CURS 2 — 11.10.2016

GEOMETRIE COMPUTAȚIONALĂ

$v = (v_1, v_2, v_3)$

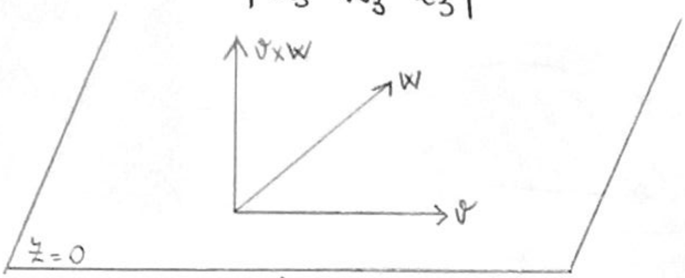
$w = (w_1, w_2, w_3)$

$$v \times w = \begin{vmatrix} v_1 & w_1 & e_1 \\ v_2 & w_2 & e_2 \\ v_3 & w_3 & e_3 \end{vmatrix}$$

OBSERVAȚIE: (IMPORTANTĂ) $v = (v_1, v_2, 0)$

$w = (w_1, w_2, 0)$

Atunci
$$v \times w = \begin{vmatrix} v_1 & w_1 & e_1 \\ v_2 & w_2 & e_2 \\ v_3 & w_3 & e_3 \end{vmatrix} \xrightarrow[\text{ultima coloană}]{\text{dezv. după}} \left(0,\ 0,\ \begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix}\right)$$



plan orizontal

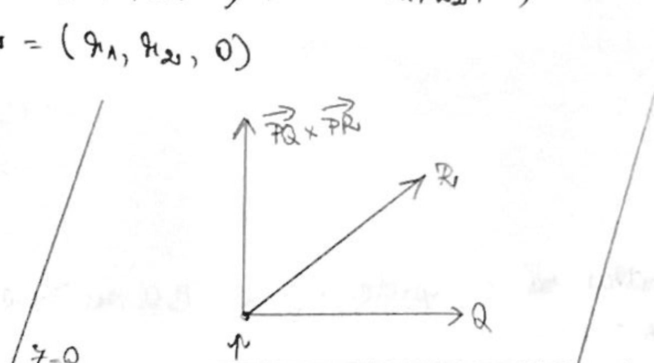
OBSERVAȚIE

$$\Delta(P,Q,R) = \begin{vmatrix} 1 & 1 & 1 \\ p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \end{vmatrix} \xrightarrow[c_3 \to c_3 - c_1]{c_2 \to c_2 - c_1} \begin{vmatrix} 1 & 0 & 0 \\ p_1 & q_1 - p_1 & r_1 - p_1 \\ p_2 & q_2 - p_2 & r_2 - p_2 \end{vmatrix} = \begin{vmatrix} q_1 - p_1 & r_1 - p_1 \\ q_2 - p_2 & r_2 - p_2 \end{vmatrix}$$

dem (Lemă):

$P = (p_1, p_2, 0)$, $Q = (q_1, q_2, 0)$

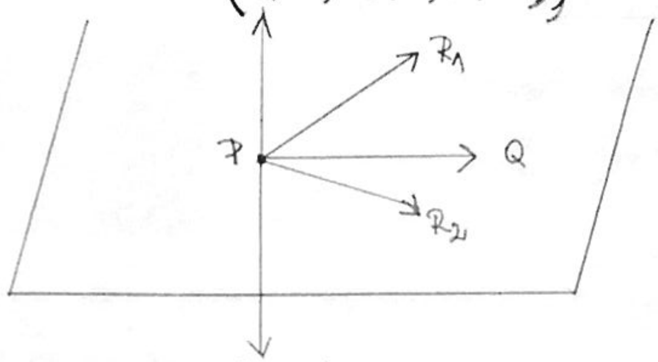
$R = (r_1, r_2, 0)$



$$\vec{PQ} \times \vec{PR} = \overbrace{(q_1 - p_1,\ q_2 - p_2,\ 0)}^{\vec{PQ}} \times \overbrace{(r_1 - p_1,\ r_2 - p_2,\ 0)}^{\vec{PR}} \overset{\text{OBS}}{=\!=} \left(0,\ 0,\ \begin{vmatrix} q_1 - p_1, & r_1 - p_1 \\ q_2 - p_2, & r_2 - p_2 \end{vmatrix}\right)$$

$$= (0, 0, \Delta(P,Q,R))$$

$\vec{PQ} \times \vec{PR_1} = (0, 0, \Delta(P, Q, R_1))$



$\vec{PQ} \times \vec{PR_2} = (0, 0, \Delta(P, Q, R_2))$

Pentru R_1 (stânga) : $\Delta(P, Q, R_1) > 0$
Pentru R_2 (dreapta) : $\Delta(P, Q, R_2) < 0$

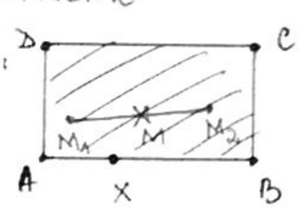
mulțime convexă

ACOPERIRI CONVEXE

- Algoritmi lenți (naivi). Două abordări
  1) găsirea punctelor extreme
  2) determinarea muchiilor frontierei de acoperiri

1) PUNCTE EXTREME

Exemplu 1:



→ A este punct extrem (pentru că NU putem găsi P, Q cu $P \neq Q$ pe dreapta sau în interior a.î. $A \in [PQ]$)

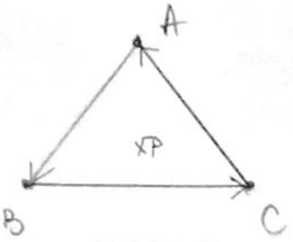
→ X NU este punct extrem ($X \in [AB]$)

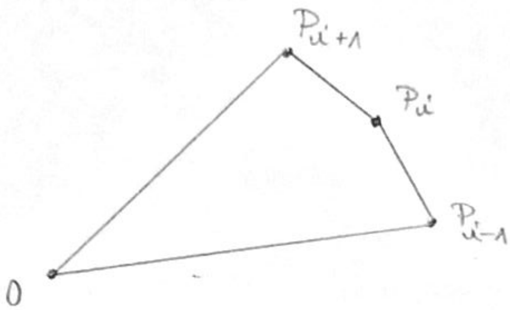
→ M NU este punct extrem ($M \in M_1M_2$ → conform figurii)

Exemplu 2

→ Disc = cerc

→ TOATE punctele cercului sunt puncte extreme

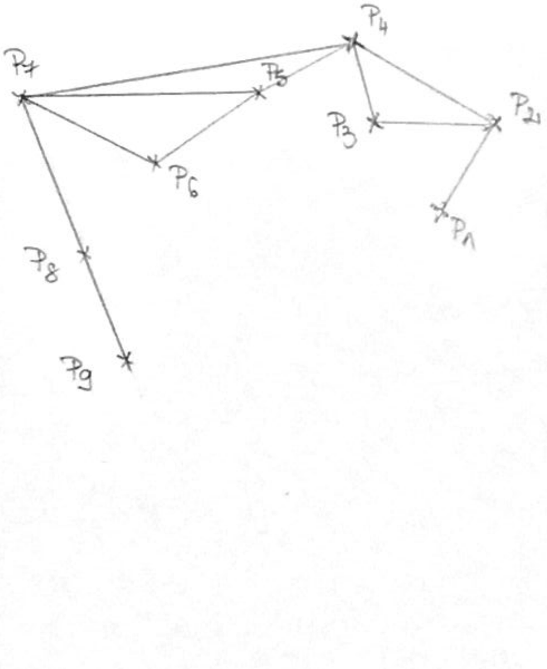




P_i punct extrem $\Leftrightarrow$ NU este în interiorul / pe laturile $\Delta O P_{i-1} P_{i+1}$ $\Leftrightarrow$

$\Leftrightarrow$ $P_{i-1} P_i P_{i+1}$ este viraj la stânga

Exemplu GRAHAM'S SCAN



(P_7, P_8, P_9 sunt coliniare)

α : $P_1 P_2 \cancel{P_3} P_4 \cancel{P_5} \cancel{P_6} P_7$

$P_1 P_2 P_4 P_7 \cancel{P_8} P_9 P_1$