- It is important, for theoretical reasons, to check that the signs of both a and b coefficients are according researchers' conjectures.
- Finally, the total effect (c) is the sum of the two previous effects: $c = c' + ab$.

$$Y = (i_Y + bi_M) + (ab + c')X + (e_Y + be_M) \rightarrow Y = i_{Y*} + cX + e_{Y*}$$

Inference about the Total Effect:

- Inference is straightforward in this case since it can be done with the simple linear model Y on X: $Y = \beta_0 + \beta_1 X + \epsilon$.
- The regression coefficient (slope) of the model corresponds to the total effect of X.
- Thus, $H_0 : \beta_1 = 0$ is equivalent to $H_0 : T^c = 0$.
- In order to make a decision concerning the previous hypothesis we can use a t-test:

$$t = \frac{b_1}{se_{b1}} \sim t_{n-2}; \quad p = Prob\left(|t_{n-2}| \geq |t| | H_0\right)$$

- We can provide with a confidence interval for the true total effect as follows:

$$b1 \pm t_{n-2;\alpha/2} \times se_{b1}$$

Inference about the Direct Effect:

- The direct effect estimates the change in Y explained by X while keeping fixed M. So, an statistical test associated with $T^{c'}$ can be run by means of the model: $Y = \beta 0 + \beta 1X + \beta 2M$.
- The test for the partial effect of $\beta 1$ is equivalent to test $H_0 : T^{c'} = 0$.
- In order to make a decision concerning the previous hypothesis we can use a t-test:
$$t = \frac{b_1}{se_{b1}} \sim t_{n-2}; \quad p = Prob\left(|t_{n-2}| \geq |t||H_0\right)$$
- We can provide with a confidence interval for the true direct effect as follows:
$$b1 \pm t_{n-2;\alpha/2} \times se_{b1}$$

Inference about the Indirect Effect:

- The indirect effect quantifies the difference between two cases that differ by a unit on X as a result of X's influence on M, which in turn influences Y, that is the causal chain: $X \to M \to Y$.
- Researchers are interested in making a decision concerning $H_0 : T^a T^b = 0$ or in estimating a confidence interval for the true indirect effect.
- There are different procedures concerning statistical inference for indirect effects, in this course we'll briefly see two: normal theory and bootstrap approach.
- Normal Theory Approach:
  - Also known as *Sobel test*, it is based on the same assumptions than those previously presented in regression models.
  - Knowing the sampling distribution of the test statistic and its standard error, it is possible to obtain the significance level and thus making a statistical decision.
    $$Z = \frac{a \times b}{se_{ab}} \sim N(0,1); \quad p = Prob\left(|Z| \geq |z| | H_0\right); \text{ where } se_{ab} = \sqrt{a^2 se_b^2 + b^2 se_a^2 + se_a^2 se_b^2}$$
  - Additionally, a $(1-\alpha)\%$ confidence interval can be obtained:
    $$a \pm Z_{\alpha/2} \times se_{ab}$$

Inference about the Indirect Effect:

- Bootstrap Confidence Interval approach:
  - Included amongst the so-called *resampling methods*, it is useful when the sampling distribution of the statistic is unknown or really hard to be analitically derived.
  - The original sample is treated as a representation of the population and it allows analysts to generate a sampling distribution for a statistic of interest by means of a (re)sampling with replacement.
  - In the mediation analysis we'll use Bootstrap for approximating a confidence interval for the true indirect effect (T^{ab}).
  - The steps followed to get a $(1-\alpha)$% Bootstrap percentile confidence interval:

  1. Randomly draw a sample of n cases with replacement (Bootstrap sample).
  2. Estimate the indirect effect (ab^*) in this new sample.
  3. Repeat (1) and (2) a total of k times, saving ab^* each time.
  4. Sort the k indirect effects estimated in ascending order.
  5. Given a confidence $1-\alpha$, obtain the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles from the sorted distribution.

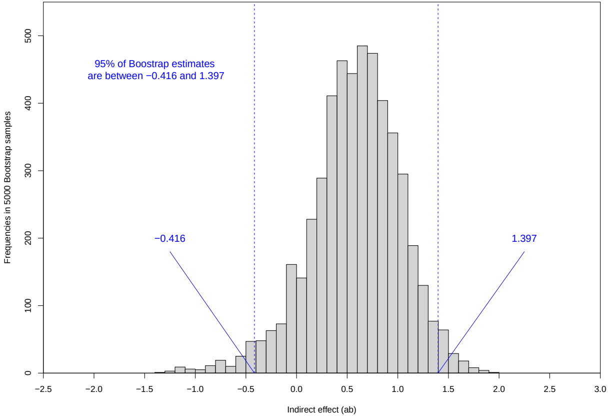
  - Other approaches for the CI building via Bootstrap are bias-correction (Boostrap BC) and bias-correction and acceleration (Bootstrap BCa).

Inference about the Indirect Effect:

- Let's assume a sample of $n = 10$. Then, two possible Bootstrap samples would be the following:

Original Sample				Bootstrap sample 1				Bootstrap sample 2			
Case	X	M	Y	Case	X	M	Y	Case	X	M	Y
1	0.00	1.50	3.00	2	0.00	2.00	2.75	4	0.00	2.50	4.50
2	0.00	2.00	2.75	1	0.00	1.50	3.00	10	1.00	5.00	5.00
3	0.00	1.00	3.00	6	1.00	4.50	4.00	3	0.00	1.00	3.00
4	0.00	2.50	4.50	8	1.00	3.00	3.50	1	0.00	1.50	3.00
5	0.00	4.00	4.75	7	1.00	2.50	2.50	7	1.00	2.50	2.50
6	1.00	4.50	4.00	9	1.00	1.50	2.00	3	0.00	1.00	3.00
7	1.00	2.50	2.50	6	1.00	4.50	4.00	1	0.00	1.50	3.00
8	1.00	3.00	3.50	9	1.00	1.50	2.00	7	1.00	2.50	2.50
9	1.00	1.50	2.00	2	0.00	2.00	2.75	9	1.00	1.50	2.00
10	1.00	5.00	5.00	6	1.00	4.50	4.00	1	0.00	1.50	3.00
a		1.10		a		1.310		a		1.375	
b		0.586		b		0.554		b		0.525	
ab		0.646		ab^*		0.726		ab^*		0.722	

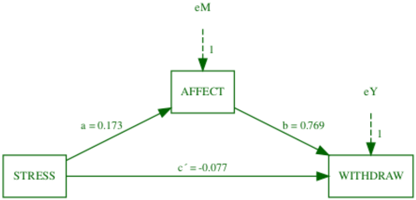
Inference about the Indirect Effect:



Mediation models

Simple mediation: Example

- Study on economic stress in entrepreneurs (100 women and 162 men) who responded an on-line survey concerning their business performance and their cognitive and emotional reactions to the economic climate (Pollack et al., 2012)*.
- The authors conjecture is that economic stress (X) leads to a higher intention to withdraw from entrepreneurial activities (Y) via a depressed affect (M). More specifically, stress increases feeelings of hopelessness which in turn leads to a higher intention to abandon business responsibilities. pre-main prep time: 2 ms



* Pollack, J.M., VanEpps, E.M., Hayes, A.F. (2012). The moderating effect of social ties on entrepreneurs' depressed affective and withdrawal intentions in response to economic stress. *Journal of Organizational Behavior*, *33*, 789-810.

		M (Affect)				Y (Withdrawal)		
Antecedent		Coeff.	SE	p		Coeff.	SE	p
X (Stress)	a	0.173	0.030	<.001	c'	-0.077	0.052	.146
M (Affect)		-	-	-	b	0.769	0.103	<.001
Constant	i_1	0.802	0.143	<.001	i_2	1.447	0.252	<.001
		$R^2 = 0.116$				$R^2 = 0.180$		
		$F(1, 260) = 33.999, p < .001$				$F(2, 259) = 28.495, p < .001$		

Adapted from Hayes (2022)*.

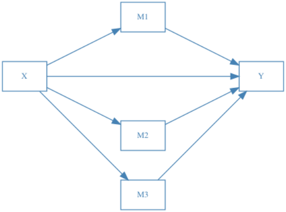
- Interpreting *ab*: Two entrepreneurs differing by one unit in their stress are estimated to differ by 0.133 units in the intention to withdraw from their business as a result of the tendency for those under relatively more economic stress to feel more depressed affect (sign of $a > 0$), which in turn translates into greater withdrawal intentions ($b > 0$).
- The 95% Bootstrap CI is $0.071; 0.201$. Thus, we conclude that there's a significantly different from zero indirect effect.
- The direct effect, $c' = -0.077$, is not significant ($t(259) = -1.467, p = .144; 95\% CI = [-0.180, 0.026]$).
- The total effect, $c = c' + ab = -0.077 + 0.133 = 0.056$ is not statistically different from zero ($t(260) = 1.035, p = .302; 95\% CI = [-0.051, 0.163]$).

* Hayes, A.F. (2022). *Introduction to Mediation, Moderation, and Conditional Process Analysis* (3rd. Ed). New York: Guilford Press.

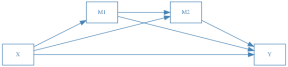
Mediation models

Multiple mediation

- The simple mediation model is useful for understanding aspects as the total effect partitioning or the rudiments of inferential procedures for mediation analysis. However it might oversimplify the causal mechanism of the studied phenomenon.
- It is justified to operate with multiple mechanisms at once since it allows to study more complex causal mechanisms to describe reality, to take into account the presence of *epiphenomenons* (X is correlated with a cause of Y but does not itself causally influence the outcome), and to assess competing theoretical mechanisms that might explain real phenomena.
- We'll focus on two forms: parallel and serial multiple mediator models. pre-main prep time: 1 ms pre-main prep time: 1 ms



Parallel mediation



Serial mediation

Parallel multiple mediator model:

pre-main prep time: 0 ms

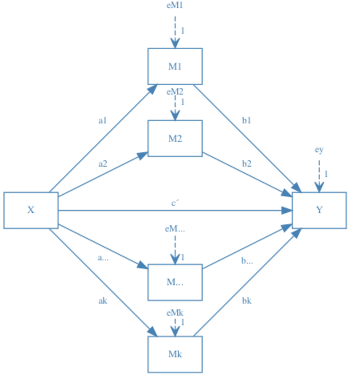
- A parallel multiple mediator model with k mediator has $k+1$ consequent variables.
- The model can be specified as follows:

$$M_i = i_{M_i} + a_i X + e_{M_i} \text{ for } i = 1 \text{ to } k$$

$$Y = i_Y + c' X + \sum_{i=1}^{k} b_i M_i + e_Y$$

- There are $k+1$ pathways corresponding to 1 direct and k specific indirect effects. The total indirect effect is thus computed as $\sum_{i=1}^{k} a_i b_i$.
- Therefore, the total effect of X on Y is:

$$c = c' + \sum_{i=1}^{k} a_i b_i$$



Mediation models

Multiple mediation

Mediation models

Moderation Models

Parallel multiple mediator model:

- Inference for direct and total effects can be straightforwardly done by means of tests adequate for OLS regression.
- Inference about specific indirect effects can be done by employing normal theory (it is not recommended):

$$a_i \times b_i \pm z_{\alpha/2} \times se_{a_i b_i}\text{; where } se_{a_i b_i} = \sqrt{a_i^2 se_{b_i}^2 + b_i^2 se_{a_i}^2 + se_{a_i}^2 se_{b_i}^2}.$$

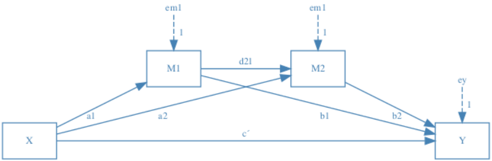
- As in simple mediation models, it is also possible to test specific indirect effects by carrying out Bootstrap CIs.
- It might be also interesting running pairwise comparisons between specific indirect effects. Again, we can use normal theory ($(a_i b_i - a_j b_j) \pm z_{\alpha/2 \times se_{a_i b_i - a_j b_j}}$; for further details see Hayes, 2022) or Bootstrapping, amongst other procedures.
- Finally, there are tests and confidence intervals for the total indirect effect (normal theory, Bootstrapping, and Monte Carlo approach).

Mediation models

Multiple mediation

Serial multiple mediator model:

- Unlike parallel multiple mediator model, serial multiple mediator models includes as an assumption that a mediator (or mediators) causally influences another mediator.
- For instance, the following diagram depicts a two-mediator model:
pre-main prep time: 1 ms



- So the model shown above includes three consequent variables and it can be specified as follows:

$$\begin{aligned} M_1 &= i_{M_1} + a_1 X + e_{M_1} \\ M_2 &= i_{M_2} + a_2 X + d_{21} M_1 e_{M_2} \\ Y &= i_Y + c' X + b_1 M_1 + b_2 M_2 + e_Y \end{aligned}$$

Mediation models
Moderation Models

Serial multiple mediator model:

- Note that the model with two mediators includes 4 pathways affecting Y: $X \rightarrow Y$, $X \rightarrow M_1 \rightarrow Y$, $X \rightarrow M_2 \rightarrow Y$, and $X \rightarrow M_1 \rightarrow M_2 \rightarrow Y$.
- The model with two mediators have three specific indirect effects and one direct effect. So, the total effect is $c = c' + a_1 b_1 + a_2 b_2 + a_1 d_{21} b_2$.
- Normal theory can be used for making decisions and estimating CIs for indirect effects. The formulae for indirect effect through a single mediator have been already presented. Standard error for the indirect effect of $a_1 d_{21} b_2$ is:

$$se_{a_1 d_{21} b2} = \sqrt{a_1^2 d_{21}^2 se_{b_2}^2 + a_1^2 b_2^2 se_{d_{21}}^2 + d_{21}^2 b_2^2 se_{a_1}^2}.$$

- Bootstrapping can also be used for estimating CIs for the indirect effects (specific, total, or pairwise comparisons) of a serial multiple mediator model.

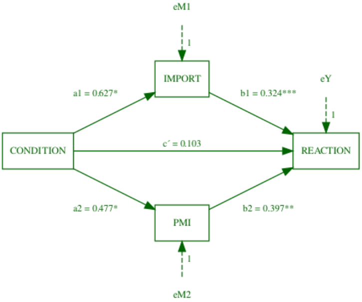
- Study on presumed media influence (Tal-Or et al., 2010)* in a sample of students (80 women and 43 men) who were randomly assigned and told to read one of the two newspaper articles (front page or interior page conditions, X) describing an economic crisis that might affect the price and supply of sugar in Israel. Then asked about their intention to buy sugar (Y) and about how much they believed others would be prompted to buy sugar as a result of this article (presumed media influence, $M2$). Additionally, they included some questions on perceived issue importance ($M1$).
- Thus authors hypothesized that article in the front page (X) would lead to a belief that others ($M2$) will be influenced to think of a likely shortage and so would buy sugar (Y). And to discard that this would be due to an epiphenomenon they included the perceived importance ($M1$).

* Tal-Or, N., Cohen, J., Tsfati, Y., Gunther, A.C. (2010). Testing causal direction in the influence of presumed media influence. *Communication Research*, *37*, 801—824.

Mediation models

Multiple mediation: Example

pre-main prep time: 1 ms

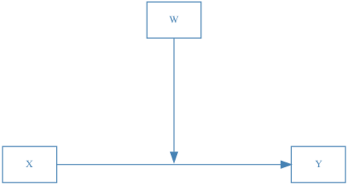


- Total effect was $c = 0.496$, $t(121) = 1.786, p = .08$, and $95\% CI = [-0.054; 1.045]$.
- Indirect effects were: $a_1 b_1 = 0.203$ (Bootstrap $95\% CI = [0.006; 0.455]$), $a_2 b_2 = 0.189$ (Bootstrap $95\% CI = [0.005; 0.4267]$). So, total indirect effect was 0.392 (Bootstrap $95\% CI = [0.091; 0.739]$).
- Direct effect: $c' = 0.103$, $t(119) = 0.432, p = .666$, and $95\% CI = [-0.370; 0.577]$.

Moderation models

Introduction

- The effect of X on some variable Y is said to be moderated by W if its size, sign, or strength depends on W. Then, W is called a moderator of X's effect on Y.
- Another way of describing moderation is that W and X interact in their influence on Y.
- Conceptual diagram is shown below: pre-main prep time: 2 ms

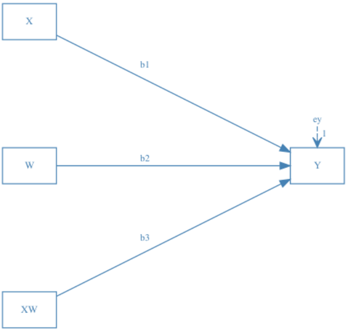


- Considering that model we can estimate it by using a multiple linear regression model (OLS).

Moderation models

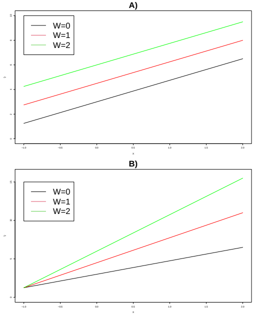
Introduction

- The statistical diagram including the effects and terms in a basic moderation model is: pre-main prep time: 1 ms



- It is important to distinguish between conditional and unconditional effects. To that aim, consider the model (A) $Y = i_Y + b_1 X + b_2 W + e_Y$ where the OLS estimates of the coefficients are: $i_Y = 3$, $b_1 = .75$, and $b_2 = 1.5$. Thus, $\hat{Y} = 3 + .75X + 1.5W$.
- Now consider a second model (B) where $Y = i_Y + b_1 X + b_2 W + b_3 XW + e_Y$ where the OLS estimates of the coefficients are: $i_Y = 3$, $b_1 = .75$, $b_2 = 1.5$, and $b_3 = 1.25$.

X	W	A	B
-1.00	0.00	1.25	1.25
-1.00	1.00	2.75	1.50
-1.00	2.00	4.25	1.75
0.00	0.00	3.00	3.00
0.00	1.00	4.50	4.50
0.00	2.00	6.00	6.00
1.00	0.00	4.75	4.75
1.00	1.00	6.25	7.50
1.00	2.00	7.75	10.25
2.00	0.00	6.50	6.50
2.00	1.00	8.00	10.50
2.00	2.00	9.50	14.50



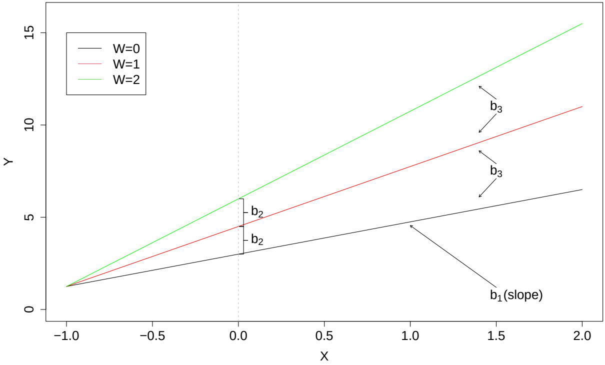
Moderation models

Basic moderation model

- In model A, the effects b_1 and b_2 are unconditional on W and X, respectively. So this model is not well-suited for testing moderation.
- In contrast, the effect of a unit-change in X on $\hat{Y}$ is $\theta_{X \to Y} = b_1 + b_3 W$ and is called the *conditional effect of X on Y*. In the example, $\theta_{X \to Y} = 0.75 + 1.25W$ and $\theta_{W \to Y} = 1.5 + 1.25W$:

	$\theta_{X \to Y} = b_1 + b_3 W$			$\theta_{W \to Y}\|X = b_2 + b_3 X$	
W	$b_1 + b_3 W$	$\theta_{X \to Y}\|W$	X	$b_2 + b_3 X$	$\theta_{W \to Y}\|X$
0.00	b_1	0.75	-1.00	$b_2 - b_3$	0.25
1.00	$b_1 + b_3$	2.00	0.00	b_2	1.50
2.00	$b_1 + 2b_2$	3.25	1.00	$b_2 + b_3$	2.75
3.00	$b_1 + 3b_2$	4.50	2.00	$b_2 + 2b_3$	4.00

- How are the coefficients interpreted?
  - b_3 estimates how much the differences in Y between two cases that differ by a unit in X changes as W changes by one unit.
  - b_1 represents the association between X and Y conditioned on $W = 0$.
  - b_2 represents the conditional effect of W on Y when $X = 0$.
- In the ANOVA jargon b_1 and b_2 are the *simple effects*.



W=0
W=1
W=2
Y
X
0
5
10
15
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
b_3
b_3
b_2
b_2
b_1(slope)

Moderation models

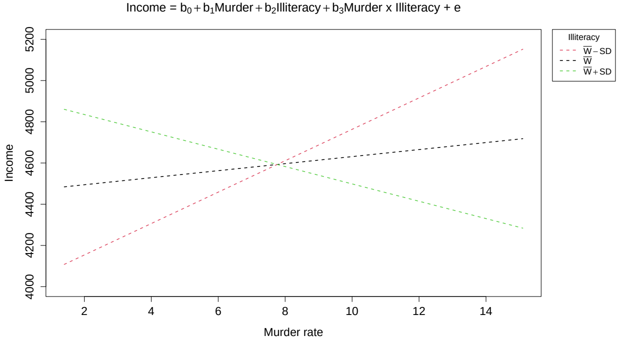
Basic moderation model

- Some caution should be taken when interpreting *conditional effects*, b_1 and b_2, since sometimes it is neither meaningful nor possible having $W = 0$ or $X = 0$.
- *Centering* both X and W makes interpretable b_1 and b_2: $W' = W - \bar{W}$ and $X' = X - \bar{X}$.
- Therefore, the model would be:

$$\hat{Y} = i_Y + b_1 X' + b_2 W' + b_3 X' W'.$$

- How do we interpret the conditional effects?
  - b_1 represents the association between X and Y conditioned on $W = \bar{W}$.
  - b_2 represents the expected change on Y explained by W when X is kept fixed on its mean.
- Interpreting b_3 might be challenging sometimes so it is suitable to visualize moderation by means of adequate plots (inspecting the relationship between X and Y given some previously selected W values).

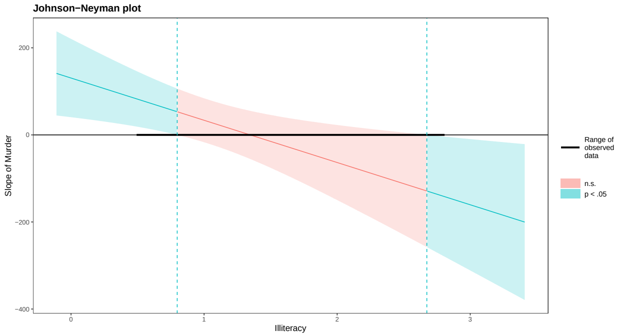
- If aiming to determine the boundaries of the moderator where is a significant effect of X on Y.
- One possible strategy for that is the *pick-a-point* approach. For instance, when W is numeric, researchers commonly pick $\bar{W} - SD$ (relatively low), $\bar{W}$ (moderate), and $\bar{W} + SD$ (relatively large). Another possibility is taking 10*th*, 50*th*, and 90*th* percentiles of the W's empirical distribution.



Moderation models

Basic moderation model

- To avoid the arbitrariness of the previous method, and given a continuous W, the Johnson-Neyman procedure can be used.
- This approach allows researchers to estimate the points of the moderator for which the conditional effect of X on Y transitions between statistically significant and not significant (i.e. a *region of significance*), given a significance level (α).



Moderation models

Example

- Study on climate change disasters and humanitarianism (Chapman Lickel, 2016)* in a sample (80 women and 43 men) where participants were randomly assigned and told to read one of the two news stories about a famine in Africa caused by severe droughts (attributing droughts to climate change or not providing information in this regard, X: 0=natural conditions or 1=climate change). Then asked about several justifications for not providing aid to the victims (justification, Y) and about how much they believed whether climate change was a real phenomenon (skepticism about climate change, W).
- The aim is to examine whether providing with a specific context to a disaster influences the decision on providing help to the victims and, additionally, if this effect depends on individuals' beliefs about climate change.
- Some models to be tested:

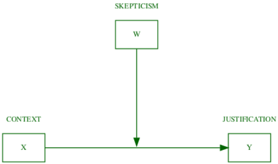
$$
\begin{aligned}
Y &= b_0 + b_1 X + \epsilon \text{ Model 1} \\
Y &= b_0 + b_1 X + b_2 W + \epsilon \text{ Model 2} \\
Y &= b_0 + b_1 X + b_2 W + b_3 (XW) + \epsilon \text{ Model 3}
\end{aligned}
$$

* Chapman, D.A., Lickel, B. (2016). Climate change and disasters: How framing affects justifications for giving or withholding aid to disaster victims. *Social Pychological and Personality Science*, *7*, 13—20.

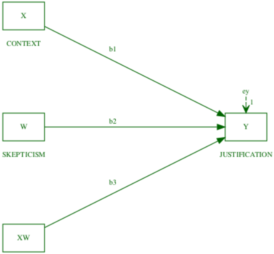
Moderation models

Example

pre-main prep time: 1 ms pre-main prep time: 1 ms



Conceptual diagram



Statistical diagram

Moderation models

Example

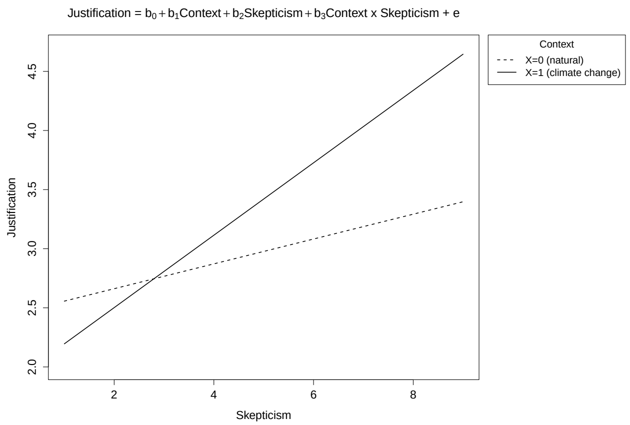
	Dependent variable:		
	Justification		
	(1)	(2)	(3)
Context	0.134	0.118	−0.562**
	(0.128)	(0.115)	(0.218)
Skepticism		0.201***	0.105***
		(0.028)	(0.038)
Context:Skepticism			0.201***
			(0.055)
Constant	2.802***	2.132***	2.452***
	(0.089)	(0.124)	(0.149)
Observations	211	211	211
R^2	0.005	0.198	0.246
Adjusted R^2	0.0005	0.190	0.235
Residual Std. Error	0.929 (df = 209)	0.837 (df = 208)	0.813 (df = 207)
F Statistic	1.100 (df = 1; 209)	25.677*** (df = 2; 208)	22.543*** (df = 3; 207)
Note:			*p<0.1; **p<0.05; ***p<0.01

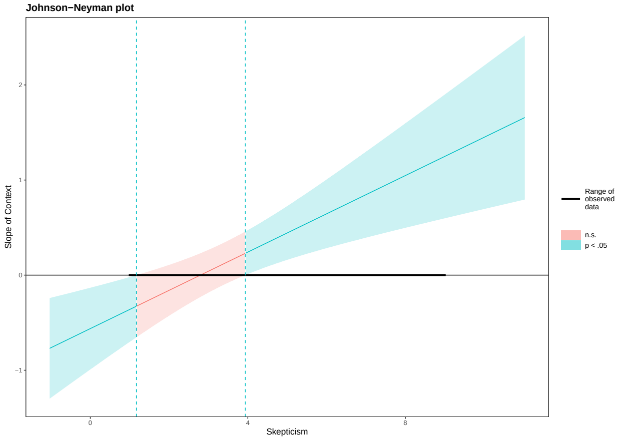
Moderation models

Example

Mediation models

Moderation Models





Johnson−Neyman plot
Slope of Context
Skepticism
2
1
0
−1
0
4
8
Range of observed data
n.s.
p < .05