Social and Economic Networks
Advanced Problems: Week 5

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1.

Consider a variation on the SIS model where instead of having a probability of $\nu\theta d$ of becoming infected when being of degree d and having a neighbor infection rate of θ, the probability is $\nu\theta$. This would be the case if, for instance, infection transmission depends on the fraction of neighbors being infected rather than the absolute number.

Show that in this case, $\rho(d)$ is independent of d and that $\rho = \theta = \frac{\lambda-1}{\lambda}$ if $\lambda > 1$ and is 0 otherwise.

2.

Consider the SIS model where the infection rate is modified to be $v\theta d+\epsilon$ where $\varepsilon > 0$ is a rate at which a node "mutates" to become infected, regardless of contact with infected nodes. Develop an expression for the steady-state θ as a function the degree distribution, and provide a solution for regular degree distributions.

3.

Given $1 \geq q \geq 0$, Morris (2000) defines a set of nodes S to be *q-cohesive* with respect to a network g if each node in S has at least a fraction q of its neighbors in S. That is, S is (at least) q-cohesive relative to g if

$$\min_{i \in S} \frac{|N_i(g) \cap S|}{d_i(g)} \geq q, \tag{1}$$

where $0/0$ is set to 1.

Consider a network (N, g) and suppose that a node/agent adopts a new technology if and only if at least a fraction of at least q of his or her neighbors do. Start with some set A having already adopting the new technology, and then iterate on the set of agents who adopt (consider any neighbors of agents in A who then have a fraction of at least q of their neighbors in A so that they will adopt the technology, and then keep iterating until nobody adopts at some step). Show that $B \cup A$ is the eventual set of nodes adopting the new technology (where $B \cap A = \emptyset$) if and only if the complement of $B \cup A$, denoted C, is more than $1 - q$-cohesive and $D \cup C$ is not q'-cohesive for any $q' > 1 - q$ for any nonempty subset D of B.