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考虑电价的不确定性

Optimizing profits from hydroelectricity production

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Abstract

This paper presents a deterministic and a stochastic mathematical model for maximizing the profits obtained by selling electricity produced through a cascade of dams and reservoirs in a deregulated market. The first model is based on deterministic electricity prices while the other integrates price stochasticity through the management of a tree of potential price scenarios. Numerical results based on historical data demonstrate the superiority of the stochastic model over the deterministic one. It is also shown that price volatility impacts the profits obtained by the stochastic model.

Keywords: Hydroelectricity; Production; Market; Mathematical programming; Stochastic programming

1. Introduction

As more and more countries deregulate their electricity market, new challenges appear for hydroelectricity producers. To maximize their profits, they now need to consider market price uncertainty in their production plan. This is much more complex than maximizing the total production when prices are constant.

In this paper, production plans in a deregulated market are optimized for a hydroelectricity producer with multiple power plants along a river. The problem under study thus involves both hydrological and electricity market issues. Hydrological issues are related to the management of hydroelectric facilities, in particular management of the water that must be released at each site. Energy market issues are concerned with the setting of electricity prices based on demands from buyers and offers from producers. This is explained in the following.

1.1. Problem setting

The problem can be best described by dividing it into four main parts: the hydrological model, the operations of the units (turbines), the electricity market, and the objective.

1.1.1. Hydrological model

There are four important sites on the river under study. The first site d_1 is a dam that retains the water of a large reservoir just behind it. The three other sites d_2, d_3 and d_4, contain a hydroelectric power plant, a dam and a reservoir. Each power plant has two turbines that run independently. The head reservoir has a capacity of 400 h m^3 while the

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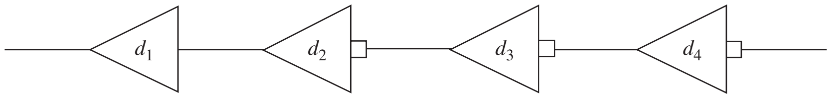

Fig. 1. Hydroelectric plants network.

reservoirs that follow have a capacity of 80, 100, and 40 h m^3, respectively. The sites are organized in sequence, namely (d_1, d_2, d_3, d_4). Hence, the water released at the upstream dam of site d_i goes into the reservoir of the downstream site d_{i+1}. This simple organization is illustrated in Fig. 1. In this figure, a triangle stands for a reservoir with a dam and a little square represents a power plant.

The time taken by the water flow to go from one site to the next is referred to as the "river routing effects". This complex physical phenomenon can be approximated through river routing coefficients. These coefficients correspond to the fraction of water released upstream that arrives at the downstream site every hour after the release. In our case, it takes about 3 h for all water to arrive at a downstream dam after its release from an upstream reservoir. The river routing coefficients thus generalize upon a single water flow time, as it is found in numerous applications (see, for example, [1–3]). Natural inflows are also taken into account, like those coming from snow melting, rain, runoff water, and natural river flow (for the head reservoir). These inflows are stochastic, but are handled here in a deterministic way, by using hourly averages provided by a natural inflow forecast model. This deterministic approximation is acceptable because deviations from natural inflow forecasts do not have a significant impact on the reservoir levels under a short-term planning horizon of 24 h (see Section 1.1.4). 以确定的方式处理入库

To satisfy operational constraints, the water level at each dam must also lie between a minimum and a maximum value, which are the same all year round. Each dam possesses a mechanism to spill a large quantity of water, if necessary. The spill flow is controlled quite precisely by adjusting gate openings. However, spilling should be avoided as much as possible, given that no electricity is produced in this case.

1.1.2. Operations of units

The amount of electricity produced by each unit (turbine) is a function of the water head, unit flow, and unit type. These curves are determined empirically and have, in general, a polynomial form. The first dam does not have any unit but the dams that follow (d_2, d_3, and d_4) have two identical units, each of capacity 95, 125, and 60 MW, respectively, for a total of 560 MW. When a unit is started, it suffers some wear due to the huge water pressure applied to it. So, there is a cost associated with starting a unit, and this cost is also taken into account. A study dedicated to start-up costs can be found in [4]. 备用

Besides producing electricity, a unit can also be "reserved", which means that some of its power capacity is put aside to provide electricity in case of a shortcoming somewhere over the network. A revenue is earned through this practice depending on the type of reserve under consideration, which can be either "10 min spin" (10S), "10 min non-spin" (10N) or "30 min non-spin" (30N). These types are related to the time required to bring the energy into use and the physical behavior of the facilities that provide it. When 10S reserve is called for, the unit must increase its output immediately and reach full capacity within 10 min. The 10N and 30N reserve only require that full power be reached within 10 and 30 min, respectively, without explicit conditions on the start of the power increase. When providing 10S reserve, the unit needs to spin at the right speed to synchronize itself with the electric network. If the unit is already producing electricity (using a fraction of its capacity), it is already synchronized. Otherwise, a cost is incurred due to the electricity that is needed to spin the unit at the right speed. Note that hydroelectric facilities are flexible enough to provide each type of reserve, which is not necessarily the case for thermal units which need more time to increase their power.

As the reserves that are asked for by the market operator are rarely consumed, we will assume in the following that no water is used for this purpose. More information on reserves can be found in [5].

1.1.3. Electricity market

In our application, the electricity prices come from a deregulated market. The latter widely differs from traditional monopolistic markets where a single player controls the production and the electricity prices are set through government regulations. It was previously believed that a monopolistic market was natural in the case of electricity production due to the necessity to balance loads and supplies at all time, but recent findings show that deregulation can be efficient and reduce electricity costs.

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In a deregulated market, a central organization (market operator) dispatches electricity production among different producers by considering offers and demands from the participants. Producers and buyers submit selling and purchase bids, respectively, where a bid corresponds to some quantity of energy and a price per MWh. Supply and demand curves are then created and the intersection of the two curves determines the market clearing price (MCP). All selling bids under the MCP and all purchase bids over the MCP are accepted. Any producer with at least one accepted bid will receive the MCP for each MWh that he produces. Any buyer with at least one accepted bid will pay the MCP for each MWh that he uses. In addition to the energy market, there are three other markets associated with the three reserve types. However, the 30N reserve is not considered here because its price is always lower than or equal to the 10N reserve price, and there is no cost for providing these two types of reserves. The price for each reserve type is calculated by considering bids from producers and the amount of reserve required for the electric network to be safe.

Since the bidding process introduces an additional level of complexity, it is not explicitly addressed here. Rather, we assume that any quantity of electricity offered by a producer is sold at a price derived from historical market data (averages for the deterministic model presented in Section 2; distributions that follow the evolution of prices over the day for the stochastic model in Section 3). A consequence of this assumption is that a producer might have to sell electricity below his bidding price. In this study, prices from the Ontario electricity market in Canada have been used. In this market, any offer must be submitted at least 2 h in advance (it should be noted that the Ontario electricity market has changed to a day-ahead commitment market in June 2006). In a 2 h-ahead market, a producer who wants to provide electricity at 4:00 PM, for example, must announce it at or before 2:00 PM. Also, offers must be submitted independently for each unit (as opposed to the whole hydroelectric plant or all facilities of a producer).

1.1.4. Objective

The objective is to find a 24-h production plan, for each unit in the power plants along a river, that maximizes the expected profit resulting from electricity and reserve sales, while satisfying the above-mentioned physical and operational constraints. The final result is a table indicating, for each hour and each unit, how the power should be distributed between electricity production and the reserves over a 24-h planning horizon.

This paper distinguishes itself from others due to the complexity of deriving plans that account for the electricity production and the reserves of each unit of a hydrological system with a series of facilities along a river. In particular, the cascade effects along the river prevent the facilities to be considered in isolation, as it is often done with thermal units.

1.2. Related work

An excellent survey on the various optimization techniques used in the field of energy can be found in [6]. When considering more specifically the hydroelectric case, the literature can be divided into three main problem classes: reservoir management problems, hydrothermal plant coordination problems and hydroelectric production problems.

Reservoir management problems are concerned with the management of water levels in reservoirs. The objective varies widely from one application to another and there are also different operational constraints that must be satisfied. This is a rather mature field of research with a rich literature (a state-of-the-art review can be found in [7]). These problems become very complex when a sequence of reservoirs must be managed, because any water released at an upstream reservoir contributes to the inflows of the downstream reservoirs. The scheduling of the water releases is thus an important issue, due to the lag time between the release and the water arrival at the downstream reservoir. Another important issue is the stochastic nature of the natural inflows. In this case, stochastic dynamic programming is a natural problem-solving approach, but its computational burden is often excessive for realistic problems with many reservoirs. Different aggregation techniques to reduce problem size have thus been proposed in the literature [8–10]. Other methods, like stochastic programming (SP) [11,12] have also been successfully applied to these problems.

The goal of hydrothermal coordination problems is to improve the coordination between hydroelectric and thermal plants. This is an important issue, because hydroelectric plants can be operated in a quite flexible way, but are limited by the amount of water available. Conversely, thermal plants are less flexible but can produce electricity on a more steady basis. A variety of optimization techniques have been proposed to solve these problems. Examples for the deterministic case are mathematical programming [13,14], neural networks [15], Lagrangian relaxation [16–19], and metaheuristics, in particular genetic algorithms [20,21]. When stochastic issues are considered, three main optimization techniques are reported, namely, SP [22], stochastic dynamic programming [23–25], and stochastic Lagrangian relaxation [26,27].

The literature on hydroelectric production can be divided into two subclasses, depending if a single reservoir or many reservoirs are considered. The latter problems are much more difficult to solve due to the "cascade" effects from one reservoir to the next. For the single reservoir case, the authors in [28] describe a problem-solving approach based on dynamic programming; stochastic inflows are then integrated and handled through stochastic dynamic programming [29]. In the case of multiple reservoirs, mixed integer programming [2,30,31], dynamic programming [32], and stochastic dynamic programming (for stochastic variants) [33–39] have all been reported in the literature.

With regard to multiple reservoirs, the problem considered by Pritchard and Zakeri in [38] is the most similar to ours. In their work, the authors develop a stochastic dynamic programming model to maximize the profits resulting from electricity sales in a deregulated market, where electricity prices are modeled through non-homogeneous Markov chains. To solve their problem with stochastic dynamic programming, the reservoir levels are first discretized. Then, the river routing effects are taken into account by introducing artificial intermediate reservoirs for each time period that falls in the lag time between the water release at an upstream reservoir and the water arrival at the next reservoir. The size of the state space grows quickly with the number of reservoirs considered (including artificial ones). As a consequence, only small problems with two reservoirs, plus two intermediate ones, are addressed in their work. Another shortcoming is that their approach is quite restricted with regard to the type of price generation processes that can be handled. Also, some practical issues are not addressed (but could be integrated at the expense of an increase in complexity), like start-up costs and reserves.

The problem-solving approach proposed by Pritchard and Zakeri remains valuable, especially if one considers that bidding issues are integrated into their model. However, it is not appropriate in our context because nine intermediate reservoirs would have to be introduced in addition to the four real reservoirs (i.e., three reservoirs between each consecutive pair of reservoirs). With only 10 discretization levels per reservoir, a state space with more than 10^{13} states per period would be obtained. The alternative model proposed in this paper grows linearly with the number of reservoirs, through the management of a tree of price scenarios (see Section 3). A shortcoming of this approach is that it prevents us from considering mid- or long-term time horizons. However, this is not required in our application because only the next 24 h are taken into account. As shown in Section 5, this is easily achieved. We also have some flexibility with regard to the stochastic process used to generate electricity and reserve prices. This would allow our system to be fed, for example, with prices obtained from an external forecast module.

In the following, the deterministic and stochastic models are described in Sections 2 and 3. Then, the electricity price model is presented in Section 4. Production plan examples and numerical results are reported in Section 5. Finally, Section 6 concludes.

2. Deterministic model

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In the deterministic case, electricity and reserve prices are assumed to be known over all periods of the production horizon. In the following, the mathematical model is first described. Then, we explain how non-linear water head effects are taken into account.

2.1. Mathematical model

In the mathematical model proposed below, the parameters are in capital letters while the variables are in lower case letters. Note also that some constraints are redundant but are presented here because they reduce the computation time of the solver.

2.1.1. Parameters

- T: number of periods for production planning.
- D: number of dams.
- U_d: number of units at dam d, $d = 1, \ldots, D$.
- P_t^{E}: price for each MW h of electricity produced in period t (\$/MW h), $t = 1, \ldots, T$.
- $P_t^{10\mathrm{S}}$: price for each MW h of electricity offered for 10S reserve in period t (\$/MW h), $t = 1, \ldots, T$.
- $P_t^{10\mathrm{N}}$: price for each MW h of electricity offered for 10N reserve in period t (\$/MW h), $t = 1, \ldots, T$.

- $MW_{d,u}(v, f)$: power of unit u of dam d as a function of flow f and volume v at dam d (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- H_d^{ref}: reference water head of dam d (m), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $H_d^{\min}$: minimum water head of dam d (m), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $H_d^{\max}$: maximum water head of dam d (m), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $H_d(v)$: water head of dam d as a function of volume v (m), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $H'_d(v)$: derivative of the function $H_d(v)$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $MW_{d,u}^{\text{ref}}(f)$: power of unit u of dam d as a function of flow f when the water head is at reference value (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $(MW_{d,u}^{\text{ref}})'(f)$: derivative of the function $MW_{d,u}^{\text{ref}}(f)$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $I_{d,u}$: number of piecewise parts in the linear approximation of $MW_{d,u}^{\text{ref}}(f)$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $R_{d,u,i}$: rate (slope) of part i in the linear approximation of $MW_{d,u}^{\text{ref}}(f)$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $i = 1, \ldots, I_{d,u}$.
- $UB_{d,u,i}$: upper bound or limit of part i in the linear approximation of $MW_{d,u}^{\text{ref}}(f)$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $i = 1, \ldots, I_{d,u} - 1$.
- $V_{d,t}^{\min}$: minimum volume of dam d at the end of period t (m^3), $d = 1, \ldots, D$; $t = 1, \ldots, T$.
- $V_{d,t}^{\max}$: maximum volume of dam d at the end of period t (m^3), $d = 1, \ldots, D$; $t = 1, \ldots, T$.
- V_d^{init}: initial volume of dam d (m^3), $d = 1, \ldots, D$.
- $F_{d,u}^{\min}$: minimum flow that can be processed by unit u of dam d (m^3/s), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $F_{d,u}^{\max}$: maximum flow that can be processed by unit u of dam d (m^3/s), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $F_d^{\text{spill}_{\max}}$: maximum spill flow of dam d (m^3/s), $d = 1, \ldots, D$.
- $F_d^{out_{\max}}$: minimum outflow of dam d (m^3/s), $d = 1, \ldots, D$.
- $F_d^{out_{\max}}$: maximum outflow of dam d (m^3/s), $d = 1, \ldots, D$.
- $MW_{d,u}^{\min}$: minimum power of unit u of dam d at reference head (when flow is $F_{d,u}^{\min}$) (MW), $d=1, \ldots, D$; $u=1, \ldots, U_d$.
- $MW_{d,u}^{\max}$: maximum power of unit u of dam d at reference head (when flow is $F_{d,u}^{\max}$) (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $NI_{d,t}$: mean natural inflow forecast at dam d for period t (m^3/s), $d = 1, \ldots, D$; $t = 1, \ldots, T$.
- RR: number of periods that are considered when calculating the river routing effects.
- $FR_{d,p}$: fraction ($\in [0, 1]$) of water, released in period $t - p$ at dam $d - 1$, that arrives at dam d in period t (same value for all t), $d = 1, \ldots, D$; $p = 0, \ldots, RR$.
- $MW_{d,u}^{\text{spin}}$: power needed by unit u of dam d to spin when 10S reserve is offered without producing electricity (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $SC_{d,u}$: start-up cost of unit u of dam d (\$), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$.
- $FVR_d(v)$: future value of water in the reservoir of dam d as a function of volume v (\$), $d = 1, \ldots, D$.
- K_d: number of piecewise parts in the linear approximation of $FVR_d(v)$, $d = 1, \ldots, D$.
- $R_{d,k}^{fvr}$: rate (slope) of part k in the linear approximation of $FVR_d(v)$, $d = 1, \ldots, D$; $k = 1, \ldots, K_d$.
- $UB_{d,k}^{fvr}$: upper bound of part k in the linear approximation of $FVR_d(v)$, $d = 1, \ldots, D$; $k = 1, \ldots, K_d - 1$.
- E_d: estimate of the energy that can be produced with each m^3 of water at dam d (MW h/m^3), $d = 1, \ldots, D$. This value gets larger as we move upstream (upward) because more electricity can be produced with the same amount of water, due to the cascade effect.
- FP: estimate of the average price at which electricity can be sold in the future (\$/MW h).
- FVR_d^{init}: future value of water in the reservoir of dam d, initially (i.e., $FVR_d^{\text{init}} = FVR_d(V_d^{\text{init}})$), $d = 1, \ldots, D$.

2.1.2. Decision variables

- $f_{d,t}^{\text{spill}} \geqslant 0$: flow spilled at dam d in period t (m^3/s), $d = 1, \ldots, D$; $t = 1, \ldots, T$.
- $f_{d,u,t}^{E} \geqslant 0$: flow going through unit u of dam d in period t to produce electricity (m^3/s), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.
- $f_{d,u,t,i}^{E} \geqslant 0$: contribution of piecewise part i, in the linear approximation of $MW_{d,u}^{\text{ref}}(f)$, to the flow going through unit u of dam d in period t to produce electricity (m^3/s), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $i = 1, \ldots, I_{d,u}$ (see Section 2.1.3).

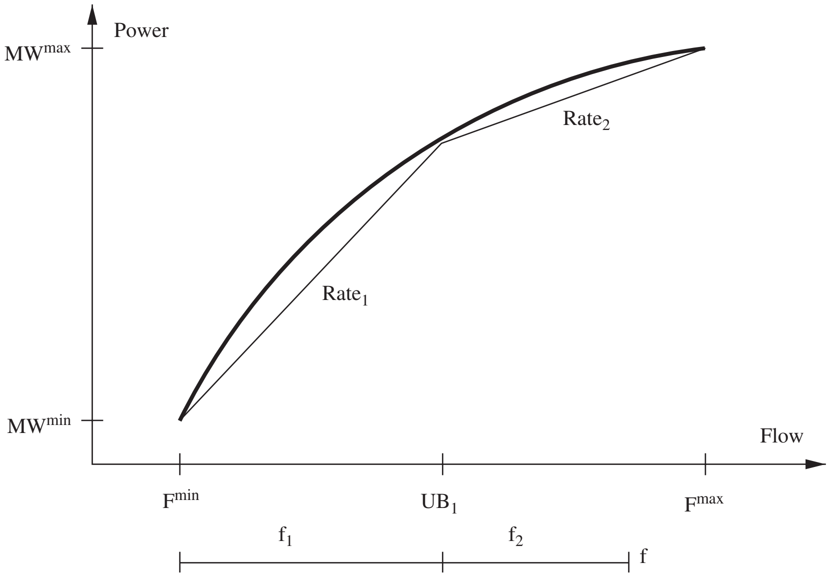

Fig. 2. Example of a power generation function at reference head.

- $y^{E}_{d,u,t} \in \{0, 1\}$: 1 if electricity is produced by unit u of dam d in period t, 0 otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 0, 1, \ldots, T$.
- $y^{\text{spin}}_{d,u,t} \in \{0, 1\}$: 1 if unit u of dam d spins by supplying electricity to it in period t, 0 otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.
- $mw^{E}_{d,u,t} \geqslant 0$: power produced by unit u of dam d in period t (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.
- $mw^{10S}_{d,u,t} \geqslant 0$: power reserved for 10S for unit u of dam d in period t (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.
- $mw^{10N}_{d,u,t} \geqslant 0$: power reserved for 10N for unit u of dam d in period t (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

2.1.3. Constraints

In our model, each unit (turbine) is considered independently. For a specific unit, the power generation function depends on two variables: the water volume v at the dam and the flow f going through the unit. More precisely, the power generation function $MW(v, f)$ that we consider is

$$MW(v, f) = \frac{H(v)}{H^{\text{ref}}} MW^{\text{ref}}(f), \tag{1}$$

where

- $H(v)$ is the water head when volume at the dam is v (this function can generally be approximated by a polynomial in v).
- $MW^{\text{ref}}(f)$ is the power generation function based on a reference head H^{ref} that gives the power produced when flow f is going through the unit.

The model assumes a constant reference water head. However, Section 2.2 explains how non-linear water head effects can be considered using a successive linear programming method. The power generation function at reference head $MW^{\text{ref}}(f)$ is typically not linear, since the marginal power increase tends to diminish when the flow increases. We thus approximate it with a concave piecewise linear function like the one found in [40]. An example is presented in Fig. 2. We note that there is no power until a flow $F^{\min}$ is reached. At this flow value, the power produced is $MW^{\min}$. At the maximal flow value $F^{\max}$, the power is $MW^{\max}$.

The general idea to approximate $MW^{\text{ref}}(f)$ is to divide the function into I intervals with bounds $F^{\min} = UB_0, UB_i$, $i = 1, \ldots, I-1$ and $F^{\max} = UB_I$. With each interval is also associated a slope R_i, $i = 1, \ldots, I$. We then define I

variables f_i, $i = 1, \ldots, I$, and set the flow f according to Eq. (2) for a given unit u and period t at dam d. Thus, if some electricity E is produced ($y^E = 1$) the flow f must be at least $F^{\min}$. The bounds on the f_i values are found in Eqs. (4))–(6). The value of $MW^{\text{ref}}(f)$ is then approximated by summing up the contribution of each interval, which corresponds to the flow associated with the interval times the slope and, if some electricity is produced ($y^E = 1$), $MW^{\min}$ is added. This calculation is done in Eq. (7).

Eq. (9) ensures that the whole capacity of a unit ($MW^{\max}$) is allocated between electricity production and the reserves. The relation between producing electricity and spinning is found in Eqs. (10) and (11). Recall that a unit must produce some electricity ($y^E = 1$) or must spin, by applying a current on it ($y^{\text{spin}} = 1$), to provide 10S reserve. This is because the unit needs to be synchronized with the network to quickly respond to a call from the network operator.

热备用

- $f^E_{d,u,t}$ is the sum of all piecewise parts plus $F^{\min}_{d,u}$, if electricity is produced ($y^E_{d,u,p} = 1$), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$f^E_{d,u,t} = y^E_{d,u,t} \cdot F^{\min}_{d,u} + \sum_{i=1,\ldots,I_{d,u}} f^E_{d,u,t,i}. \tag{2}$$

- $f^E_{d,u,t}$ equals 0 if $y^E_{d,u,t}$ equals 0 and is between 0 and $F^{\max}_{d,u}$ otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$0 \leqslant f^E_{d,u,t} \leqslant F^{\max}_{d,u} \cdot y^E_{d,u,t}. \tag{3}$$

- Bounds on $f^E_{d,u,t,i}$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $i = 1, \ldots, I_{d,u}$.

$$i = 1: \quad 0 \leqslant f^E_{d,u,t,i} \leqslant UB_{d,u,1} - F^{\min}_{d,u}, \tag{4}$$

$$i = 2, \ldots, I_{d,u} - 1: \quad 0 \leqslant f^E_{d,u,t,i} \leqslant UB_{d,u,i} - UB_{d,u,i-1}, \tag{5}$$

$$i = I_{d,u}: \quad 0 \leqslant f^E_{d,u,t,i} \leqslant F^{\max}_{d,u} - UB_{d,u,I_{d,u}-1}. \tag{6}$$

- Evaluation of $mw^E_{d,u,t} = MW^{\text{ref}}_{d,u}(f^E_{d,u,t})$ through its linear approximation, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$mw^E_{d,u,t} = y^E_{d,u,t} \cdot MW^{\min}_{d,u} + \sum_{i=1,\ldots,I_{d,u}} f^E_{d,u,t,i} \cdot R_{d,u,i}. \tag{7}$$

- Spill flow of dam d in period t is between 0 and a maximum value, $d = 1, \ldots, D$; $t = 1, \ldots, T$.

$$0 \leqslant f^{\text{spill}}_{d,t} \leqslant F^{\text{spill}_{\max}}_{d}. \tag{8}$$

- All power of unit u of dam d is used or reserved at period t, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$mw^E_{d,u,t} + mw^{10\text{S}}_{d,u,t} + mw^{10\text{N}}_{d,u,t} = MW^{\max}_{d,u}. \tag{9}$$

- Power for 10S can only be reserved if unit u of dam d spins or produces electricity, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$mw^{10\text{S}}_{d,u,t} \leqslant MW^{\max}_{d,u} \cdot (y^E_{d,u,t} + y^{\text{spin}}_{d,u,t}). \tag{10}$$

- Unit u of dam d cannot produce electricity and spin at the same time, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$.

$$y^E_{d,u,t} + y^{\text{spin}}_{d,u,t} \leqslant 1. \tag{11}$$

2.1.4. Intermediate variables 中间变量

The values of the following intermediate variables depend on the parameters and the original decision variables. They are used here to simplify the formulation of the constraints that follow. These variables and their constraints represent the hydrological laws that govern the system.

The outflow at a dam is calculated through Eq. (12). It is the spilling flow plus the sum of flows going through each unit. For the model to be completely defined, historical outflows for period $-(RR-1), \ldots, 0$ are used, where RR is the number of periods required for all water released at the upstream dam to reach the downstream reservoir ($RR = 3$ h in our case). These values are needed in Eq. (14) to calculate the inflows of the first periods of the planning horizon.

The inflows are calculated through Eqs. (13) and (14). For a specific hour, the inflow of dam d is equal to the natural inflows at this hour plus the flow coming from the upstream dam $d-1$ (taking into account the lag time between the water release at the upstream dam $d-1$ and the water arrival at the downstream dam d). The summation over p considers the fraction of water FR that arrives at the same hour ($p = 0$), 1 h later ($p = 1$), 2 h later ($p = 2$) and 3 h later ($p = 3$), for $RR = 3$.

Natural inflows can be integrated into models that take into account weather forecasts [41–46]. However, stochastic natural inflows are not considered here because their impact on a short-term horizon is limited. That is, reservoir levels are not likely to be significantly modified by unexpected natural inflows over an horizon of 24 h. In fact, these levels are mostly modified through plant operations (flows for producing electricity and spilling flows). Also, natural inflow forecasts are generally reliable enough over a 24-h horizon to be considered deterministic, as opposed to long-term forecasts.

Finally, Eqs. (15) and (16) calculate the water volume of a reservoir at the end of period t, which is the volume at the end of period $t-1$ plus the water gain (inflows) minus the water loss (outflows).

- $out_{d,t}$: flow that leaves the reservoir of dam d in period t (m^3/s), $d = 1, \ldots, D$; $t = 1, \ldots, T$. It is also assumed that a number of "historical" outflow values are available for periods $-(RR-1), \ldots, 0$.

$$out_{d,t} = f_{d,t}^{\text{spill}} + \sum_{u=1,\ldots,U_d} f_{d,u,t}^{E}. \tag{12}$$

- $in_{d,t}$: flow that enters the reservoir of dam d in period t (m^3/s), $d = 1, \ldots, D$; $t = 1, \ldots, T$.

$$d = 1: \quad in_{d,t} = NI_{d,t}, \tag{13}$$

$$d = 2, \ldots, D: \quad in_{d,t} = NI_{d,t} + \sum_{p=0,\ldots,RR} out_{d-1,t-p} \cdot FR_{d,p}. \tag{14}$$

- $v_{d,t}$: water volume of dam d at the end of period t (m^3), $d = 1, \ldots, D$; $t = 1, \ldots, T$.

$$t = 1: \quad v_{d,t} = V_d^{\text{init}} + (in_{d,t} - out_{d,t}) \cdot 3600, \tag{15}$$

$$t = 2, \ldots, T: \quad v_{d,t} = v_{d,t-1} + (in_{d,t} - out_{d,t}) \cdot 3600. \tag{16}$$

2.1.5. Constraints on intermediate variables

- Water volume of dam d at the end of period t is between a minimum and a maximum value, $d=1, \ldots, D$; $t=1, \ldots, T$.

$$V_{d,t}^{\min} \leqslant v_{d,t} \leqslant V_{d,t}^{\max}. \tag{17}$$

- Outflow of dam d in period t is between a minimum and a maximum value, $d = 1, \ldots, D$; $t = 1, \ldots, T$.

$$F_d^{out\min} \leqslant out_{d,t} \leqslant F_d^{out\max}. \tag{18}$$

2.1.6. Objective

The goal of this model is to maximize the profit of a single production day for a hydroelectricity producer. This profit is the production revenues minus the production costs and minus the future value of any water used during that day. It is shown in Eq. (19).

The revenue obtained from electricity and reserve sales in Eq. (20), is simply the power used for electricity production and for the reserves multiplied by their corresponding prices. In the deterministic case, average electricity and reserve prices are directly fed into the model. In Eq. (21), a cost for spinning is incurred when a unit offers 10S reserve without producing electricity at the same time. Furthermore, Eqs. (22)–(25) state that a cost is incurred when the unit needs a start-up. Thus, this cost is added when there is some production at the current period t ($y_{d,u,t}^E = 1$) but no production at the previous period $t-1$ ($y_{d,u,t-1}^E = 0$). Note that Eq. (22) assumes that there is no production before period 1 (i.e., at period 0). Note also that Eqs. (24) and (25) are not mandatory as the optimization process minimizes the start-up costs. However, intensive computational tests have shown that their inclusion reduces the computation time.

It is important to consider the value of the water used for production. Otherwise, the model will tend to use all water available and empty the reservoirs at the end of the day, because it will not consider the adverse impact of this strategy on future revenues. The future value of water in a reservoir (FVR) is a function that estimates the profit that can be made with this water in the future. This function is normally concave as the marginal value of the water decreases when its availability increases. For example, it is not possible to exceed the capacity of the turbines when the prices are high, even if a lot of water is available, as spilling would be unavoidable. More importantly, the average electricity sales prices tend to decrease when a lot of water is available. The FVR is thus approximated with a concave piecewise linear function.

We assume that the FVR function is the same at the beginning and at the end of the day, due to the short-term horizon. With this assumption, the value of the water used for production during the day is expressed as *FVR*(*initial volume*) −*FVR*(*final volume*). The initial FVR can be calculated exactly with the true function before the optimization, based on the initial volume. In the optimization model, the final FVR is approximated with the piecewise linear function represented in Eq. (26). Each variable v_k^{fin} represents one of the K parts of the piecewise approximation, in a way similar to the production function approximation. The bounds on v_k^{fin} are found in Eqs. (28)–(30). Finally, the sum of all v_k^{fin} variables must be equal to the final volume, adjusted to take into account the river routing effects, as shown in Eq. (27). That is, the final volume of dam d is equal to the volume $v_{d,T}$ at the end of the last period T plus the inflows released at dam $d-1$ before or at period T that arrive at dam d at periods $T+1, T+2, \ldots, T+RR$. These delayed inflows should be considered, otherwise they would be forgotten in the objective.

In practice, the FVR should be evaluated with a long-term model that would integrate the stochastic nature of natural inflows and prices. This could be done with stochastic dynamic programming over an horizon of several months. To obtain a tractable problem, however, some aggregation of the reservoirs should be performed to reduce problem size [8–10].

- Objective function ($)

$$Maximize \sum_{\substack{d=1,\ldots,D \\ u=1,\ldots,U_d \\ t=1,\ldots,T}} \{rev_{d,u,t} - c^{\text{spin}}_{d,u,t} - c^{\text{start}}_{d,u,t}\} - \sum_{d=1,\ldots,D} val_d. \tag{19}$$

- $rev_{d,u,t}$: revenue from selling electricity and reserve for unit u of dam d in period t ($), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$.

$$rev_{d,u,t} = (mw^E_{d,u,t} \cdot P^E_t + mw^{10S}_{d,u,t} \cdot P^{10S}_t + mw^{10N}_{d,u,t} \cdot P^{10N}_t). \tag{20}$$

- $c^{\text{spin}}_{d,u,t}$: cost of providing electricity to unit u of dam d to make it spin in period t ($), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$.

$$c^{\text{spin}}_{d,u,t} = y^{\text{spin}}_{d,u,t} \cdot MW^{\text{spin}}_{d,u} \cdot P^E_t. \tag{21}$$

- $c^{\text{start}}_{d,u,t} \geqslant 0$: start-up cost of unit u of dam d at period t ($), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=2,\ldots,T$.

$$t=1: \quad c^{\text{start}}_{d,u,t} = SC_{d,u} \cdot y^E_{d,u,t}, \tag{22}$$

$$t=2,\ldots,T: \quad c^{\text{start}}_{d,u,t} \geqslant SC_{d,u} \cdot (y^E_{d,u,t} - y^E_{d,u,t-1}), \tag{23}$$

$$c^{\text{start}}_{d,u,t} \leqslant SC_{d,u} \cdot y^E_{d,u,t}, \tag{24}$$

$$c^{\text{start}}_{d,u,t} \leqslant SC_{d,u} \cdot (1 - y^E_{d,u,t-1}). \tag{25}$$

- val_d: net value of water used for production at dam d during planning horizon (equal to the value at the start of the planning horizon minus the value at the end) ($), $d=1,\ldots,D$.

$$val_d = FVR^{\text{init}}_d - \sum_{k=1,\ldots,K_d} R^{fvr}_{d,k} \cdot v^{fin}_{d,k}. \tag{26}$$

- $v^{fin}_{d,k} \geqslant 0$: contribution of piecewise part k to the final volume of dam d, in the linear approximation of FVR_d (*final_volume*) (m^3), $d=1,\ldots,D$; $k=1,\ldots,K_d$.

The sum of the $v_{d,k}^{fin}$ values is equal to the final volume, which is then adjusted to account for the river routing effects at the end of the horizon, $d = 1, \ldots, D$. The summation after $v_{d,T}$ corresponds to inflows that arrive at dam d at periods $T+1, T+2, \ldots, T+RR$.

$$\sum_{k=1,\ldots,K_d} v_{d,k}^{fin} = v_{d,T} + 3600 \sum_{\substack{r=1,\ldots,RR\\ p=r,\ldots,RR}} out_{d-1,T+r-p} \cdot FR_{d,p}. \tag{27}$$

- Bounds on $v_{d,k}^{fin}$, $d = 1, \ldots, D$; $k = 1, \ldots, K_d$.

$$k=1: \quad 0 \leqslant v_{d,k}^{fin} \leqslant UB_{d,k}^{fvr}, \tag{28}$$

$$k=2,\ldots,K_d-1: \quad 0 \leqslant v_{d,k}^{fin} \leqslant UB_{d,k}^{fvr} - UB_{d,k-1}^{fvr}, \tag{29}$$

$$k=K_d: \quad 0 \leqslant v_{d,k}^{fin} \leqslant V_{d,1}^{\max} - UB_{d,K_d-1}^{fvr}. \tag{30}$$

2.2. *Water head effects*

This section describes how non-linear water head effects can be evaluated using a successive linear programming method that is often reported in the literature on water management [35,47–49]. The method starts from the initial solution obtained by solving the original model. Then, a new model is created with the same variables and constraints except that the electricity production equation (7) is replaced by

$$\begin{aligned} mw_{d,u,t}^{E} = {} & MW_{d,u}(\widehat{v}_{d,t}^{avg}, \widehat{f}_{d,u,t}^{E}) + \frac{H_d'(\widehat{v}_{d,t}^{avg})}{H_d^{\text{ref}}} MW_{d,u}^{\text{ref}}(\widehat{f}_{d,u,t}^{E}) \cdot (v_{d,t}^{avg} - \widehat{v}_{d,t}^{avg}) \\ & + \frac{H_d(\widehat{v}_{d,t}^{avg})}{H_d^{\text{ref}}} (MW_{d,u}^{\text{ref}})'(\widehat{f}_{d,u,t}^{E}) \cdot (f_{d,u,t}^{E} - \widehat{f}_{d,u,t}^{E}), \end{aligned} \tag{31}$$

where $\widehat{f}_{d,u,t}^{E}$ is the value of $f_{d,u,t}^{E}$ in the initial solution, $v_{d,t}^{avg}$ is the average volume of period t given by

$$t=1: \quad v_{d,t}^{avg} = \frac{V_d^{\text{init}} + v_{d,t}}{2}, \tag{32}$$

$$t=2,\ldots,T: \quad v_{d,t}^{avg} = \frac{v_{d,t-1} + v_{d,t}}{2} \tag{33}$$

and $\widehat{v}_{d,t}^{avg}$ is the value of $v_{d,t}^{avg}$ in the initial solution.

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Formula (31) is in fact the order 1 Taylor's polynomial approximation of function $MW_{d,u}(v, f)$ evaluated at $\widehat{v}_{d,t}^{avg}$, $\widehat{f}_{d,u,t}^{E}$. This approximation is only valid when $v_{d,t}^{avg}$ and $f_{d,u,t}^{E}$ are close enough to $\widehat{v}_{d,t}^{avg}$ and $\widehat{f}_{d,u,t}^{E}$, respectively. We thus impose the following additional constraints for $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$:

$$\widehat{v}_{d,t}^{avg} - \lambda \cdot (V_{d,t}^{\max} - V_{d,t}^{\min}) \leqslant v_{d,t}^{avg} \leqslant \widehat{v}_{d,t}^{avg} + \lambda \cdot (V_{d,t}^{\max} - V_{d,t}^{\min}), \tag{34}$$

$$\widehat{f}_{d,u,t}^{E} - \lambda \cdot (F_{d,u}^{\max} - F_{d,u}^{\min}) \leqslant f_{d,u,t}^{E} \leqslant \widehat{f}_{d,u,t}^{E} + \lambda \cdot (F_{d,u}^{\max} - F_{d,u}^{\min}), \tag{35}$$

where $\lambda \in [0, 1]$ is a parameter that defines how close $v_{d,t}^{avg}$ and $f_{d,u,t}^{E}$ are from $\widehat{v}_{d,t}^{avg}$ and $\widehat{f}_{d,u,t}^{E}$, respectively. Furthermore, as the capacity of a unit now depends on the water head, Eq. (9) is replaced by

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$$mw_{d,u,t}^{E} + mw_{d,u,t}^{10S} + mw_{d,u,t}^{10N} = \boxed{\frac{H_d(\widehat{v}_{d,t}^{avg})}{H_d^{\text{ref}}} MW_{d,u}^{\max} + \frac{H_d'(\widehat{v}_{d,t}^{avg})}{H_d^{\text{ref}}} MW_{d,u}^{\max} \cdot (v_{d,t}^{avg} - \widehat{v}_{d,t}^{avg}),} \tag{36}$$

where the right-hand side of the equation is the order 1 Taylor's polynomial approximation of function $MW_{d,u}(v, F_{d,u}^{\max}) = (H_d(v)/H_d^{\text{ref}}) MW_{d,u}^{\max}$. We also replace Eq. (10) by

$$mw_{d,u,p}^{10S} \leqslant \frac{H_d^{\max}}{H_d^{\text{ref}}} MW_{d,u}^{\max} \cdot (y_{d,u,t}^{E} + y_{d,u,t}^{\text{spin}}) \tag{37}$$

because $mw^{10S}_{d,u,t}$ can take values up to $(H^{\max}_d/H^{\mathrm{ref}}_d)MW^{\max}_{d,u}$ when the water head is at its maximum value. To speed-up the computations, we also add constraints to eliminate binary variables that have already been optimized (for $d=1,\ldots,D;\ u=1,\ldots,U_d;\ t=1,\ldots,T$):

$$y^E_{d,u,t}=\widehat{y}^E_{d,u,t}, \tag{38}$$

$$y^{\mathrm{spin}}_{d,u,t}=\widehat{y}^{\mathrm{spin}}_{d,u,t}. \tag{39}$$

The overall optimization process is the following:

1. Find an initial solution s to the original model
2. $\lambda \leftarrow \lambda_0$
3. $iter \leftarrow 1$
4. While $iter \leqslant N_{iter}$ and $\lambda > \lambda_{\min}$
   (a) Find a solution s' with the modified water head model
   (b) If $F(s') > F(s)$ then 比较目标函数
      $s \leftarrow s'$
      Else if $F(s') < F(s)$
      $\lambda \leftarrow \lambda \cdot \lambda_{\mathrm{mult}}$ (with $\lambda_{\mathrm{mult}} \in (0,1)$)
      Else if $F(s') = F(s)$
      Exit while loop (the solution is optimal)
   (c) $iter \leftarrow iter + 1$

This procedure optimizes the profit while taking into account water head by first allowing large changes in volume and flow (λ large) and smaller changes later. The function F corresponds to the objective function in the linear model, except that it is calculated with the true function $MW_{d,u}(v,f)$, instead of the piecewise linear approximation or the Taylor's approximation. In the procedure, the parameter λ keeps its value while the new solution improves upon the current one (i.e., $F(s') > F(s)$). When $F(s') < F(s)$, the value of λ is reduced according to $\lambda \cdot \lambda_{\mathrm{mult}}$, with $\lambda_{\mathrm{mult}} < 1$. The procedure is repeated until either N_{iter} iterations have been performed, λ is small enough or convergence is observed (i.e., $F(s') = F(s)$). Section 5.2 analyzes the impact of water head on solution quality.

2.3. Parameter values

The numerical results are based on a real hydrological network, but some data have been modified for confidentiality reasons. As already mentioned, we consider a large reservoir at the head of the river, followed by three smaller reservoirs each with an associated hydroelectric power plant. Each power plant has two similar units (turbines). The parameter values used for the experiments are the followings:

- Number of periods (hours): $T = 24$.
- Number of dams: $D = 4$.
- Number of units (turbines) at each dam: $U_1 = 0$, $U_2 = 2$, $U_3 = 2$, $U_4 = 2$.
- Prices for electricity and reserves at period t (historical averages):

$$P^E_t = \overline{P}^E_t, \quad P^{10S}_t = \overline{P}^{10S}_t, \quad P^{10N}_t = \overline{P}^{10N}_t, \quad t = 1,\ldots,24.$$

- Reference water head of dam $d = 2,3,4$: $H^{\mathrm{ref}}_{2,u} = 45.5$, $H^{\mathrm{ref}}_{3,u} = 49$, $H^{\mathrm{ref}}_{4,u} = 35$, $u = 1,2$.
- Minimum water head of dam $d = 2,3,4$: $H^{\min}_{2,u} = 44.25$, $H^{\min}_{3,u} = 47.5$, $H^{\min}_{4,u} = 33.9$, $u = 1,2$.
- Maximum water head of dam $d = 2,3,4$: $H^{\max}_{2,u} = 46.75$, $H^{\max}_{3,u} = 50.5$, $H^{\max}_{4,u} = 36.1$, $u = 1,2$.
- Water head of dam $d = 2,3,4$ as a function of volume v:

$$H_2(v) = -0.0002179v2 + 0.0486781v + 44.25.$$

$$H_3(v) = -0.0001823v2 + 0.0482292v + 47.5.$$

$$H_4(v) = -0.0009066v2 + 0.0912637v + 33.9.$$

- Derivative of function H:

$$H'_d(v) = \mathrm{d}(H_d(v))/\mathrm{d}v, \quad d = 1, \ldots, 4.$$

- Power function of unit u of dam $d = 2, 3, 4$ (where MW^{ref} is the power at the reference head value):

$$MW_{d,u}(v, f) = H_d(v)/H_d^{\mathrm{ref}} MW_{d,u}^{\mathrm{ref}}(f).$$

平方

$$MW_{2,u}^{\mathrm{ref}}(f) = -0.0021491 f2 + 1.0198214 f - 25.6604314, \quad u = 1, 2.$$

$$MW_{3,u}^{\mathrm{ref}}(f) = -0.0015629 f2 + 1.0412698 f - 43.1562882, \quad u = 1, 2.$$

$$MW_{4,u}^{\mathrm{ref}}(f) = -0.0018621 f2 + 0.8344132 f - 27.3971054, \quad u = 1, 2.$$

- Derivative of MW^{ref}:

$$(MW_{d,u}^{\mathrm{ref}})'(f) = \mathrm{d}(MW_{\mathrm{d},u}^{\mathrm{ref}}(f))/\mathrm{d}f, \quad d = 1, \ldots, 4.$$

- Number of piecewise parts in the linear approximation of MW^{ref}: $I_{d,u} = 4$, $d = 1, \ldots, 4$; $u = 1, 2$
- Slopes in the linear approximation of MW^{ref} for dam $d = 2, 3, 4$:

$$R_{2,u} = \{0.761929, 0.525528, 0.321364, 0.138690\}, \quad u = 1, 2.$$

$$R_{3,u} = \{0.791206, 0.611472, 0.431739, 0.259820\}, \quad u = 1, 2.$$

$$R_{4,u} = \{0.610961, 0.461993, 0.313025, 0.164057\}, \quad u = 1, 2.$$

- Bounds in the linear approximation of MW^{ref} for dam $d = 2, 3, 4$:

$$UB_{2,u} = \{90, 140, 185\}, \quad u = 1, 2.$$

流量断点

$$UB_{3,u} = \{110, 165, 225\}, \quad u = 1, 2.$$

$$UB_{4,u} = \{80, 120, 160\}, \quad u = 1, 2.$$

- Minimum volume of dam d at the end of period t: $V_{d,t}^{\min} = 0$, $d = 1, \ldots, 4$, $t = 1, \ldots, 24$.
- Maximum volume of dam $d = 1, 2, 3, 4$ at the end of period t: $V_{1,t}^{\max} = 400\,\mathrm{h\,m}^3$, $V_{2,t}^{\max} = 80\,\mathrm{h\,m}^3$, $V_{3,t}^{\max} = 100\,\mathrm{h\,m}^3$, $V_{4,t}^{\max} = 40\,\mathrm{h\,m}^3$, $t = 1, \ldots, 24$.
- Initial volume of dam $d = 1, 2, 3, 4$: $V_1^{\mathrm{init}} = 200\,\mathrm{h\,m}^3$, $V_2^{\mathrm{init}} = 40\,\mathrm{h\,m}^3$, $V_3^{\mathrm{init}} = 50\,\mathrm{h\,m}^3$, $V_4^{\mathrm{init}} = 20\,\mathrm{h\,m}^3$.
- Minimum flow that can be processed by unit u of dam $d = 2, 3, 4$: $F_{2,u}^{\min} = 30$, $F_{3,u}^{\min} = 50$, $F_{4,u}^{\min} = 40$, $u = 1, 2$.
- Maximum flow that can be processed by unit u of dam $d = 2, 3, 4$: $F_{2,u}^{\max} = 225$, $F_{3,u}^{\max} = 275$, $F_{4,u}^{\max} = 200$, $u = 1, 2$.
- Maximum spill flow of dam d: $F_d^{\mathrm{spill}_{\max}} = 1000$, $d = 1, \ldots, 4$.
- Minimum outflow of dam d: $F_d^{out_{\min}} = 0$, $d = 1, \ldots, 4$.
- Maximum outflow of dam d: $F_d^{out_{\max}} = 1000$, $d = 1, \ldots, 4$.
- Minimum power of unit u of dam $d = 2, 3, 4$: $MW_{2,u}^{\min} = 3$, $MW_{3,u}^{\min} = 5$, $MW_{4,u}^{\min} = 3$, $d = 2, \ldots, 4$; $u = 1, 2$.
- Maximum power of unit u of dam $d = 2, 3, 4$: $MW_{2,u}^{\max} = 95$, $MW_{3,u}^{\max} = 125$, $MW_{4,u}^{\max} = 65$, $u = 1, 2$.
- Mean natural inflow forecast at dam $d = 1, 2, 3, 4$ for period t (historical averages): $NI_{1,t} = 40$, $NI_{2,t} = 16$, $NI_{3,t} = 12$, $NI_{4,t} = 10$, $t = 1, \ldots, 24$.
- Number of periods (hours) for calculating the river routing effects: $RR = 3$.
- Fraction of water, released in period $t - p$ at dam $d - 1$ that arrives at dam $d = 2, 3, 4$ in period t, $p = 0, 1, 2, 3$:

$$FR_{2,0} = 0, \quad FR_{2,1} = 0.3, \quad FR_{2,2} = 0.4, \quad FR_{2,3} = 0.2.$$

径流演进3小时，汇流系数

$$FR_{3,0} = 0.1, \quad FR_{3,1} = 0.5, \quad FR_{3,2} = 0.3, \quad FR_{3,3} = 0.1.$$

$$FR_{4,0} = 0, \quad FR_{4,1} = 0.4, \quad FR_{4,2} = 0.4, \quad FR_{4,3} = 0.2.$$

- Power needed by unit u of dam $d=2,3,4$ to make it spin to offer 10S reserve: $MW^{spin}_{2,u}=1.9$, $MW^{spin}_{3,u}=2.5$, $MW^{spin}_{4,u}=1.3$, $u=1,\dots,2$.
- Start-up cost of unit $u=2,3,4$: $SC_{2,u}=285$, $SC_{3,u}=375$, $SC_{4,u}=195$, $u=1,2$ (see below).
- Number of piecewise parts in the approximation of future water value function FVR_d at dam d: $K_d=1$, $d=1,\dots,4$ (see below). 蓄能和库容的关系被当作线性函数
- Estimate of electricity that can be produced for each m^3 of water at dam $d=1,2,3,4$: $E_1=0.000364$, $E_2=0.000363$, $E_3=0.000227$, $E_4=0.000096$ (see below). 每立方米水发电量
- Slope of the linear approximation of future water value function at dam d: $R^{fvr}_{d,1}=E_d\cdot FP$, $d=1,\dots,4$ (see below).
- Estimate of the average price at which electricity can be sold in the future: $FP=64$\$/MW h, unless specified otherwise (see below).
- Initial water value at dam d: $FVR^{init}_d=FVR_d(V^{init}_d)$ (calculated with the linear approximation obtained with $R^{fvr}_{d,1}$), $d=1,\dots,4$.

The future value of water is obtained here by considering that each m^3 of water at a given dam has a fixed value. This value is $E_d\cdot FP$, where E_d is an estimate of the electricity that can be produced with each m^3 of water at dam d and FP is an estimate of the average electricity sales price in the future. This value gets larger as we move upstream (upward) because more electricity can be produced with the same amount of water, due to the cascade effects (i.e., E_d is larger for upstream reservoirs). The value of FP is evaluated approximately at 64 \$/MW h from historical production and price data. In Section 5.4, the sensitivity of this value on solution quality is analyzed for the two models.

The start-up cost has been set to 3\$ per nominal MW of output according to the study in [4]. For example, for a 95 MW unit, the cost is 285\$ per start-up. For all units, it is always better to stop a unit than let it run at minimum flow for an hour because the start-up cost (285\$, 375\$, 195\$ for units of dam 2, 3, and 4, respectively) is lower than the opportunity cost of running at minimum flow. The latter can be approximated by the amount of additional electricity that could have been produced in the future multiplied by the average electricity price in the future: $(F^{min}_{d,u}\cdot 3600\cdot(E_d-E_{d+1})-MW^{min}_{d,u})\cdot FP$, which is 748.03\$, 1189.12\$, 692.74\$ for units of dam 2, 3, and 4, respectively. However, if one chooses to produce at $MW^{max}_{d,u}$ for periods p and $p+2$ instead of consecutive periods p and $p+1$, the difference in electricity prices between periods $p+1$ and $p+2$ must be at least $SC_{d,u}/MW^{max}_{d,u}$ (about 3.10\$ for all units) to justify the additional start-up cost.

3. Stochastic model

The mathematical model proposed in this section is based on SP which is one of the main optimization methods that integrates random variables. SP has often been applied in the electricity domain [11,12,22,50], including cascades of hydroelectric power plants [51–53]. To the best of our knowledge, however, this is the first time that SP is applied on such a detailed model. A fundamental reference on SP can be found in [54], while a survey on applications in the energy domain is in [6].

In the proposed model, electricity prices are assumed to follow a given probability distribution and production decisions are taken at different periods without knowing with certainty all price outcomes over the 24-h horizon (no anticipation). Also, price outcomes for the next periods depend on the price outcomes from previous periods, thus leading to a multi-stage decision process encoded in a tree structure.

In the following, we first describe the tree structure proposed in this work. Then, the process for generating a particular tree is explained. Finally, the mathematical model is described.

3.1. Tree structure

A tree structure is used to represent different possible price scenarios (outcomes, realizations). Each node in the tree is associated with a stage, consisting of a subset of periods. From that node, different children nodes can be reached with a certain conditional probability, depending on the probability distribution associated with the electricity prices. Production decisions for the next stage are taken at each node, without knowing with certainty the corresponding price outcomes.

Fig. 3 shows such a tree over an horizon of 24 periods (t). The horizon is divided into three different subsets of periods, with each subset corresponding to a different stage in the tree. Stage 0 corresponds to the root where no price

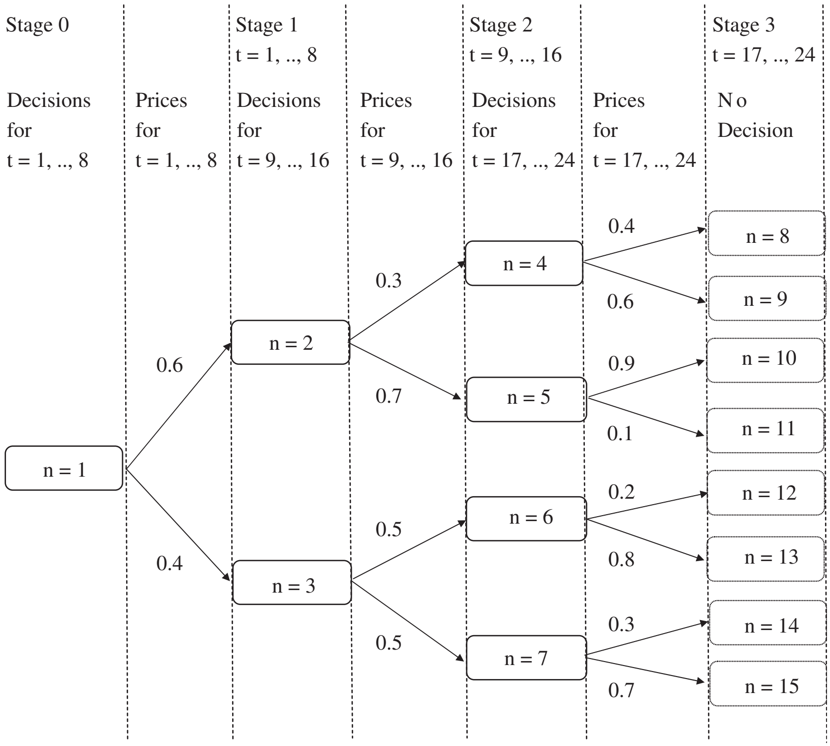

Fig. 3. A scenario tree.

has realized yet. The first stage is associated with the set of periods {1,…,8}, the second stage with {9,…,16} and the third stage with {17,…,24}. In the latter case, complete price scenarios over the 24-h horizon have been obtained. The tree illustrated in this example thus contains 15 nodes (n), but only eight different price scenarios.

At each stage, including the root but excluding the leaves, production decisions are taken for the next stage, without knowing with certainty the electricity and reserve prices for the corresponding set of periods. As we can see, production decisions are first taken at the root for periods 1–8, without knowing with certainty the corresponding prices for these periods. Then, depending on the particular price outcome observed, a child node is reached at the next stage and new decisions are taken for periods 9–16, etc. Note that no decision is taken at the leaves of the tree when the prices are completely known.

Once again, even if the price outcomes for the next stage are not known with certainty when the production decisions are taken, their conditional probability is known (these probabilities are shown on the arcs of Fig. 3). It is thus possible to compute the conditional expected profit associated with the production decisions for the next stage. The overall expected profit can be calculated as the summation over all internal nodes in the tree of the absolute probability of being in that node, as obtained by multiplying the conditional probabilities on the path leading from the root to the node, times the conditional profit expectation for the next stage. For example, the absolute probability of being in node $n = 7$ is equal to $0.4 \cdot 0.5 = 0.2$. Clearly, the leaves must be left apart, given that no decision is taken there.

The model presented in Section 3.3 does not explicitly consider the conditional probability, but rather the absolute probability, of being in node n when a decision for period t is taken, noted by $PR_{t,n}$. In the previous example, $PR_{t,7}=0.2$ for periods $t = 17, \ldots, 24$. These probabilities are found in objective (57). Also, to simplify the formulation, the model does not handle sets of periods explicitly, but rather handles each period individually. A node associated with period t can be obtained via set S_t. More precisely, S_t contains the set of nodes where a decision is taken for period t. In our example, $S_t = \{1\}$ for $t = 1, \ldots, 8$, $S_t = \{2, 3\}$ for $t = 9, \ldots, 16$ and $S_t = \{4, 5, 6, 7\}$ for $t = 17, \ldots, 24$. Parameter IN is the number of nodes where decisions are taken, that is all internal nodes (in our example, $IN = 7$). The topology of the tree is contained in $AN_{n,t,p}$ which corresponds to the ancestor of node n, where a decision for period $t - p$ is taken. For example, $AN_{2,9,1} = 1$ and $AN_{4,17,2} = 2$. Note that the ancestor node of n can be node n itself if t and $t - p$ are part of the same stage or set of periods (e.g., $AN_{2,10,1} = 2$ and $AN_{4,20,2} = 4$). For consistency, $AN_{n,t,0} = n$ for $n = 1, \ldots, IN$.

3.2. Scenario tree construction

The shape of the scenario tree depends on the sets of periods associated with each stage. For example, a much deeper tree is obtained by associating a single period with each stage. Also, the branching factor in the tree corresponds to the number of different price outcomes considered at each stage. Thus, the following parameters determine the shape of the tree:

- N_b: number of branches at each stage (branching factor) and
- N_s: number of sets of periods or stages, with t_j the last period in stage $j = 1, \ldots, N_s$.

Recall that decisions must be taken at least two periods (hours) before their actual implementation according to the market rules. Thus, the periods $t_j - 2$ (for $j = 1, \ldots, N_s - 1$) are those where decisions for the next stage are taken and determine the children nodes that will be reached. Let us assume, for example, that nodes b, c, and d are associated with stage j (from smallest to largest prices at period $p_j - 2$) and are the three children nodes of a given node a. We denote the price associated with node $s = b, c, d$ by P'_s and the price of period $t_j - 2$ in some newly generated price vector by P^{new}. Then, from node a, we go to:

- node b if $P^{\text{new}} \leqslant P'_b$,
- node c if $P'_b < P^{\text{new}} \leqslant P'_c$, and
- node d otherwise.

The literature on scenario tree construction is rich and the reader is referred to [55] for a recent overview. In our case, a sampling method based on the stochastic process defined in Section 4 was used to generate prices for the scenario tree. This simple approach was chosen for convenience, given that our focus is on modeling issues. First, N_v vectors of prices of cardinality T are generated. Assuming a branching factor N_b and a partition of the T periods into N_s sets of periods, the vectors are then sorted based on the price at period $t_1 - 2$ (two periods before the last period in the first stage), starting with the smallest price. We then take the average price for each period from 1 to t_1 based on the first $\lfloor N_v/N_b \rfloor$ vectors, which are those with the smallest prices, and we associate these averages and the corresponding subset of price vectors with the first child node. We do the same for the next child node by averaging over the next $\lfloor N_v/N_b \rfloor$ vectors. This is repeated until every child node of the root is done. Then, we go to the next stage and repeat the whole procedure again for every node associated with that stage: we sort the subset of vectors associated with the current node based on the price of period $t_2 - 2$ (two periods before the last period in the second stage) and associate a (sub)subset of vectors and its corresponding averages with every child. The procedure stops at the leaves of the scenario tree. The tree is created in a such way that the average price associated with each period corresponds to the expected value of the stochastic process that is used to generate the prices. Note also that the price P'_s mentioned above (with node s associated with stage j), is obtained by taking the maximum price at period $t_j - 2$ over the set of price vectors associated with node s.

Fig. 4 illustrates the procedure. Here, we have six periods ($T = 6$), two sets of periods ($N_s = 2$) and a branching factor of 2 ($N_b = 2$), with $t_1 = 3$, $t_2 = 6$. Eight vectors of prices are generated ($N_v = 8$), where each vector corresponds to a single line in part (a). In part (b), the price vectors are sorted based on their value at period 1 ($t_1 - 2$). Then, the first four price vectors are averaged over periods 1–3 and are associated with node $s = 2$. The same is done with the four last price vectors to obtain node $s = 3$. Note that the price P'_2 for $s = 2$ is 4. Thus, if the actual price at period 1 is less than or equal to 4, node $s = 2$ is reached, otherwise node $s = 3$ is reached. In part (c), the price vectors 1 to 4 are sorted according to their values at period 4 ($t_2 - 2$). The same is done for vectors 5 to 8. The averages are then computed and associated with nodes $s = 4, \ldots, 7$ over the four pairs of vector prices. The prices P'_4 and P'_6 for nodes $s = 4$ and 6 are 2 and 5, respectively. The resulting tree is presented in part (d).

It is worth noting that the stochastic process used for modeling electricity prices is not restricted to be Markovian, like the one proposed later in Section 4 (where the price at the current period depends only on the price observed in the previous period). As a matter of fact, a tree structure can be used to model more general dependencies.

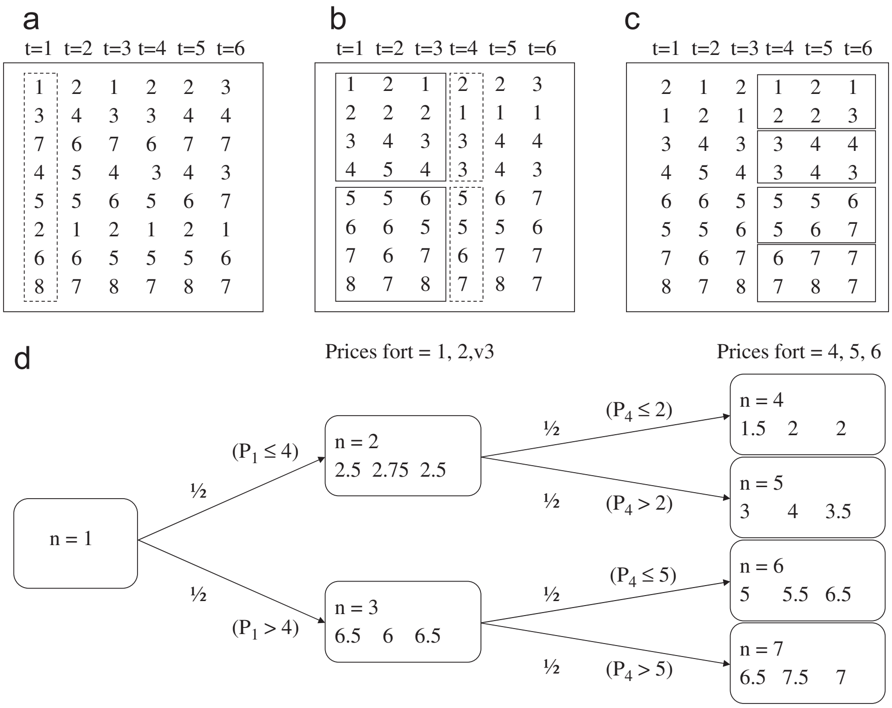

Fig. 4. Scenario tree construction example.

3.3. Mathematical model

The mathematical model will now be formally introduced. The stochastic model exhibits similarities with the deterministic model, as its main components are the same. Decision and intermediate variables are also the same, but an additional index is added to take into account different price outcomes (see below).

3.3.1. Parameters

Here are the new parameters for the stochastic model (see Section 2.1.1 for the other ones).

- IN: number of internal nodes in the scenario tree.
- S_t: set of nodes where a decision is taken for period t, $t = 1, \ldots, T$.
- $AN_{n,t,p}$: ancestor of node n, where a decision is taken for period $t - p$, $t = 1, \ldots, T$; $n \in S_t$; $p = 0, \ldots, t - 1$. Note that the ancestor node can be the same as node n if t and $t - p$ are in the same set of periods.
- $PR_{t,n}$: absolute probability to be in node n when a decision is taken for period t, $t = 1, \ldots, T$; $n \in S_t$. This probability is obtained by multiplying the conditional probabilities on the path from the root to the corresponding scenario node in the tree of scenarios. These probabilities are the same for all periods in the same set of periods.
- $P^E_{t,n}$: conditional expected price for each MW h of energy produced at period t when a decision for period t is taken in node n (\$/MW h), $t = 1, \ldots, T$; $n \in S_t$.
- $P^{10S}_{t,n}$: conditional expectation of price for each MW h offered of 10S reserve when a decision for period t is taken in node n (\$/MW h), $t = 1, \ldots, T$; $n \in S_t$.
- $P^{10N}_{t,n}$: conditional expectation of price for each MW h offered of 10N reserve when a decision for period t is taken in node n (\$/MW h), $t = 1, \ldots, T$; $n \in S_t$.

3.3.2. Decision variables

- $f^{\text{spill}}_{d,t,n} \geqslant 0$: flow spilled at dam d in period t when a decision for period t is taken in node n (m^3/s), $d = 1, \ldots, D$; $t = 1, \ldots, T$; $n \in S_t$.

- $f^E_{d,u,t,n} \geqslant 0$: flow going through unit u of dam d to produce electricity in period t, when a decision for period t is taken in node n (m^3/s), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.
- $f^E_{d,u,t,i,n} \geqslant 0$: contribution of piecewise part i, in the linear approximation of $MW^{\mathrm{ref}}_{d,u}(f)$, to the flow going through unit u of dam d in period t to produce electricity, when a decision for period t is taken in node n (m^3/s), $d=1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $i = 1, \ldots, I_{d,u}$; $n \in S_t$.
- $y^E_{d,u,t,n} \in \{0, 1\}$: 1 if electricity is produced by unit u of dam d in period t when a decision for period t is taken in node n, 0 otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.
- $y^{\mathrm{spin}}_{d,u,t,n} \in \{0, 1\}$: 1 if unit u of dam d spins by supplying electricity to it in period t when a decision for period t is taken in node n, 0 otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.
- $mw^E_{d,u,t,n} \geqslant 0$: power produced by unit u of dam d in period t when a decision for period t is taken in node n (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.
- 旋转备用 $mw^{10\mathrm{S}}_{d,u,t,n} \geqslant 0$: power reserved for "10 min spin" reserve for unit u of dam d in period t when a decision for period t is taken in node n (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.
- $mw^{10\mathrm{N}}_{d,u,t,n} \geqslant 0$: power reserved for "10 min non-spin" reserve for unit u of dam d in period t when a decision for 非旋转备用 period t is taken in node n (MW), $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

3.3.3. Constraints on decision variables

- $f^E_{d,u,t,n}$ is the sum of all piecewise parts plus $F^{\min}_{d,u}$ if electricity is produced ($y^E_{d,u,t,n}=1$), $d=1, \ldots, D$; $u=1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

$$f^E_{d,u,t,n} = y^E_{d,u,t,n} \cdot F^{\min}_{d,u} + \sum_{i=1,\ldots,I_{d,u}} f^E_{d,u,t,i,n}. \tag{40}$$

- $f^E_{d,u,t,n}$ equals 0 if $y^E_{d,u,t,n}$ equals 0 and is between 0 and a maximum value otherwise, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

$$0 \leqslant f^E_{d,u,t,n} \leqslant F^{\max}_{d,u} \cdot y^E_{d,u,t,n}. \tag{41}$$

- Bounds on $f^E_{d,u,t,i,n}$, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $i = 1, \ldots, I_{d,u}$; $n \in S_t$.

$$i = 1: \quad 0 \leqslant f^E_{d,u,t,i,n} \leqslant UB_{d,u,1} - F^{\min}_{d,u}, \tag{42}$$

$$i = 2, \ldots, I_{d,u} - 1: \quad 0 \leqslant f^E_{d,u,t,i,n} \leqslant UB_{d,u,i} - UB_{d,u,i-1}, \tag{43}$$

$$i = I_{d,u}: \quad 0 \leqslant f^E_{d,u,t,i,n} \leqslant F^{\max}_{d,u} - UB_{d,u,I_{d,u}-1}. \tag{44}$$

- Calculation of $mw^E_{d,u,t,n} = MW^{\mathrm{ref}}_{d,u}(f^E_{d,u,t,n})$ through the linear approximation, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

$$mw^E_{d,u,t,n} = y^E_{d,u,t,n} \cdot MW^{\min}_{d,u} + \sum_{i=1,\ldots,I_{d,u}} f^E_{d,u,t,i,n} \cdot R_{d,u,i}. \tag{45}$$

- Spill flow of dam d in period t is between 0 and a maximum value, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

$$0 \leqslant f^{\mathrm{spill}}_{d,p,n} \leqslant F_d^{\mathrm{spill}_{\max}}. \tag{46}$$

- Maximum power of unit u of dam d is used or reserved in period t, $d = 1, \ldots, D$; $t = 1, \ldots, T$; $n \in S_t$.

$$mw^E_{d,u,t,n} + mw^{10\mathrm{S}}_{d,u,t,n} + mw^{10\mathrm{N}}_{d,u,t,n} = MW^{\max}_{d,u}. \tag{47}$$

- Power for 10S reserve can only be reserved if unit u of dam d produces electricity or spins, $d = 1, \ldots, D$; $u = 1, \ldots, U_d$; $t = 1, \ldots, T$; $n \in S_t$.

$$mw^{10\mathrm{S}}_{d,u,t,n} \leqslant MW^{\max}_{d,u} \cdot (y^E_{d,u,t,n} + y^{\mathrm{spin}}_{d,u,t,n}). \tag{48}$$

- Unit u of dam d cannot produce electricity and spin at the same time, $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$; $n\in S_t$.

$$y^E_{d,u,t,n}+y^{\text{spin}}_{d,u,t,n}\leqslant 1. \tag{49}$$

3.3.4. Intermediate variables

- $out_{d,t,n}$: flow that leaves the reservoir of dam d in period t, when a decision for period t is taken in node n (m^3/s), $d=1,\ldots,D$; $t=1,\ldots,T$; $n\in S_t$. It is assumed that a number of "historical" outflow values are available for periods $-(RR-1),\ldots,0$.

$$out_{d,t,n}=f^{\text{spill}}_{d,t,n}+\sum_{u=1,\ldots,U_d} f^E_{d,u,t,n}. \tag{50}$$

- $in_{d,t,n}$: flow that enters the reservoir of dam d in period t when a decision for period t is taken in node n (m^3/s), $d=1,\ldots,D$; $t=1,\ldots,T$; $n\in S_t$.

$$d=1:\quad in_{d,t,n}=NI_{d,t}, \tag{51}$$

$$d=2,\ldots,D:\quad in_{d,t,n}=NI_{d,t}+\sum_{p=0,\ldots,RR} out_{d-1,t-p,AN_{n,t,p}}\cdot FR_{d,p}. \tag{52}$$

- $v_{d,t,n}$: volume of dam d at the end of period t, when a decision for period t is taken in node n (m^3), $d=1,\ldots,D$; $t=1,\ldots,T$; $n\in S_t$.

$$t=1:\quad v_{d,t,n}=V^{\text{init}}_d+(in_{d,t,n}-out_{d,t,n})\cdot 3600, \tag{53}$$

$$t=2,\ldots,T:\quad v_{d,t,n}=v_{d,t-1,AN_{n,t,1}}+(in_{d,t,n}-out_{d,t,n})\cdot 3600. \tag{54}$$

3.3.5. Constraints on intermediate variables

- Volume of dam d at the end of period t is between a minimum and maximum value, when a decision for period t is taken in node n, $d=1,\ldots,D$; $t=1,\ldots,T$; $n\in S_t$.

$$V^{\min}_d\leqslant v_{d,t,n}\leqslant V^{\max}_d. \tag{55}$$

- Outflow of dam d in period t is between a minimum and maximum value, when a decision for period t is taken in node n, $d=1,\ldots,D$; $t=1,\ldots,T$; $n\in S_t$.

$$F^{out_{\min}}_d\leqslant out_{d,t,n}\leqslant F^{out_{\max}}_d. \tag{56}$$

3.3.6. Objective

- Objective function ($)

$$\textit{Maximize}\sum_{\substack{d=1,\ldots,D\\ u=1,\ldots,U_d\\ t=1,\ldots,T\\ n\in S_t}} PR_{t,n}\cdot(rev_{d,u,t,n}-c^{\text{spin}}_{d,u,t,n}-c^{\text{start}}_{d,u,t,n})-\sum_{\substack{d=1,\ldots,D\\ n\in S_T}} PR_{T,n}\cdot val_{d,n}. \tag{57}$$

- $rev_{d,u,t,n}$ conditional expectation of revenue from selling electricity and reserve for unit u of dam d in period t, when a decision for period t is taken in node n ($), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$; $n\in S_t$.

$$rev_{d,u,t,n}=(mw^E_{d,u,t,n}\cdot P^E_{t,n}+mw^{10\text{S}}_{d,u,t,n}\cdot P^{10\text{S}}_{t,n}+mw^{10\text{N}}_{d,u,t,n}\cdot P^{10\text{N}}_{t,n}). \tag{58}$$

- $c^{\text{spin}}_{d,u,t,n}$: conditional expectation of cost for providing electricity to unit u of dam d to make it spin in period t, when a decision for period t is taken in node n ($), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$; $n\in S_t$.

$$c^{\text{spin}}_{d,u,t,n}=y^{\text{spin}}_{d,u,t,n}\cdot MW^{\text{spin}}_{d,u}\cdot P^E_{t,n}. \tag{59}$$

- $c^{\text{start}}_{d,u,t,n} \geqslant 0$: starting cost of unit u of dam d in period t, when a decision for period t is taken in node n (\$), $d=1,\ldots,D$; $u=1,\ldots,U_d$; $t=1,\ldots,T$; $n \in S_t$.

$$t=1: \quad c^{\text{start}}_{d,u,t,n} = SC_{d,u} \cdot y^E_{d,u,t,n}, \tag{60}$$

$$t=2,\ldots,T: \quad c^{\text{start}}_{d,u,t,n} \geqslant SC_{d,u} \cdot (y^E_{d,u,t,n} - y^E_{d,u,t-1,AN_{n,t,1}}), \tag{61}$$

$$c^{\text{start}}_{d,u,t,n} \leqslant SC_{d,u} \cdot y^E_{d,u,t,n}, \tag{62}$$

$$c^{\text{start}}_{d,u,t,n} \leqslant SC_{d,u} \cdot (1 - y^E_{d,u,t-1,AN_{n,t,1}}). \tag{63}$$

- $val_{d,n}$: value of net water used for production at dam d during planning horizon when a decision for period T is taken in node n (equal to $FVR_d(V^{\text{init}}_d) - FVR_d(\mathit{final\_volume})$) (\$), $d=1,\ldots,D$; $n \in S_T$.

$$val_{d,n} = FVR^{\text{init}}_d - \sum_{k=1,\ldots,K_d} R^{fvr}_{d,k} \cdot v^{fin}_{d,k,n}. \tag{64}$$

- $v^{fin}_{d,k,n} \geqslant 0$: contribution of piecewise part k, in the linear approximation of $FVR_d(\mathit{final\_volume})$, to the final volume of dam d when a decision for period T is taken in node n (m^3), $d=1,\ldots,D$; $k=1,\ldots,K_d$; $n \in S_T$.
The summation of $v^{fin}_{d,k,n}$ over k is equal to the final volume adjusted for the river routing effects at the end of horizon, $d=1,\ldots,D$; $n \in S_T$. The summation after $v_{d,T,n}$ corresponds to the inflows that arrive at dam d in periods $T+1, T+2, \ldots, T+RR$

$$\sum_{k=1,\ldots,K_d} v^{fin}_{d,k,n} = v_{d,T,n} + 3600 \sum_{\substack{r=1,\ldots,RR\\ p=r,\ldots,RR}} out_{d-1,T+r-p,AN_{n,T,r-1}} \cdot FR_{d,p}. \tag{65}$$

- Bounds on $v^{fin}_{d,k,n}$, $d=1,\ldots,D$; $k=1,\ldots,K_d$; $n \in S_T$.

$$k=1: \quad 0 \leqslant v^{fin}_{d,k,n} \leqslant UB^{fvr}_{d,k}, \tag{66}$$

$$k=2,\ldots,K_d-1: \quad 0 \leqslant v^{fin}_{d,k,n} \leqslant UB^{fvr}_{d,k} - UB^{fvr}_{d,k-1}, \tag{67}$$

$$k=K_d: \quad 0 \leqslant v^{fin}_{d,k,n} \leqslant V^{\max}_{d,1} - UB^{fvr}_{d,K_d-1}. \tag{68}$$

3.4. Water head effects

Water head effects for the stochastic model are evaluated in the same way as the deterministic model. That is, the initial solution is obtained from solving the original model. Then a new model is created with the same variables and constraints except that the electricity production Eq. (45) is replaced by

$$\begin{aligned} mw^E_{d,u,t,n} = {} & MW_{d,u}(\widehat{v}^{avg}_{d,t,n}, \widehat{f}^E_{d,u,t,n}) + \frac{H'_d(\widehat{v}^{avg}_{d,t,n})}{H^{\text{ref}}_d} MW^{\text{ref}}_{d,u}(\widehat{f}^E_{d,u,t,n}) \cdot (v^{avg}_{d,t,n} - \widehat{v}^{avg}_{d,t,n}) \\ & + \frac{H_d(\widehat{v}^{avg}_{d,t,n})}{H^{\text{ref}}_d} (MW^{\text{ref}}_{d,u})'(\widehat{f}^E_{d,u,t,n}) \cdot (f^E_{d,u,t,n} - \widehat{f}^E_{d,u,t,n}), \end{aligned} \tag{69}$$

where $\widehat{f}^E_{d,u,t,n}$ is the value of $f^E_{d,u,t,n}$ in the initial solution, $v^{avg}_{d,t,n}$ is the average volume in period t when a decision for period t is taken in scenario node n, as given by

$$t=1: \quad v^{avg}_{d,t,n} = \frac{V^{\text{init}}_d + v_{d,t,n}}{2}, \tag{70}$$

$$t=2,\ldots,T: \quad v^{avg}_{d,t,n} = \frac{v_{d,t-1,AN_{n,t,1}} + v_{d,t,n}}{2} \tag{71}$$

and $\widehat{v}^{avg}_{d,t,n}$ is the value of $v^{avg}_{d,t,n}$ in the initial solution. Furthermore, Eq. (47) is replaced by

$$mw^{E}_{d,u,t,n} + mw^{10S}_{d,u,t,n} + mw^{10N}_{d,u,t,n} = \frac{H_d(\widehat{v}^{avg}_{d,t,n})}{H^{ref}_d} MW^{max}_{d,u} + \frac{H'_d(\widehat{v}^{avg}_{d,t,n})}{H^{ref}_d} MW^{max}_{d,u} \cdot (v^{avg}_{d,t,n} - \widehat{v}^{avg}_{d,t,n}) \tag{72}$$

and Eq. (48) by

$$mw^{10S}_{d,u,t,n} \leqslant \frac{H^{max}_d}{H^{ref}_d} MW^{max}_{d,u} \cdot (y^{E}_{d,u,t,n} + y^{spin}_{d,u,t,n}). \tag{73}$$

We also impose the following additional constraints for $d = 1, \ldots, D;\ u = 1, \ldots, U_d; t = 1, \ldots, T; n \in S_t$:

$$\widehat{v}^{avg}_{d,t,n} - \lambda \cdot (V^{max}_{d,t} - V^{min}_{d,t}) \leqslant v^{avg}_{d,t,n} \leqslant \widehat{v}^{avg}_{d,t,n} + \lambda \cdot (V^{max}_{d,t,n} - V^{min}_{d,t,n}), \tag{74}$$

$$\widehat{f}^{E}_{d,u,t,n} - \lambda \cdot (F^{max}_{d,u} - F^{min}_{d,u}) \leqslant f^{E}_{d,u,t,n} \leqslant \widehat{f}^{E}_{d,u,t,n} + \lambda \cdot (F^{max}_{d,u} - F^{min}_{d,u}), \tag{75}$$

$$y^{E}_{d,u,t,n} = \widehat{y}^{E}_{d,u,t,n}, \tag{76}$$

$$y^{spin}_{d,u,t,n} = \widehat{y}^{spin}_{d,u,t,n}. \tag{77}$$

The overall optimization process is the same as the one used for the deterministic model (see Section 2.2).

3.5. Parameter values

This section describes the parameter values that are found only in the stochastic model (see Section 2.3 for the other parameter values). To create a scenario tree, the branching factor N_b and the number of sets of periods (or levels) N_s must first be specified. Then, the t_j's (last period in each set) are chosen to get, as much as possible, the same number of periods at each level. That is

$$j = 1, \ldots, N_s - 1: \quad t_j = \lfloor T \cdot j / N_s \rfloor, \tag{78}$$

$$j = N_s: \quad t_j = T. \tag{79}$$

For the exact meaning of N_b, N_s and t_j, see Section 3.1. The value of *IN*, the number of internal nodes in the scenario tree, is then

$$IN = N_b^{N_s} - 1. \tag{80}$$

To associate average prices with the nodes in the tree of scenarios, the number of price vectors N_v generated with the Monte Carlo simulation was set to 100 000 (see Section 3.2).

4. Electricity price model

Electricity is quite different from other commodities because it cannot be efficiently stored and thus requires a balance between supplies and demands. That is why electricity prices exhibit spikes and seasonal behaviors. There are two main classes of electricity price models: continuous and discrete time models. Continuous time models are particularly useful for pricing financial derivatives related to electricity prices [56] or to model long-term price behaviors [57]. Discrete time models are mostly used for hourly electricity price forecasts over short-term horizons [58].

Let $X(\tau) = P(\tau + \Delta) - P(\tau)$ be the difference in electricity prices between time τ and time $\tau + \Delta$, where Δ is a fixed value. A continuous time model expresses $X(\tau)$ as a function of some underlying continuous time stochastic process. In [57], for example, $X(\tau) = X_1(\tau) + X_2(\tau)$ where $X_1(\tau)$ and $X_2(\tau)$ are two independent Ornstein–Uhlenbeck processes. In these models, the parameter values of the continuous process must first be estimated based on historical data. Then, the resulting model is discretized in some way to fit an appropriate optimization model. Due to the rather intricate problem-solving approach associated with continuous models and our short-term focus, a discrete time model is chosen here.

Table 1
Regression results for each hour

Hour i	avg_i^{dep}	avg_i^{ind}	a_i	b_i	$R2_i$	σ_i
1	3.59	3.62	0.455	0.867	0.761	0.187
2	3.51	3.58	0.212	0.921	0.856	0.148
3	3.47	3.51	0.181	0.938	0.870	0.141
4	3.46	3.47	0.280	0.916	0.879	0.134
5	3.47	3.45	0.369	0.898	0.840	0.151
6	3.54	3.47	0.303	0.933	0.774	0.182
7	3.71	3.55	0.335	0.950	0.751	0.210
8	3.90	3.74	0.419	0.931	0.792	0.208
9	3.99	3.91	0.626	0.860	0.791	0.201
10	4.07	4.00	0.625	0.861	0.775	0.202
11	4.12	4.07	0.396	0.913	0.789	0.199
12	4.13	4.12	0.425	0.899	0.790	0.198
13	4.12	4.13	0.379	0.906	0.789	0.199
14	4.09	4.12	0.333	0.913	0.807	0.190
15	4.04	4.09	0.274	0.923	0.830	0.187
16	4.03	4.04	0.369	0.906	0.843	0.178
17	4.06	4.03	0.663	0.844	0.794	0.191
18	4.07	4.06	0.544	0.869	0.685	0.251
19	4.10	4.10	0.317	0.923	0.804	0.207
20	4.13	4.11	0.756	0.821	0.783	0.203
21	4.10	4.13	0.588	0.849	0.786	0.195
22	3.91	4.07	0.491	0.839	0.775	0.183
23	3.74	3.89	0.552	0.819	0.761	0.177
24	3.62	3.73	0.217	0.913	0.788	0.179
Average	3.87	3.87	0.421	0.892	0.796	0.188
Minimum	3.46	3.45	0.181	0.819	0.685	0.134
Maximum	4.13	4.13	0.756	0.950	0.879	0.251

In discrete time models, the price at period t is function of the prices from previous periods $t-1$, $t-2$, etc. We chose a periodic autoregressive process (PAR), because the latter allows this function to depend on t (in our case, the function changes according to the time of the day). PAR models have been successfully applied to economic time series with a periodic behavior [59,60]. They have also been applied for electricity load forecasts [61] and electricity price modeling [62]. An important reference on periodic autoregressive processes can be found in [63].

In the equation that follows, the logarithm of the electricity price for the current period (hour) ϕ_t^E depends on the logarithm of the price of the previous hour through the following PAR model:

$$\phi_t^E = a_{(t \bmod 24)+1} + b_{(t \bmod 24)+1} \cdot \phi_{t-1}^E + \varepsilon_t, \quad t = 1, \ldots, T, \tag{81}$$

where $T = 24$ h and ε_t is a random variable that follows a normal distribution with zero mean and a standard deviation of $\sigma_{(t \bmod 24)+1}$. As suggested in [63], the parameters a_i and b_i, $i = 1, \ldots, 24$ (one for each hour) are obtained through linear regression. Basically, the price logarithm of the previous hour is the independent variable while the price logarithm of the current hour is the dependent one. Prices from IESO (Ontario Independent Electricity System Operation), available on their website [64] from May 2002 to June 2006, were used in this study. To eliminate price spikes that are due to abnormal or artificially created market conditions and focus on normal price behaviors, we did not consider cases where the prices radically changed from one hour to the next (i.e., if the current price is less than half that or more than twice that of the previous hour). About 3.3% of the data were eliminated according to this rule.

The results are presented in Table 1 where avg_i^{dep} is the dependent variable average (price logarithm of the current hour) for hour i, avg_i^{ind} is the independent variable average (price logarithm of the previous hour) for hour i, a_i and b_i are the estimated parameters of the model, $R2_i$ is the determination coefficient for hour i and σ_i is the standard deviation of the estimation error (standard deviation of the normal random variable ε_t) when $(t \bmod 24) + 1 = i$.

The determination coefficients $R2_i$ are high considering that the electricity prices are rather volatile. Even in the worst case ($i = 18$), the estimation is good at $R2_{18} = 0.685$. The average is 0.796 which means that almost 80% of

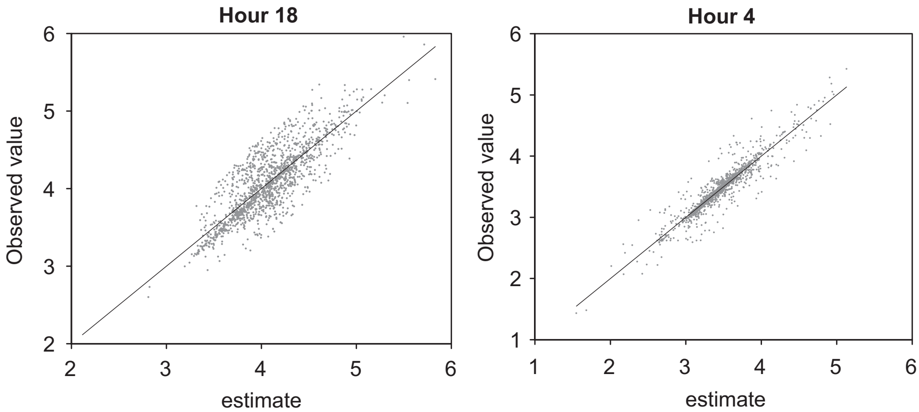

Fig. 5. Linear relationship observed in the worst case ($i=18$, $R2_{18}=0.685$) and in the best case ($i=4$, $R2_4=0.879$).

Table 2
Average electricity and reserve prices

t	$\overline{P}_t^E$	$\overline{P}_t^{10S}$	$\overline{P}_t^{10N}$	σ_t^E
1	37.00	4.52	0.25	6.98
2	34.72	4.54	0.24	7.99
3	33.68	4.46	0.23	8.75
4	33.38	4.41	0.25	9.17
5	34.03	4.23	0.27	9.91
6	36.89	4.11	0.45	12.21
7	43.91	4.53	1.01	16.89
8	52.32	4.28	2.29	22.00
9	56.81	5.54	3.81	23.73
10	61.19	5.58	4.38	25.49
11	64.42	6.40	5.05	28.00
12	65.47	5.68	4.28	28.99
13	65.34	7.50	5.98	29.59
14	64.05	4.59	3.24	29.43
15	61.81	4.58	3.35	28.94
16	61.12	4.36	3.53	28.31
17	62.78	5.07	4.35	27.45
18	64.26	7.26	6.57	29.83
19	64.96	9.50	8.78	31.33
20	65.89	7.40	6.71	29.45
21	63.49	5.95	4.89	27.27
22	53.46	2.28	1.17	21.71
23	45.38	2.56	0.58	17.17
24	40.89	3.58	0.30	16.06
Average	52.80	5.12	3.00	21.53
Minimum	33.38	2.28	0.23	6.98
Maximum	65.89	9.50	8.78	31.33

the variation in the current price, on average, can be explained by the price of the previous hour. Fig. 5 shows the relationship observed in the worst case ($i=18$, $R2_{18}=0.685$) and in the best case ($i=4$, $R2_4=0.879$).

It is possible to show by induction (see Appendix A) that:

$$\phi_t^E = \sum_{i=1,\ldots,t} a_i \prod_{j=i+1,\ldots,t} b_j + \phi_0^E \prod_{k=1,\ldots,t} b_k + \sum_{i=1,\ldots,t} \varepsilon_i \prod_{j=i+1,\ldots,t} b_j \tag{82}$$

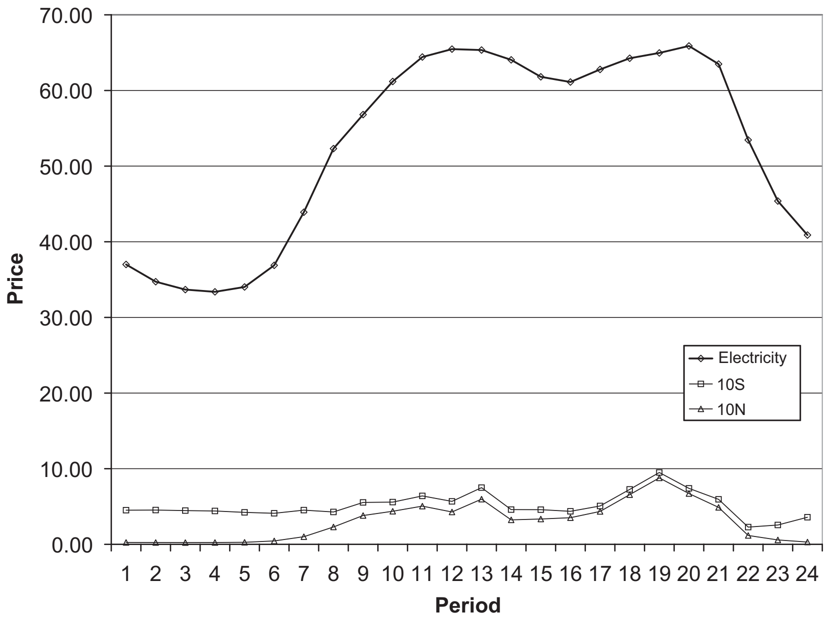

Fig. 6. Average electricity and reserve prices.

which means that ϕ_t^E follows a normal distribution with an expected value and a variance given by

$$E[\phi_t^E] = \sum_{i=1,\ldots,t} \sum a_i \prod_{j=i+1,\ldots,t} b_j + \phi_0^E \prod_{k=1,\ldots,t} b_k, \tag{83}$$

$$Var[\phi_t^E] = \sum_{i=1,\ldots,t} \sigma^2 \prod_{j=i+1,\ldots,t} b_j^2. \tag{84}$$

The true electricity price process ψ_t^E is the exponentiation of ϕ_t^E, that is

$$\psi_t^E = \mathrm{e}^{\phi_t^E}. \tag{85}$$

This process follows a lognormal distribution with an expected value and a variance given by

$$E[\psi_t^E] = E[\phi_t^E] + Var[\phi_t^E]/2, \tag{86}$$

$$Var[\psi_t^E] = (\mathrm{e}^{Var[\phi_t^E]} - 1)\mathrm{e}^{2E[\phi_t^E] + Var[\phi_t^E]}. \tag{87}$$

In the experiments of Section 5, the process is started by setting ϕ_0^E to the historical average of hour 24, which is the hour just before hour 1. Formulas (81) and (85) are then used recursively to generate prices for the following periods. The expected electricity prices $\overline{P}_t^E = E[\psi_t^E]$ are calculated for each period with Eqs. (83), (84), and (86). For the two reserve types 10S and 10N, deterministic prices obtained from historical averages are used ($\overline{P}_t^{10S}$ for 10S and $\overline{P}_t^{10N}$ for 10N, $t = 1, \ldots, T$). Table 2 and Fig. 6 show the average electricity and reserve prices in \$/MW h used in our experiments. Table 2 also presents the standard deviation σ_t^E of the electricity prices, which is the square root of (87).

5. Numerical results

This section first presents two production plan examples. Then, the results of four different experiments are summarized: we study the impact of the water head effects on solution quality; we compare the deterministic and stochastic

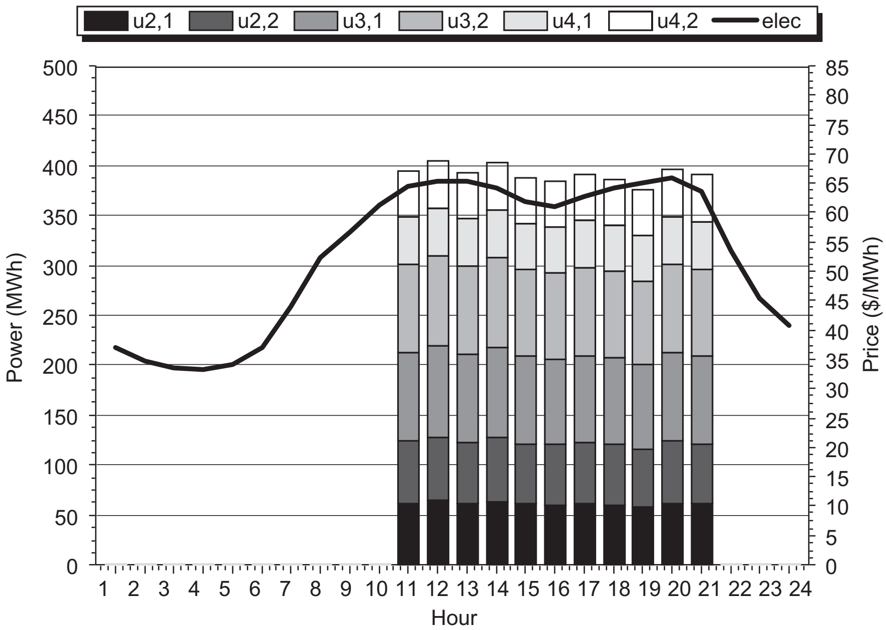

Fig. 7. Deterministic model production example.

models and empirically demonstrate the superiority of the latter model with regard to solution quality; we perform a sensitivity analysis with regard to the future value of water; finally, we examine the impact of price volatility.

Both the deterministic and stochastic models were solved with CPLEX [65]. The maximal size of the CPLEX search tree was limited to 1024 MB. All numerical tests were executed on a 3 GHz Pentium 4, with 1 GB of memory.

5.1. Production plan examples

This section presents a production plan obtained with the deterministic model and another one with the stochastic model. Fig. 7 shows the deterministic production plan. For each hour, the figure gives the average electricity price forecast (elec) and the quantity of electricity offered by each unit ($u_{d,j}$ is the jth unit of dam d). We observe that the quantity offered depends heavily on the electricity price, as the two curves have a similar shape. Note also that the start-up costs tend to create contiguous blocks of production. Indeed, without start-up costs, it would be better to produce at hour 10 ($\overline{P}_{10}^{E} = 61.19$) instead of hour 16 ($\overline{P}_{16}^{E} = 61.12$). But with two start-up costs, this strategy becomes too expensive. It would even be more profitable to produce electricity at hour 16 and sell reserve at hour 10, because the reserve prices are higher at hour 10 ($\overline{P}_{10}^{10\mathrm{S}} = 5.58$, $\overline{P}_{10}^{10\mathrm{N}} = 4.38$ and $\overline{P}_{16}^{10\mathrm{S}} = 4.36$, $\overline{P}_{16}^{10\mathrm{N}} = 3.53$).

Fig. 8 presents the production plan generated by the stochastic model with a branching factor of 2 ($N_{\mathrm{b}} = 2$) and two sets of periods ($N_{\mathrm{s}} = 2$). In the graphics, the quantity of electricity offered and the conditional expected price (elec) are shown. Part (a) of the figure shows the decisions that are taken in the first 12 hours. Note that these decisions are the same as those taken with the deterministic model. Then, different decisions are taken at hour 10 for hours 13–24, depending on the electricity price at hour 10 (P_{10}). If the price is low ($P_{10} \leqslant 56.40$), the decisions shown in part (b) are taken (node $s = 2$). On the other hand, if the price is high ($P_{10} > 56.40$), decisions shown in part (c) are taken (node $s = 3$). The boundary price 56.40 was calculated during the creation of the scenario tree (see Section 3.2). As in the deterministic case, the quantity offered depends heavily on the electricity price. In the low price scenario, it is better to keep the water for future periods, while in the high price scenario, is it better to produce more. The expected profit in the stochastic case is 213 100\$, as opposed to 197 230\$ in the deterministic case. The value of the stochastic solution (VSS) is thus 213 100\$ − 197 230\$ = 15 870\$ and, even in this simple case, the expected profit is 8.0% higher when the stochastic model is used.

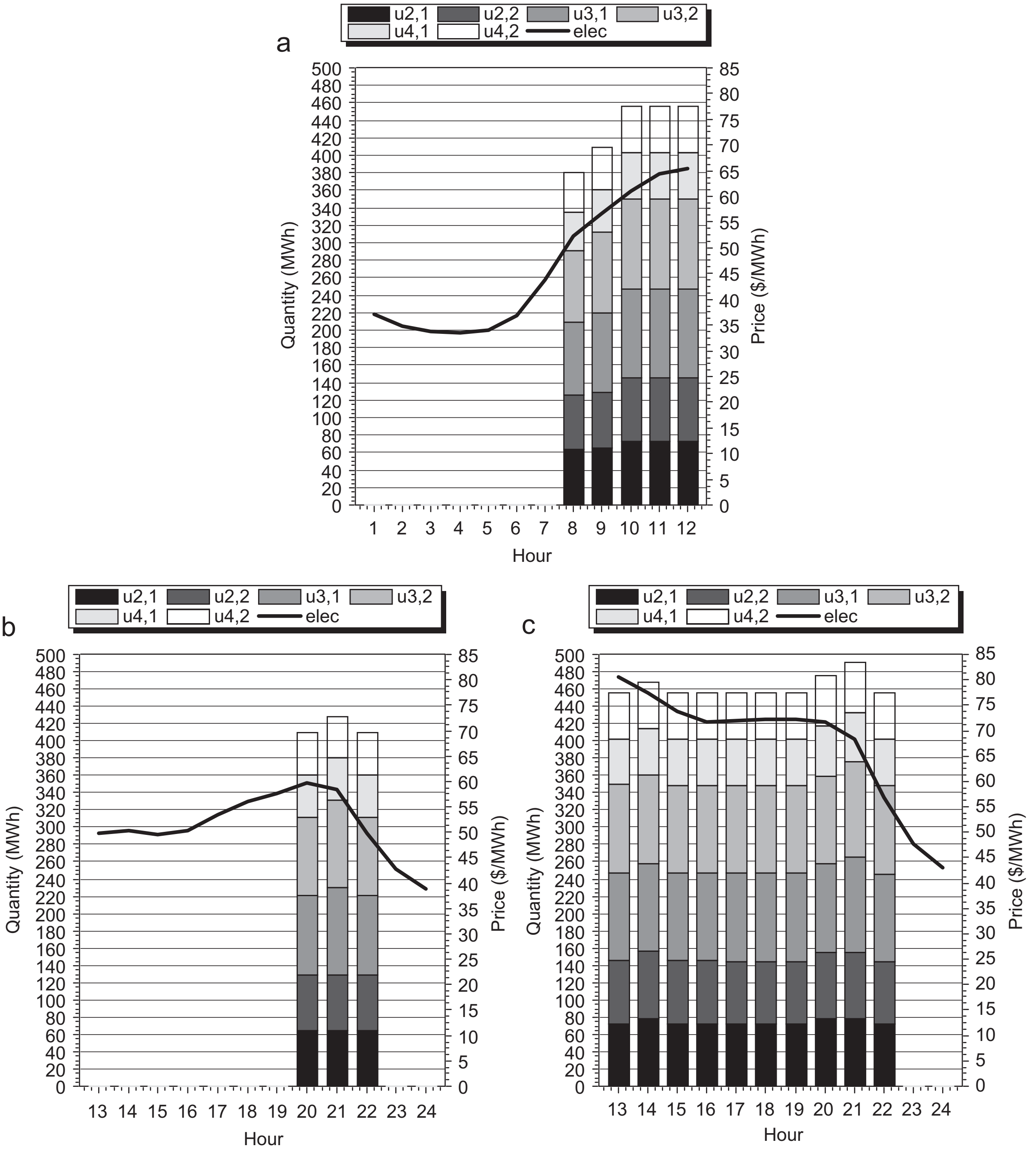

Fig. 8. Stochastic model production example.

5.2. Water head effects

This experiment analyzes the impact of water head on solution quality. The parameters of the successive linear programming method have been set to $\lambda_0 = 0.5$, $\lambda_{\text{mult}} = 0.75$, $\lambda_{\min} = 0.00001$ (see Section 2.2). Furthermore, in the case of the stochastic model, we have $N_b = 3$ and $N_s = 7$, as these values provided the best results (see Section 5.3). Table 3 presents the improvement with regard to the solution value obtained after N_{iter} iterations with each model,

Table 3
Impact of iteration number

N_{iter}	Deterministic (%)	Stochastic (%)
5	0.00	0.00
10	0.51	0.29
15	0.63	0.42
20	0.67	0.46
25	0.68	0.47
30	0.68	0.48
40	0.68	0.48
50	0.68	0.48
100	0.68	0.48

when compared to the solution value obtained only with the reference head. In all cases, the production plans are evaluated with the exact production function $MW_{d,u}(v, f)$ (in fact, all numerical comparisons reported here are based on the exact production function).

We first observe that the improvements obtained with the deterministic and stochastic models are 0.68% and 0.48%, respectively. The improvement over the initial solution is larger in the case of the deterministic model, simply because there is more room for improvement. It should be noted that more electricity is produced on average with the deterministic plan, while a larger part of the profits comes from reserve sales in the stochastic plan. The improvement reaches a plateau after 30 iterations, as only a fraction of a dollar is gained in the following iterations. Note that no improvement is observed in the first five iterations. It means that the value of λ_0 is too large and that the approximation is not valid. Improvements are only observed when λ is less than 0.1. In what follows, we thus use $\lambda_0 = 0.1$ and $N_{iter} = 30$. As the models that are solved from one iteration to the next do not contain any binary variables (see Sections 2.2 and 3.4), they can be solved quickly. Indeed, it takes 0.06 and 86 s to solve the initial deterministic and stochastic models, respectively, while only 0.005 and 6 s are required on average to perform one iteration of the successive linear programming approach in the deterministic and stochastic cases, respectively.

5.3. Branching factor and number of levels

This experiment studies the impact of the branching factor and number of levels in the tree of scenarios on solution quality. The latter was evaluated through a Monte Carlo approach. That is, we first generated optimal production plans with CPLEX for trees of scenarios of different shapes by using different values for parameters N_b and N_s. Then, each production plan obtained was evaluated on 1 000 000 new price vectors generated with our stochastic price generation scheme (see Section 4) by following the path of production decisions in the tree of scenarios based on the prices at periods $t_j - 2$, for $j = 1, \ldots, N_s - 1$, as explained in Section 3.2.

Table 4 reports the results for different values of N_b and N_s. The numbers in the table correspond to the percentage of improvement over the deterministic model (which corresponds to $N_b = 1$ and $N_s = 1$). The computation was stopped as soon as the relative gap dropped below 0.01%. The relative gap corresponds to the difference between the upper bound obtained by relaxing the integrality constraints and the best integer solution found, divided by the upper bound. A gap of zero indicates an optimal solution. The time limit was set to 2 h, but the relative gap dropped below 0.01% in less than 10 min even on the largest instances (see below).

The empty entries in the table correspond to models that were too large to be created and solved with 1 GB of memory. Given that the stochastic model with $N_b = 1$ or $N_s = 1$ is, in fact, a deterministic model, the improvement is equal to 0% in these cases. Otherwise, a significant difference is observed in the performance of the deterministic and stochastic models. Furthermore, this difference increases when the stochastic model is more refined (i.e., when the values of parameters N_b and N_s increase). In fact, the improvement over the deterministic model reaches 15.9% with $N_b = 3$ and $N_s = 7$.

Table 5 reports the computation times in seconds on each instance and Table 6 indicates the problem size for some instances in terms of the number of binary variables, continuous variables, and constraints. It should be noted that the complexity of the problem is directly related to the presence of binary variables (i.e., without them, the problem would

Table 4
Improvement of the stochastic model over the deterministic model

N_s	N_b							
	1 (%)	2 (%)	3 (%)	4 (%)	5 (%)	6 (%)	7 (%)	8 (%)
1	0	0	0	0	0	0	0	0
2	0	8.0	7.4	8.3	8.1	8.4	8.3	8.4
3	0	11.2	11.5	11.8	11.9	12.0	12.1	12.0
4	0	11.8	12.8	13.2	13.2	13.3	13.4	13.5
5	0	12.9	14.2	14.6	14.8			
6	0	13.7	15.2	15.6				
7	0	14.6	15.9					
8	0	15.0						

Table 5
Computation time of the stochastic model (in seconds)

N_s	N_b							
	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	2	1	2	2	3	3	4
3	1	3	4	6	11	15	21	30
4	1	4	10	26	58	112	204	326
5	1	5	49	215	557			
6	1	11	111	532				
7	1	31	461					
8	1	44						

Table 6
Problem size of some instances

N_s	N_b	Binary	Real	Constraints
1	1	288	2264	2924
2	2	432	3400	4396
3	3	1248	9848	12 748
4	4	6120	48 452	62 720
5	5	67 488	533 656	691 116
6	4	65 520	521 432	674 300
7	3	65 592	519 636	672 672

be easy). To solve mixed integer linear problems, CPLEX branches on binary variables. This procedure can require a computation time that grows exponentially with the number of binary variables, in the worst case.

5.4. Future value of water

This subsection analyzes the sensitivity of the models to the future value of water. Fig. 9 reports the improvement obtained with the stochastic model over the deterministic one for different future average electricity sales prices (FP). The parameters for the stochastic model were set to $N_b = 3$, $N_s = 7$. A stopping criterion based on a relative gap of 0.01% was also used here. With this stopping criterion, all problems were solved in less than half an hour of computation time.

We can see that the improvement first increases with the value of FP and then decreases. When the future value of water is low (FP $\leqslant$ 20\$), it is better to produce at full capacity with both models. That is why the improvement is null. However, as the future value of water increases, a more clever production strategy must be developed to efficiently

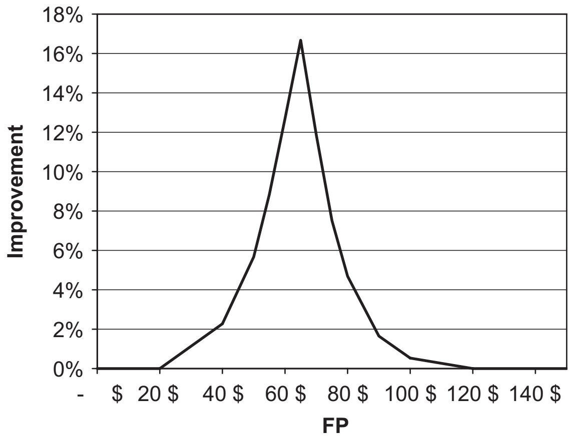

Fig. 9. Improvement as a function of the future value of water (FP).

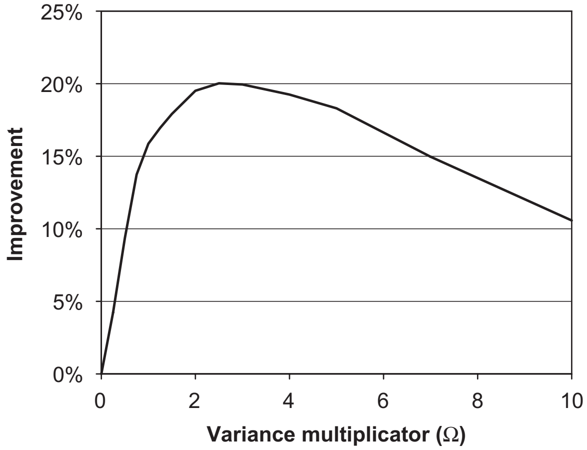

Fig. 10. Improvement as a function of price volatility.

distribute the water between current production and future production. In these cases, the stochastic model performs better than the deterministic one. Then, when the future value of water becomes really high (i.e., FP$\geqslant$120$), there is no difference again between the two models, because it is better to keep all water for the future. The stochastic model is thus useful for FP values between 20$ and 120$, with a peak at about 65$. Analysis of historical production data shows that FP is about 64$. This is the reference value that we used in the other experiments, but this analysis shows that good results would have also be obtained for values that are not too far from 64$.

5.5. Price volatility

The last experiment evaluates the impact of price volatility (or variance) on the performance of the stochastic model. In this regard, the variance of the normal random variable ε_t (see Section 4) is multiplied by parameter Ω. The results are shown in Fig. 10 where the improvement of the stochastic model over the deterministic model is reported for different Ω values.

In these experiments, the parameters for the stochastic model were set to $N_b = 3$ and $N_s = 7$. With a stopping criterion based on a relative gap of 0.01%, all problems were solved in less than 20 min of computation time. The results clearly show that the improvement of the stochastic model over the deterministic one increases with price volatility, until a critical point is reached where the improvement starts to decrease ($\Omega = 2.5$). Of course, the variance in a real market will never be as high as what is obtained with the largest Ω values tested. In fact, values between 0.75 and 1.25 better represent what is observed in the Ontario's electricity market.

6. Conclusion

This paper has introduced two mathematical models to maximize the profits obtained from hydroelectricity sales. The first model is based on deterministic prices while the second one integrates stochastic prices by considering a number of different price realizations that are organized into a tree structure. The numerical results shows that the stochastic model is superior to the deterministic one with regard to solution quality. Also, the performance of the stochastic model improves when prices are more volatile (higher variance).

Future developments will now be aimed at integrating the bidding process observed in deregulated markets within the stochastic model. By considering these bids, price volatility could be better exploited. Indeed, with a clever bidding strategy, electricity can be sold when prices are unexpectedly high, even if that was not planned in the original plan. The same thing is true for prices that happen to be unexpectedly low. Thus, additional profits can be expected from this integration.

Acknowledgments

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Appendix A. Proof of formula (82)

Theorem 1. *Let* $\phi_t^E = a_t + b_t \cdot \phi_{t-1}^E + \varepsilon_t$. *Then* $\phi_t^E = \sum_{i=1,\dots,t} a_i \prod_{j=i+1,\dots,t} b_j + \phi_0^E \prod_{i=1,\dots,t} b_k + \sum_{i=1,\dots,t} \varepsilon_i \prod_{j=i+1,\dots,t} b_j$.

Proof. For $t = 1$, the formula is obvious. Suppose it is true for $t-1$ and let us show it is also true for t.

$$\begin{aligned}
\phi_t^E &= a_t + b_t \cdot \phi_{t-1}^E + \varepsilon_t \\
&= a_t + b_t \cdot \left(\sum_{i=1,\dots,t-1} a_i \prod_{j=i+1,\dots,t-1} b_j + \phi_0^E \prod_{k=1,\dots,t-1} b_k + \sum_{i=1,\dots,t-1} \varepsilon_i \prod_{j=i+1,\dots,t-1} b_j \right) + \varepsilon_t \\
&= \sum_{i=1,\dots,t} a_i \prod_{j=i+1,\dots,t} b_j + \phi_0^E \prod_{k=1,\dots,t} b_k + \sum_{i=1,\dots,t} \varepsilon_i \prod_{j=i+1,\dots,t} b_j. \qquad (88)
\end{aligned}$$

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