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t8code - modular adaptive mesh refinement in the exascale era

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Summary

In this note we present our software library t8code for scalable dynamic adaptive mesh refinement (AMR) officially released in 2022 (Holke et al., 2022). t8code is written in C/C++, open source, and readily available at www.dlr-amr.github.io/t8code. The library provides fast and memory efficient parallel algorithms for dynamic AMR to handle tasks such as mesh adaptation, load-balancing, ghost computation, feature search and more. t8code can manage meshes with over one trillion mesh elements (Holke et al., 2021) and scales up to one million parallel processes (Holke, 2018). It is intended to be used as mesh management back end in scientific and engineering simulation codes paving the way towards high-performance applications of the upcoming exascale era.

Introduction

AMR has been established as a successful approach for scientific and engineering simulations over the past decades (Babuvška & Rheinboldt, 1978; Bangerth et al., 2007; Dörfler, 1996; Teunissen & Keppens, 2019). By modifying the mesh resolution locally according to problem specific indicators, the computational power is efficiently concentrated where needed and the overall memory usage is reduced by orders of magnitude. However, managing adaptive meshes and associated data is a very challenging task, especially for parallel codes. Implementing fast and scalable AMR routines generally leads to a large development overhead motivating the need for external mesh management libraries like t8code.

t8code is written in C/C++, open source, and the latest release can be obtained at https://dlr-amr.github.io/t8code (Holke et al., 2022). It uses efficient space-filling curves (SFC) to manage the data in structured refinement trees. While in the past being successfully applied to quadrilateral and hexahedral meshes (Burstedde et al., 2011; Weinzierl, 2019), t8code extends these SFC techniques in a modular fashion, such that arbitrary element shapes are supported. We achieve this modularity through a novel decoupling approach that separates high-level (mesh global) algorithms from low-level (element local) implementations. All high-level algorithms can then be applied to different implementations of element shapes and refinement patterns. A mix of different element shapes in the same mesh is also supported.

Currently, t8code provides implementations of Morton type SFCs with $1 : 2^d$ refinement for vertices ($d = 0$), lines ($d = 1$), quadrilaterals, triangles ($d = 2$), hexahedra, tetrahedra, prisms, and pyramids ($d = 3$). The latter having a $1 : 10$ refinement rule with tetrahedra emerging as child elements (Knapp, 2020). Additionally, implementation of other refinement patterns and SFCs is possible according to the specific requirements of the application.

The purpose of this note is to provide a brief overview and a first point of entrance for software developers working on codes storing data on (distributed) meshes.

For further information beyond this short note and also for code examples, we refer to our Documentation and Wiki (Holke et al., 2022) and our other technical papers on t8code (Becker, 2021; Burstedde & Holke, 2016, 2017; Dreyer, 2021; Elsweijer, 2021; Holke, 2018; Holke et al., 2021; Knapp, 2020; Lilikakis, 2022).

Fundamental Concepts

t8code is based on the concept of tree-based adaptive mesh refinement. Starting point is an unstructured input mesh, which we call coarse mesh that describes the geometry of the computational domain. The coarse mesh elements are refined recursively in a structured pattern, resulting in refinement trees of which we store only minimal information of the finest elements (the leafs of the tree). We call this resulting fine mesh the forest.

By enumerating the children in the refinement pattern we obtain a space-filling curve logic. Via these SFCs, all elements in a refinement tree are assigned an index and are stored in linear order of these indices. Information such as coordinates or element neighbors do not need to be stored explicitly, but can be recovered from the index and the appropriate information of the coarse elements. The less elements the input mesh has, the more memory and runtime are saved through the SFC logic. t8code supports distributed coarse meshes of arbitrary size and complexity, which we tested for up to 370 million input elements~ (Burstedde & Holke, 2017).

The forest mesh is distributed, that is, at any time, each parallel process only stores a unique portion of the forest mesh, the boundaries of which are calculated from the SFC indices; see Fig. Figure 1.

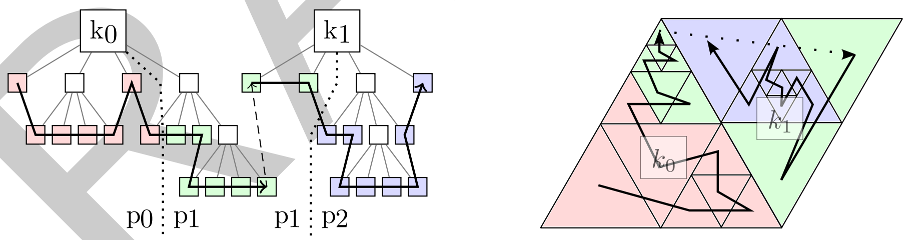

Figure 1: Left: Quad-tree of an exemplary forest mesh consisting of two trees (k_0, k_1) distributed over three parallel processes P0 to P2. The SFC is represented by a black curve tracing only the finest elements (leaf nodes) of each tree. Right: Sketch of the associated triangular mesh refined up to level three.

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