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GRUPA: 141

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(Curs)

# - Temă 2 -

1) $(\mathbb{R}^4,+,\cdot)/_{\mathbb{R}}$ ; $U=\{x\in\mathbb{R}^4 / x_2-x_3=0 ; x_1+x_4=0\}$

a) REPER ÎN U

b) $W\subseteq\mathbb{R}^4$ a.î. $\mathbb{R}^4=U\oplus W$

c) $p: U\oplus W\to U\oplus W$ ; $p(0,1,2,-1)=?$

$s$ - SIMETRIE ; $s(0,1,2,-1)=?$

a) $U: \begin{cases} x_2-x_3=0 \Rightarrow x_2=x_3 \\ x_1+x_4=0 \Rightarrow x_1=-x_4 \end{cases} \Rightarrow U=\{(x_1,x_2,x_2,-x_1)/x_1,x_2\in\mathbb{R}\}=$

$=\{x_1(1,0,0,-1)+x_2(0,1,1,0)/x_1,x_2\in\mathbb{R}\}$

$A=\begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1\end{pmatrix} \Rightarrow rang(A)=2 \Rightarrow \dim_{\mathbb{R}} U=4-2=2$

$U=S(A)$

$\mathcal{R}=\{(1,0,0,-1);(0,1,1,0)\}$ SG. PT. U
$|\mathcal{R}|=\dim_{\mathbb{R}}U=2 \Rightarrow \mathcal{R}=SLI$
$\Rightarrow \mathcal{R}$ REPER ÎN U

b) EXTINDEM $\mathcal{R}$ LA UN REPER ÎN $\mathbb{R}^4$

$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0\end{vmatrix} = (-1)^{4+1+1}\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0\end{vmatrix} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 1 & 0\end{vmatrix} \neq 0$

$W=\langle(0,1,1,0);(0,1,0,0)\rangle \Rightarrow \mathbb{R}^4=U\oplus W$

c) $x=(0,1,2,-1)$. DESCOMPUN $x$ ÎN RAPORT CU $U\oplus W$.

$\mathcal{R}=\{(1,0,0,1);(0,1,1,0);(1,0,0,0);(0,1,0,0)\}$

$x=(0,1,2,-1)=\underbrace{a(1,0,0,-1)+b(0,1,1,0)}_{u\in U}+\underbrace{c(1,0,0,0)+d(0,1,0,0)}_{w\in W}$

$(0,1,2,-1)=(a+c, b+d, b, -a)$

$\begin{cases} a+c=0 \Rightarrow c=-1 \\ b+d=1 \Rightarrow d=-1 \\ b=2 \\ -a=-1 \Rightarrow a=1\end{cases} \Rightarrow (1,2,-1,-1)$ COORD. $x$ ÎN RAPORT CU $\mathcal{R}$

$u=1(1,0,0,-1)+2(0,1,1,0)=(1,2,2,-1)$

$w=-1(1,0,0,0)+(-1)(0,1,0,0)=(-1,-1,0,0)$

$x = (0,1,2,-1) = (1,2,2,-1) + (-1,-1,0,0)$

$p(x) = p(u+w) = u = (1,2,2,-1)$

$s = 2p - id_V = 2(1,2,2,-1) - (0,1,2,-1) = (2,3,2,-1)$

2) $f: \mathbb{R}^3 \to \mathbb{R}^3$; $f(x) = (x_1 + x_2 - x_3 ; 2x_1 + x_2 + x_3 ; x_1)$

a) $f$ LINIARĂ   c) $\operatorname{Ker} f$ - ? $\operatorname{Im} f$ - ?

b) MATRICE $f$ cu $R_0$   d) REPER ÎN $\operatorname{Ker}$ și $\operatorname{Im}$

a) $f(ax+by) = af(x) + bf(y)$ → TREBUIE SĂ DEMONSTRĂM

$ax + by = (ax_1 + by_1 ; ax_2 + by_2 ; ax_3 + by_3)$

$f(ax+by) = (ax_1 + by_1 + ax_2 + by_2 - ax_3 - by_3 ; 2ax_1 + 2by_1 + ax_2 + by_2 + ax_3 + by_3 ; ax_1 + by_1) = a(x_1 + x_2 - x_3 ; 2x_1 + x_2 + x_3 ; x_1) + b(y_1 + y_2 - y_3 ; 2y_1 + y_2 + y_3 ; y_1) = af(x) + bf(y)$

b) $Y = AX$

$$\begin{pmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_2 + x_3 \\ x_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} ; \det(A) = 1+1 = 2 \neq 0 \Rightarrow$$

$\Rightarrow \operatorname{rang}(A) = 3$

d+c) • $\operatorname{Ker} f = \{x \in \mathbb{R}^3 \mid f(x) = 0_{\mathbb{R}^3}\}$

$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ 2x_1 + x_2 + x_3 = 0 \\ x_1 = 0 \end{cases} \Rightarrow x_2 = x_3 (= 0)$$

$\operatorname{Ker} f = S(A) \Rightarrow \dim_{\mathbb{R}} \operatorname{Ker} f = 3 - 3 = 0$

$\operatorname{Ker} f = \{(0,0,0)\} \Rightarrow R_1 = \{(0,0,0)\}$

• $\operatorname{Im} f = \{y \in \mathbb{R}^3 \mid (\exists) x \in \mathbb{R}^3 \text{ a.î. } f(x) = y\}$

$$(*) \begin{cases} x_1 + x_2 - x_3 = y_1 \\ 2x_1 + x_2 + x_3 = y_2 \\ x_1 = y_3 \end{cases}$$ ESTE COMPATIBIL $(\Delta_A = 2 \neq 0)$ ; $\Delta_c = \begin{vmatrix} 1 & 1 & y_1 \\ 2 & 1 & y_2 \\ 1 & 0 & y_3 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow y_3 + y_2 - y_1 - 2y_3 = 0 \Rightarrow y_2 - y_1 - y_3 = 0 \Rightarrow y_2 = y_1 + y_3$

$\operatorname{Im} f = \{(y_1, y_2, y_3) \in \mathbb{R}^3 \mid y_2 = y_1 + y_3\} = \{\underbrace{(y_1, y_1 + y_3, y_3)}_{y_1(1,1,0) + y_3(0,1,1)} \mid y_1, y_3 \in \mathbb{R}\}$

$R_2 = \{(1,1,0);(0,1,1)\}$ SG. PT. $\operatorname{Im} f$

$\operatorname{rang}\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = 2 (\text{MAX}) \Rightarrow R_2 = \text{SLI}$

$\Rightarrow R_2$ REPER ÎN $\operatorname{Im} f$

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1) $(\mathbb{R}^4,+,\cdot)/\mathbb{R}$ ; $V_1=\{(a,b,c,0)/a,b,c\in\mathbb{R}\}$; $V_2=\{(0,0,d,e)/d,e\in\mathbb{R}\}$

a) $\mathbb{R}^4=V_1+V_2$ b) SUMA NU E DIRECTĂ

a) $V_1=\{a(1,0,0,0)+b(0,1,0,0)+c(0,0,1,0)/a,b,c\in\mathbb{R}\}$

$V_2=\{d(0,0,1,0)+e(0,0,0,1)/d,e\in\mathbb{R}\}$

$\mathcal{R}_0=\{e_1=(1,0,0,0);e_2=(0,1,0,0);e_3=(0,0,1,0);e_4=(0,0,0,1)\}$ (REPER CANONIC ÎN $\mathbb{R}^4$)

$\text{rang}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{pmatrix}=3 \Rightarrow \left.\begin{matrix}\{e_1,e_2,e_3\}=SLI\\ <\{e_1,e_2,e_3\}>=V_1\end{matrix}\right\}\Rightarrow \mathcal{R}_1=\{e_1,e_2,e_3\}$ REPER ÎN $V_1$

$\text{rang}\begin{pmatrix}0&0\\0&0\\1&0\\0&1\end{pmatrix}=2 \Rightarrow \left.\begin{matrix}\{e_3,e_4\}=SLI\\ <\{e_3,e_4\}>=V_2\end{matrix}\right\}\Rightarrow \mathcal{R}_2=\{e_3,e_4\}$ REPER ÎN $V_2$

$\left.\begin{matrix}\mathcal{R}_0=\mathcal{R}_1\cup\mathcal{R}_2\\ \text{Dar } \mathcal{R}_1\cap\mathcal{R}_2=\{e_3\}\neq\emptyset\end{matrix}\right\}\Rightarrow \mathbb{R}^4=V_1+V_2$

$\dim_{\mathbb{R}}(V_1+V_2)=\dim_{\mathbb{R}}V_1+\dim_{\mathbb{R}}V_2-\dim_{\mathbb{R}}(V_1\cap V_2)=$
$=3+2-1=4=\dim_{\mathbb{R}}\mathbb{R}^4$

b) SUMA NU E DIRECTĂ PT. CĂ $V_1\cap V_2\neq\emptyset$, $\dim_{\mathbb{R}}(V_1\cap V_2)=1$ (de la punctul a)

2) $(\mathbb{R}^4,+,\cdot)/\mathbb{R}$ ; $U_1=\{X\in\mathbb{R}^4/X_1+X_2+X_3+X_4=0\}$

$U_2=\{X\in\mathbb{R}^4/X_1=X_2=X_3=0\}$

a) $\mathbb{R}^4=U_1\oplus U_2$

b) SĂ SE DESCOMPUNĂ $X=(1,2,0,1)$ ÎN RAPORT CU SUMA DIRECTĂ

a) $U_1: X_1+X_2+X_3+X_4=0$

$A_1=(1\ 1\ 1\ 1)\Rightarrow \text{rang}(A_1)=1$

$U_1=S(A_1)\Rightarrow \dim_{\mathbb{R}}U_1=4-1=3$

$$U_2: \begin{cases} x_1=0 \\ x_2=0 \\ x_3=0 \end{cases} \qquad A_2=\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \operatorname{rang}(A_2)=3 \ ; U_2=S(A_2)$$

$\dim_{\mathbb{R}} U_2 = 4-3=1$

$$\underset{U_1\cap U_2}{\overset{U}{\|}}: \begin{cases} x_1+x_2+x_3+x_4=0 \Rightarrow x_4=0 \\ x_1=0 \\ x_2=0 \\ x_3=0 \end{cases} \Rightarrow \qquad U=\{0_{\mathbb{R}^4}\} \Rightarrow$$

$\Rightarrow \dim_{\mathbb{R}}(U_1+U_2) = \dim_{\mathbb{R}} U_1 + \dim_{\mathbb{R}} U_2 + \dim_{\mathbb{R}} U = 3+1=4=\dim_{\mathbb{R}} \mathbb{R}^4 \Rightarrow$

$\Rightarrow \mathbb{R}^4 = U_1 \oplus U_2$

b) $U_1=\{(-x_2-x_3-x_4, x_2, x_3, x_4) \mid x_2,x_3,x_4\in\mathbb{R}\} = \{x_2(-1,1,0,0)+x_3(-1,0,1,0)+$
$+x_4(-1,0,0,1) \mid x_2,x_3,x_4\in\mathbb{R}\}$

$\left.\begin{array}{l} \mathcal{R}_1=\{(-1,1,0,0);(-1,0,1,0);(-1,0,0,1)\} \text{ SG PT. } U_1 \\ |\mathcal{R}_1| = \dim_{\mathbb{R}} U_1 = 3 \Rightarrow \mathcal{R}_1 = \text{SLI} \end{array}\right\} \Rightarrow \mathcal{R}_1$ REPER

$U_2=\{(0,0,0,x_4) \mid x_4\in\mathbb{R}\} = \{x_4(0,0,0,1) \mid x_4\in\mathbb{R}\}$

$\left.\begin{array}{l} \mathcal{R}_2=\{(0,0,0,1)\} \text{ SG PT } U_2 \\ |\mathcal{R}_2| = \dim_{\mathbb{R}} U_2 = 1 \Rightarrow \mathcal{R}_2 = \text{SLI} \end{array}\right\} \Rightarrow \mathcal{R}_2$ REPER

$x=(1,2,0,1)=\overbrace{a(-1,1,0,0)+b(-1,0,1,0)}^{u\in U_1}+\overbrace{c(-1,0,0,1)+d(0,0,0,1)}^{v\in U_2}=$
$=(-a-b-c, a, b, c+d)$

$$\begin{cases} -a-b-c=1 \Rightarrow c=-3 \\ a=2 \\ b=0 \\ c+d=1 \Rightarrow d=4 \end{cases} \Rightarrow (2,0,-3,4) \text{ COORD. LUI } x \text{ ÎN RAPORT CU } \mathcal{R}_1\cup\mathcal{R}_2$$

$u=2(-1,1,0,0)+0(-1,0,1,0)=(-2,2,0,0)\in U_1$
$v=-3(-1,0,0,1)+4(0,0,0,1)=(3,0,0,1)\in U_2$

$(1,2,0,1)=(-2,2,0,0)+(3,0,0,1)$