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GRUPA: 141

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(Curs)

= Tema 3 =

1) $g:\mathbb{R}^3\times\mathbb{R}^3\to\mathbb{R};\ g(x,y)=x_2y_1+x_1y_2+2x_3y_1+2x_1y_3$

a) g FORMĂ BILINI., SIM.
b) $G=?$, ÎN RAP. CU $\mathcal{R}_0$
c) $\operatorname{Ker} g=?$
d) $Q:\mathbb{R}^3\to\mathbb{R};\ Q=?$
e) SĂ SE ADUCĂ Q LA O FORMĂ CANONICĂ. $Q\ ?$ " + DEF. "

a) • g BILINI. $\Leftrightarrow$ $\begin{cases}1)\ g(\alpha x+\beta y,z)=\alpha g(x,z)+\beta g(y,z)\\ 2)\ g(x,\alpha y+\beta z)=\alpha g(x,y)+\beta g(x,z)\end{cases}$; $(\forall)\ x,y,z\in\mathbb{R}^3$, $(\forall)\ \alpha,\beta\in\mathbb{R}$

1) $g(\alpha x+\beta y,z)=(\alpha x_2+\beta y_2)z_1+(\alpha x_1+\beta y_1)z_2+2(\alpha x_3+\beta y_3)z_1+2(\alpha x_1+\beta y_1)z_3=$
$=\alpha x_2z_1+\beta y_2z_1+\alpha x_1z_2+\beta y_1z_2+\alpha 2x_3z_1+\beta 2y_3z_1+\alpha 2x_1z_3+\beta 2y_1z_3=$
$=\alpha(x_2z_1+x_1z_2+2x_3z_1+2x_1z_3)+\beta(y_2z_1+y_1z_2+2y_3z_1+2y_1z_3)=$
$=\alpha g(x,z)+\beta\cdot g(y,z)$ ✓

Identic și pt. 2)

• g SIMETRICĂ $\Leftrightarrow g(x,y)=g(y,x);\ (\forall)\ x,y\in\mathbb{R}^3$

$x_2y_1+x_1y_2+2x_3y_1+2x_1y_3=y_2x_1+y_1x_2+2y_3x_1+2y_1x_3$ ✓

b)
$$G=\begin{pmatrix}0&1&2\\1&0&0\\2&0&0\end{pmatrix}\quad a_{13}\ (\neq 0)$$

c) $\operatorname{Ker} g=\{x\in\mathbb{R}^3 \mid g(x,y)=0,\ (\forall)\ y\in\mathbb{R}^3\}$

$\begin{cases}g(x,e_1)=0\\ g(x,e_2)=0\\ g(x,e_3)=0\end{cases}\Rightarrow\begin{cases}x_2+2x_3=0\Rightarrow x_2=-2x_3\\ x_1=0\\ 2x_1=0\end{cases}$; $\det(G)=0\Rightarrow g$ NU E NEDEGENERATĂ

$\operatorname{Ker} g=\{(0,-2x_3,x_3)\}$

d) $Q(x)=g(x,x)=x_2x_1+x_1x_2+2x_3x_1+2x_1x_3=2x_1x_2+4x_1x_3=$
$=2(x_1x_2+2x_1x_3)$

e) $y_1=\frac{1}{2}(x_1+x_3);\ y_2=\frac{1}{2}(x_1-x_3);\ y_3=x_2$ (PT. CĂ NICIUN $G_{ii}\neq 0$)

$$Q(x) = 2y_1^2 - 2y_2^2 + 2y_1y_3 + 2y_2y_3 = 2\left(y_1 + \frac{1}{2}y_3\right)^2 - 2y_2^2 + 2y_2y_3 - \frac{1}{2}y_3^2 =$$
$$= 2\left(y_1 + \frac{1}{2}y_3\right)^2 - 2\left(y_2 - \frac{1}{2}y_3\right)^2 + \frac{1}{2}y_3^2 - \frac{1}{2}y_3^2 =$$
$$= 2\left(y_1 + \frac{1}{2}y_3\right)^2 - 2\left(y_2 - \frac{1}{2}y_3\right)^2$$

$$\begin{cases} t_1 = y_1 + \frac{1}{2}y_3 \\ t_2 = y_2 - \frac{1}{2}y_3 \end{cases} \Rightarrow \begin{cases} t_1 = \frac{1}{2}(x_1 + x_3) + \frac{1}{2}x_2 = \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ t_2 = \frac{1}{2}(x_1 - x_3) - \frac{1}{2}x_2 = \frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 \\ \cancel{t_3 = x_2} \end{cases}$$

$$Q(x) = 2t_1^2 - 2t_2^2$$

$(1,1) \to$ SIGNATURA $\Rightarrow Q$ NU $\in$ "+ DEF." $(+2,0)$

2) $f : \mathbb{R}^3 \to \mathbb{R}^3$ AP. LINI.

$$A = \begin{pmatrix} 5 & 1 & 3 \\ 0 & 7 & 6 \\ 0 & 1 & 8 \end{pmatrix};\ [f]_{R_0, R_0} = A;\ R_0 = \text{REPER CANONIC}$$

a) VALORI PROPRII = ?

b) UN REPER ÎN CARE MATRICEA ASOCIATĂ LUI f E DIAGONALĂ

a) $$P(\lambda) = \begin{vmatrix} 5-\lambda & 1 & 3 \\ 0 & 7-\lambda & 6 \\ 0 & 1 & 8-\lambda \end{vmatrix} = (5-\lambda)\begin{vmatrix} 7-\lambda & 6 \\ 1 & 8-\lambda \end{vmatrix} = 0$$

$\lambda_1 = 5 \in \mathbb{R}$

$$(7-\lambda)(8-\lambda) - 6 = 0$$
$$56 - 7\lambda - 8\lambda + \lambda^2 - 6 = 0$$
$$\lambda^2 - 15\lambda + 50 = 0$$
$$\Delta = 15^2 - 4 \cdot 50 = 25 \Rightarrow \sqrt{\Delta} = 5$$
$$\lambda_2 = \frac{15+5}{2} = 10 \in \mathbb{R}$$
$$\lambda_3 = \frac{15-5}{2} = 5 \in \mathbb{R}$$

$$\begin{cases} \lambda_1 = 5;\ m_1 = 2 \\ \lambda_2 = 10;\ m_2 = 1 \end{cases}$$

b) $$V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid f(x) = 5x\} \Rightarrow \begin{cases} x_2 + 3x_3 = 0 \\ 2x_2 + 6x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} 0 & 1 & 3 \\ 0 & 2 & 6 \\ 0 & 1 & 3 \end{vmatrix} = 0;\ \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \dim V_{\lambda_1} = 3 - 1 = 2 = m_{\lambda_1} \quad \begin{cases} x_2 + 3x_3 = 0 \Rightarrow 3x_3 = -x_2 \Rightarrow x_2 = -3x_3 \\ 2x_2 + 6x_3 = 0 \Rightarrow 2x_2 - 2x_2 = 0 \end{cases}$$

$$V_{\lambda_1} = \{(0, -3x_3, x_3) \mid x_3 \in \mathbb{R}\} = \{x_3(0,-3,1) \mid x_3 \in \mathbb{R}\} \Rightarrow R_1 = \{(0,-3,1)\}$$

$$V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid f(x) = 10x\} \Rightarrow \begin{cases} -5x_1 + x_2 + 3x_3 = 0 \\ -3x_2 + 6x_3 = 0 \\ x_2 - 2x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} -5 & 1 & 3 \\ 0 & -3 & 6 \\ 0 & 1 & -2 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \dim V_{\lambda_2} = 3 - 2 = 1 = m_{\lambda_2} \quad \begin{cases} -5x_1 + x_2 = -3x_3 \Rightarrow -5x_1 = -5x_3 \Rightarrow x_1 = x_3 \\ x_2 = 2x_3 \end{cases} \Rightarrow V_{\lambda_2} = \{x_3(1,2,1) \mid x_3 \in \mathbb{R}\}$$

$$\Rightarrow R_2 = \{(1,2,1)\} \Rightarrow R = R_1 \cup R_2$$