The Journal of Open Source Software

t8code - modular adaptive mesh refinement in the exascale era

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DOI: 10.xxxxxx/draft

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Editor: Open Journals

Reviewers:

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Submitted: 01 January 1970
Published: unpublished

Summary

In this note we present our software library t8code for scalable dynamic adaptive mesh refinement (AMR) officially released in 2022 (Holke et al., 2022). t8code is written in C/C++, open source, and readily available at www.dlr-amr.github.io/t8code. The library provides fast and memory efficient parallel algorithms for dynamic AMR to handle tasks such as mesh adaptation, load-balancing, ghost computation, feature search and more. t8code can manage meshes with over one trillion mesh elements (Holke et al., 2021) and scales up to one million parallel processes (Holke, 2018). It is intended to be used as mesh management backend in scientific and engineering simulation codes paving the way towards high-performance applications of the upcoming exascale era.

Statement of need

Adaptive mesh refinement (AMR) has been established as a successful approach for scientific and engineering simulations over the past decades (Babuvška & Rheinboldt, 1978; Bangerth et al., 2007; Dörfler, 1996; Teunissen & Keppens, 2019). By modifying the mesh resolution locally according to problem specific indicators, the computational power is efficiently concentrated where needed and the overall memory usage is reduced by orders of magnitude. However, managing adaptive meshes and associated data is a very challenging task, especially for parallel codes. Implementing fast and scalable AMR routines generally leads to a large development overhead motivating the need for external mesh management libraries like t8code.

Currently, t8code's AMR capabalities supports vertices ($d = 0$), lines ($d = 1$), quadrilaterals, triangles ($d = 2$), hexahedra, tetrahedra, prisms, and pyramids ($d = 3$). The latter having a $1 : 10$ refinement rule with tetrahedra emerging as child elements (Knapp, 2020). Additionally, implementation of other refinement patterns and element shapes is possible according to the specific requirements of the application. t8code aims to provide a comprehensive mesh management framework for a wide range of use cases in science and engineering applications.

Fundamental Concepts

t8code is based on the concept of tree-based adaptive mesh refinement. Starting point is an unstructured input mesh, which we call coarse mesh that describes the geometry of the computational domain. The coarse mesh elements are refined recursively in a structured pattern, resulting in refinement trees of which we store only minimal information of the finest elements (the leafs of the tree). We call this resulting fine mesh the forest.

By enumerating the children in the refinement pattern we obtain a space-filling curve logic. Via these SFCs, all elements in a refinement tree are assigned an index and are stored in linear order of these indices. Information such as coordinates or element neighbors do not need to be stored explicitly, but can be recovered from the index and the appropriate information of the coarse elements. The forest mesh can be distributed, that is, at any time, each parallel process only stores a unique portion of the forest mesh, the boundaries of which are calculated from the SFC indices; see Figure 1.

While in the past being successfully applied to quadrilateral and hexahedral meshes (Burstedde et al., 2011; Weinzierl, 2019), t8code extends these SFC techniques in a modular fashion, such that arbitrary element shapes are supported. We achieve this modularity through a novel decoupling approach that separates high-level (mesh global) algorithms from low-level (element local) implementations. All high-level algorithms can then be applied to different implementations of element shapes and refinement patterns. A mix of different element shapes in the same mesh is also supported.

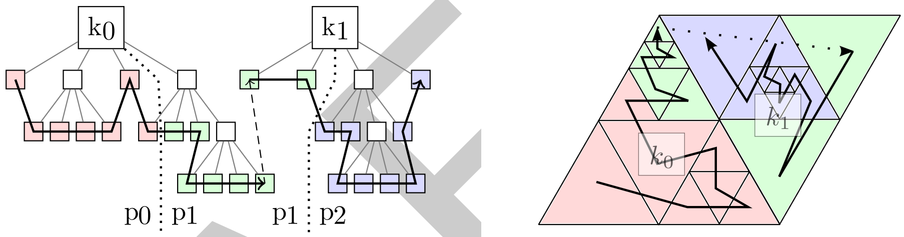

Figure 1: Left: Quad-tree of an exemplary forest mesh consisting of two trees (k_0, k_1) distributed over three parallel processes P0 to P2. The SFC is represented by a black curve tracing only the finest elements (leaf nodes) of each tree. Right: Sketch of the associated triangular mesh refined up to level three.

Performance

t8code supports distributed coarse meshes of arbitrary size and complexity, which we tested for up to 370 million input elements (Burstedde & Holke, 2017). Moreover, we present some of our benchmark results from various performance studies conducted on the JUQUEEN (*JUQUEEN Supercomputer*, n.d.) and the JUWELS (*JUWELS Supercomputer*, n.d.) supercomputers at the Jülich Supercomputing Center. t8code's Ghost and Partition routines are exceptionally fast with proper scaling of up to 1.1 trillion mesh elements; see Table 1, (Holke et al., 2021). In **??** we show a strong scaling result for a tetrahedral mesh achieving ideal strong scaling efficiency for the Ghost algorithm. Furthermore, in a prototype code (Dreyer, 2021) implementing a high-order discontinuous Galerkin method (DG) for advection-diffusion equations on dynamically adaptive hexahedral meshes we obverse a 12 times speed-up compared to non-AMR meshes with only an overall 10 to 15% runtime contribution of t8code; see autoref{fig:t8code_runtimes}.

# process	# elements	# elem. / process	Ghost	Partition
49,152	1,099,511,627,776	22,369,621	2.08 s	0.73 s
98,304	1,099,511,627,776	11,184,811	1.43 s	0.33 s

Runtimes on JUQUEEN for the ghost layer and partitioning operations for a distributed mesh consisting of 1.1 trillion elements.

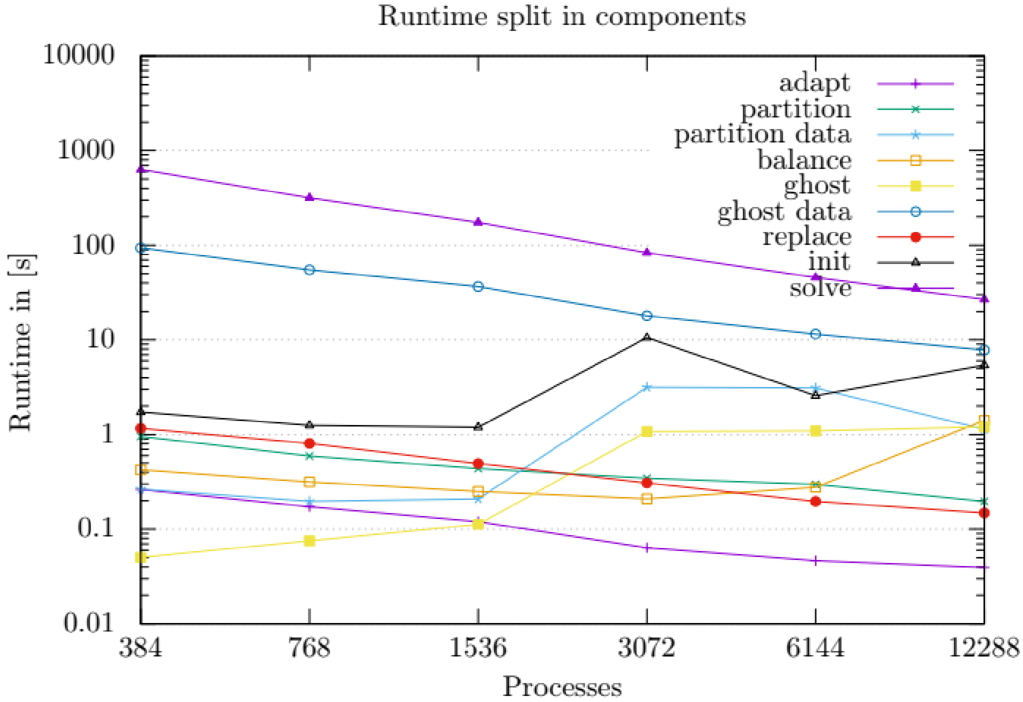

Figure 2: Runtimes on JUQUEEN of the different components of our DG prototype code coupled with t8code. Note that all features associated with dynamical mesh adaptation utilize only around 15% of the total runtime largely independent of the number of processes.

Conclusion

In this note, we introduce our open source AMR library t8code. We give a brief overview of the fundamental design principles and high-level operations. Due to the high modularity, t8code can be easily extended for a wide range of use cases. Performance results confirm that t8code is a solid choice for mesh management in high-performance applications in the upcoming exascale era.

For further information beyond this short note and also for code examples, we refer to our Documentation and Wiki (Holke et al., 2022) and our other technical papers on t8code (Becker, 2021; Burstedde & Holke, 2016, 2017; Dreyer, 2021; Elsweijer, 2021; Holke, 2018; Holke et al., 2021; Knapp, 2020; Lilikakis, 2022).

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