

Tema Nr 21) ⁷ Începerea cu un caz particular: $n=5$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ l_1 & 1 & 0 & 0 & 0 \\ 0 & l_2 & 1 & 0 & 0 \\ 0 & 0 & l_3 & 1 & 0 \\ 0 & 0 & 0 & l_4 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} r_1 & \Delta_1 & 0 & 0 & 0 \\ 0 & r_2 & \Delta_2 & 0 & 0 \\ 0 & 0 & r_3 & \Delta_3 & 0 \\ 0 & 0 & 0 & r_4 & \Delta_4 \\ 0 & 0 & 0 & 0 & r_5 \end{pmatrix}$$

Din enunț, A este de tipul:

$$A = \begin{pmatrix} a_1 & b_1 & 0 & 0 & 0 \\ c_1 & a_2 & b_2 & 0 & 0 \\ 0 & c_2 & a_3 & b_3 & 0 \\ 0 & 0 & c_3 & a_4 & b_4 \\ 0 & 0 & 0 & c_4 & a_5 \end{pmatrix}$$

(Cazul general reușe în mod similar.)

De asemenea, $A = L \cdot R \Rightarrow A =$

$$\begin{pmatrix} r_1 & \Delta_1 & 0 & 0 & 0 \\ l_1 \cdot r_1 & l_1 \cdot \Delta_1 + r_2 & \Delta_2 & 0 & 0 \\ 0 & l_2 \cdot r_2 & l_2 \cdot \Delta_2 + r_3 & \Delta_3 & 0 \\ 0 & 0 & l_3 \cdot r_3 & l_3 \cdot \Delta_3 + r_4 & \Delta_4 \\ 0 & 0 & 0 & l_4 \cdot r_4 & l_4 \cdot \Delta_4 + r_5 \end{pmatrix}$$

De aici, putem deduce:

- $b_i = \Delta_i$; $i = \overline{1, n-1}$
- $r_i = l_i \cdot r_i$; $i = \overline{1, n-1}$
- $a_1 = r_1$; $a_i = l_{i-1} \cdot \Delta_{i-1} + r_i$; $i = \overline{2, n}$

Schema numerică oferită:

$$\left\{ \begin{array}{l} r_1 = a_1 \quad (\text{din enunț sus } \checkmark) \\ \text{for } i = 1 : (n-1) \text{ do} \end{array} \right.$$

$$l_i = r_i / \Delta_i \quad (\Leftrightarrow r_i = l_i \cdot \Delta_i \quad \checkmark)$$

$$\Delta_i = b_i \quad (\text{din enunț sus } \checkmark)$$

$$r_{i+1} = a_{i+1} - l_i \cdot \Delta_i \quad (\Leftrightarrow a_{i+1} = r_{i+1} + l_i \cdot \Delta_i \quad \checkmark)$$

end for

$$b) \begin{cases} Ax = b \\ A = L \cdot R \end{cases} \Rightarrow \begin{cases} L \cdot Rx = b \\ L \cdot y = b \end{cases}$$

Începem cu un caz particular: $n = 5$

$$L \cdot y = b \Leftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ l_1 & 1 & 0 & 0 & 0 \\ 0 & l_2 & 1 & 0 & 0 \\ 0 & 0 & l_3 & 1 & 0 \\ 0 & 0 & 0 & l_4 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} \Leftrightarrow \begin{pmatrix} y_1 \\ l_1 y_1 + y_2 \\ l_2 y_2 + y_3 \\ l_3 y_3 + y_4 \\ l_4 y_4 + y_5 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} y_1 = b_1 \\ y_i = b_i - l_{i-1} \cdot y_{i-1}, \quad i = \overline{2, n} \end{cases} \quad (\text{pt. cazul general})$$

$$Rx = y \Leftrightarrow \begin{pmatrix} r_1 & \Delta_1 & 0 & 0 & 0 \\ 0 & r_2 & \Delta_2 & 0 & 0 \\ 0 & 0 & r_3 & \Delta_3 & 0 \\ 0 & 0 & 0 & r_4 & \Delta_4 \\ 0 & 0 & 0 & 0 & r_5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} \Leftrightarrow \begin{pmatrix} r_1 x_1 + \Delta_1 x_2 \\ r_2 x_2 + \Delta_2 x_3 \\ r_3 x_3 + \Delta_3 x_4 \\ r_4 x_4 + \Delta_4 x_5 \\ r_5 x_5 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} x_n = y_n / r_n \\ x_i = (y_i - \Delta_i \cdot x_{i+1}) / r_i; \quad i = \overline{1, n-1} \end{cases} \quad (\text{pt. cazul general})$$

[illegible]