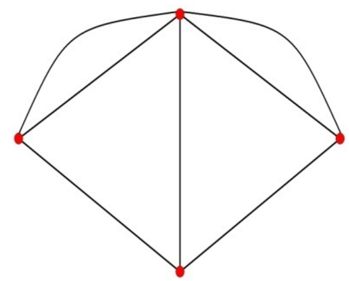
Social and Economic Networks: Models and Analysis



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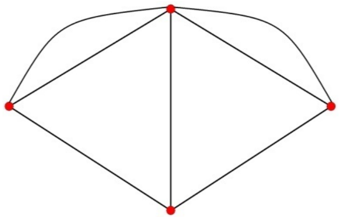
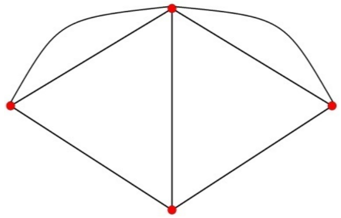
5.1: Diffusion

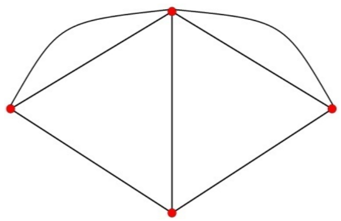


Outline



- Part I: Background and Fundamentals
  - Definitions and Characteristics of Networks (1,2)
  - Empirical Background (3)
- Part II: Network Formation
  - Random Network Models (4,5)
  - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
  - Diffusion and Learning (7,8)
  - Games on Networks (9)

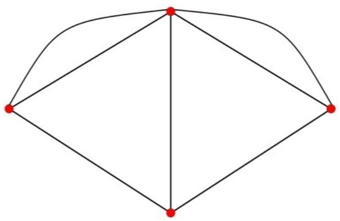
Networks and Behavior



- How does network structure impact behavior?

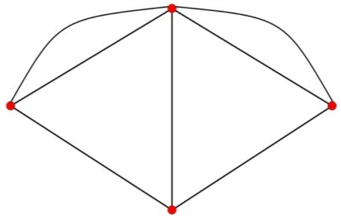
- Simple infections, contagion – diffusion
- Opinions, information – learning
- Choices, decisions – games on networks

Diffusion



- disease
- Ideas basic information (know or not know)
- Buy a product or not (come back to complementarities later...)

Diffusion

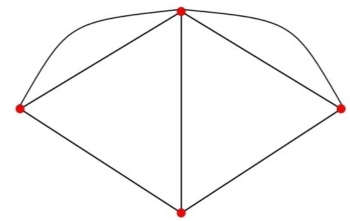


- Questions and Background

- Bass Model – no networks

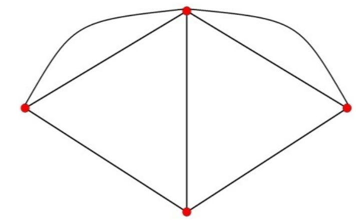
- Bring in interaction structure

S-Shape Adoption



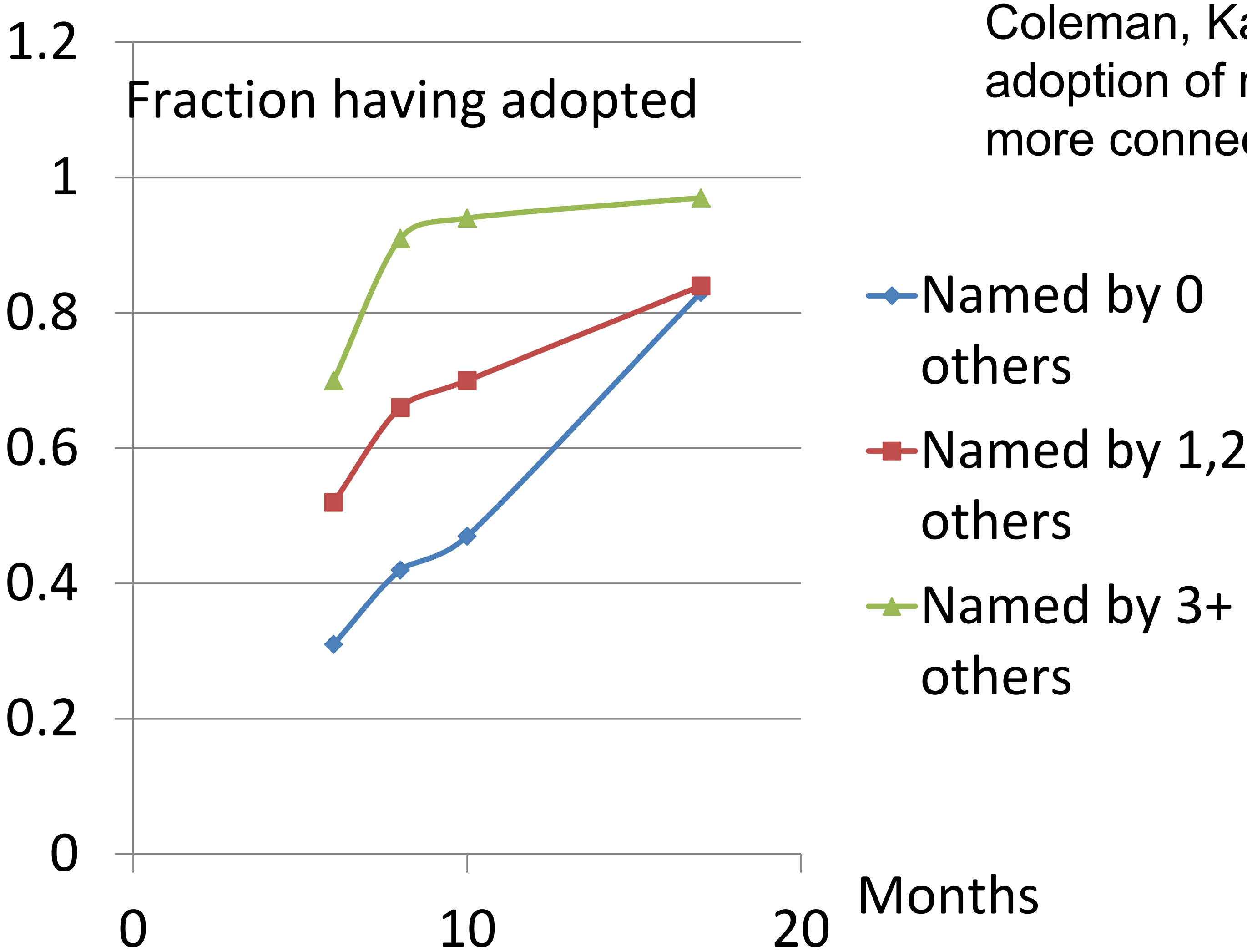
- Diffusion over time and space
  - Griliches economic story: variation in cost effectiveness by geography
- Initial adopters
  - Who are they?   High degree? Innovators?
- Increase in speed
  - Word of mouth, observations of neighbors
- Eventual slowdown
  - Saturation

Diffusion: Coleman, Katz, Menzel (1966)



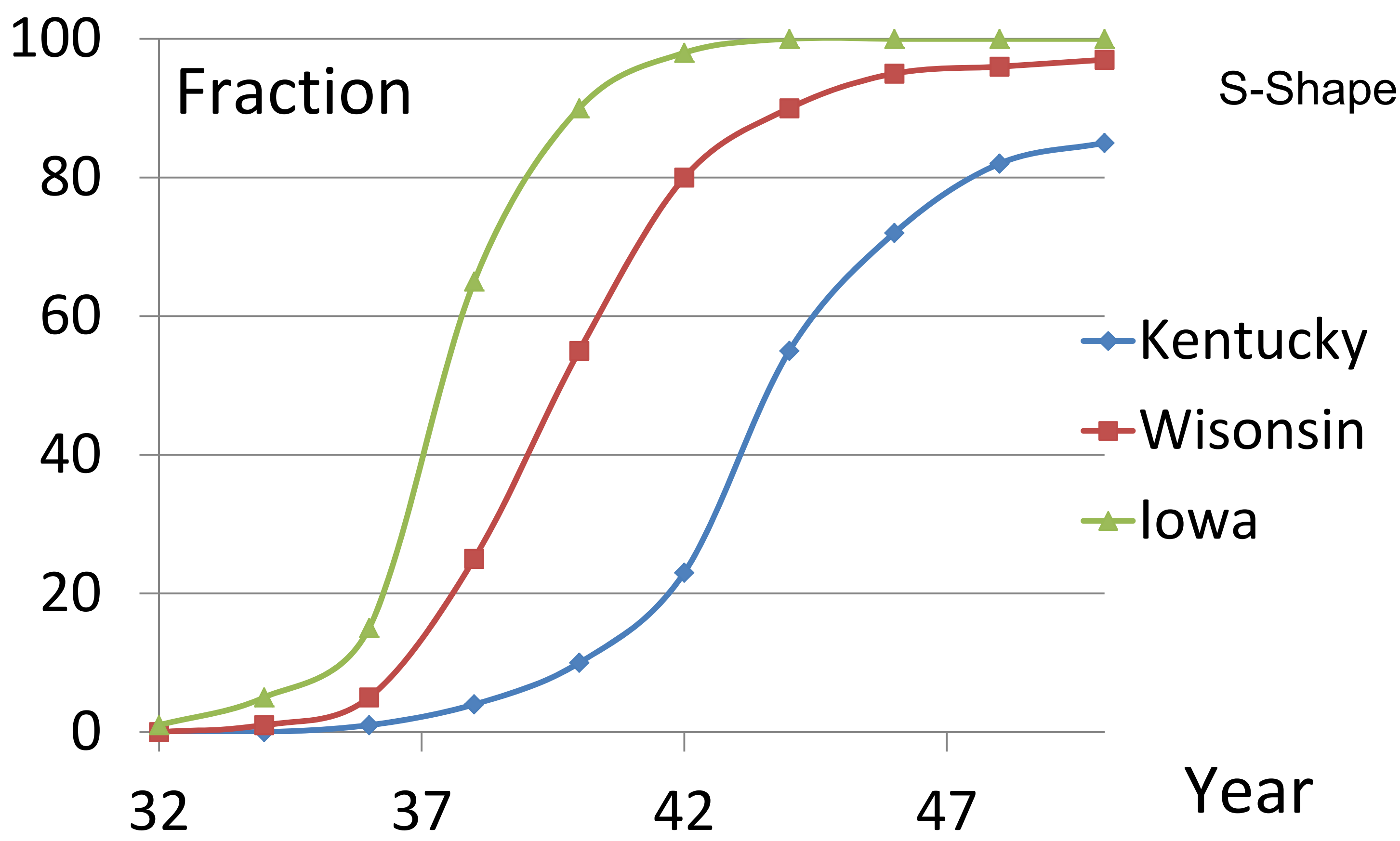
adoption (prescribing) of new drug by doctors:
more connected are earlier adopters

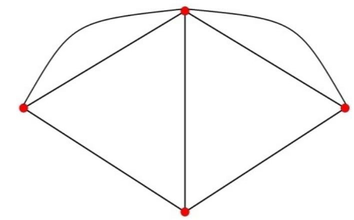
Fraction Adopting by:	names by 0 others (36)	named by 1 or 2 others (56)	named by 3+ others (33)
6 months	.31	.52	.70
8 months	.42	.66	.91
10 months	.47	.70	.94
17 months	.83	.84	.97



Coleman, Katz, Menzel (1966)
adoption of new drug by doctors
more connected earlier adopters

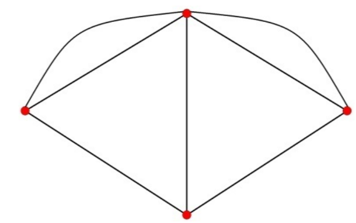
Griliches (1957): Hybrid Corn Diffusion





S-Shape, Spatial Pattern...

Questions:



- Extent of diffusion?
- How does it depend on the particulars of the process as well as the network?
- Time patterns? S-shape?
- Welfare analysis?

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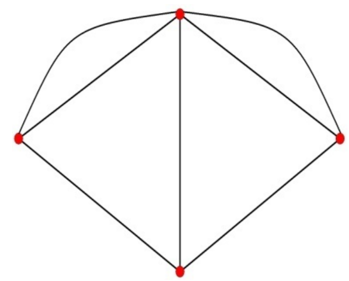

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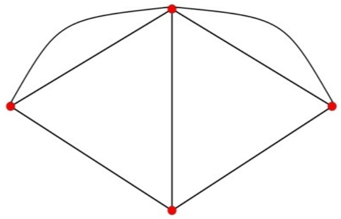
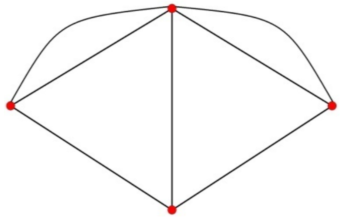
5.2: Bass Model

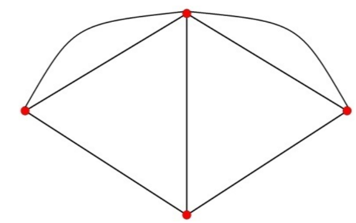


Diffusion



- Questions and Background
- Bass Model – no networks
- Bring in interaction structure

Bass Model

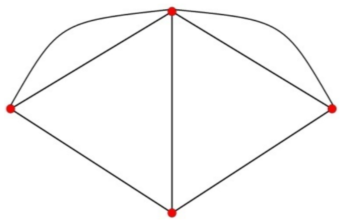


- A benchmark model with no explicit social structure

- Two actions/states/behaviors 0 and 1

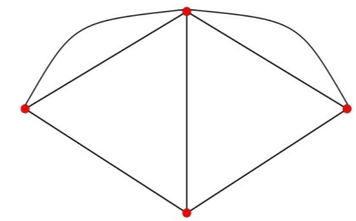
- F(t) fraction of the population who have adopted action 1 at time t

Bass Model



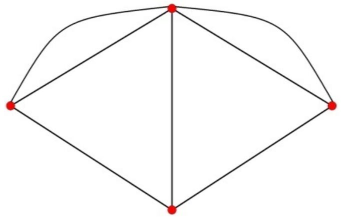
- p rate of spontaneous innovation/adoption
- q rate of imitation of adoption
- $dF(t)/dt = (p + q\,F(t))(1-F(t))$

Solution:



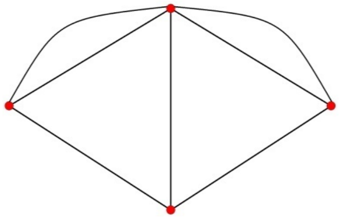
- p rate of spontaneous innovation/adoption
- q rate of imitation of adoption
- $dF(t)/dt = (p + q\,F(t))(1-F(t))$
- $F(t) = (1-e^{-(p+q)t})/\,(1+qe^{-(p+q)t}/p)$

Getting the S-shape



- Gives S-shape (if q>p) and tends to 1 in the limit
- Initially only p matters, then q takes over
- Eventually change slows as F(t) approaches 1

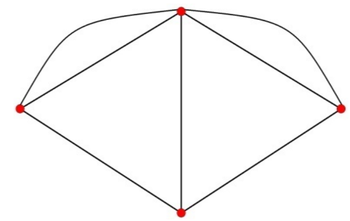
Getting the S-shape



$$dF(t)/dt = (p + q\, F(t))(1-F(t))$$

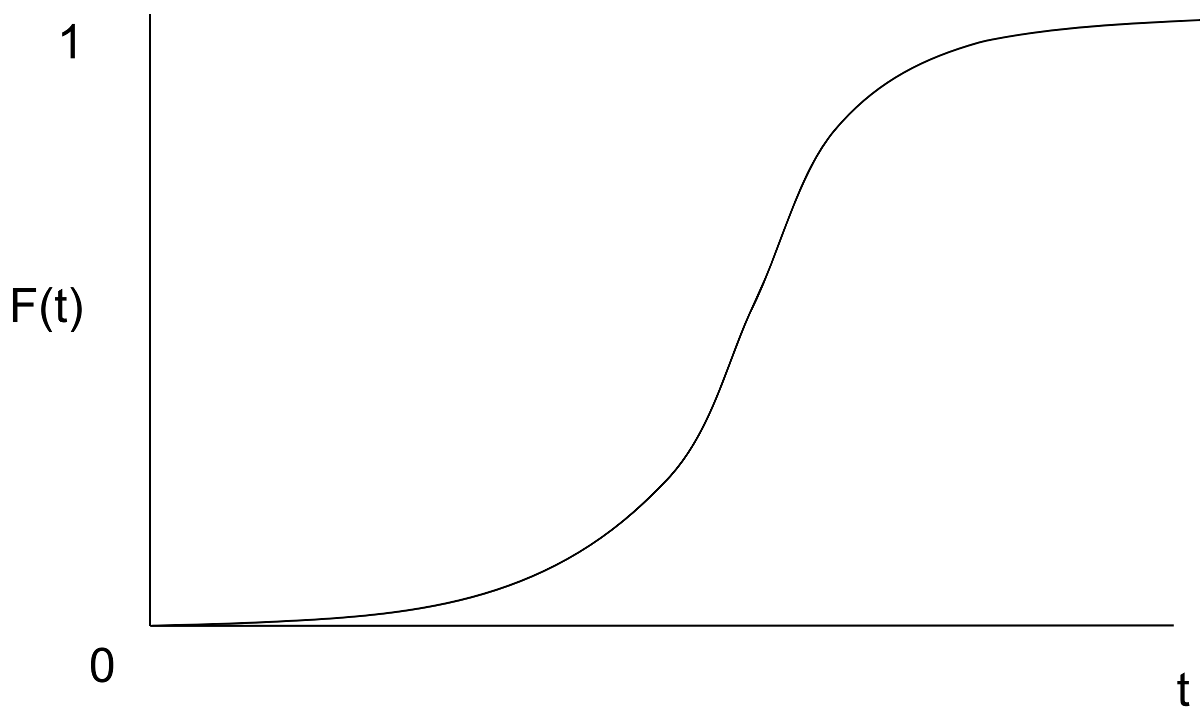
- when F(t) nears 1 , $dF(t)/dt = 0$
- when $F(t)=0$, $dF(t)/dt = p$

Getting the S-shape

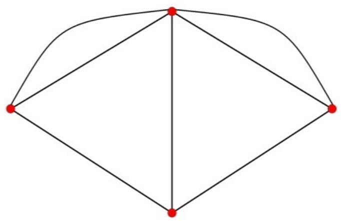


$$dF(t)/dt = (p + q\, F(t))(1-F(t))$$

- when $F(t)$ nears 1 , $dF(t)/dt = 0$
- when $F(t)=0$, $dF(t)/dt = p$
- when $F(t)=\varepsilon$, $dF(t)/dt = (p + q\, \varepsilon)\, (1- \varepsilon)$
- to get initial convexity: need $(p + q\, \varepsilon)\, (1- \varepsilon) > p$
- $q\ (1- \varepsilon) > p$ so initially need $q > p$


1
F(t)
0
t

Next: Beyond Bass - Component Structure



- Reach of diffusion is bounded by the component structure
- Some players or nodes are immune
- Some links fail to transmit…
- Answers questions of when get diffusion, and its extent (neither answered by simple Bass)

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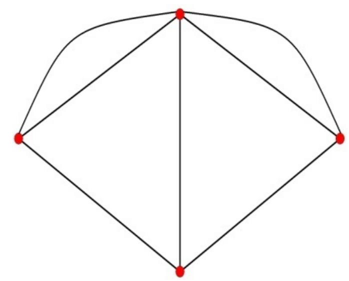

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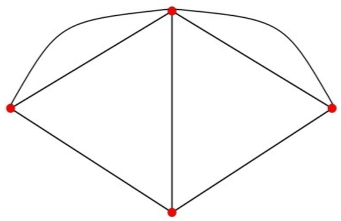
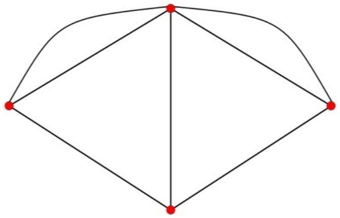
5.3: Diffusion on Random Networks

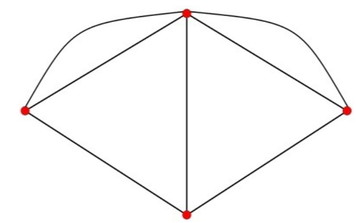


Diffusion



- Questions and Background
- Bass Model – no networks
- Bring in interaction structure

Random Networks and Diffusion



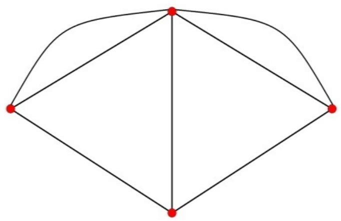
- Idea, disease, computer virus spreads via connections in the network

- Nodes are linked if one would ``infect'' the other

- Will an infection take hold?

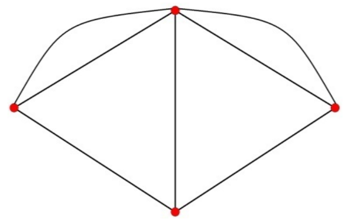
- How many nodes/people will it reach?

Questions:



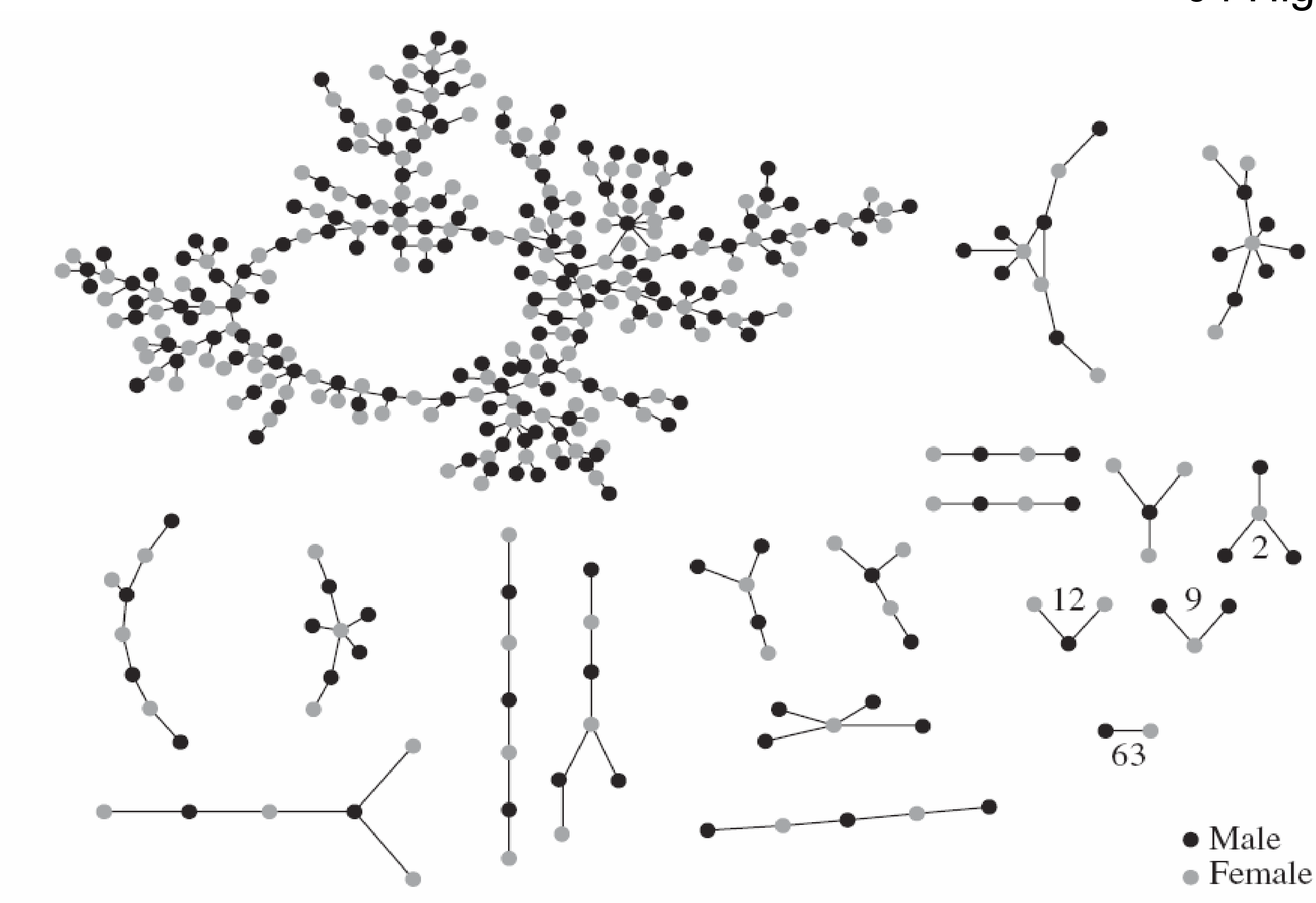
- When do we get diffusion?
- What is the extent of diffusion?
- How does it depend on the particulars of the process as well as the network?
- Who is likely to be infected earliest?

Component Structure

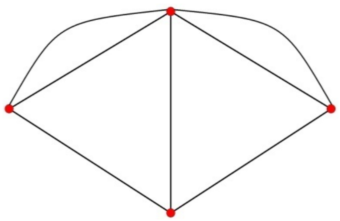


- Reach of contagion is determined by the component structure
- Some players or nodes are immune, Some links fail to transmit…
- What do components look like of those who are susceptible and given links that work

Bearman, Moody, and Stovel's
04 High School Romance

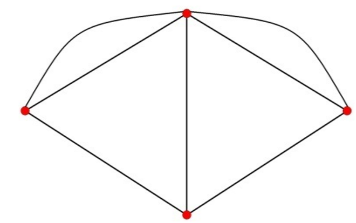


Extent of Diffusion



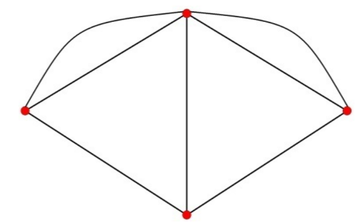
- Get nontrivial diffusion if someone in the giant component is infected/adopts
- Size of the giant component determines likelihood of diffusion and its extent
- Random network models allow for giant component calculations

Extent of Diffusion



- Simple example of such a calculation
- Work with Erdos-Renyi random network
- How big is the giant component??

Size of the Giant Component:



- How big is the giant component when there is one?

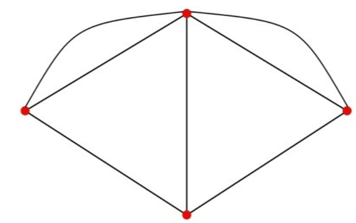
- Size of the giant component when

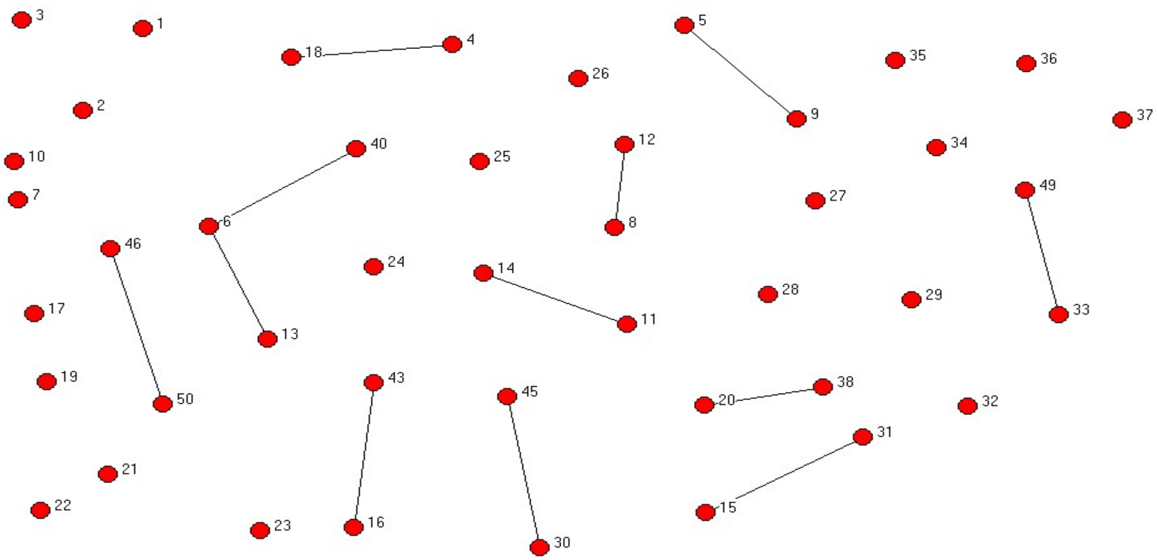
  $1/n < p < \log(n)/n$

[know that if $p \ll 1/n$ all isolated, and if

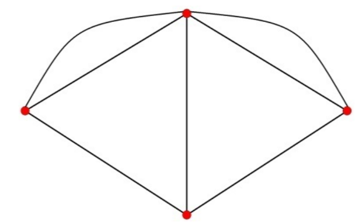
$\log(n)/n \ll p$ then all path connected]

Poisson p=.01, 50 nodes

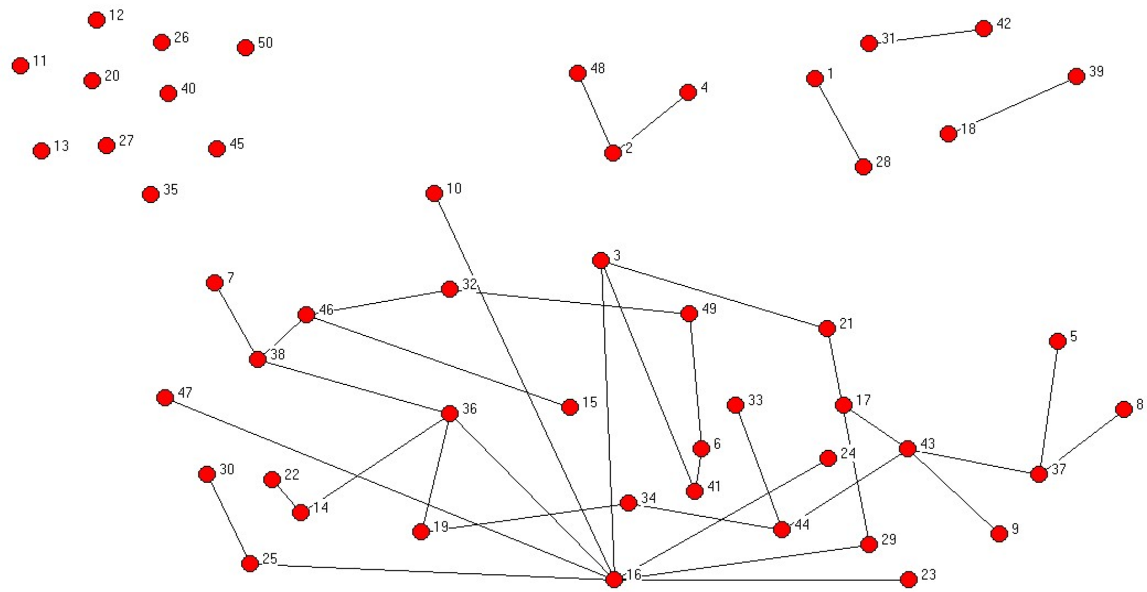




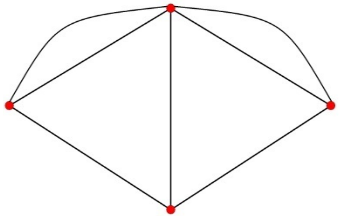
Poisson p=.03, 50 nodes

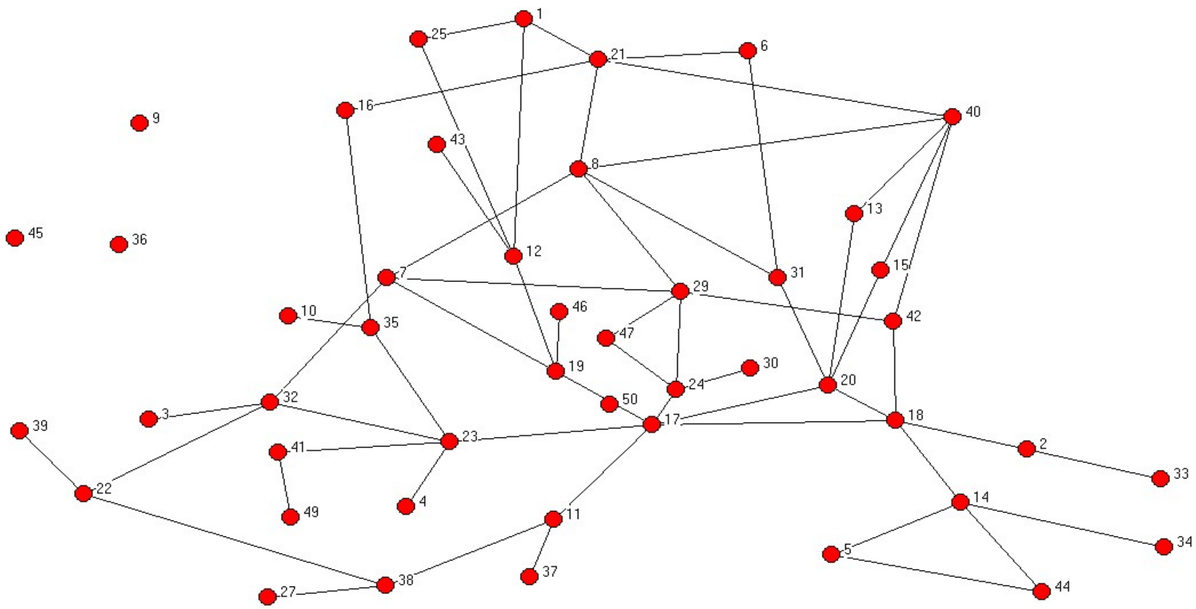


.02 is the threshold for emergence of cycles and a giant component

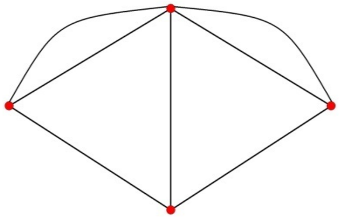


Poisson p=.05, 50 nodes

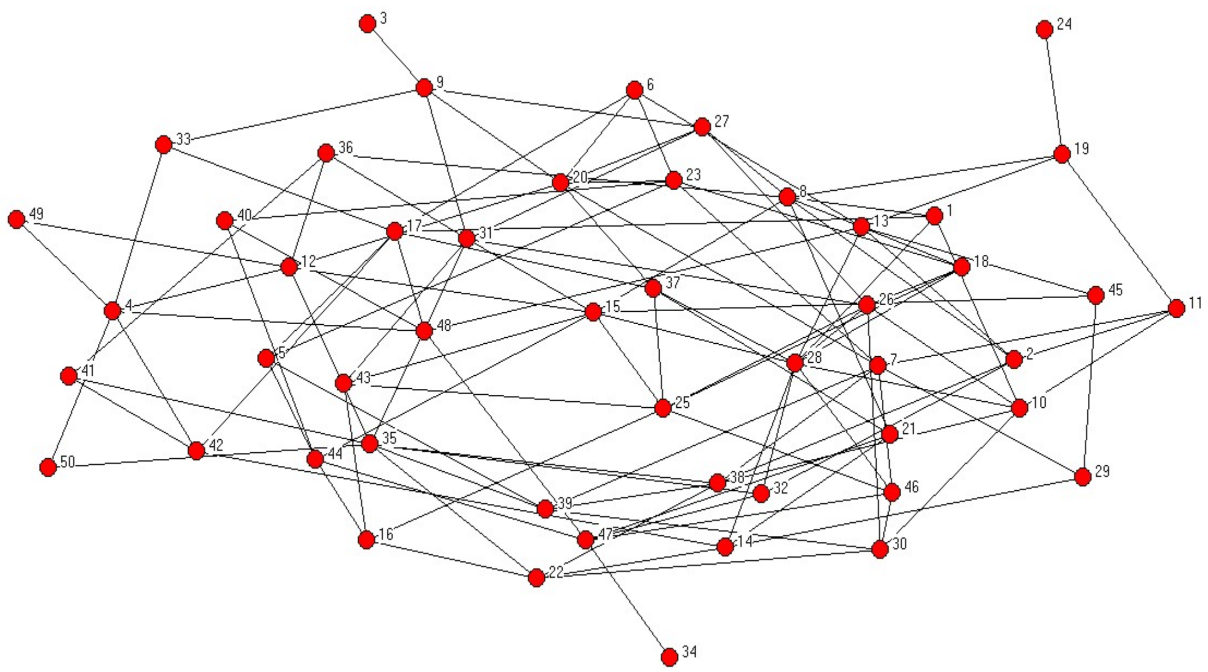




Poisson p=.10, 50 nodes



.08 is the threshold for connection



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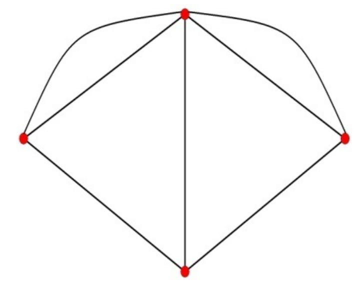

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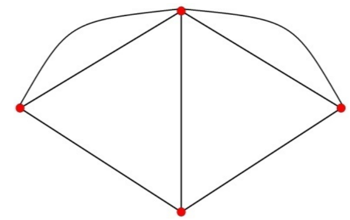
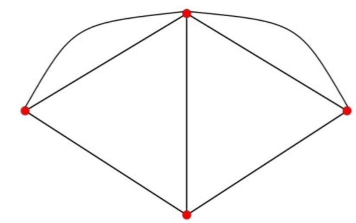
5.4: Giant Component
Poisson Case



Calculating the Size of the Giant Component

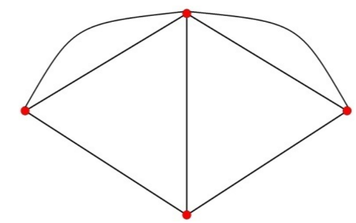


- **q** is fraction of nodes in largest component

- look at any node: chance it is in the giant component is **q**

- chance that this node is outside of the giant component is *the chance that all of its neighbors are outside of the giant component*

Calculating the Size of the Giant Component

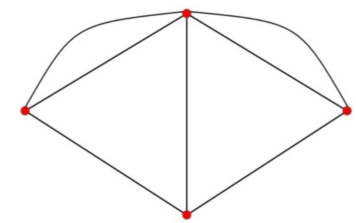


- Probability that a node is outside of the giant component = 1-q

  = probability that all of its neighbors are outside

  = $(1\text{-}q)^d$ where d is the node's degree

Giant Component Size

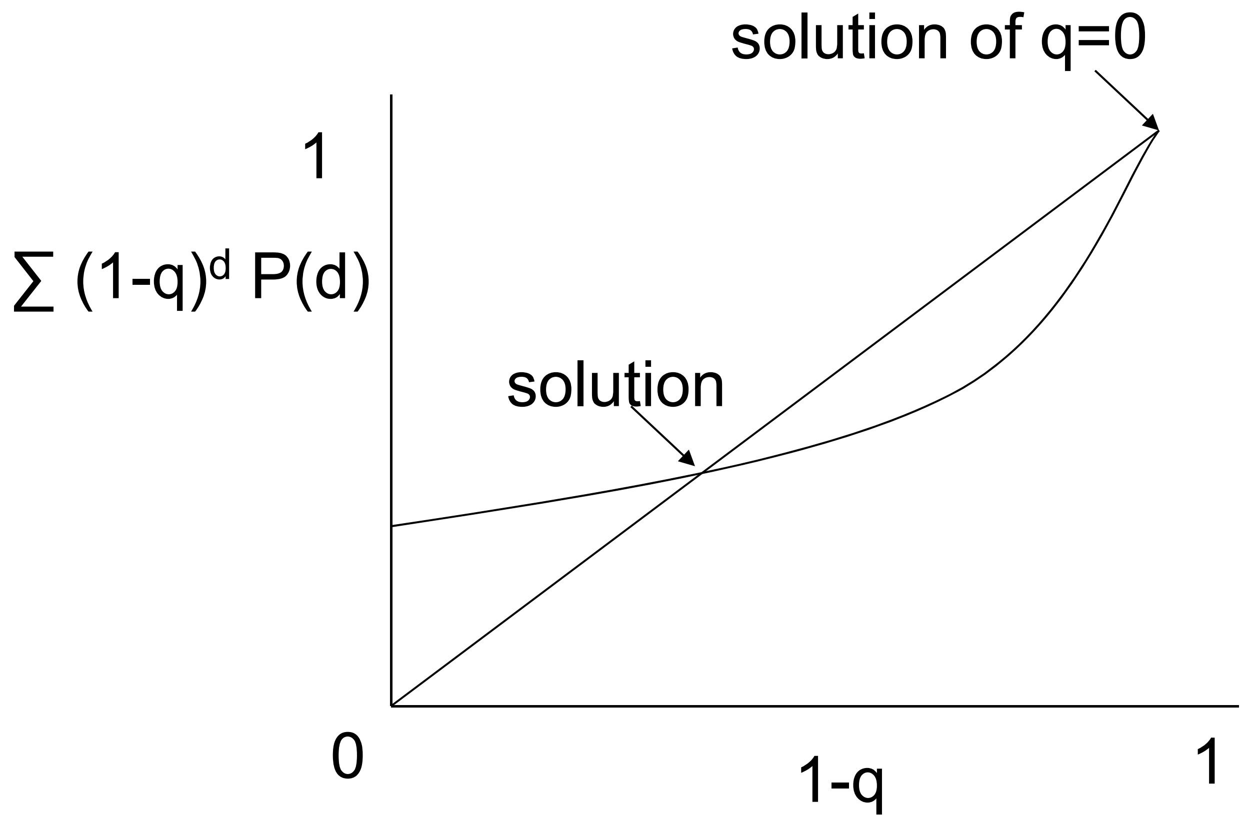


- So, probability 1-q that a node is outside of the giant component is

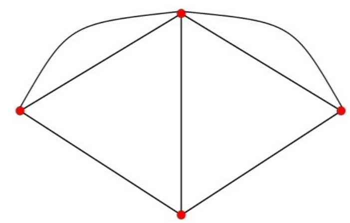
$$1-q = \sum (1-q)^d P(d)$$

Where P(d) is the chance that the node has d neighbors

- Solve for q…


solution of q=0
1
∑ (1-q)^d P(d)
solution
0
1-q
1

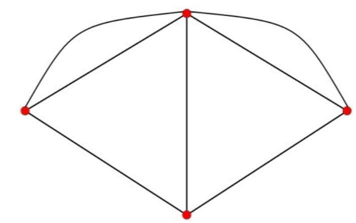
Giant Component Size: Poisson Case



Solve $1\text{-}q = \sum (1\text{-}q)^d P(d)$

when $P(d) = [\, (n\text{-}1)^d / d! \,]\ p^d e^{-(n-1)p}$

Giant Component Size: Poisson Case

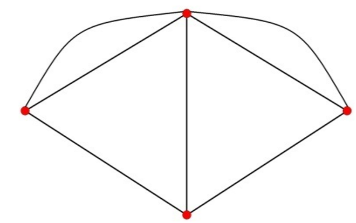


Solve $1-q = \sum (1-q)^d P(d)$

when $P(d) = [\, (n-1)^d / d! \,] \; p^d e^{-(n-1)p}$

so $1-q = e^{-(n-1)p} \sum [(1-q)\,(n-1)p]^d / d!$

Useful Approximations



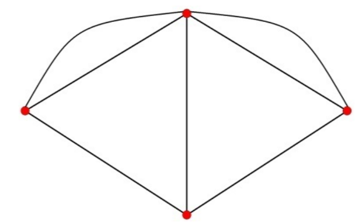
Taylor series approximation:

$$e^x = 1 + x + x^2/2! + x^3/3! \ldots$$

$$= \sum \; x^d / d!$$

$$[\, f(x) = f(a) + f'(a)(x-a)/1! + f''(a)(x-a)^2/2! \ldots]$$

Giant Component Size: Poisson Case



Solve $1\text{-}q = \sum (1\text{-}q)^d P(d)$

when $P(d) = [\ (n\text{-}1)^d / d!\ ]\ p^d e^{-(n-1)p}$

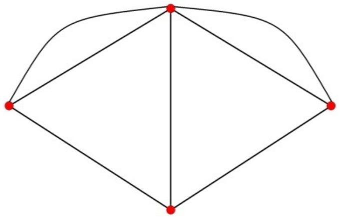
so $1\text{-}q = e^{-(n-1)p} \sum [(1\text{-}q)(n\text{-}1)p]^d / d!$

$= e^{-(n-1)p} e^{(n-1)p(1-q)}$

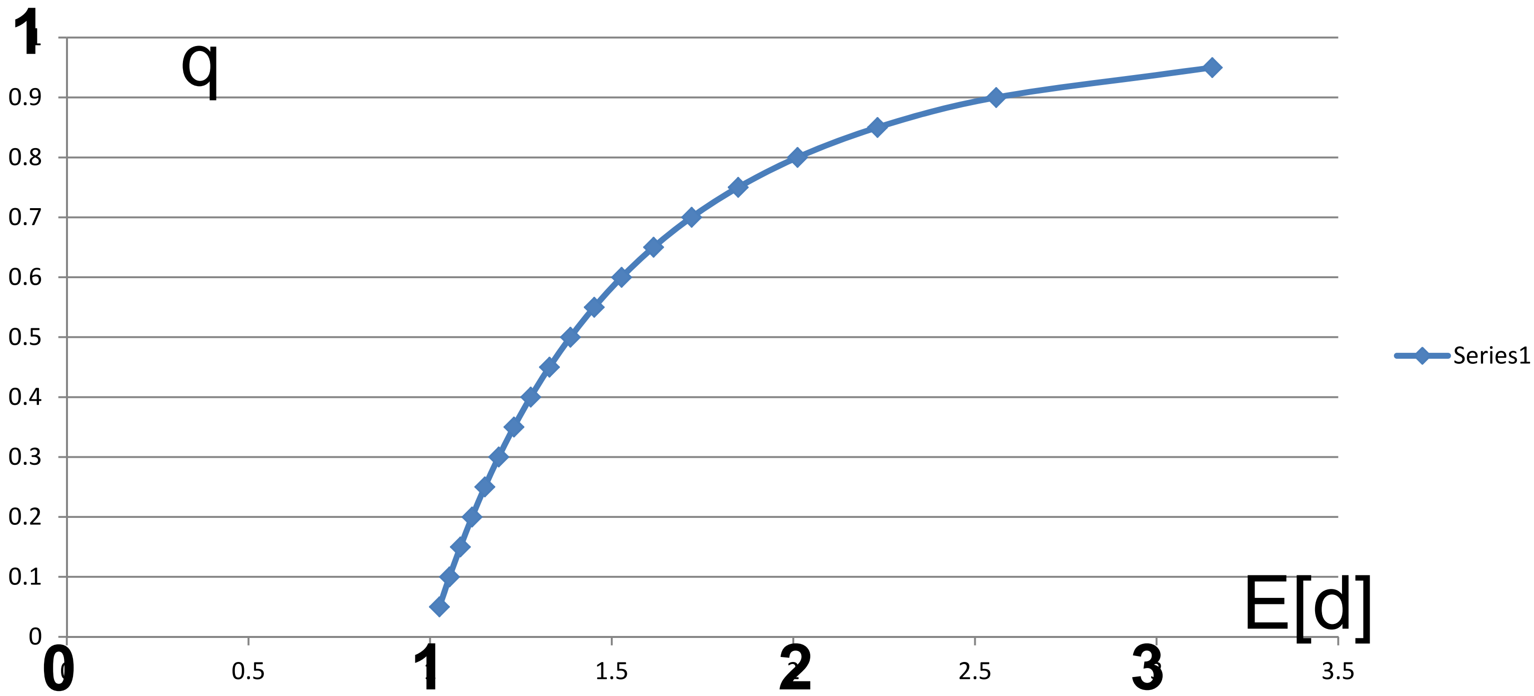
$= e^{-q(n-1)p}$

or $-\log(1\text{-}q) / q = (n\text{-}1)\, p = E[d]$

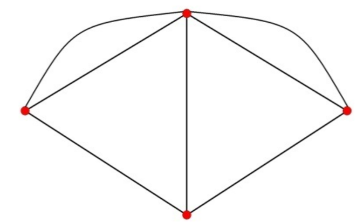
Giant Component Size:



$$-\log(1-q) / q = E[d]$$



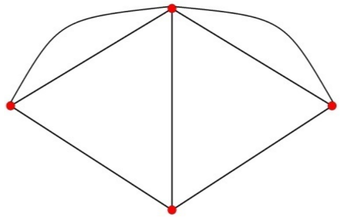
Who is infected?



- Probability of being in the giant component:
- $1-(1-q)^d$ increasing in d
- More connected, more likely to be infected

(more likely to be infected at any point in time...)

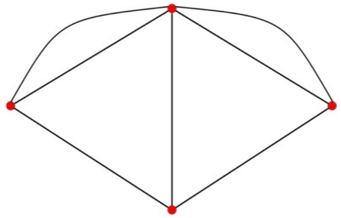
Extensions:



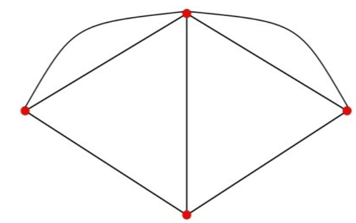
- **Immunity:** delete a fraction of nodes and study the giant component on remaining nodes

- **Probabilistic infection**
  - Random infection: have some links fail, just lower p

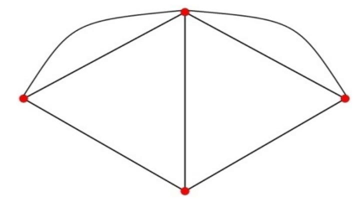
Contagion with Immunity and Link Failure



- Some node is initially exposed to infection
- π of the nodes are immune naturally
- only some links result in contagion – fraction f
- What is the extent of the infection?



- Consider a random network on n nodes
- Delete fraction π of the nodes
- Delete fraction 1-f of the links
- If starts at a node in giant component of the remaining network, then the giant component of that network is the extent of the infection; otherwise negligible



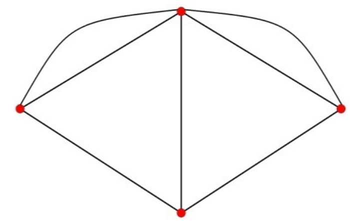
- Let q be the fraction of nodes of the remaining network in its giant component

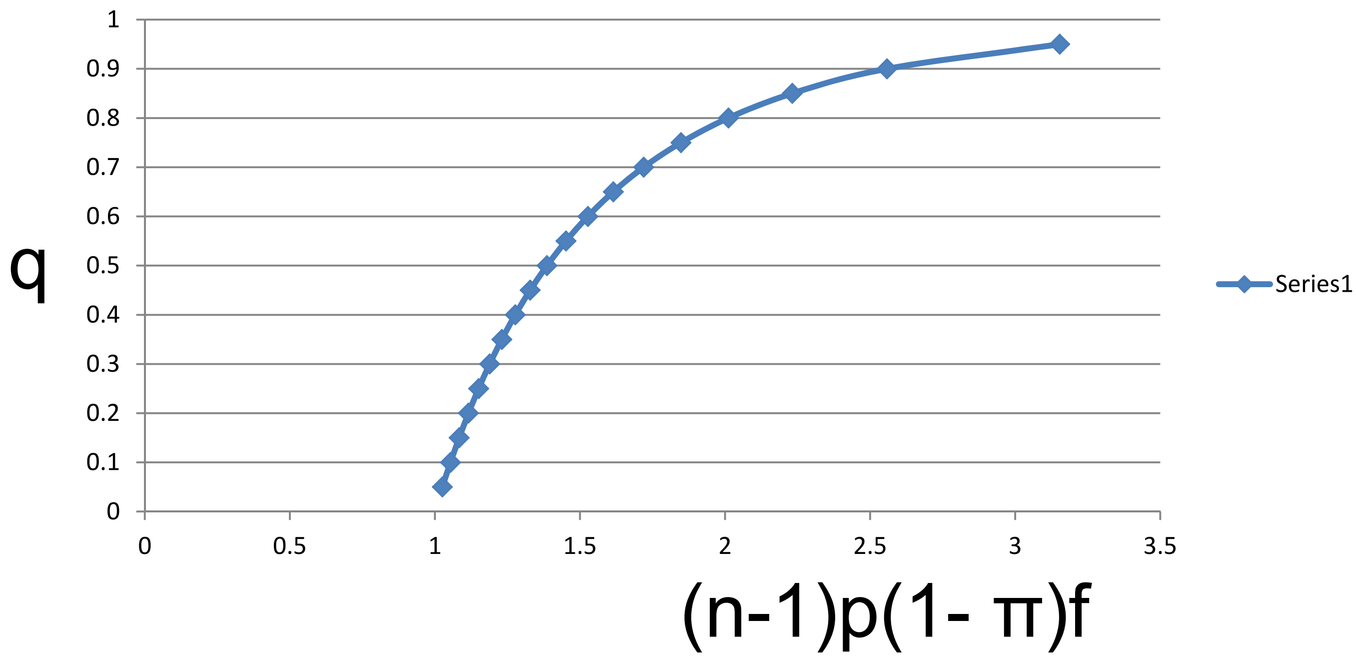
- $q(1-\pi)$ is the probability of a nontrivial contagion

- Conditional on a contagion it infects $q(1-\pi)$ of the original nodes

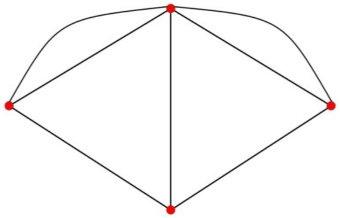
- q solves $\quad -\log(1-q)/q = (n-1)p(1-\pi)f$

Infected Fraction of Nodes:



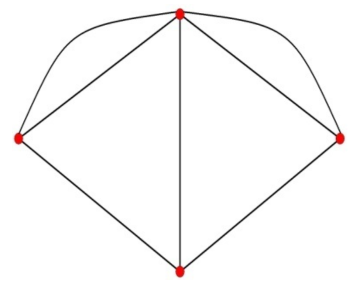


Implications:



- Infection can fail if π is high enough or f or p are low enough
- High π - immunization, low virulence
- Low f - low contagiousness
- Low p - low contact among population

Social and Economic Networks: Models and Analysis



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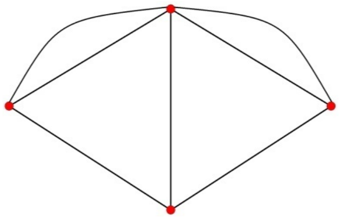
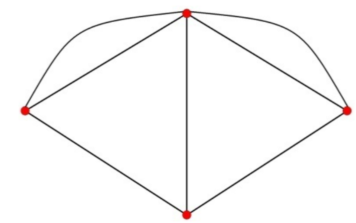
5.5: SIS Model

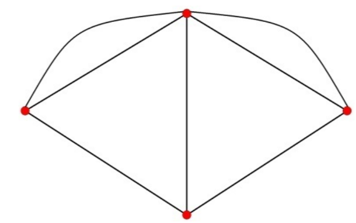


SIS Model



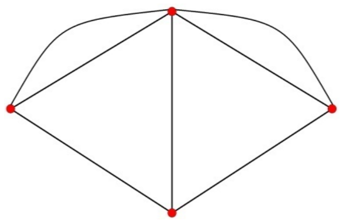
- An extensively studied model in epidemiology
- Allows nodes to change behaviors back and forth over time
- Model of catching some recurring diseases, who to vote for, etc.

SIS Model



- Nodes are infected or susceptible
- Probability that get infected is proportional to number of infected neighbors with rate $v>0$, plus spontaneous ε
- get well randomly in any period at rate $\delta>0$
- Let ρ be the percent infected

SIS Model



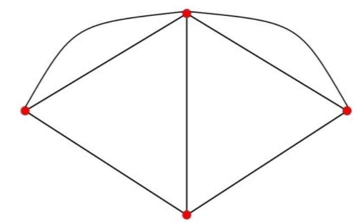
- Start with benchmark where all players mix with even probabilities

- Randomly meet an individual each period

- Large Markov chain

- Steady state mean-field: $d\rho/dt = 0$

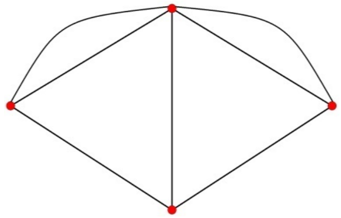
Mean-Field



$$d\rho/dt = (1-\rho)(v\rho+\varepsilon) - \rho\delta = 0$$

$$\rho = [\ (v-\delta-\varepsilon)+ ((v-\delta-\varepsilon)^2 + 4\ \varepsilon v)^{1/2}\ ] \ / \ 2v$$

``Mean-Field'' drop ε



$$d\rho/dt = (1-\rho)v\rho - \rho\delta$$

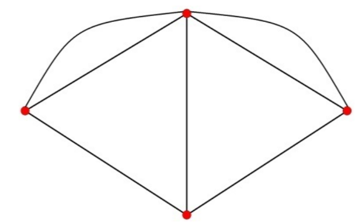
$$(1-\rho)v\rho - \rho\delta = 0$$

Two solutions:

$$\rho = 1 - \delta/v \quad (\text{if} > 0)$$

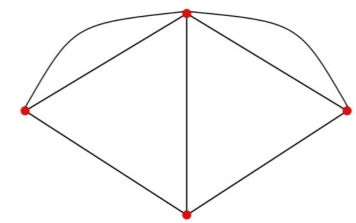
$$\rho = 0$$

Implications:



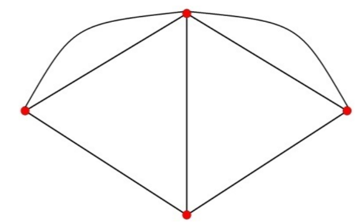
- $\rho = 1 - \delta/v$
- If $\delta > v$ then recover faster than get sick, no infection stays
- Otherwise, infection stays at some level, for low recovery rates can lead to large infections

Where's the network?



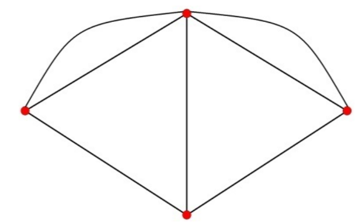
- so far uniformly random interaction
- missing heterogeneity in degree
- missing local patterns
- we can at least address the first concern...

Explore Degree Distribution Influence



- random matching with d_i matches for node i
- ρ(d) fraction of nodes of degree d infected
- θ fraction of randomly chosen neighbors infected

Chance that meet an infected node



- P(d) fraction of nodes that have d meetings

- More likely to meet someone who has high d

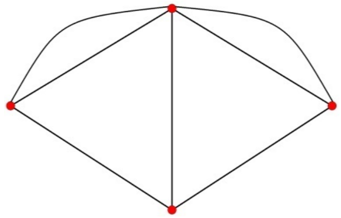
- likelihood of meeting node of degree d is

  $P(d)\ d\ /E[d]$

- So likelihood of meeting infected node is:

  $\theta = \Sigma\ \rho(d)\ P(d)\ d\ /\ E[d]$

Mean Field: Pastor-Satorras and Vespignagi 2000

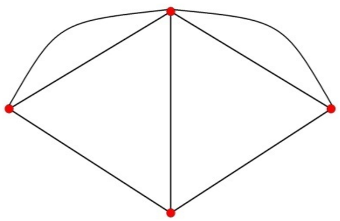


- $\theta = \Sigma\ \rho(d)\ P(d)\ d\ /\ E[d]$ fraction of infected neighbors/random partners

- Steady state: for each d

  $0 = d\rho(d)\ /dt = (\ 1-\ \rho(d)\ )v\theta d\ -\ \rho(d)\ \delta$

Solving

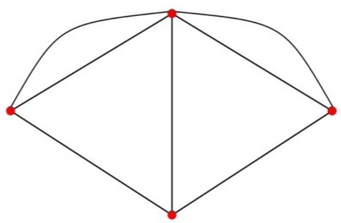


- Steady state: for each d

$$0 = d\rho(d)/dt = (1 - \rho(d))v\theta d - \rho(d)\,\delta$$

$$\rho(d) = \lambda\theta d / (\lambda\theta d + 1) \quad \text{where } \lambda = v/\delta$$

Solving



$$\rho(d) = \lambda\theta d \,/\, (\lambda\theta d + 1) \quad \text{where } \lambda = v/\delta$$

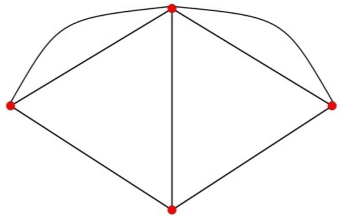
$$\theta = \Sigma\, \rho(d)\, P(d)\, d \,/\, E[d]$$

$$= \Sigma\, P(d)\, \lambda\theta d^2 \,/[\, (\lambda\theta d + 1)\, E[d]]$$

- Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 \,/[\, (\lambda\theta d + 1)\, E[d]]$$

Solving



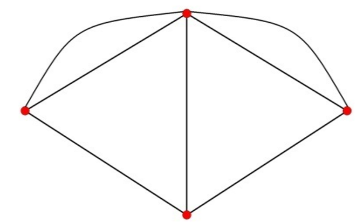
Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 /[\,(\lambda\theta d + 1)\, E[d]]$$

What can we say about how this depends on the ``network structure''?

How does infection rate of neighbors θ depend on $P(d)$, $E(d)$?

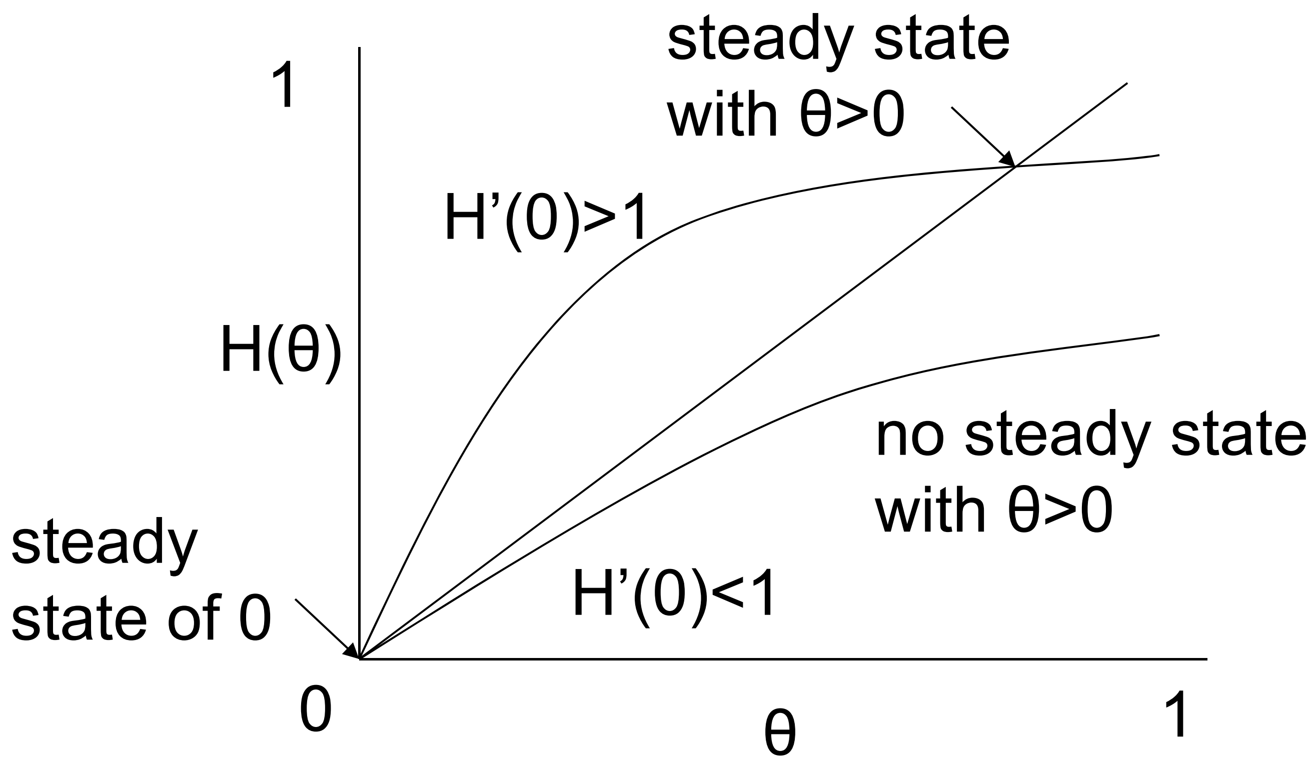
Solving



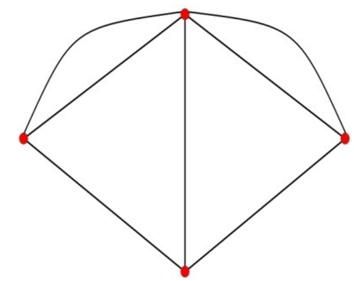
Steady state infection rate of people you meet is the solution to

$$\theta = H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 / [\,(\lambda\theta d + 1)\, E[d]\,]$$

See what $H(\theta)$ looks like and how it depends on $P(d)$, $E[d]$ etc.


steady state
with θ>0
1
H'(0)>1
H(θ)
no steady state
with θ>0
steady
state of 0
H'(0)<1
0
θ
1

Social and Economic Networks: Models and Analysis



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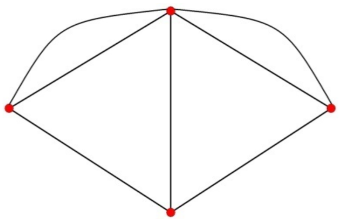
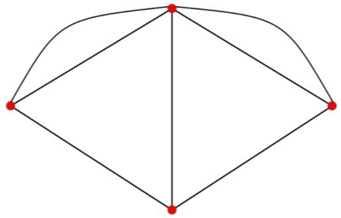
5.6: Solving the SIS Model



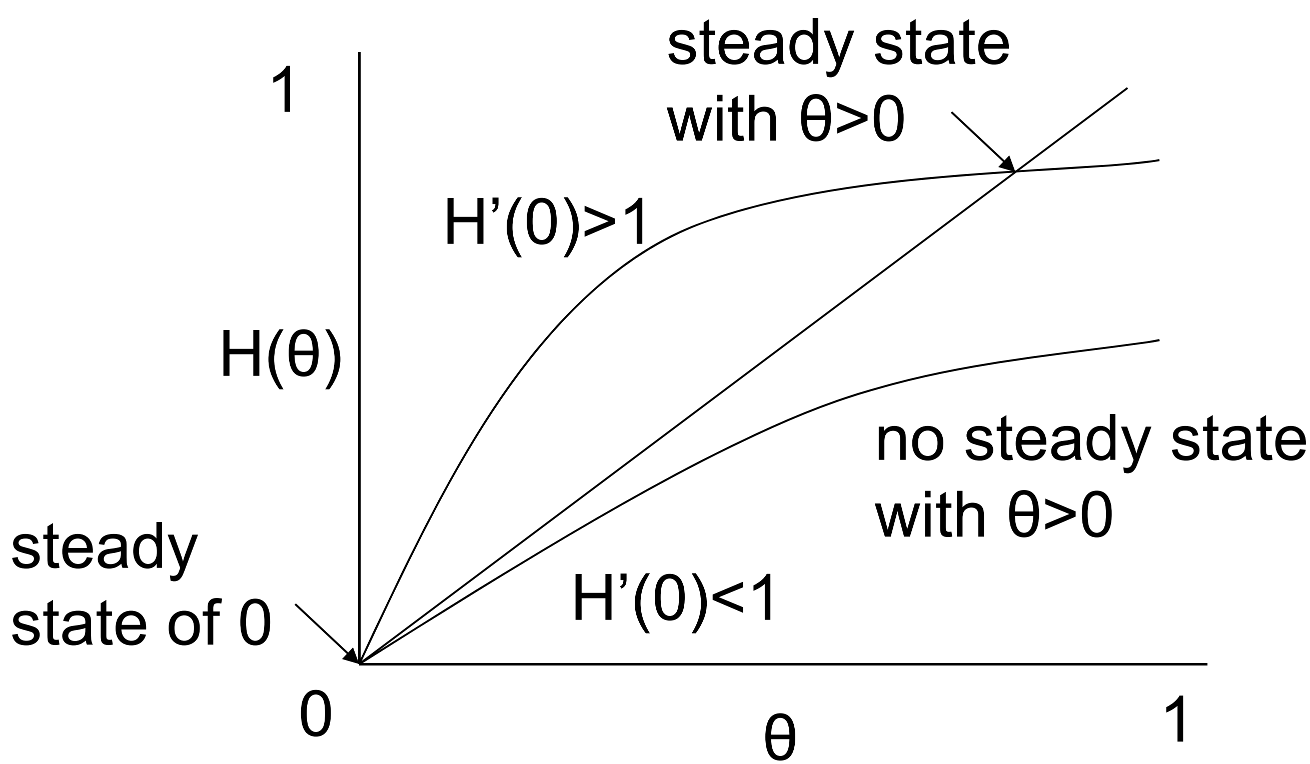
Solving



Steady state infection rate of people you meet is the solution to

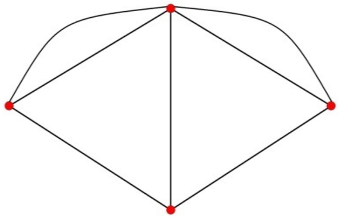
$$\theta = H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$$

See what $H(\theta)$ looks like and how it depends on $P(d)$, $E[d]$ etc.



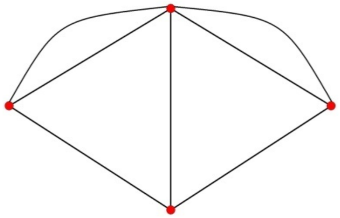
steady state
with θ>0
1
H'(0)>1
H(θ)
no steady state
with θ>0
steady
state of 0
H'(0)<1
0
θ
1

Properties of H



- $H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$

Properties of H

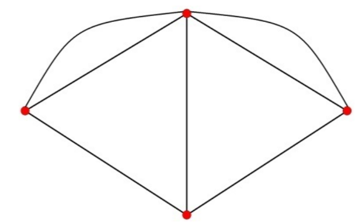


- $H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$

- $H'(\theta) = \Sigma\, P(d)\, \lambda d^2 / [\, (\lambda\theta d + 1)^2\, E[d]] \;\; > \;\; 0$

  so H is increasing

Properties of H



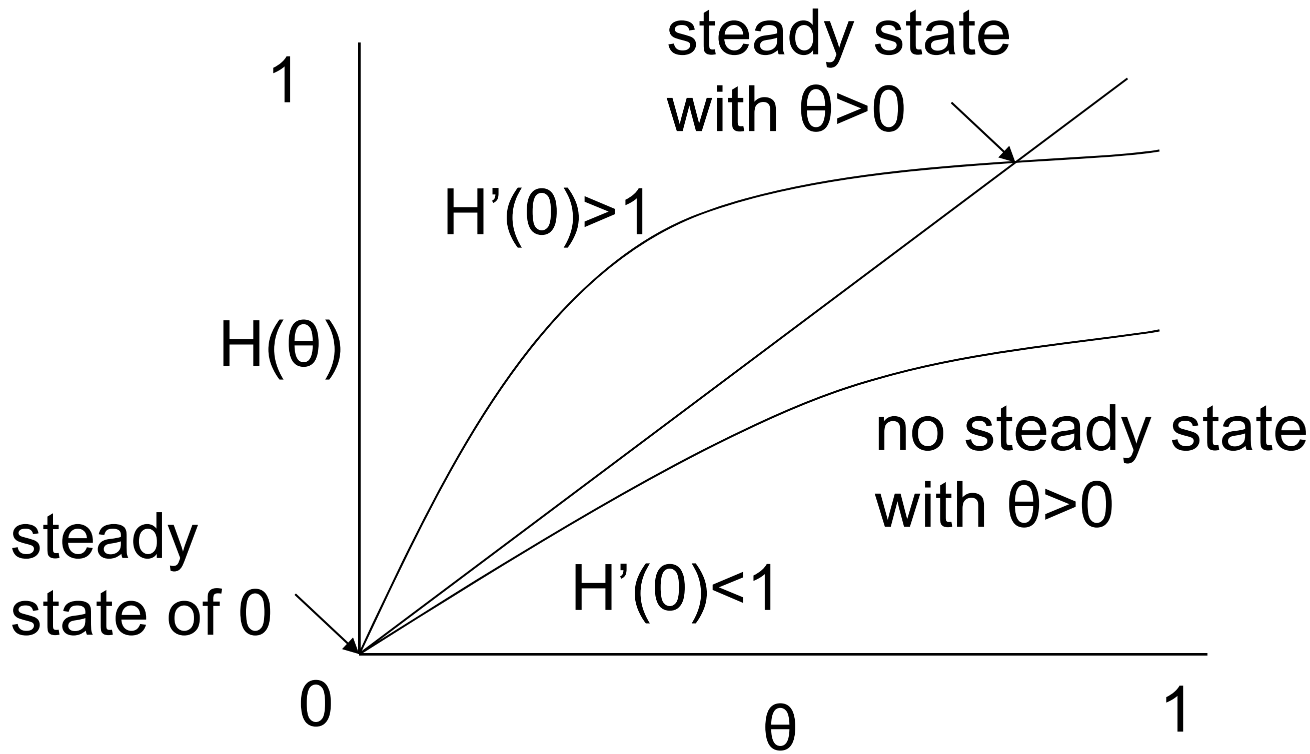
- $H(\theta) = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$

- $H'(\theta) = \Sigma\, P(d)\, \lambda d^2 / [\, (\lambda\theta d + 1)^2\, E[d]] \;\; > \;\; 0$

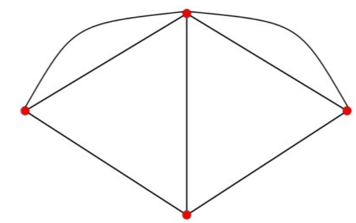
  so H is increasing

- $H''(\theta) = -\, 2 \;\; \Sigma\, P(d)\, \lambda^2 d^3 / [\, (\lambda\theta d + 1)^3\, E[d]] \;\; < \;\; 0$

  so H is strictly Concave


steady state
with θ>0
1
H'(0)>1
H(θ)
no steady state
with θ>0
steady
state of 0
H'(0)<1
0
θ
1

Nonzero Steady State: Lopez-Pintado (08)



- Look at $H'(0)$:

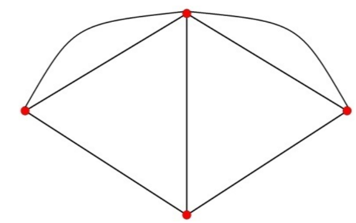
$$H'(\theta) = \Sigma\, P(d)\, \lambda d^2 / [\, (\lambda\theta d + 1)^2\, E[d]]$$

$$H'(0) = \Sigma\, P(d)\, \lambda d^2 / E[d]$$

$$= \boldsymbol{\lambda E[d^2]/E[d]}$$

(recall $\lambda = v/\delta$)

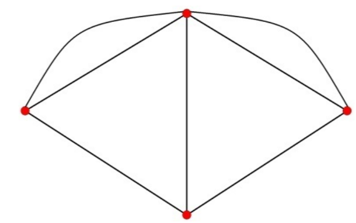
Theorem: Conditions for Steady State of Mean-Field SIS Process



There exists a nonzero steady-state if and only if $\boldsymbol{\lambda > E[d]/E[d^2]}$

So need infection/recovery rate to be high enough relative to average degree divided by second moment (roughly variance)

Conditions for Steady State



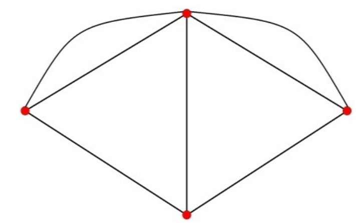
Iff **$\lambda > E[d]/E[d^2]$** have a nonzero steady state

In a **regular network**, need $\lambda > 1/E[d]$

In a **E-R network**, need $\lambda > 1/(1+E[d])$

In a **power-law network**, $E[d^2]$ diverges –
always have a nonzero steady state

Ideas:

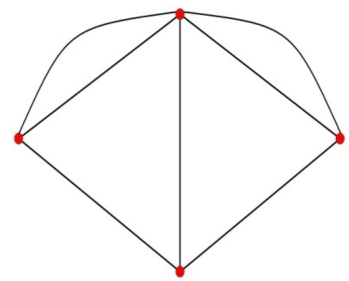


- High degree nodes are more prone to infection

- Serve as conduits

- Higher variance, more such nodes to enable infection

Social and Economic Networks: Models and Analysis



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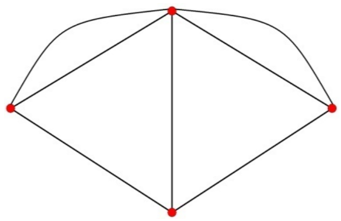
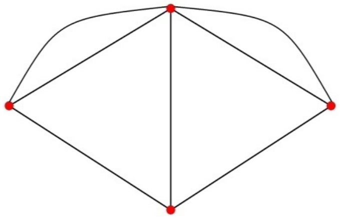
5.7: Solving the SIS Model



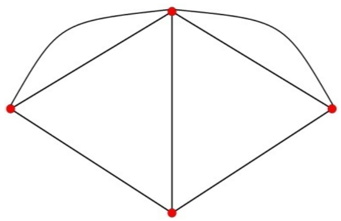
Solving for the Positive Steady State when it Exists



$$\theta = \Sigma P(d) \lambda\theta d^2 / [(\lambda\theta d + 1) E[d]]$$

$$1 = \Sigma P(d) \lambda d^2 / [(\lambda\theta d + 1) E[d]]$$

Solving for the Positive Steady State when it Exists

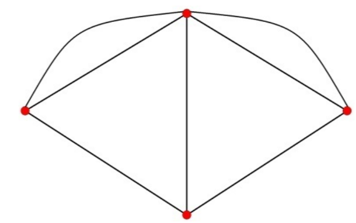


$$\theta = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$$

$$1 = \Sigma\, P(d)\, \lambda d^2 / [\, (\lambda\theta d + 1)\, E[d]]$$

Regular: $1 = \lambda E[d]/(\lambda\theta E[d] + 1)$; $\theta=1-1/(\lambda E[d])$;

Solving for the Positive Steady State when it Exists



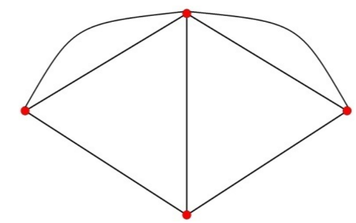
$\theta = \Sigma\ P(d)\ \lambda\theta d^2 / [\ (\lambda\theta d + 1)\ E[d]]$

$1 = \Sigma\ P(d)\ \lambda d^2 / [\ (\lambda\theta d + 1)\ E[d]]$

Regular: $1 = \lambda E[d]/(\lambda\theta E[d] + 1)$; $\theta=1-1/(\lambda E[d])$;

θ is increasing in $\lambda E[d]$ Need $\lambda E[d] > 1$

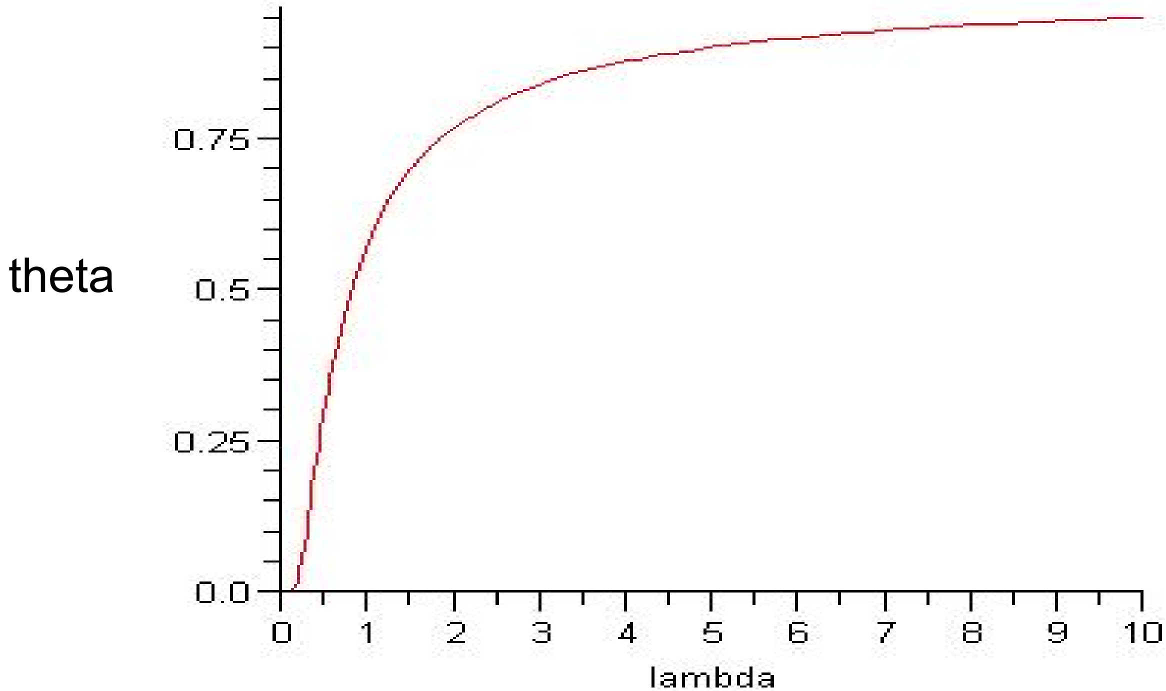
Solving for the Positive Steady State when it Exists



$$\theta = \Sigma\, P(d)\, \lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$$

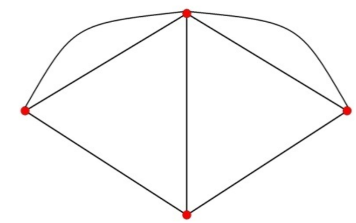
$$1 = \Sigma\, P(d)\, \lambda d^2 /[\, (\lambda\theta d + 1)\, E[d]]$$

Power: $P(d) = 2d^{-3}$ .... $\theta=1/(\lambda(e^{1/\lambda}-1))$;



neighbor infection rate for power distribution

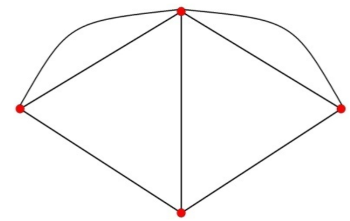
Ordering Networks
Jackson Rogers (07b)



$$\theta = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$$

How does the right side shift with P(d)?

Ordering Networks
Jackson Rogers (07b)

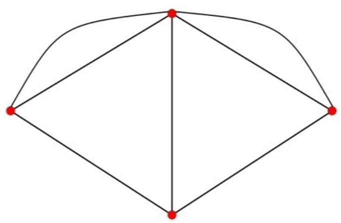


$\theta = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$

How does the right side shift with P(d)?

$\lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$ is increasing in d

Ordering Networks
Jackson Rogers (07b)

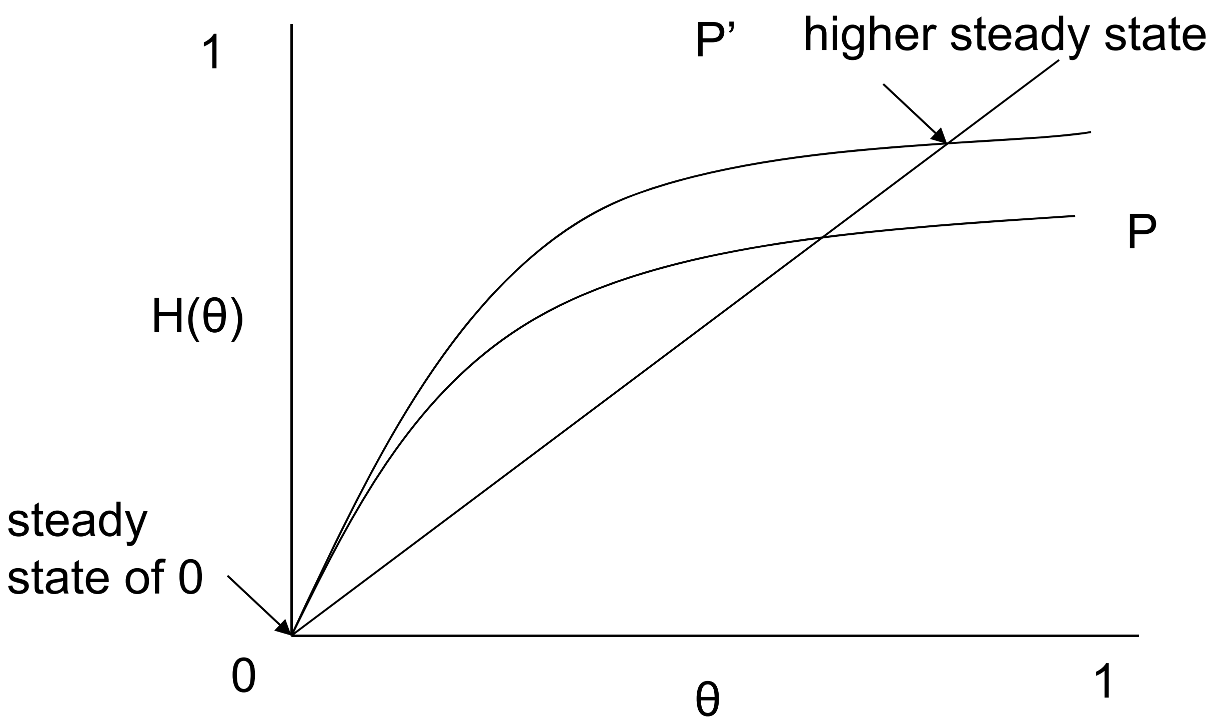


$\theta = \Sigma\, P(d)\, \lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$

How does the right side shift with P(d)?

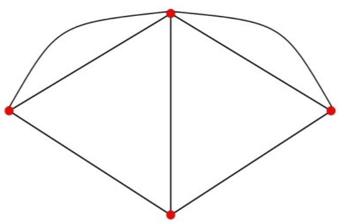
$\lambda\theta d^2 / [\, (\lambda\theta d + 1)\, E[d]]$ is increasing in d

If P' first order stochastic dominates P, then
rhs increases at every θ



1
P'
higher steady state
P
H(θ)
steady
state of 0
0
θ
1

Ordering Networks
Jackson Rogers (07b)

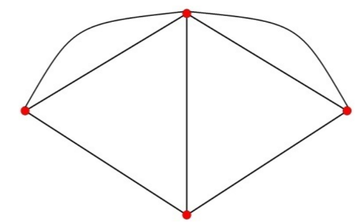


$$\theta = \Sigma\, P(d)\, \lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$$

How does the right side shift with P(d)?

$\lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$ is convex in d

Ordering Networks Jackson Rogers (07b)

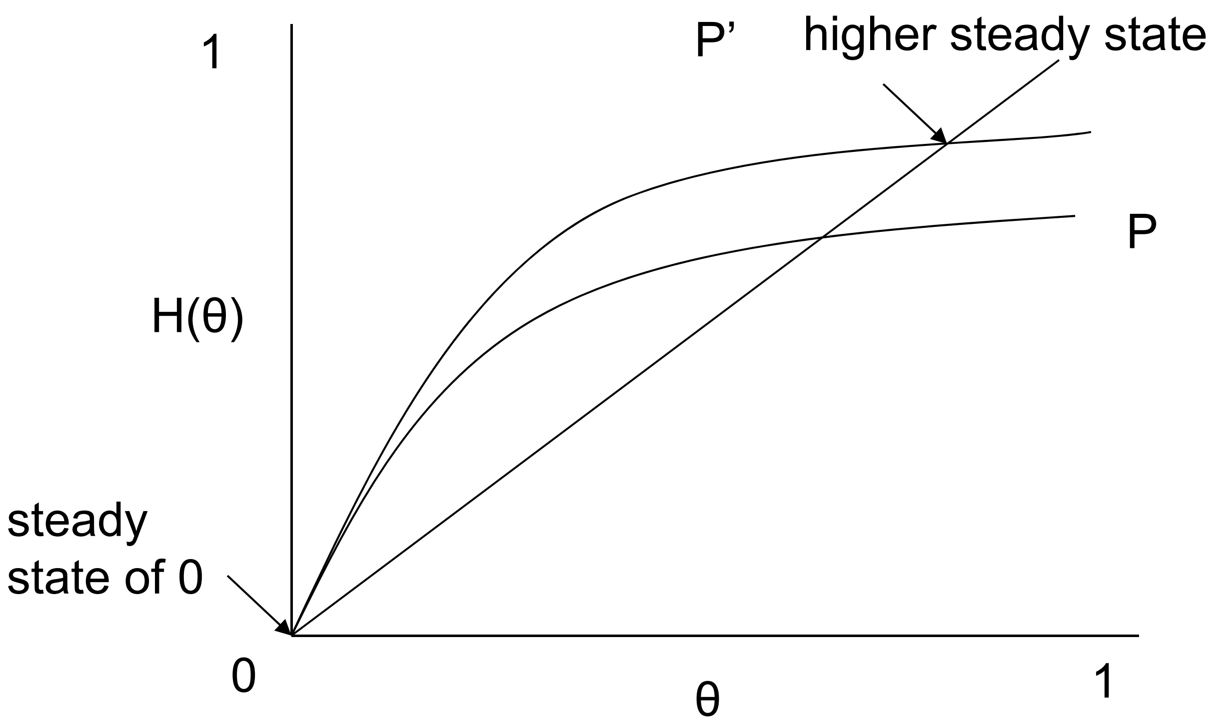


$\theta = \Sigma\, P(d)\, \lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$

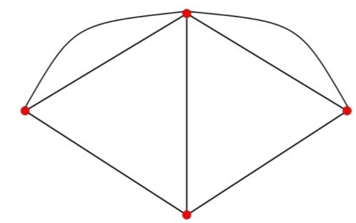
How does the right side shift with P(d)?

$\lambda\theta d^2 /[\, (\lambda\theta d + 1)\, E[d]]$ is convex in d

If P' is a mean-preserving spread of P, then
rhs increases at every θ

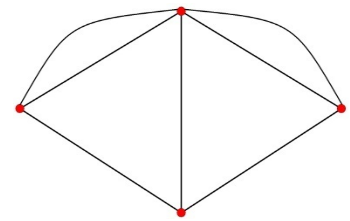

1
P'
higher steady state
P
H(θ)
steady
state of 0
0
θ
1

Ideas:



- Mean preserving spread – more high degree nodes and low degree nodes
- Higher degree nodes are more prone to infection
- Neighbors are more likely to be high degree
- So, either first order stochastic dominance, or mean-preserving spreads in P increase θ

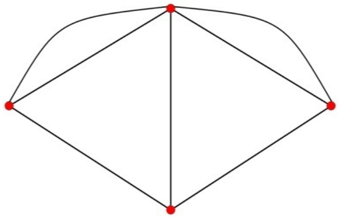
What about Average?

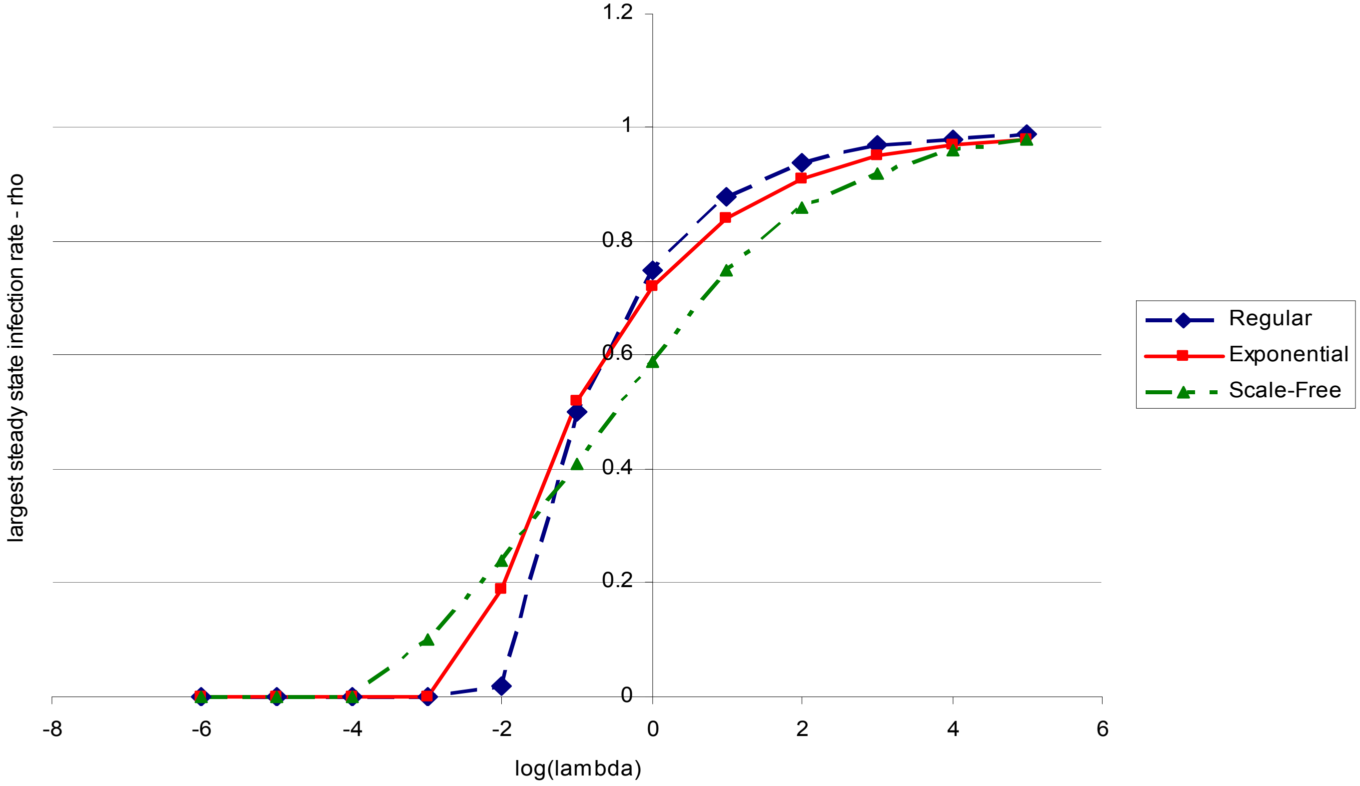


- infection rate of neighbors is not the same as infection rate of the population

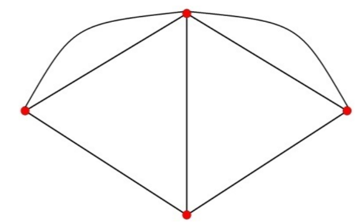
Theorem JR (2007): If P' is a mean preserving spread of P, then the highest steady state $\theta' > \theta$, but the corresponding $\rho' > \rho$ if λ is low, while $\rho' < \rho$ if λ is high

Steady States





Proof



$0 = d\rho(d)/dt = (1 - \rho(d))v\theta d - \rho(d)\,\delta$

Expecting over d:

$0 = v\theta E[d] - \Sigma\, P(d)\, \rho(d)\, v\theta d - \rho\delta$

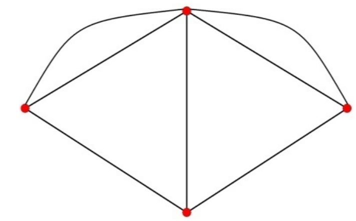
$= v\theta\, E[d] - v\theta^2\, E[d] - \rho\delta$

$\rho = \lambda\theta\, E[d]\, (1 - \theta)$

rhs is increasing in θ iff $\theta < 1/2$

θ is increasing in λ

SIS Diffusion Model

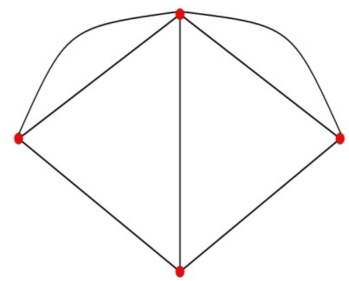


Simple and tractable model

Bring in relative meeting rates

Can order infections by properties of ``network''

Social and Economic Networks: Models and Analysis



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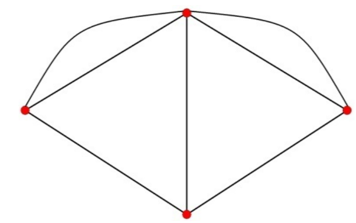
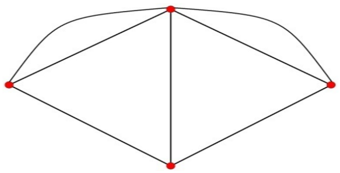
5.8:Fitting a Diffusion Model to Data

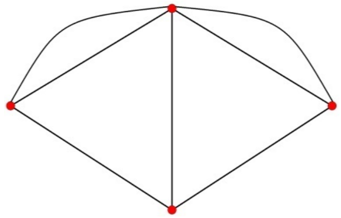


Estimating Models



- **Banerjee, Chandrasekhar, Duflo, Jackson (2013)** Study of Diffusion:
- Map network structure via surveys, observe behavior
- Model diffusion and fit the model from observed networks and behaviors

Questions

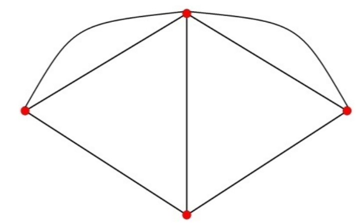


- What determines behavior:
  - Pure access to information (no strategic effects)?
  - Complementarities (strategic affects)?

- Are non-participants important in diffusion?
  - Model information passing by participants (usual contagion)
  - Information passing by non-participants too

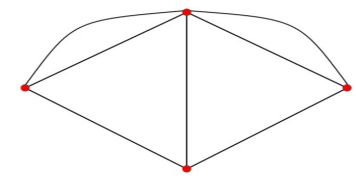
Estimate structural models of diffusion and behavior

Modeling diffusion:



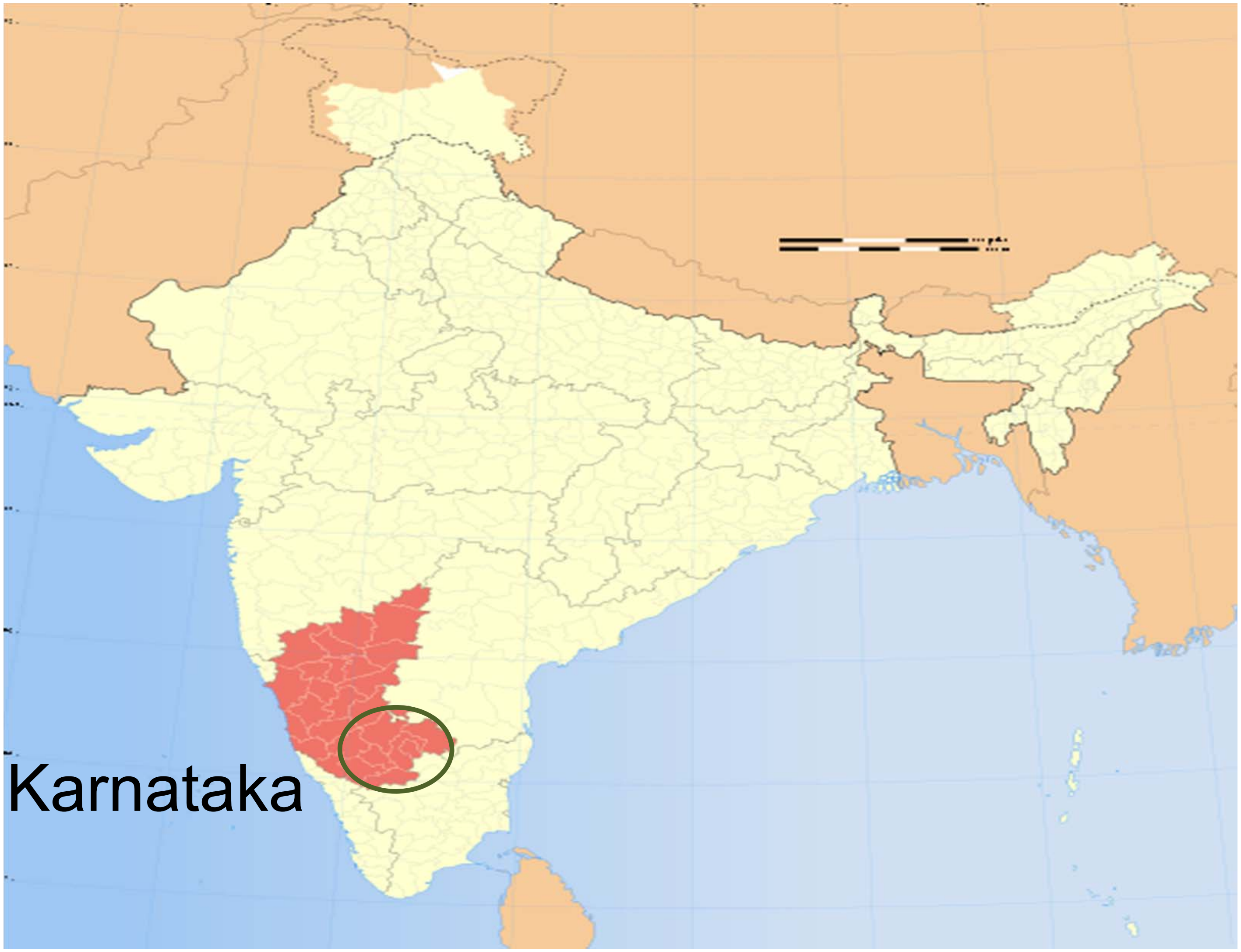
- Know the set of initially informed nodes
- Informed nodes (repeatedly) pass information randomly to their neighbors over discrete times
- Once informed (just once), nodes choose to participate depending on their characteristics and their neighbors' choices

Background

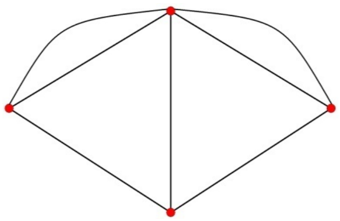


75 rural villages in Karnataka, relatively isolated from microfinance initially

- BSS entered 43 of them and offered microfinance
- We surveyed villages before entry, observed network structure and various demographics
- Tracked microfinance participation over time

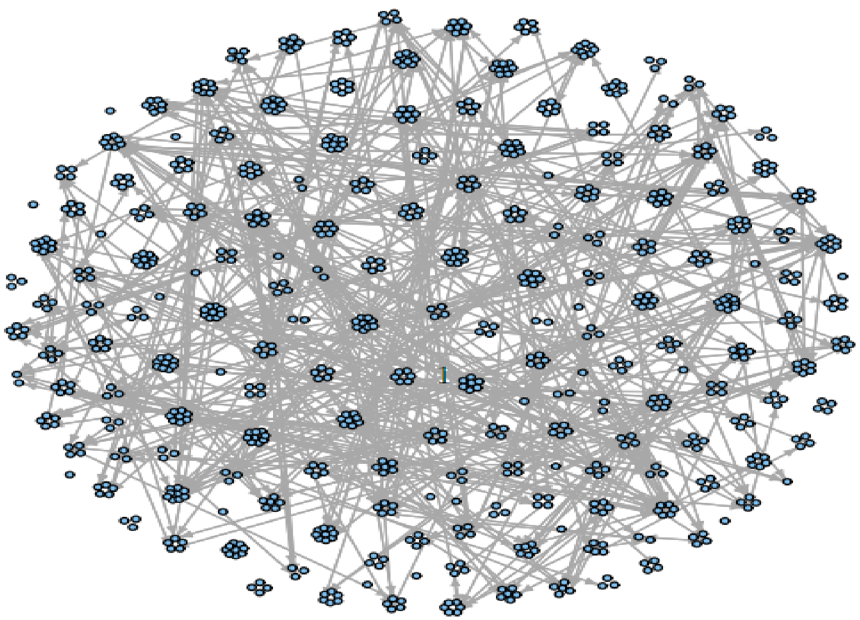

Karnataka

Background: 75 Indian Villages – Networks

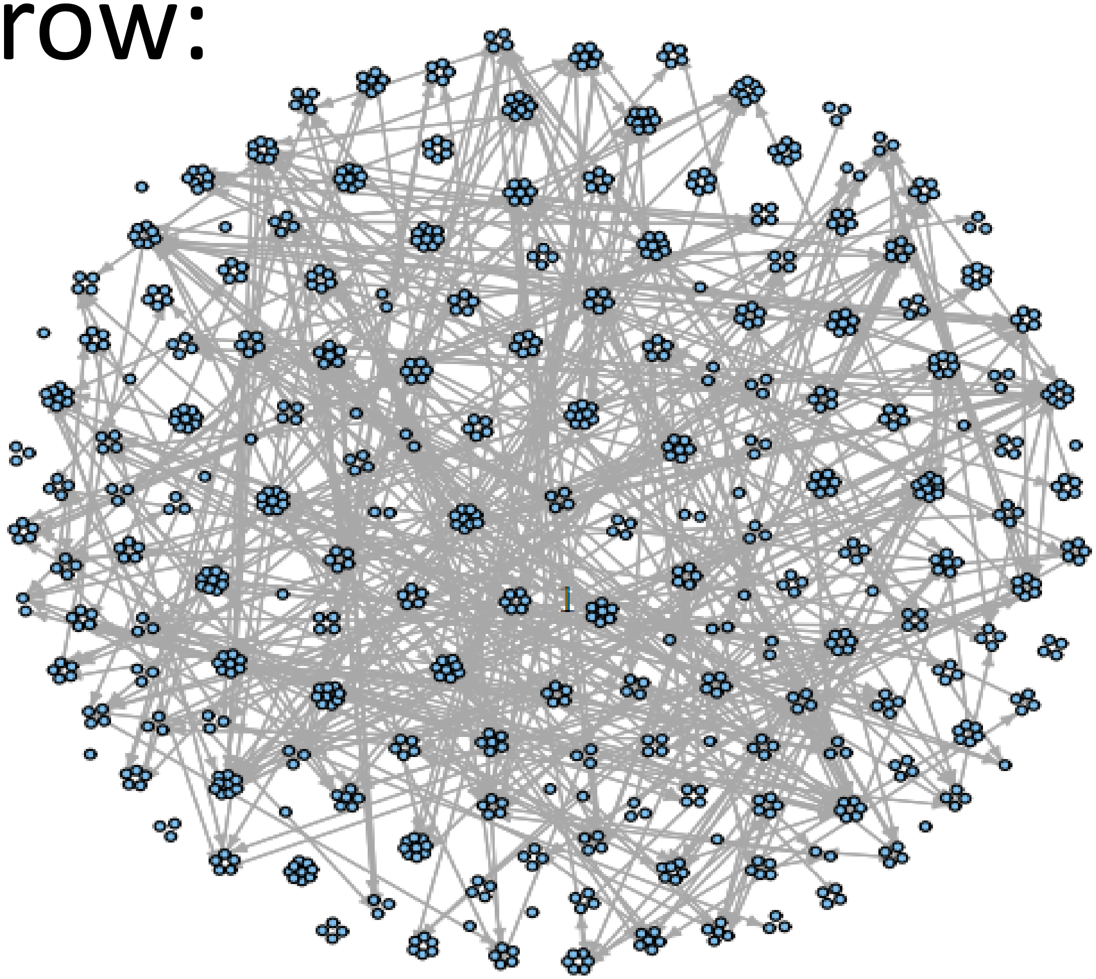


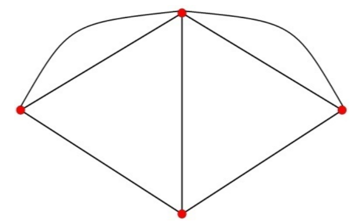
- ``Favor'' Networks:
  - both borrow and lend money
  - both borrow and lend kero-rice
- ``Social'' Networks:
  - both visit come and go
  - friends (talk together most)
- Others (temple, medical help…)

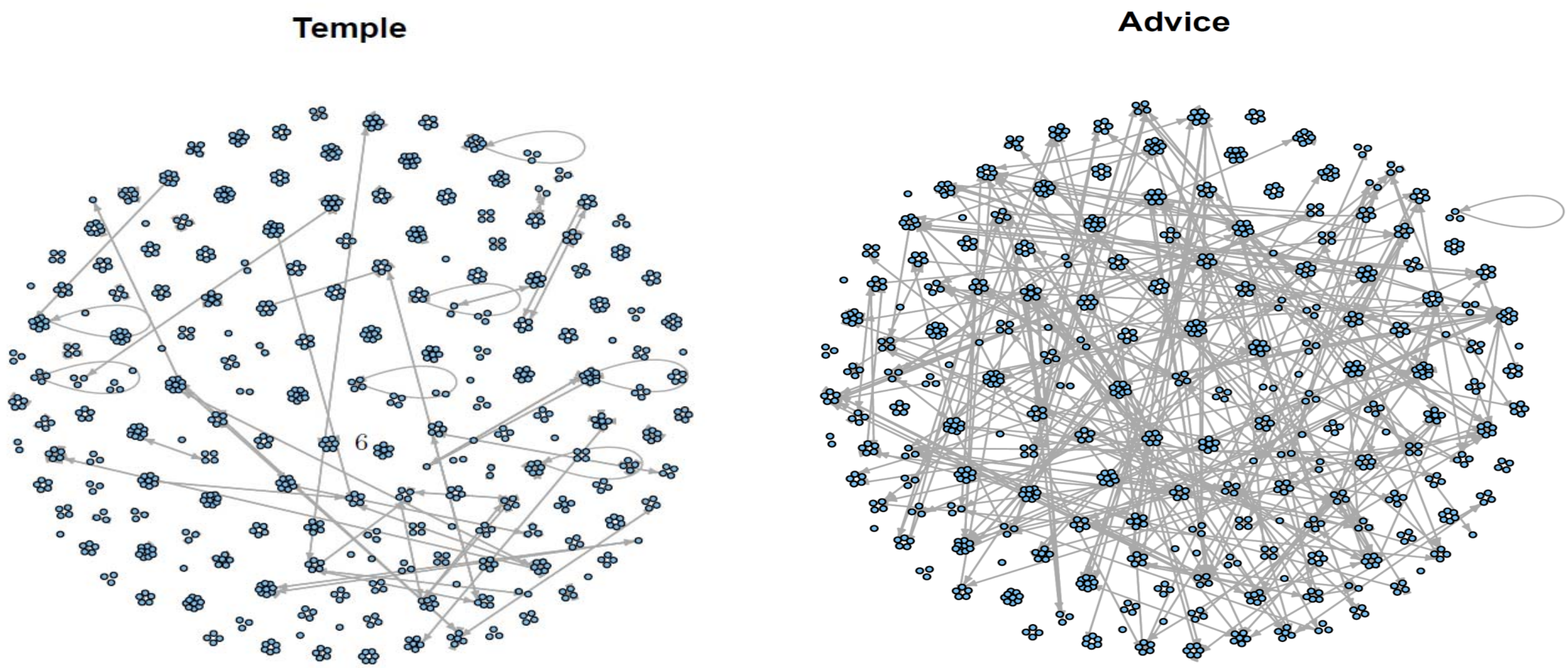
Borrow:

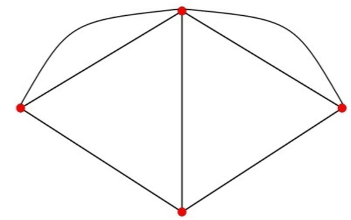


Borrow:

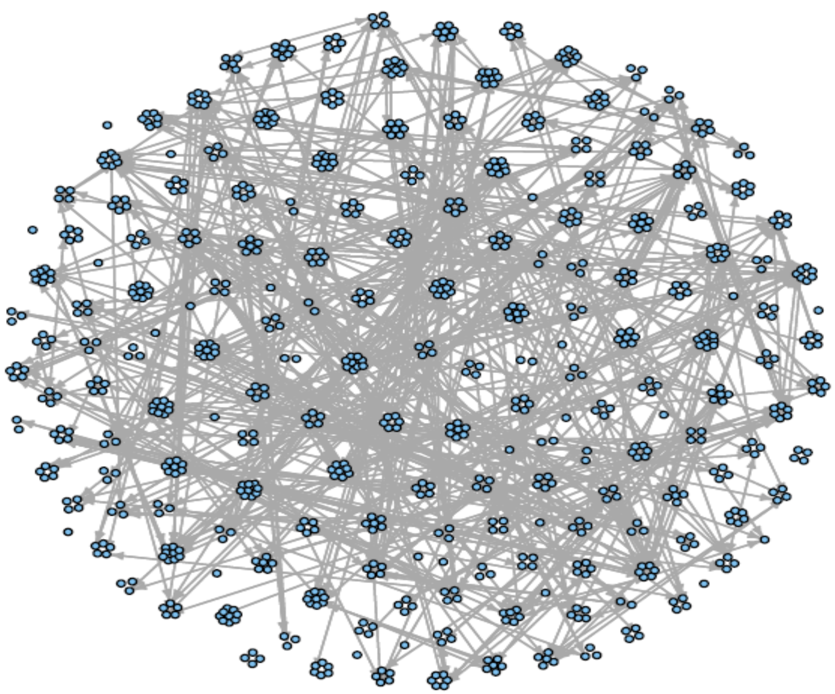




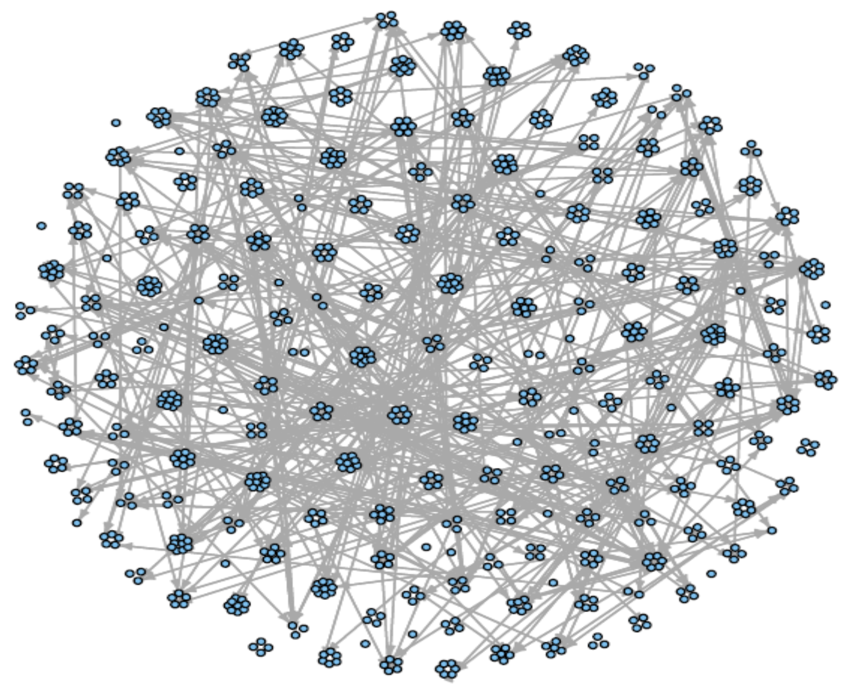

Temple
6
Advice



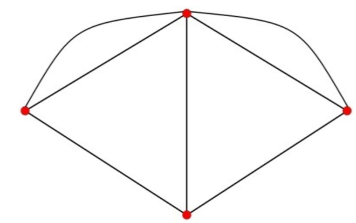
Kero−Come



Medic

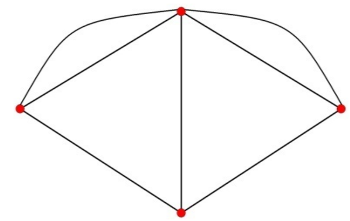


Data also include



- Microfinance participation by individual, time
- Number of households and their composition
- Demographics: age, gender, subcaste, religion, profession, education level, family...
- Wealth variables: latrine, number rooms, roof,
- Self Help Group participation rate, ration card, voting
- Caste: village fraction of ``higher castes'' (GM/FC and OBC, remainder are SC/ST)

Standard Peer-effects analysis:



Let p_i be prob i participates

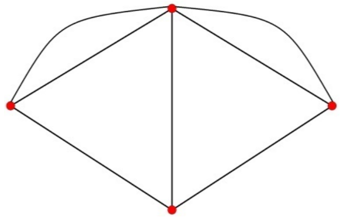
- $\text{Log}(p_i/(1-p_i))$

  $= \; b_0$

  $+ \; b_{char}$ characteristics$_i$

  $+ \; b_{Peer}$ frac$_i$ friends participate

Standard Peer-effects analysis:



Let p_i be prob i participates

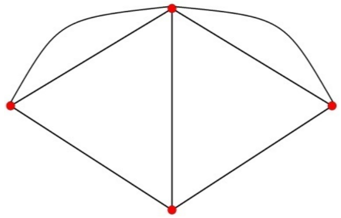
- $\text{Log}(p_i/(1-p_i))$

  $= \ b_0$

  $+ \ b_{char}$ characteristics$_i$

  $+ \ \mathbf{2.5}^{***}$ frac$_i$ friends participate

Standard Peer-effects analysis:



Let p_i be prob i participates

- $\text{Log}(p_i/(1-p_i))$

  $= \; b_0$

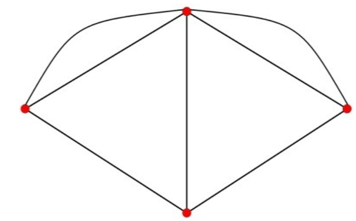
  $+ \; b_{char}$ characteristics$_i$

  $+ \; \mathbf{2.5}^{***}$ frac$_i$ friends participate

frac 0 to 1 increases $p_i/(1-p_i)$ by factor 12.2,

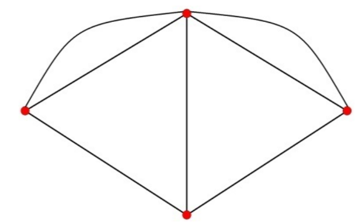
frac .1 to .3 increases $p_i/(1-p_i)$ by factor 1.65,

Modeling behavior/information diffusion:



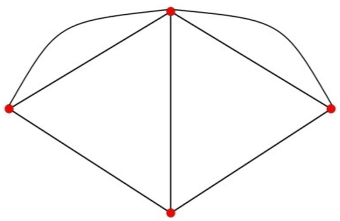
- Use network information for diffusion, not just who friends are:
  - People who hear about microfinance randomly pass to friends – diffusion in network
  - Once hear, decide whether to participate – friends might matter

Participation Decision



- Once informed, make choice of whether to participate
- Choice allowed to depend on personal characteristics and fraction of informed neighbors who participate
  - Complementarity?
  - Substitution?

Choice Decision



Let p_i be i's choice of whether to participate

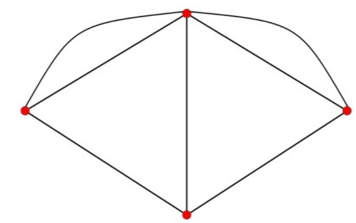
- $\text{Log}(p_i/(1-p_i))$

  $= \ b_0$

  $+ \ b_{char}$ characteristics$_i$

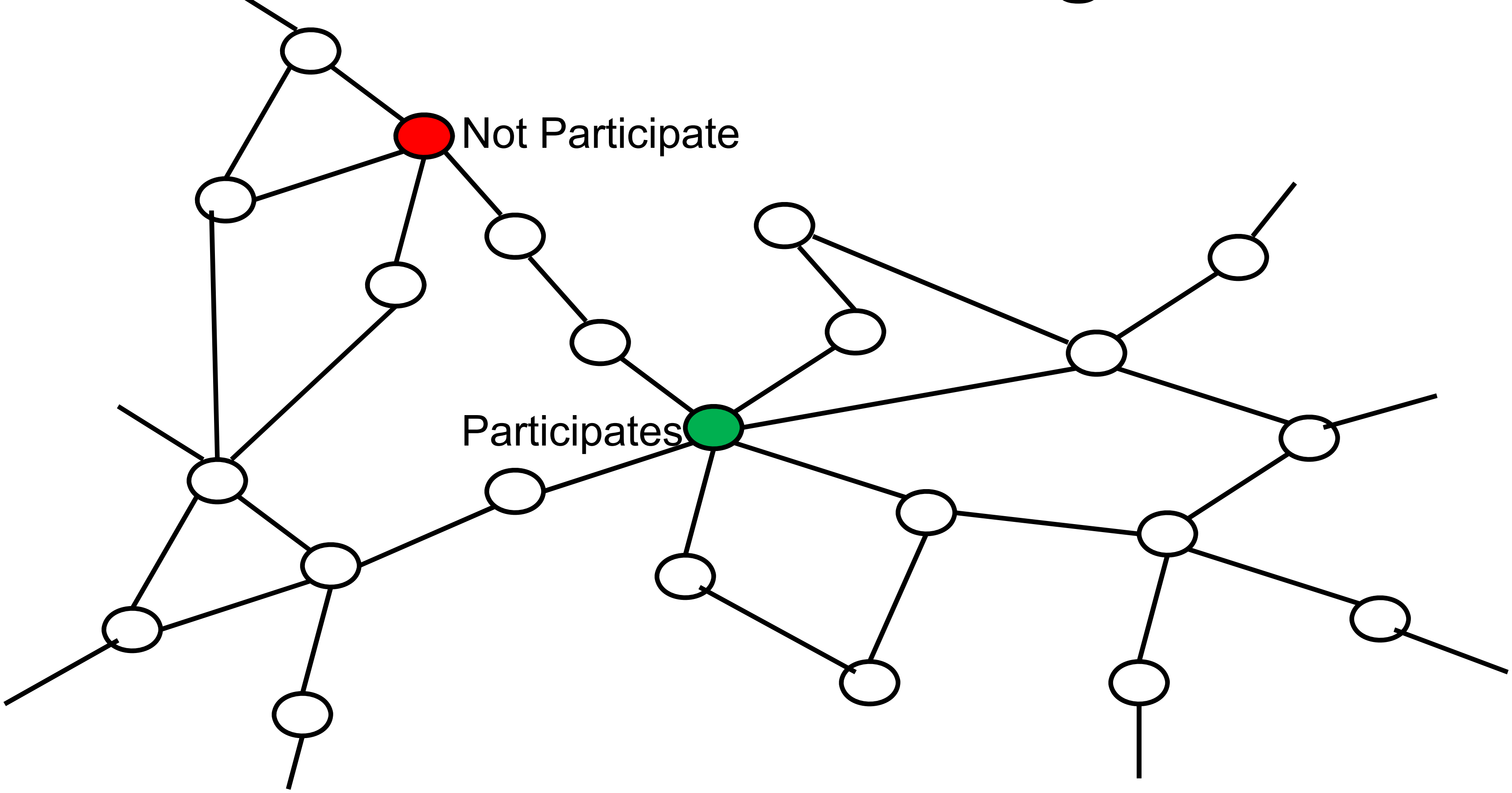
  $+ \ b_{Peer}$ frac$_i$ informing friends participating

Modeling behavior/information diffusion:

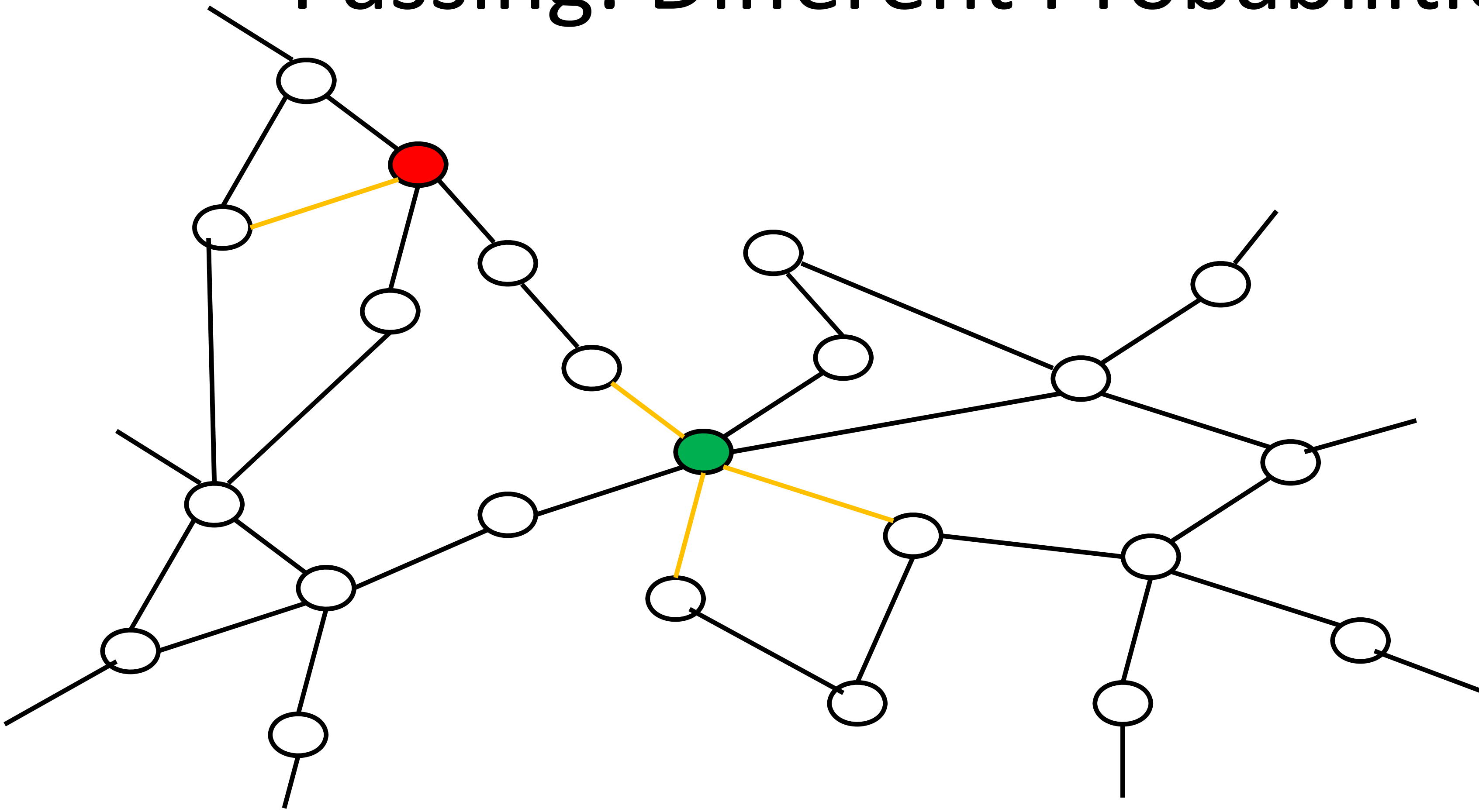


- Probability of passing to a given individual:
  - q^N if did Not participate
  - q^P if did Participate

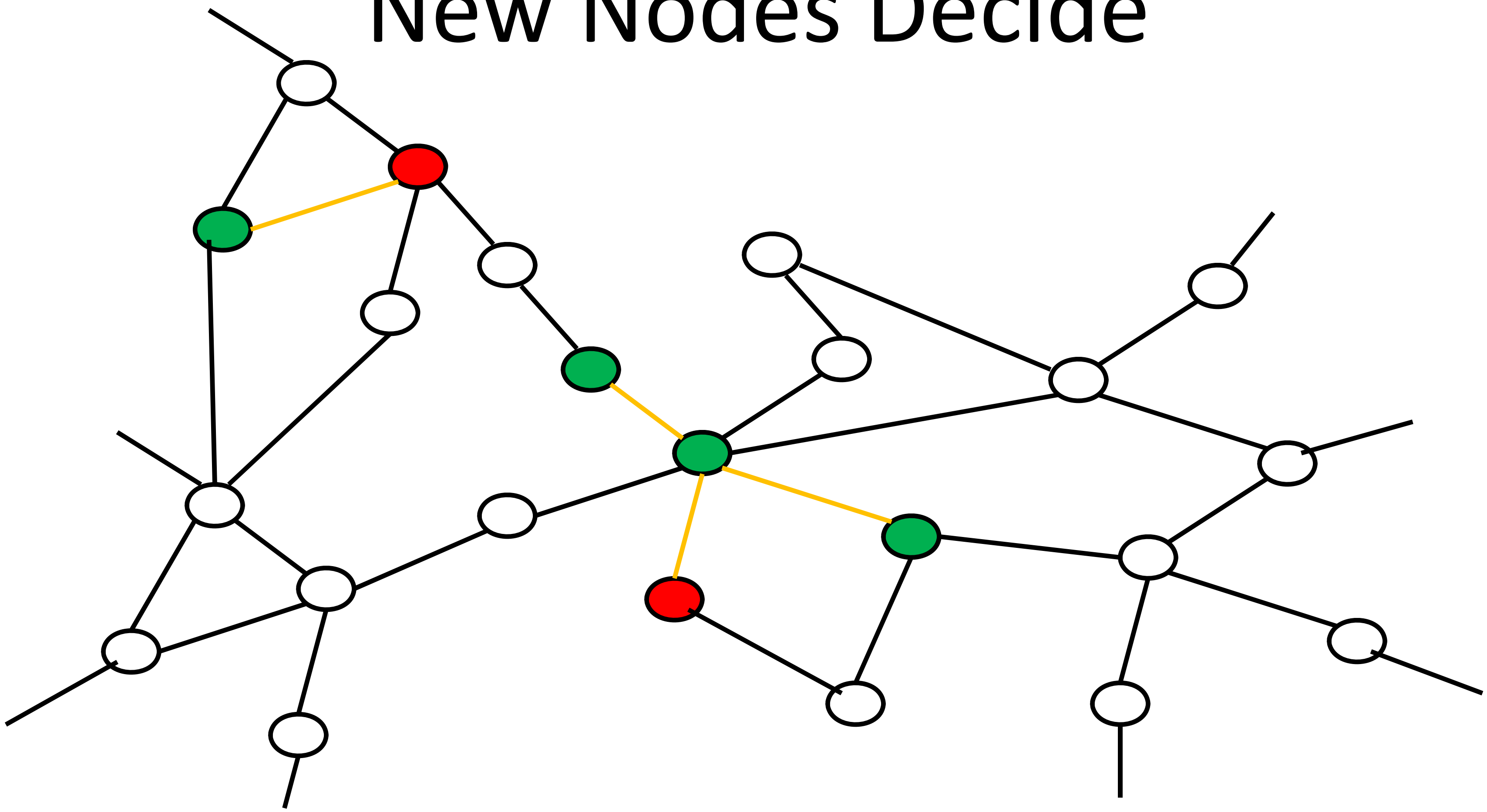
Information Passing Leaders



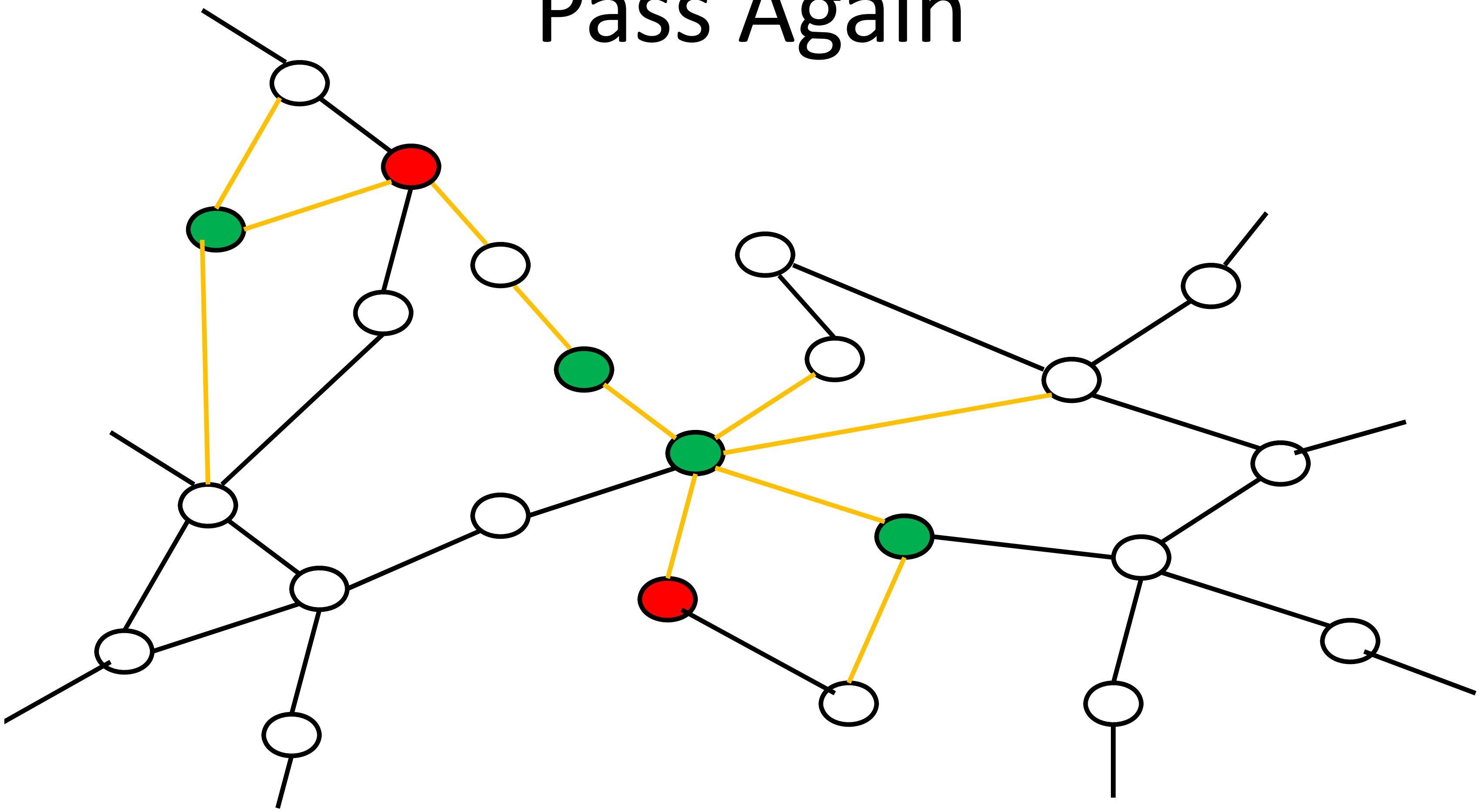
Passing: Different Probabilities



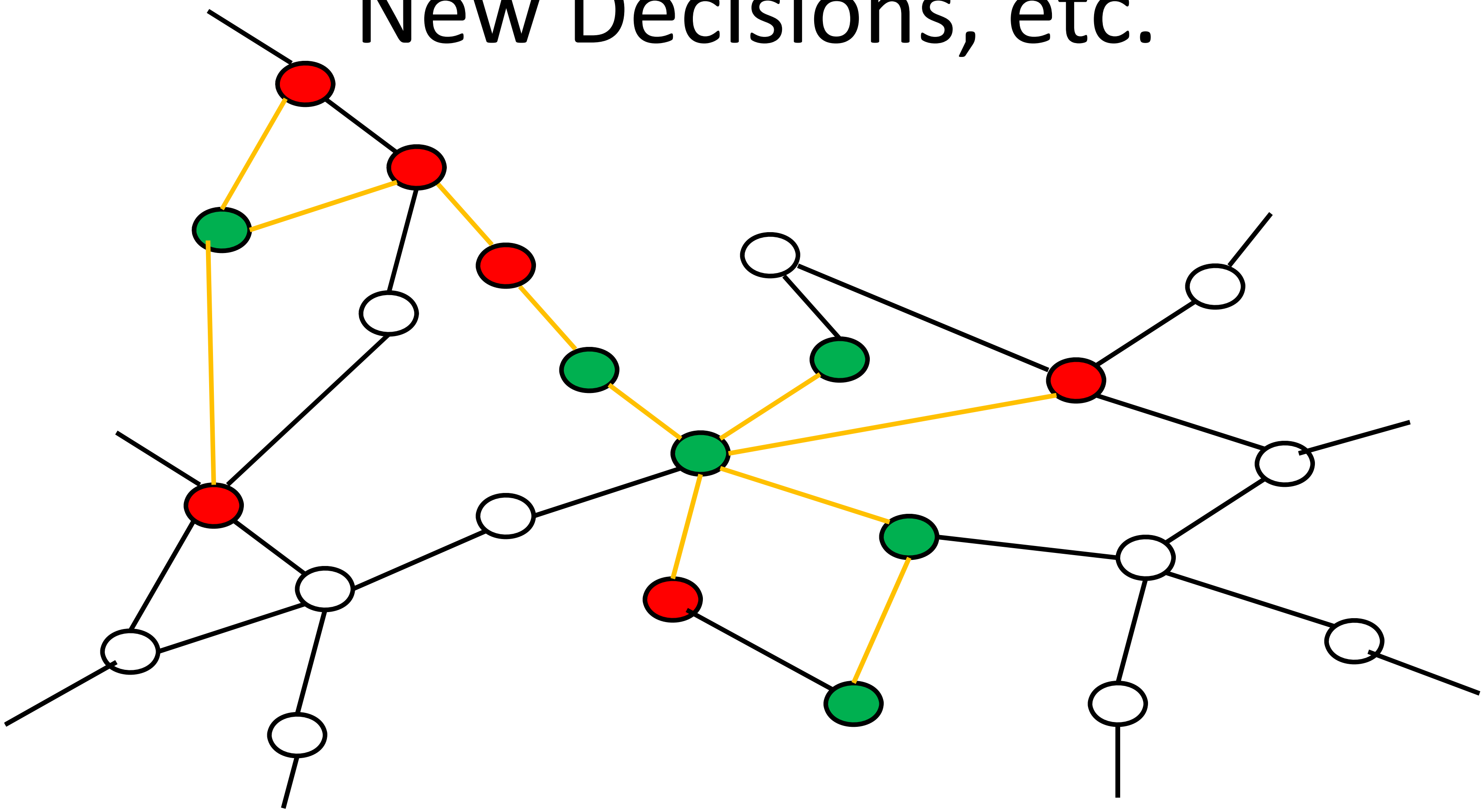
New Nodes Decide



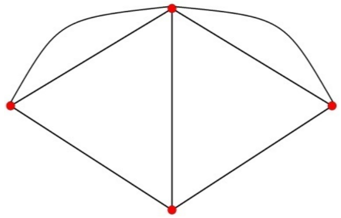
Pass Again



New Decisions, etc.



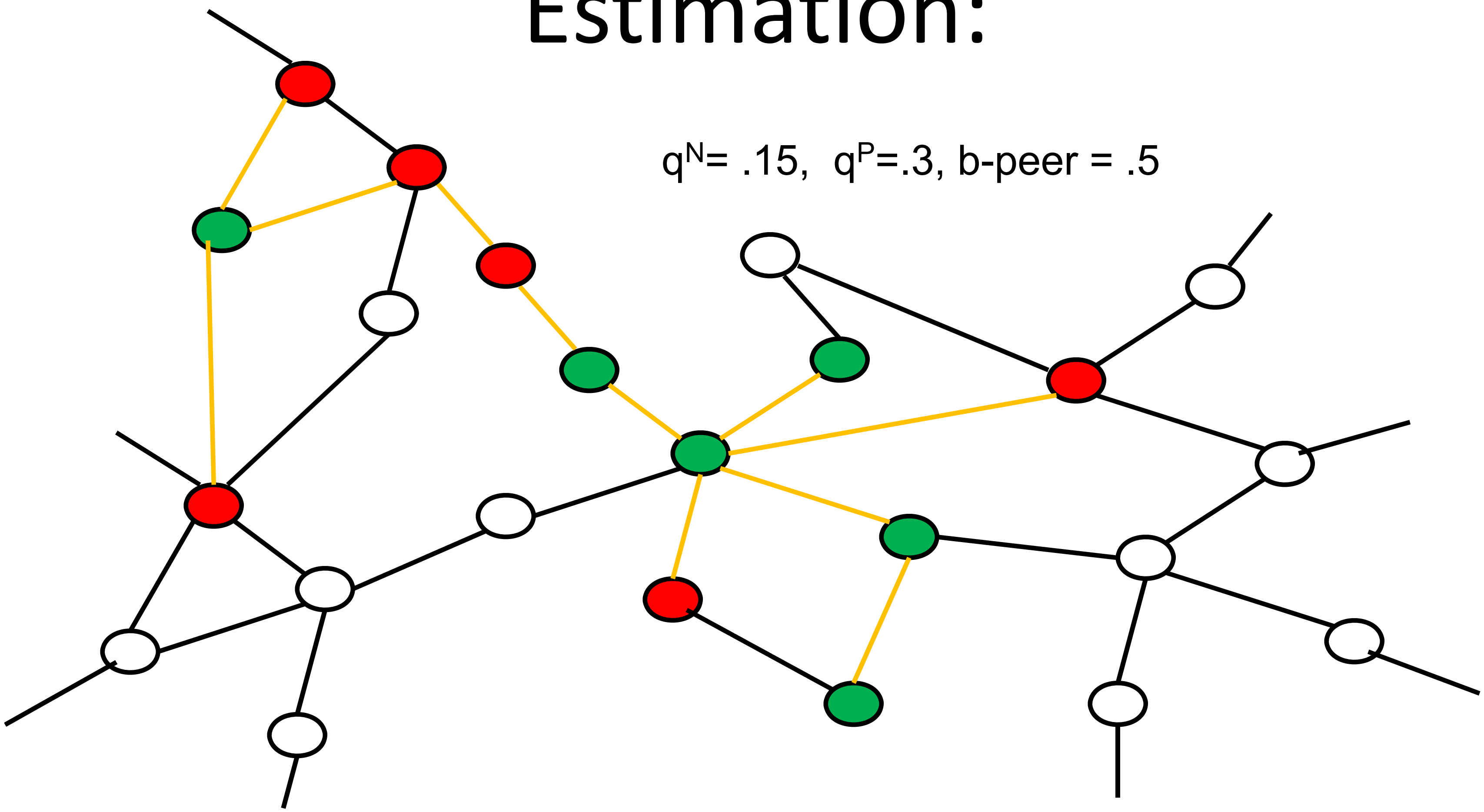
Estimation technique:



- Estimate b_0, b_{char}, from initially informed (saves on computation size of grid)
- q^N, q^P, b_{peer} - For each choice of parameters, simulate on the actual networks of the villages for time period proportional to number of trimesters in data for village (3 to 8 times)
- Choose parameters to best match simulated participation rates and various moments to observed moments (GMM)

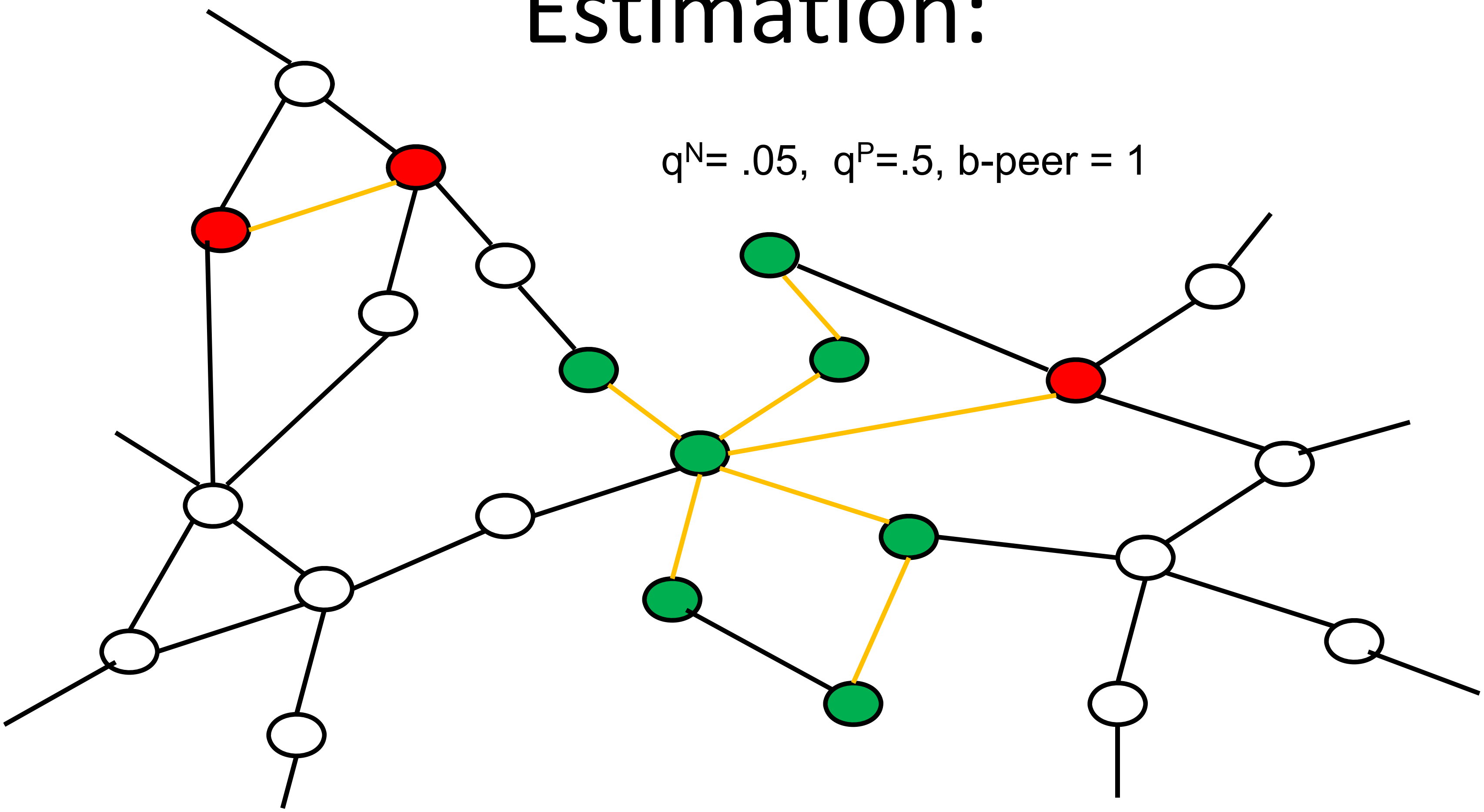
Estimation:

q^N= .15, q^P=.3, b-peer = .5

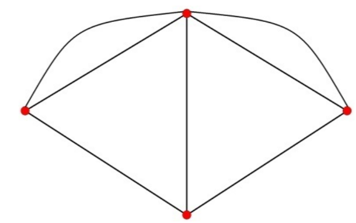


Estimation:

$q^N = .05$, $q^P = .5$, b-peer = 1



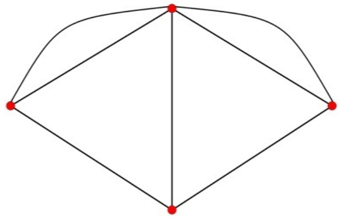
Estimated parameters:



- Information significant, peer/endorse effect not

	qN	qP	b-peer	qN – qP
Diffusion	0.05***	0.55***	-0.20	-0.50***
and peer	[0.01]	[0.13]	[0.16]	[0.13]

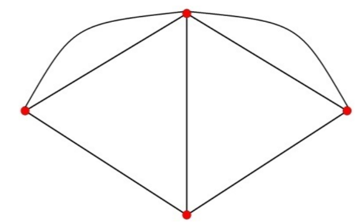
Estimated parameters:



- Information significant, peer/endorse effect not

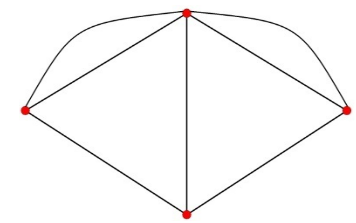
	qN	qP	b-peer	qN – qP
Diffusion	0.05***	0.55***	-0.20	-0.50***
and peer	[0.01]	[0.13]	[0.16]	[0.13]
just peer!:			2.5***	

Network Effects:



- Significant information passing parameters
- Information passing depends on whether participate: more likely if participate
- Slight complementarities, but insignificant

Information Passing

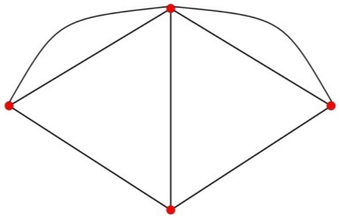


- What fraction of eventual informed agents are accounted for by information passing of non-participants?

- Hold all else constant, but rerun the model with q^N=0

- See what happens to information and participation rates

Information Passing



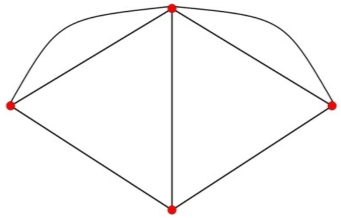
Median for model as fit:

Informed 85.8% Participation 20.7%

Median for model re-setting $q^N = 0$

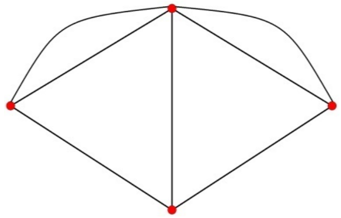
Informed 58.9% Participation 13.8%

Results from Fitting Models of Diffusion:



- Significant information passing parameters
- Insignificant, limited Peer Effects
- Information passing depends on whether participate: more likely if participate
- Nonparticipants play a substantial role (1/3 of total)

Conclusions

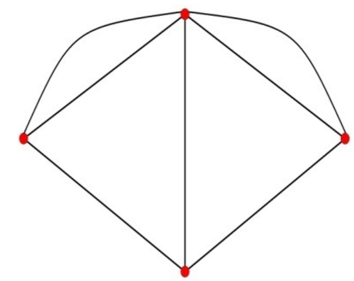


- **Models of diffusion can help us disentangle effects**

- **Important for policy**
  - **Enhance information spreading?**
  - **Help overcome/enhance peer influences**

- **Relate back to network structure**
  - **homophily**
  - **degree distribution, clustering ...**

Social and Economic Networks: Models and Analysis



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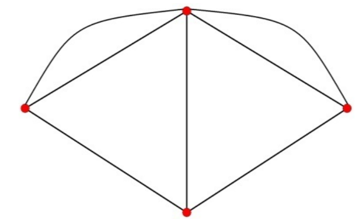
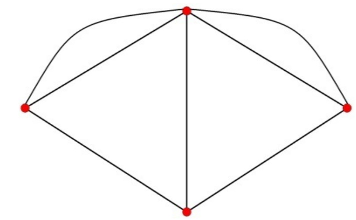
5.8b Application: Financial Contagions



Explore Contagions



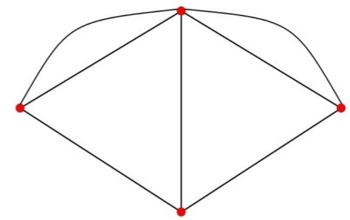
Simple model of Elliott, Golub Jackson 13:

Companies are linked to each other via various contracts: debts, promised deliveries, equity, ...

That exposes each company to others' investments and values

First, let us see how to use networks to model exposures

Explore Contagions



An organization has direct investments:

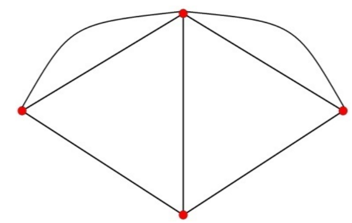
Fraction c_i of value accrues directly to them

Fraction $1-c_i$ is owed to others

Also hold obligations of d_i other organizations:

Have claims to those other organizations' investments

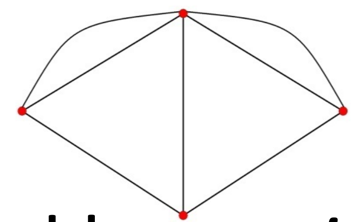
Model



- $\{1, \ldots, n\}$: Organizations (countries, firms, banks...)

- p_i: price of investments of organization i

Cross Holdings:



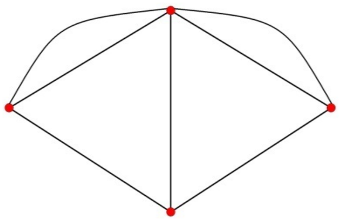
- C_{ij}: cross holdings: fraction of org *j* owned by org *i*

- $C_{ii} = 0$: (don't own yourself)

- $\hat{C}_{ii} = 1 - \sum_j C_{ji}$:

  fraction of org *i* privately held

Value of an Organization



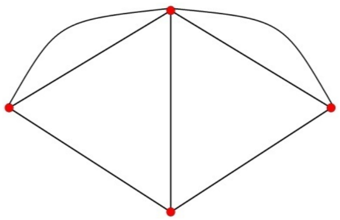
book value:

$$V_i = pi + \sum_j C_{ij} V_j$$

direct asset holdings

cross-holdings

Value of an Organization



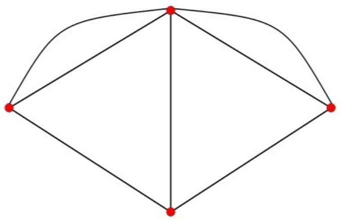
$$V_i = p_i \quad + \quad \sum_j C_{ij} V_j$$

$$V = p + \boldsymbol{C} V$$

Leontief calculation of book value

$$V = (\boldsymbol{I} - \boldsymbol{C})^{-1}\, p$$

Value of an Organization



Book value:

$$V = (\boldsymbol{I} - \boldsymbol{C})^{-1}\ p$$

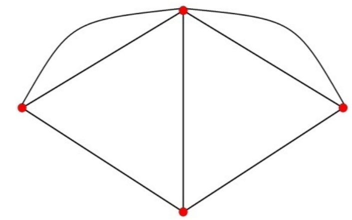
Market value – value to final (private) investors.

$$v_i = \hat{C}_{ii} V_i$$

$$v = \underbrace{\widehat{\boldsymbol{C}}\,(\boldsymbol{I} - \boldsymbol{C})^{-1}}\ p$$

$$v = \boldsymbol{A}\ p$$

Value of an Organization



Book value:

$$V = (\boldsymbol{I} - \boldsymbol{C})^{-1}\ p$$

Market value – value to final (private) investors.

$$v_i = \hat{C}_{ii} V_i$$

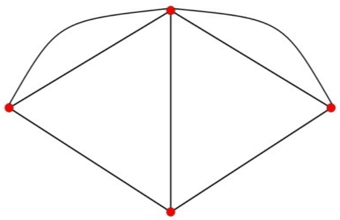
$$v = \underbrace{\widehat{\boldsymbol{C}}\ (\boldsymbol{I} - \boldsymbol{C})^{-1}}\ p$$

$$v = \boldsymbol{A}\ p$$

A_{ij} :

fraction of the investments owned by org j that ultimately accrue to private shareholders of i

Example



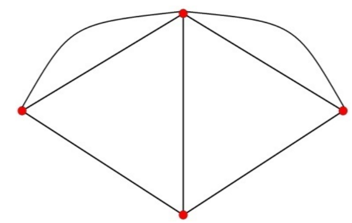
- Two organizations: $n = 2$

- Each owns half of the other: $\boldsymbol{C} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$

- Implied holdings by private investors:

$$\widehat{\boldsymbol{C}} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

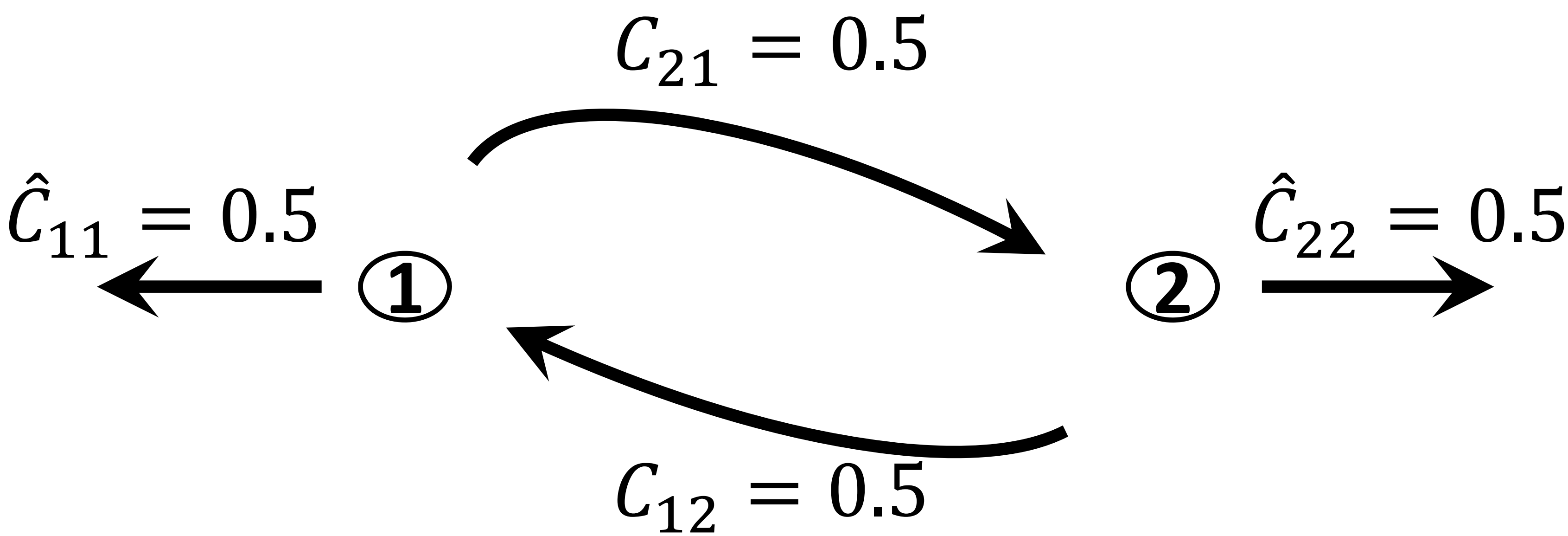
Example



- Two organizations: $n = 2$
- Each owns half of the other: $\boldsymbol{C} = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}$
- Final investors' claims on assets:

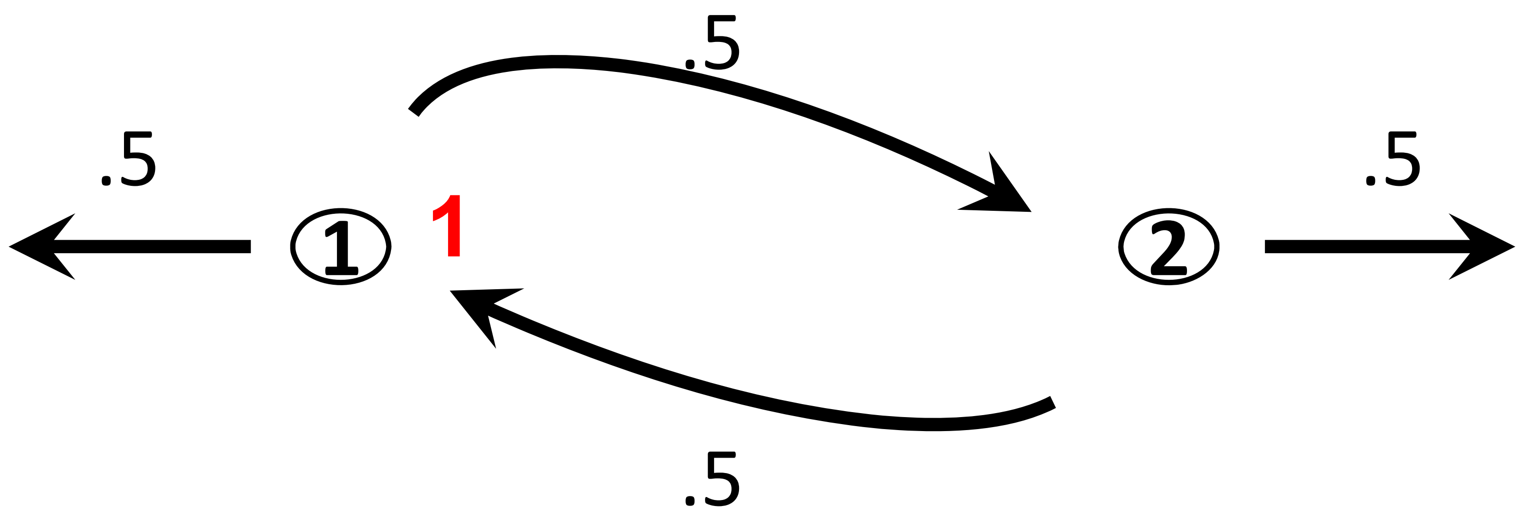
$$\widehat{\boldsymbol{C}} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \qquad \boldsymbol{A} = \widehat{\boldsymbol{C}}\,(\boldsymbol{I} - \boldsymbol{C})^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

Example



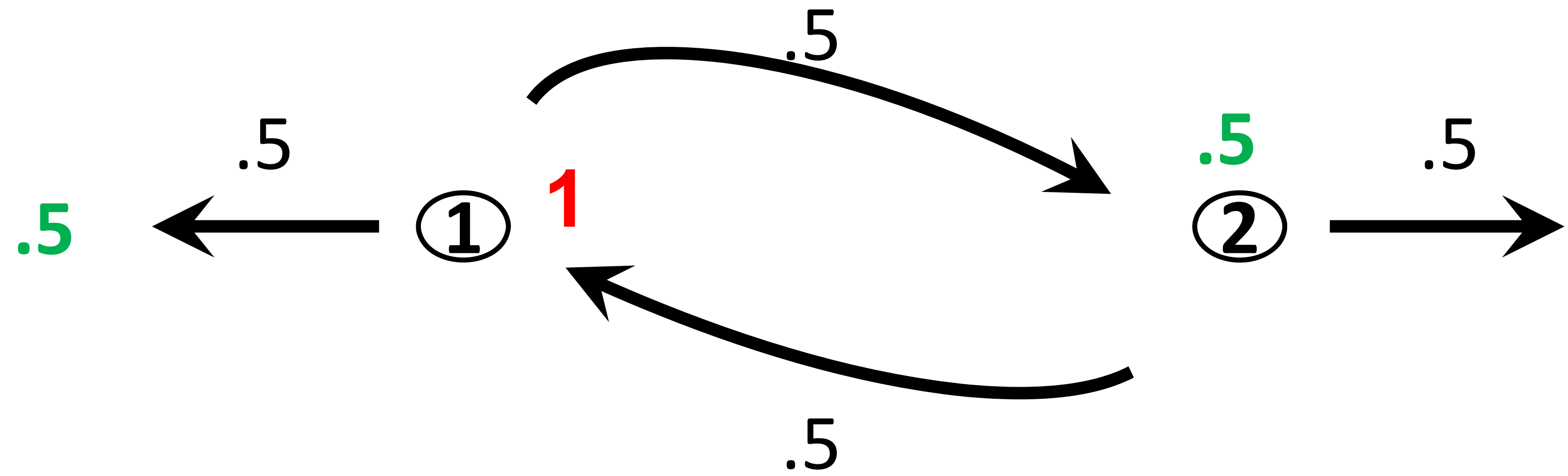
Example

What happens to $1 of investment income to 1?



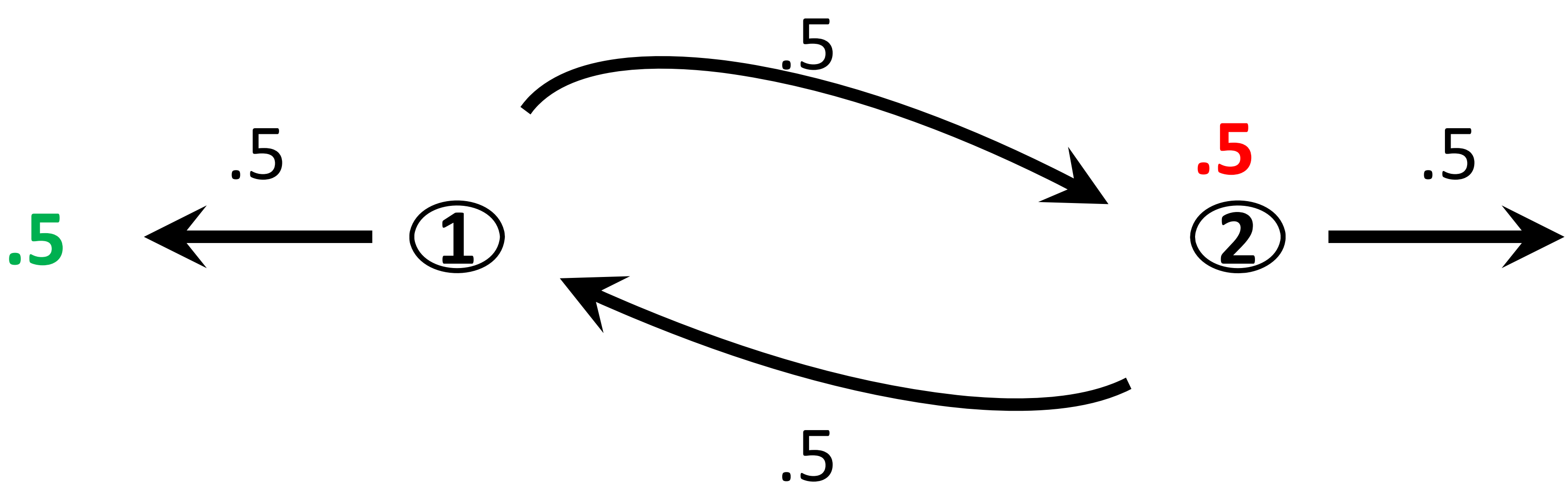
Example

What happens to $1 of investment income to 1?



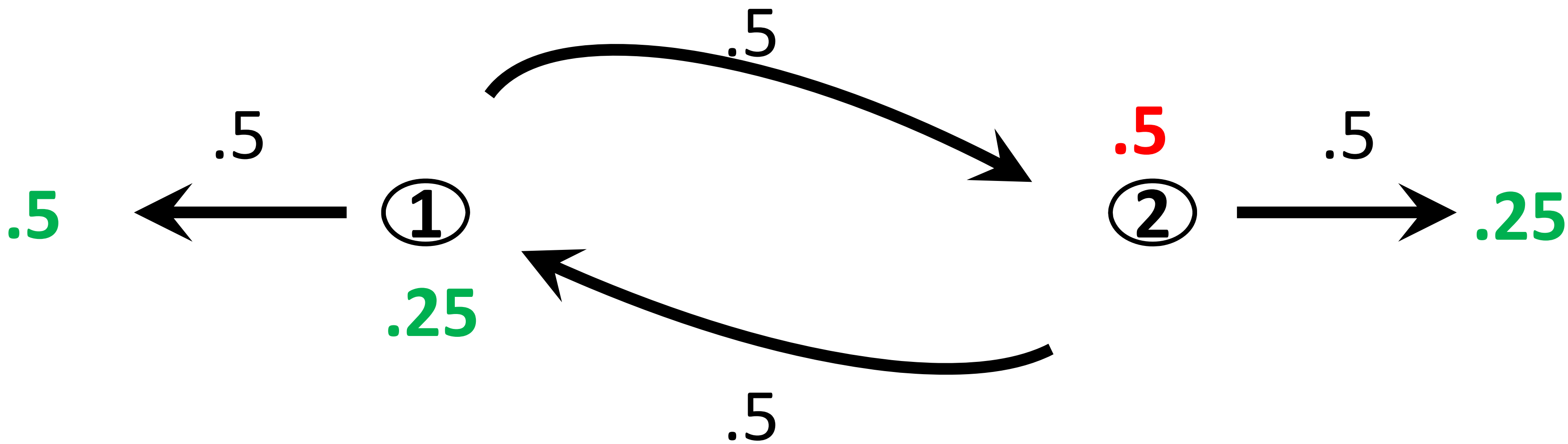
Example

What happens to $1 of investment income to 1?



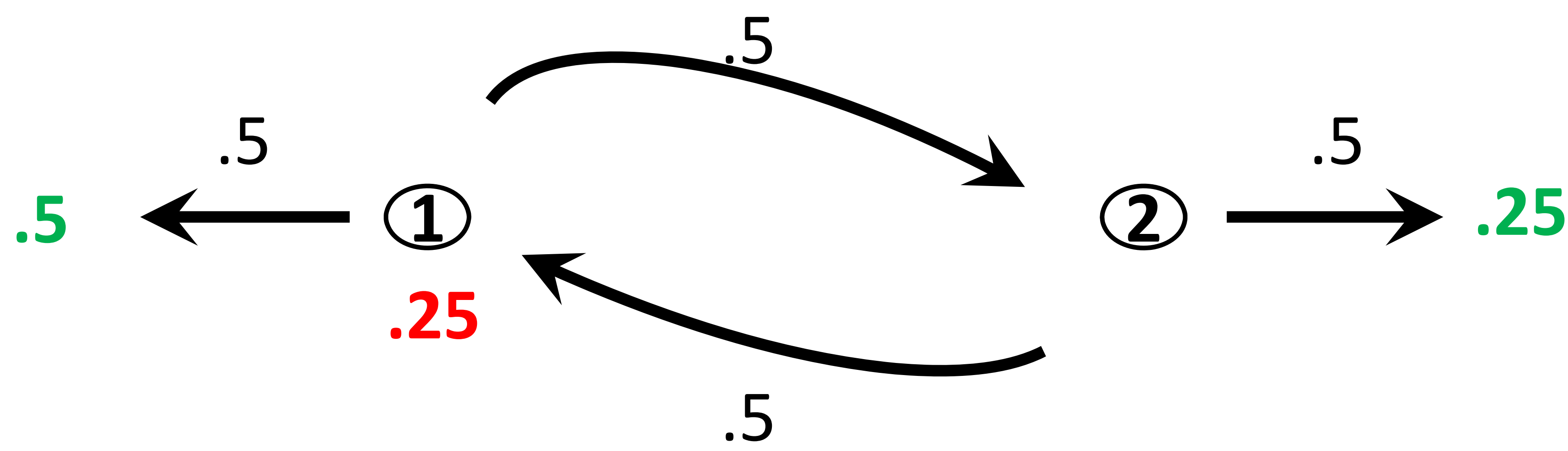
Example

What happens to $1 of investment income to 1?



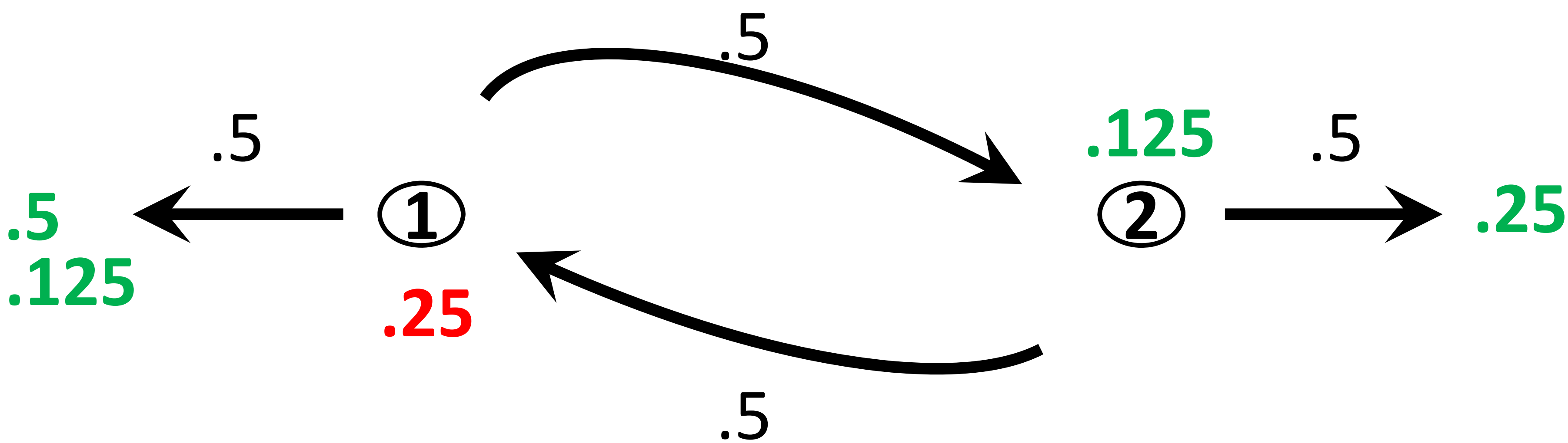
Example

What happens to $1 of investment income to 1?



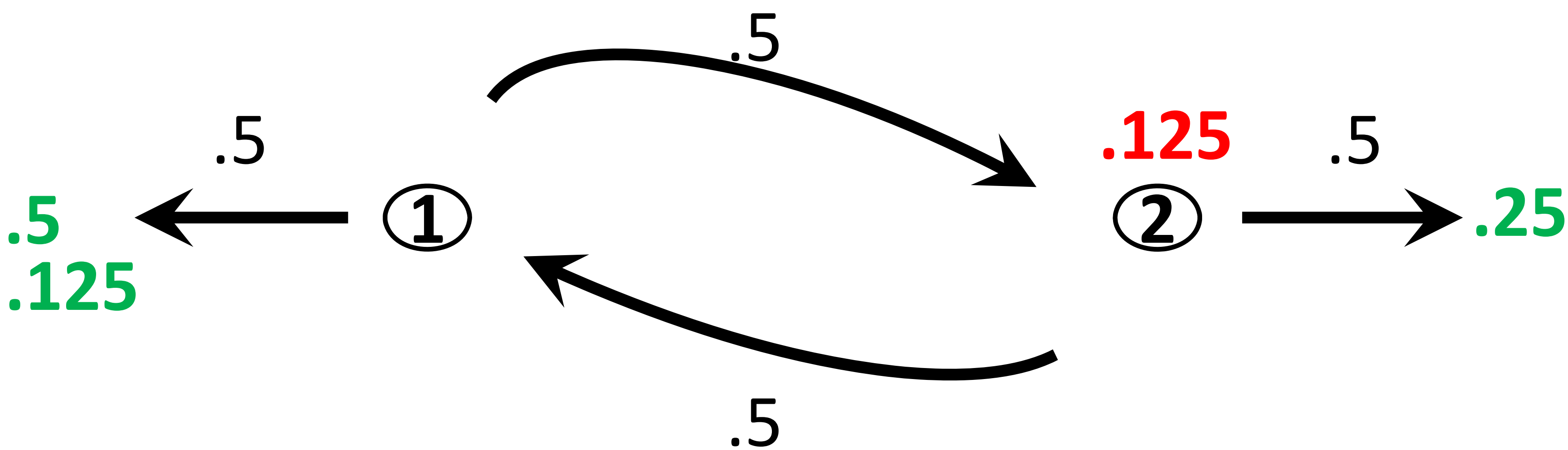
Example

What happens to $1 of investment income to 1?

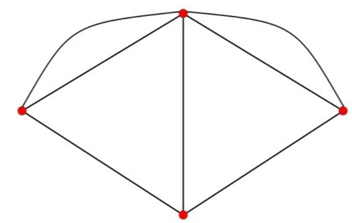


Example

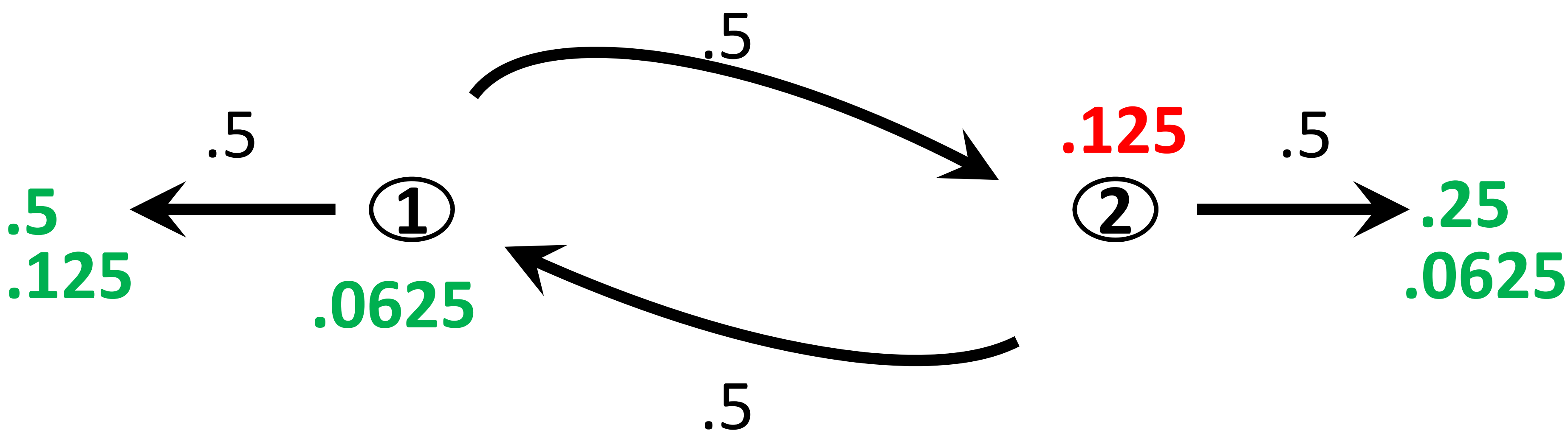
What happens to $1 of investment income to 1?



Example

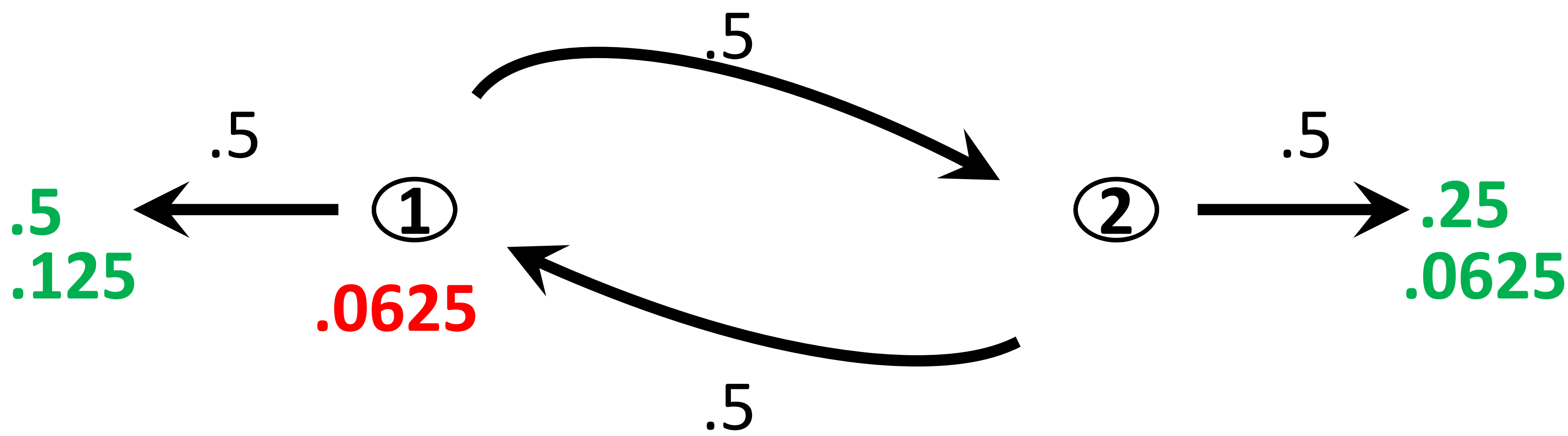


What happens to $1 of investment income to 1?



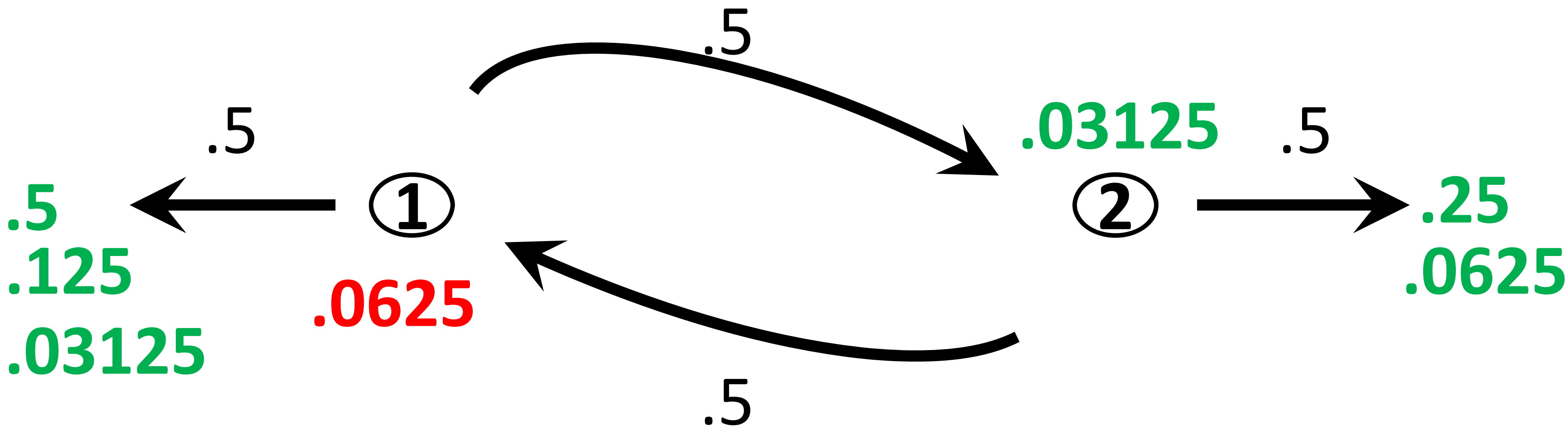
Example

What happens to $1 of investment income to 1?



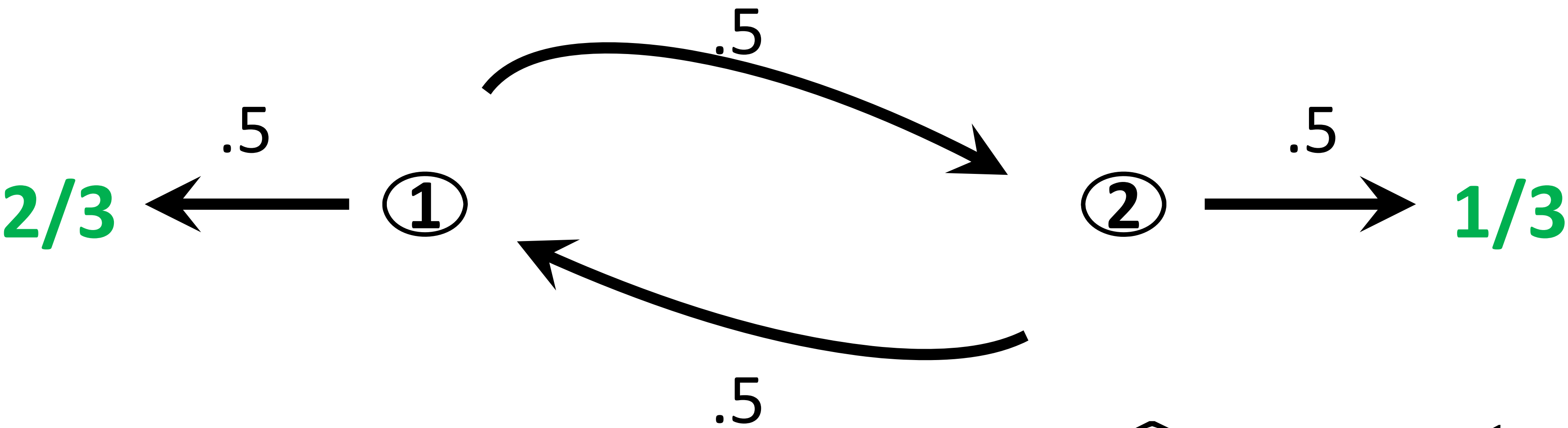
Example

What happens to $1 of investment income to 1?

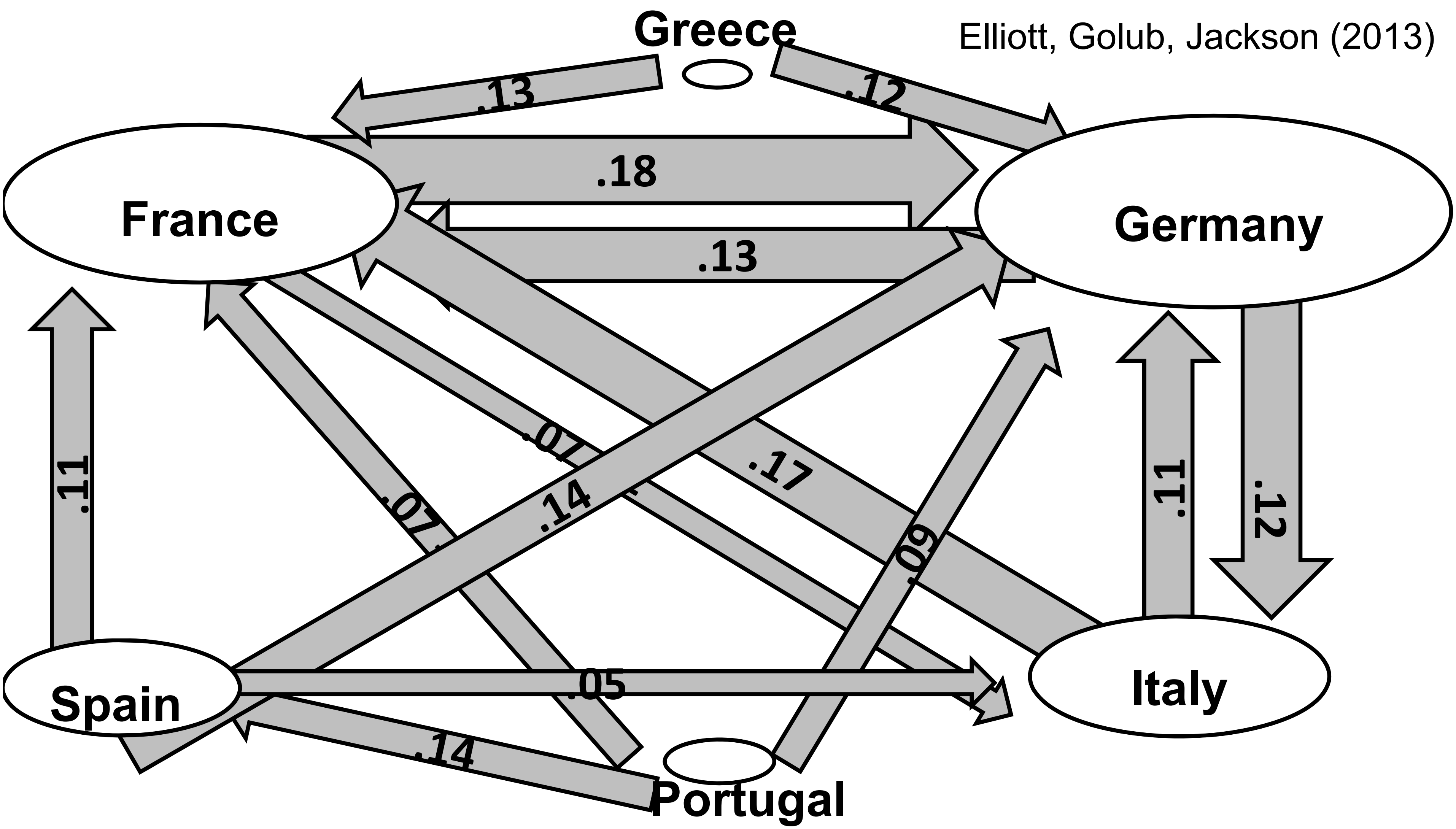


Example

What happens to $1 of investment income to 1?



$$\hat{\boldsymbol{C}}\,(\boldsymbol{I}-\boldsymbol{C})^{-1}$$


Elliott, Golub, Jackson (2013)
Greece
France
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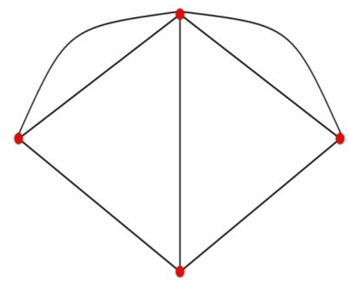
Social and Economic Networks: Models and Analysis



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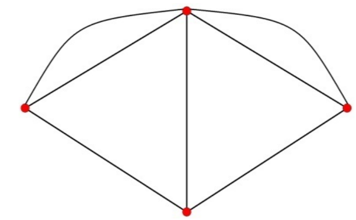
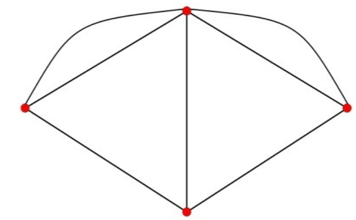
5.8c Application: Financial Contagions



Explore Contagions



Simple model of Elliott, Golub Jackson 13:

An organization has direct investments:

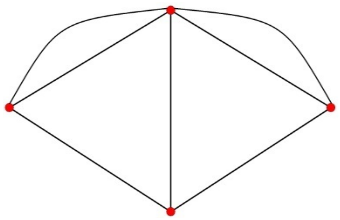
Fraction c_i of value accrues directly to them

Fraction $1-c_i$ is owed to others

Also hold obligations of d_i other organizations:

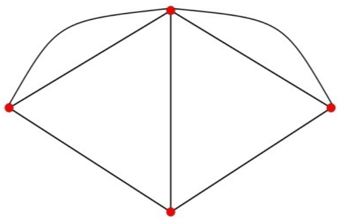
Have claims to those other organizations' investments

Simulation Setup



- n=100 organizations
- Random network g with $\Pr(g_{ij} = 1) = d/(n-1)$
- **d** = expected # other organizations that an organization cross holds (***d = level of diversification***)
- Fraction **c** of org cross-held (evenly split among those holding it), **1-c** held privately (***c = level of integration***)
- So, claim i has on j: $C_{ij} = cg_{ij}/d_j$

The Exercise



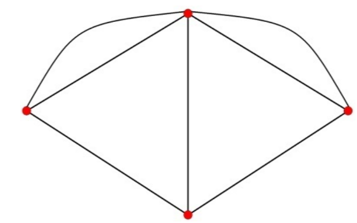
- One asset per organization (their investments), starts at value 1

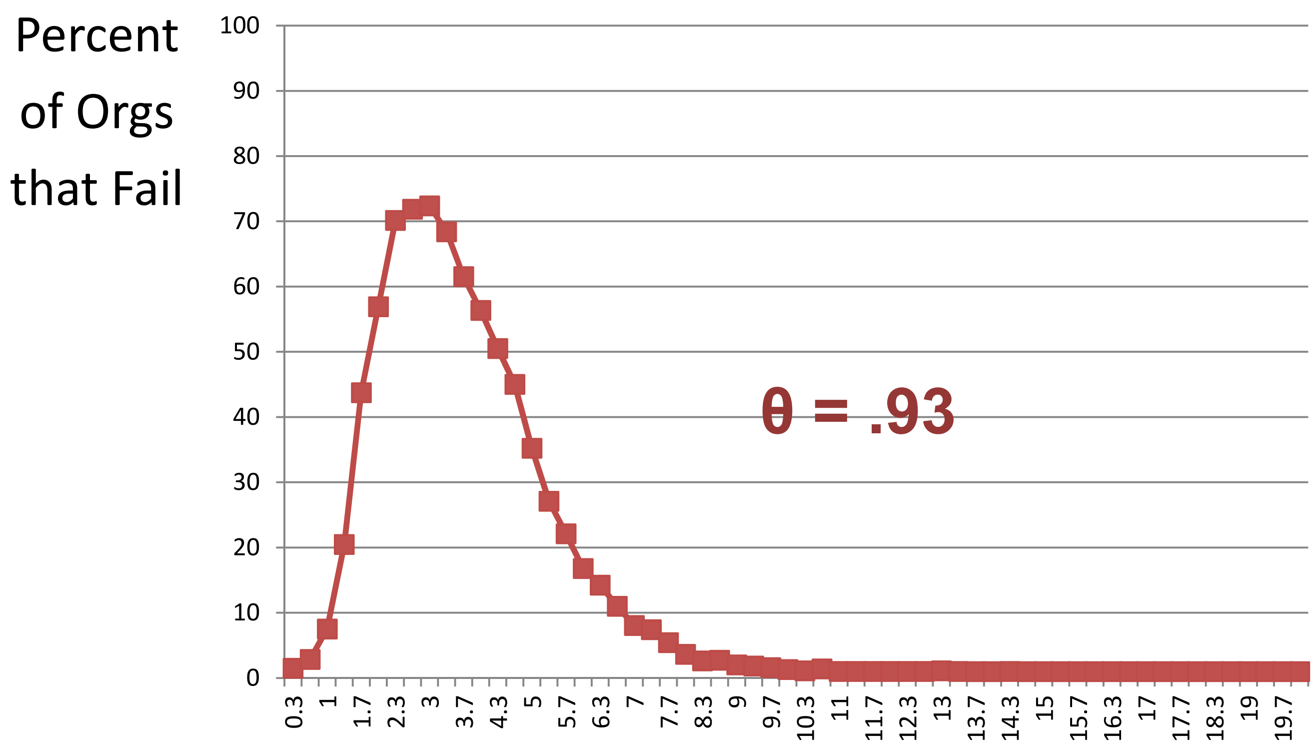
- Pick one organization’s investment to devalue to 0

- If an organization’s value
  drops below θ of its starting value, it fails.

- Look at resulting cascade

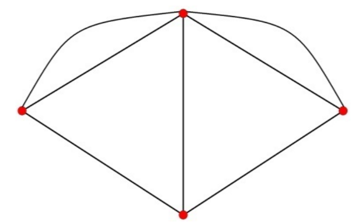
Diversification and Contagion: 93% threshold, c=.5



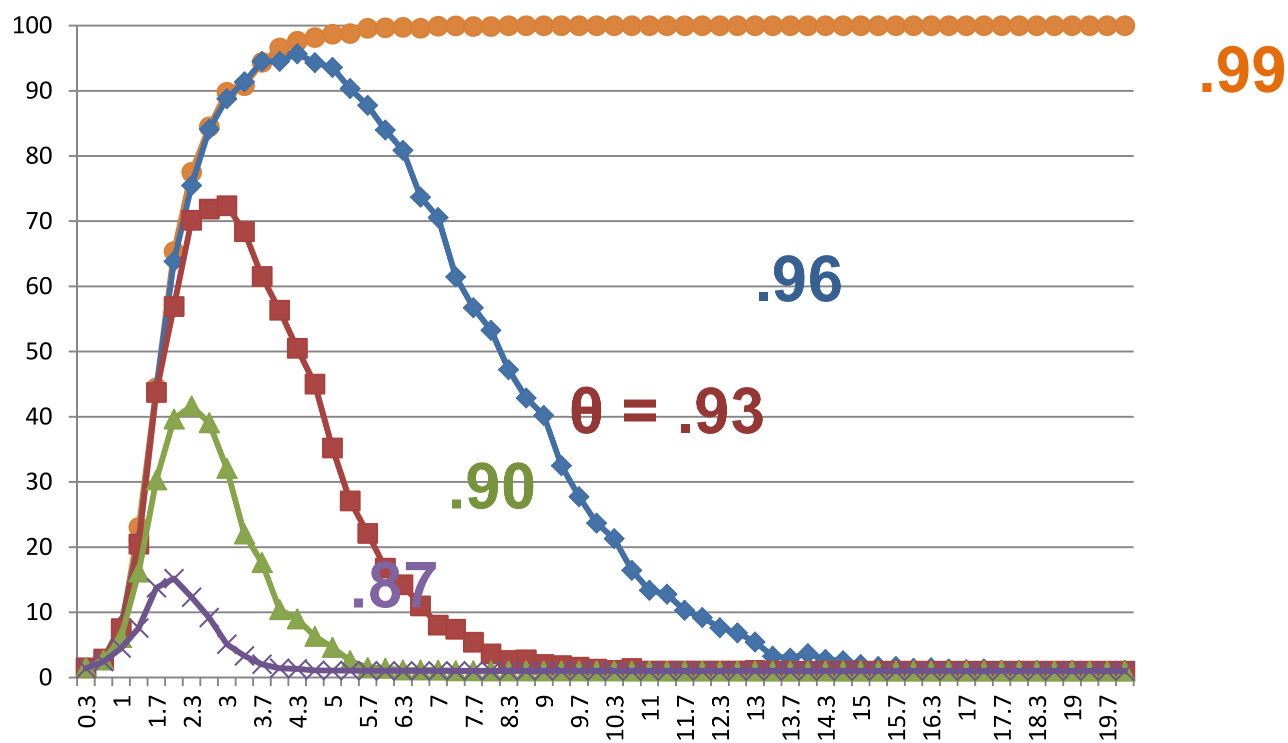


Degree: Expected # of cross-holdings

Diversification and Contagion: Various Thresholds

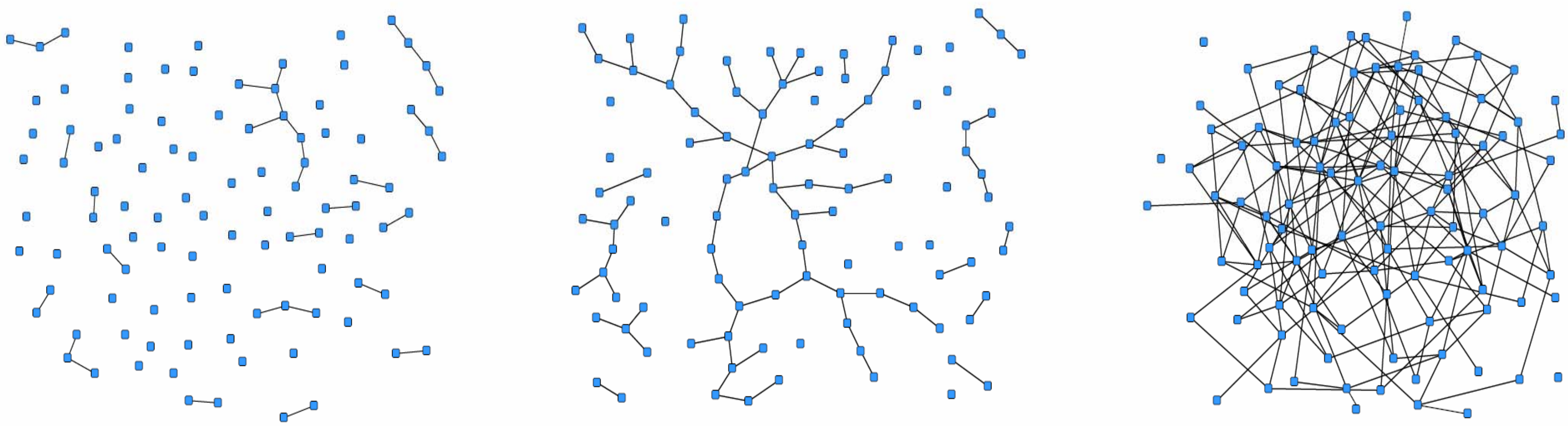


Percent of Orgs that Fail



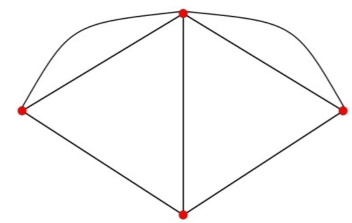
Degree: Expected # of cross-holdings

Intuition



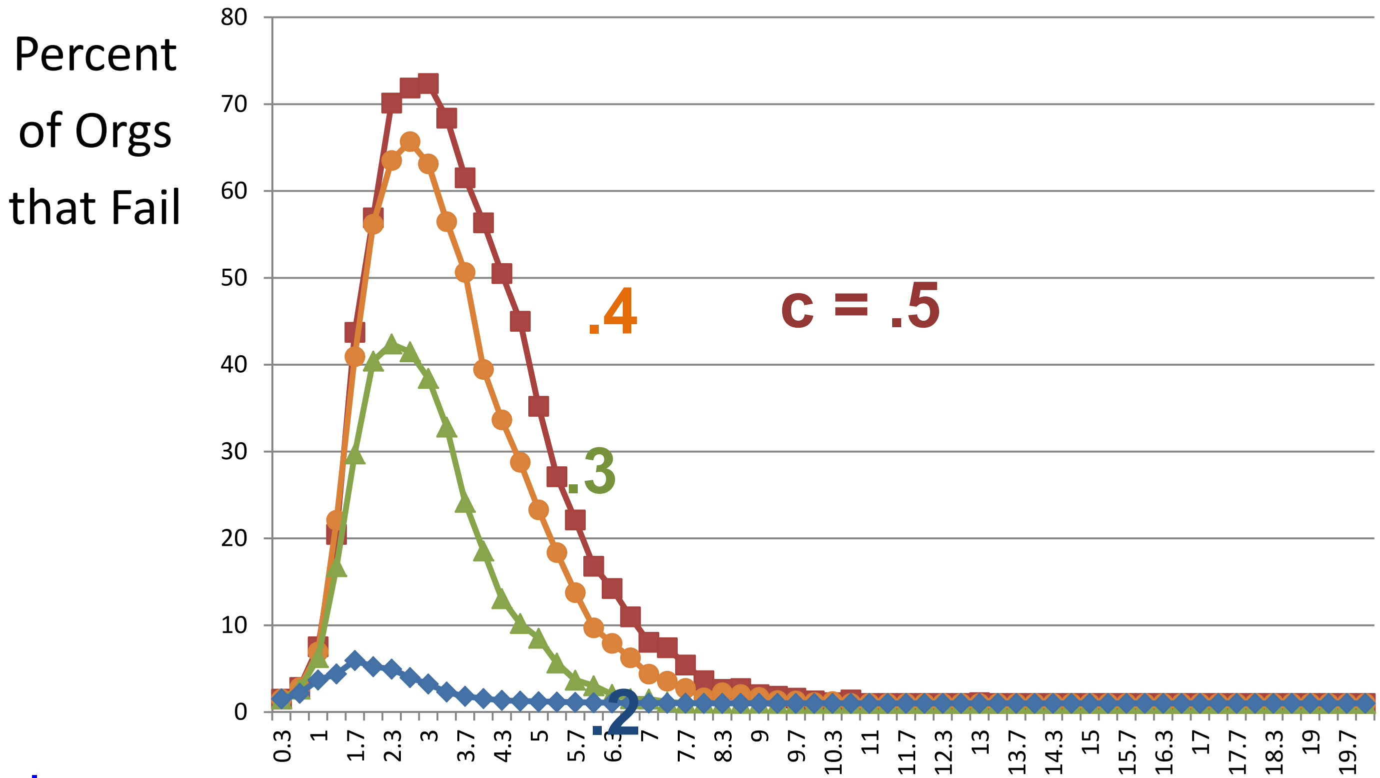
(a) Low diversification (b) Medium diversification (c) High diversification

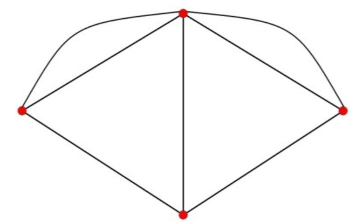
Diversification : Dangerous Middle Levels



- Low diversification:
  - fragmented network, no widespread contagion
- Medium diversification
  - Connected network, contagion is possible
  - Exposure to only a few others makes it easy to spread
- High diversification
  - Little exposure to any single other organization
  - Failures do not spread

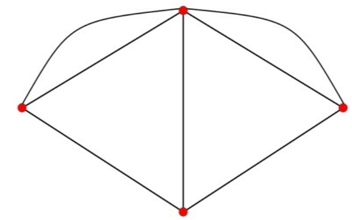
Integration: .93 Threshold



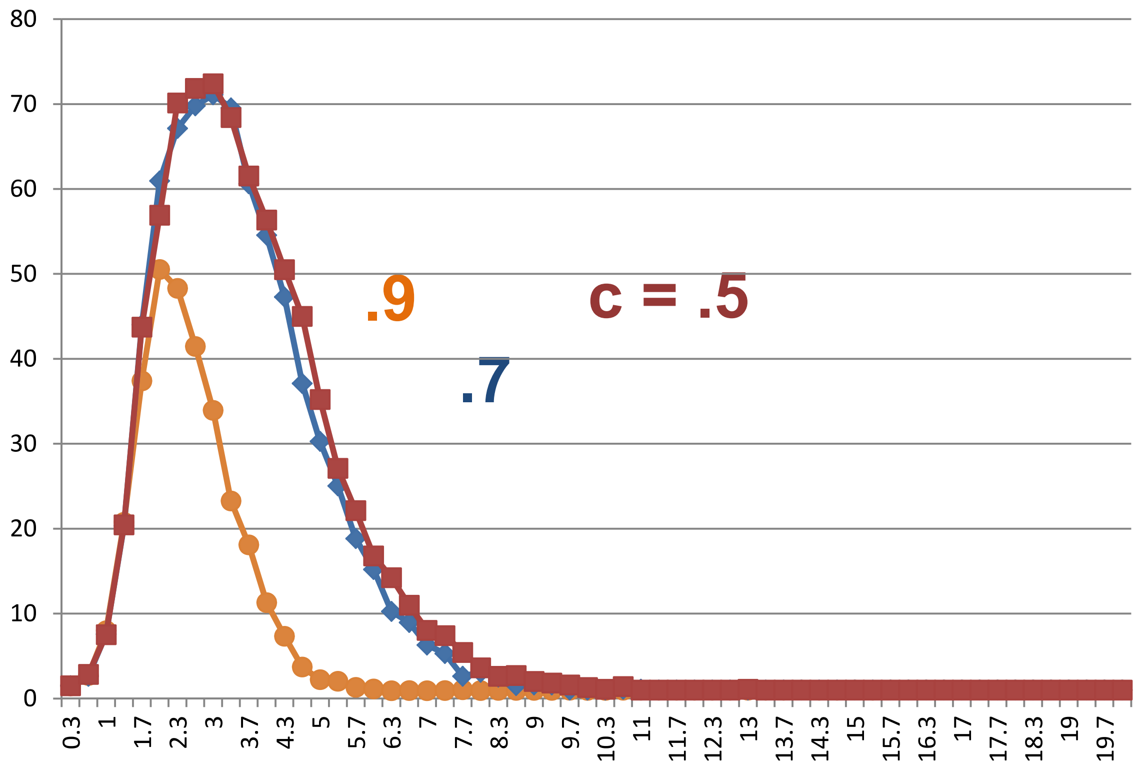


Degree: Expected # of cross-holdings

Integration: .93 Threshold

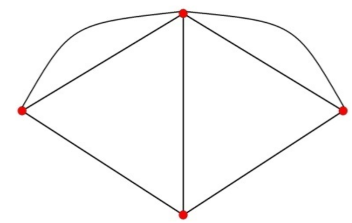


Percent of Orgs that Fail



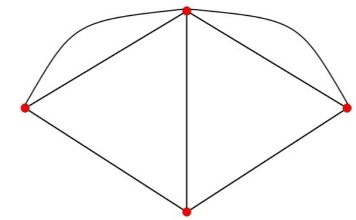
Degree: Expected # of cross-holdings

Integration



- Low integration: little exposure to others, failures don’t trigger others
- Middle integration: exposure to others substantial enough to trigger contagion
- High integration: difficult to get a *first* failure – failure of own assets does not trigger failure

Analysis

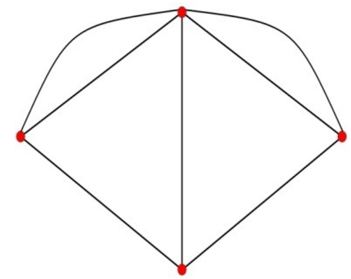


- Analyze richer networks

- Understand indirect holdings and how valuations/devaluations spread

- Understand effects of diversification, integration, size of shocks, correlation of shocks, heterogeneity in networks!...

Social and Economic Networks: Models and Analysis



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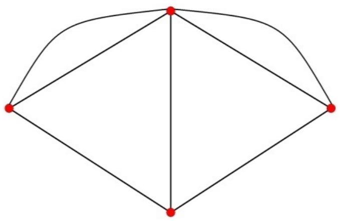
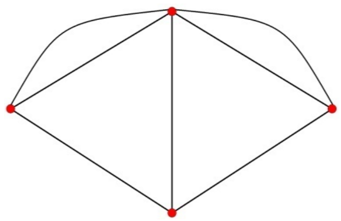
5.9:Diffusion Summary



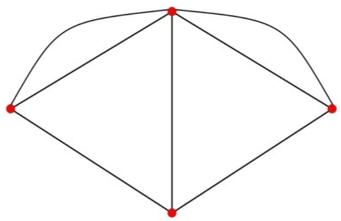
Lessons:



- Thresholds/``Phase Transitions'':
  - low density no contagion
  - middle density some probability of infection, part of population infected – reach most of population even with average degree around 3…
  - high density sure infection and all infected

- Degree affects who is infected and when

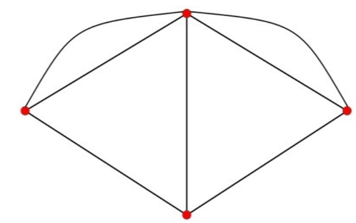
General Points



- Diffusion modeling
  - Important to model both information and peer effects:
  - Not simply an infection model: nonparticipants communicate - Distinguishes such models from epidemiology

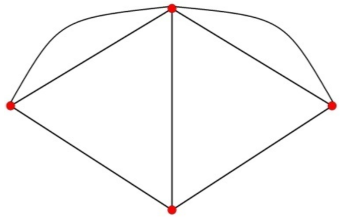
- Need more studies that identify the details of what matters in interactions: information, learning, complementarities/substitution, peer pressure, ...

Diffusion



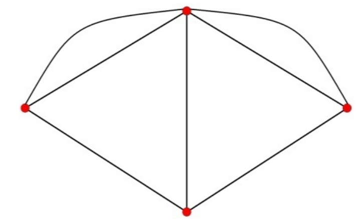
- Network structure matters
- Tractable, and simulations can go a long way to offering predictions
- experiment with changes in network structure…

Outline



- Part I: Background and Fundamentals
  - Definitions and Characteristics of Networks (1,2)
  - Empirical Background (3)
- Part II: Network Formation
  - Random Network Models (4,5)
  - Strategic Network Models (6, 11)
- Part III: Networks and Behavior
  - Diffusion and Learning (7,8)
  - Games on Networks (9)

Week 5 Wrap



- Adoption curves: s-shapes of diffusion
- S-shape: combination of imitation/complementarities and eventual saturation
- Initial contagion:
  - Depends on density and variance: high degree nodes serve as hubs and enable diffusion
- Extent of diffusion
  - relates to component structure, density – beyond one friend, (homophily...)
- Diffusion modeling
  - Can help dissect peer effects
  - underlies many relations: sheds light on financial contagions...

Week 5: References in order mentioned

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