Convex Optimization - Homework 6: Applications

David McPherson

April 21, 2016

1 Exercise 1: Two Stage Decision Making

1.1 Demand Measured prior to Purchase

The linear program formulation for maximizing profit is:

$$\max \left\{ \begin{bmatrix} 25 & 10 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} - \begin{bmatrix} 3.50 & 2.60 & -6.80 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right\} \tag{1}$$

subject to

$$s_i \leq D_i, \forall i \in \{1, 2\} \tag{2}$$

$$s_i \leq y_i, \forall i \in \{1, 2\} \tag{3}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \leq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{4}$$

where y_i is the quantity of drug i produced, s_i is the quantity of drug i sold, D_i is the quantity of drug i demanded, and x_i is the quantity of ingredient i purchased.

Implementing this formulation in CVX with a demand of $D_1 = 150$ and $D_2 = 200$ yields a profit of \$765 by producing and selling 150 units of Drug 1 and 0 units of Drug 2. To produce this, 450 units of Ingredient 1, 150 units of Ingredient 2, and 150 units of Ingredient 3 were acquired.

1.2 Profit Maximization for Three Demand Scenarios

The same formulation (with knowing the demand ahead of purchasing, producing, and selling) was used to maximize profit for each of the three demand scenarios. The maximum profit and associated drug production, selling, and ingredient purchases are filed in Table 1.

	Profit	s_1	s_2	y_1	y_2	x_1	x_2	x_3
Scenario 1	510	100	0	100	0	300	100	100
Scenario 2	765	150	0	150	0	450	150	150
Scenario 3	1020	200	0	200	0	600	200	200

Note that no amount of Drug 2 was produced, since we produce a \$0.60 loss with each sale.

1.3 Demand Measured prior to Selling

Both of the above analyses assume perfect information with demand being known before ingredients are acquired and any manufacturing takes place. However, a more realistic scenario would be one where we only learn demand after acquisition and assembly has already taken place. Now we need to optimize one choice of the x_i and y_1 such that the expected profit is maximized. We will use superscripts to denote versions of the variables that depend on the scenario. In this analysis, we have s_i^j denoting the amount of drug i sold in demand scenario j and D_i^j denoting the demand for drug i in the same scenario j. Therefore our expected profit optimization problem can be formulated as:

$$\max \left\{ [25 \quad 10] \left(0.2 \begin{bmatrix} s_1^1 \\ s_2^1 \end{bmatrix} + 0.5 \begin{bmatrix} s_1^2 \\ s_2^2 \end{bmatrix} + 0.3 \begin{bmatrix} s_1^3 \\ s_2^3 \end{bmatrix} \right) - [3.50 \quad 2.60 \quad -6.80] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right\} \tag{5}$$

subject to

$$s_i^j \le D_i^j, \forall i \in \{1,2\}, \forall j \in \{1,2,3\} \tag{6}$$

$$s_i^j \le y_i, \forall i \in \{1,2\}, \forall j \in \{1,2,3\} \tag{7}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \le \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{8}$$

Solving this formulation in CVX results in an expected profit of $515.00 with

	Profit	s_1	s_2	y_1	y_2	x_1	x_2	x_3
Scenario 1	-485	100	0					
Scenario 2	765	150	0	150	0	450	150	150
Scenario 3	765	150	0					

1.4 Demand Measured prior to Manufacture

A slightly better scenario would be if information on the drug demand was received prior to manufacture but after ingredient acquisition. In this case, we need to optimize one choice of the x_i such that the expected profit is maximized (we can tailor the y_i and s_i to the demand scenario). Now, our expected profit optimization problem can be formulated as:

$$\max \left\{ [25 \quad 10] \left(0.2 \begin{bmatrix} s_1^1 \\ s_2^1 \end{bmatrix} + 0.5 \begin{bmatrix} s_1^2 \\ s_2^2 \end{bmatrix} + 0.3 \begin{bmatrix} s_1^3 \\ s_2^3 \end{bmatrix} \right) - [3.50 \quad 2.60 \quad -6.80] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right\} \tag{9}$$

subject to

$$s_i^j \le D_i^j, \forall i \in \{1,2\}, \forall j \in \{1,2,3\} \tag{10}$$

$$s_i^j \le y_i^j, \forall i \in \{1,2\}, \forall j \in \{1,2,3\} \tag{11}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y_1^j \\ y_2^j \end{bmatrix} \leq \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \forall j \in \{1,2,3\} \tag{12}$$

Solving this formulation in CVX results in an expected profit of \$615.00 with

	Profit	s_1	s_2	y_1	y_2	x_1	x_2	x_3
Scenario 1	15	100	50	100	50			
Scenario 2	765	150	0	150	0	450	150	150
Scenario 3	765	150	0	150	0			

2 Exercise 2: Dicsount Factor Curve Fitting

2.1 Formulation

The formulation for this problem is:

$$\begin{aligned} &\min_a \sum_{i=1}^{I} |p_0 f(t_0^{(i)}) + p_1 f(t_1^{(i)})| \\ = &\min_a \sum_{i=1}^{I} |p_0(a_{0,k_{i0}} + a_{1,k_{i0}} t_0^{(i)} + a_{2,k_{i0}} t_0^{(i)2}) + p_1(a_{0,k_{i1}} + a_{1,k_{i1}} t_1^{(i)} + a_{2,k_{i1}} t_1^{(i)2})| \end{aligned} \tag{13}$$

where k_{ij} is the interval that $t_j^{(i)}$ falls in.
Subject to the constraints:

$$a_{0,k} + a_{1,k} m_{k+1} + a_{2,k} m_{k+1}^2 = a_{0,k+1} + a_{1,k+1} m_{k+1} + a_{2,k+1} m_{k+1}^2, \forall k \in \{1,2,\ldots,N-1\} \tag{14}$$

$$a_{1,k} + 2a_{2,k} m_{k+1} = a_{1,k+1} + 2a_{2,k+1} m_{k+1}, \forall k \in \{1,2,\ldots,N-1\} \tag{15}$$

$$a_{1,k} + 2a_{2,k} m_k \leq 0 a_{1,k} + 2a_{2,k} m_{k+1} \leq 0, \forall k \in \{1,2,\ldots,N\} \tag{16}$$

$$a_{0,N} + a_{1,N} T + a_{2,N} T^2 \geq 0 \tag{17}$$

2.2 CVX Solution

3 Exercise 3: Probability Density Estimation

3.1 Reward Function

3.2 Optimization Formulation

3.3 CVX Solution

3.4 Non-Increasing Requirement

3.5 Bounded Slope Requirements