Social and Economic Networks
Advanced Problems: Week 7

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1.

Provide an example of a directed network with three players for which the only equilibria to a best shot game played on that network are in mixed strategies. Identify a mixed-strategy equilibrium. Assume that players pay a cost $1 > c > 0$ for taking the action, and get a payoff of 1 if either they or at least one of their (directed) neighbor(s) take the action.

2.

Consider the following variation of a local public goods game on a network. Payoffs are given by

$$u_i(x_i, x_{N_i(g)}) = f(x_i + \sum_{j \in N_i(g)} x_j) - c(x_i), \tag{1}$$

where f is strictly concave and c is strictly convex and both have continuous derivatives, and there exists $x^* > 0$ such that $f'(x^*) = c'(x^*)$, which is the action level that an individual chooses if he or she is the only provider.

Find a pure strategy equilibrium on a complete network and show that it is the unique pure strategy equilibrium and that all players choose positive actions.

Next, consider a circle network with an even number of players, and suppose that $f'(2x^*) < c'(0)$. Describe a specialized equilibrium where only some players choose positive actions.

3.

Consider a network (N, g) and a coordination game such that action 1 is a best response for any player if and only if at least a fraction of at least q of his or her neighbors play action 1 (and presume that a player takes action 1 when indifferent). Show the following result from Morris (2000). Let a labeling of nodes be a bijection (one-to-one and onto function) ℓ from N to N. Let $\alpha_\ell(i)$ be the fraction of $\ell(i)$'s neighbors who have labels less than $\ell(i)$. Show that there is a contagion from m nodes[1] if and only if there exists a labeling ℓ such that $\alpha_\ell(i) \geq q$ for all $i \geq m + 1$.

[1] Starting from those nodes taking action 1, iteratively consider best responses of all players besides those m to the previous play. So, if any of the $n - m$ nodes have at least q friends among the m, change them to action 1, and then keep iterating until no nodes change. There is a contagion if all agents end up taking action 1.

From this show that there exists a set S that is uniformly less than $1-q$-cohesive if and only if there is a labeling ℓ such that $\alpha_\ell(i) \geq q$ for all $i \geq m+1$ where m is the cardinality of the complement of S.

A set is uniformly less than $(1-q)$-cohesive if every nonempty subset has at least q of its neighbors outside of that subset.