[redacted] LARISA
GRUPA: 141

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(curs)

= Tema 3 =

1) $q: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$; $q(x,y) = x_2y_1 + x_1y_2 + 2x_3y_1 + 2x_1y_3$

a) q FORMĂ BILINI., SIM.
b) $G = ?$, ÎN RAP. CU $\mathcal{R}_0$
c) $\ker q = ?$
d) $Q: \mathbb{R}^3 \to \mathbb{R}$; $Q = ?$
e) SĂ SE ADUCĂ Q LA O FORMĂ CANONICĂ. $Q \stackrel{?}{=} 4$ + DEF. 4

a) • q BILINI. $\Leftrightarrow$ 1) $q(\alpha x + \beta y, z) = \alpha q(x,z) + \beta q(y,z)$
2) $q(x, \alpha y + \beta z) = \alpha q(x,y) + \beta q(x,z)$; $(\forall) x,y,z \in \mathbb{R}^3$, $(\forall) \alpha, \beta \in \mathbb{R}$

1) $q(\alpha x + \beta y, z) = (\alpha x_2 + \beta y_2)z_1 + (\alpha x_1 + \beta y_1)z_2 + 2(\alpha x_3 + \beta y_3)z_1 + 2(\alpha x_1 + \beta y_1)z_3$
$= \alpha x_2z_1 + \beta y_2z_1 + \alpha x_1z_2 + \beta y_1z_2 + \alpha 2x_3z_1 + \beta 2y_3z_1 + \alpha 2x_1z_3 + \beta 2y_1z_3$
$= \alpha(x_2z_1 + x_1z_2 + 2x_3z_1 + 2x_1z_3) + \beta(y_2z_1 + y_1z_2 + 2y_3z_1 + 2y_1z_3) =$
$= \alpha q(x,z) + \beta \cdot q(y,z)$ ✓

Idem ș pt. 2)

• q SIMETRICĂ $\Leftrightarrow q(x,y) = q(y,x)$; $(\forall) x,y \in \mathbb{R}^3$
$x_2y_1 + x_1y_2 + 2x_3y_1 + 2x_1y_3 = y_2x_1 + y_1x_2 + 2y_3x_1 + 2y_1x_3$ ✓

b) $G = \begin{pmatrix} 0 & 1 & \textcircled{2} \\ 1 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}$ $a_{13} (\neq 0)$

c) $\ker q = \{x \in \mathbb{R}^3 \mid q(x,y) = 0, (\forall) y \in \mathbb{R}^3\}$

$\begin{cases} q(x,e_1) = 0 \\ q(x,e_2) = 0 \\ q(x,e_3) = 0 \end{cases} \Rightarrow \begin{cases} x_2 + 2x_3 = 0 \Rightarrow x_2 = -2x_3 \\ x_1 = 0 \\ 2x_1 = 0 \end{cases}$; $\det(G) = 0 \Rightarrow q$ NU E NEDEGENERATĂ

$\ker q = \{(0, -2x_3, x_3)\}$

d) $Q(x) = q(x,x) = x_2x_1 + x_1x_2 + 2x_3x_1 + 2x_1x_3 = 2x_1x_2 + 4x_1x_3 =$
$= 2(x_1x_2 + 2x_1x_3)$

e) $y_1 = \frac{1}{2}(x_1 + x_3)$; $y_2 = \frac{1}{2}(x_1 - x_3)$; $y_3 = x_2$ (PT. CĂ NICIUN $G_{ii} \neq 0$)

$Q(x) = 2y_1^2 - 2y_2^2 + 2y_1y_3 + 2y_2y_3 = 2(y_1 + \frac{1}{2}y_3)^2 - 2y^2 + 2y_2y_3 - \frac{1}{2}y_3^2 =$

$= 2(y_1 + \frac{1}{2}y_3)^2 - 2(y_2 - \frac{1}{2}y_3)^2 + \frac{1}{2}y_3^2 - \frac{1}{2}y_3^2 =$

$= 2(y_1 + \frac{1}{2}y_3)^2 - 2(y_2 - \frac{1}{2}y_3)^2$

$\begin{cases} z_1 = y_1 + \frac{1}{2}y_3 \\ z_2 = y_2 - \frac{1}{2}y_3 \end{cases} \Rightarrow \begin{cases} t_1 = \frac{1}{2}(x_1 + x_3) + \frac{1}{2}x_2 = \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ t_2 = \frac{1}{2}(x_1 - x_3) - \frac{1}{2}x_2 = \frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 \\ \cancel{t_3 = x_2} \end{cases}$

$Q(x) = 2t_1^2 - 2t_2^2$

$(1,1) \rightarrow$ SIGNATURA $\Rightarrow Q$ NU $\in$ "+ DET." $(+2,0)$

2) $f: \mathbb{R}^3 \to \mathbb{R}^3$ AP. LINI.

$A = \begin{pmatrix} 5 & 1 & 3 \\ 0 & 7 & 6 \\ 0 & 1 & 8 \end{pmatrix}$; $[f]_{\mathcal{R}_0, \mathcal{R}_0} = A$; $\mathcal{R}_0$ = REPER CANONIC

a) VALORI PROPRII = ?

b) UN REPER ÎN CARE MATRICEA ASOCIATĂ LUI f $\in$ DIAGONALĂ

a) $P(\lambda) = \begin{vmatrix} 5-\lambda & 1 & 3 \\ 0 & 7-\lambda & 6 \\ 0 & 1 & 8-\lambda \end{vmatrix} = (5-\lambda)\begin{vmatrix} 7-\lambda & 6 \\ 1 & 8-\lambda \end{vmatrix} = 0$

$\lambda_1 = 5 \in \mathbb{R}$

$(7-\lambda)(8-\lambda) - 6 = 0$

$56 - 7\lambda - 8\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 15\lambda + 50 = 0$

$\Delta = 15^2 - 4 \cdot 50 = 25 \Rightarrow \sqrt{\Delta} = 5$

$\lambda_2 = \frac{15+5}{2} = 10 \in \mathbb{R}$

$\lambda_3 = \frac{15-5}{2} = 5 \in \mathbb{R}$

$\begin{cases} \lambda_1 = 5; \ m_1 = 2 \\ \lambda_2 = 10; \ m_2 = 1 \end{cases}$

b) $V_{\lambda_1} = \{x \in \mathbb{R}^3 / f(x) = 5x\} \Rightarrow \begin{cases} x_2 + 3x_3 = 0 \\ 2x_2 + 6x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} 0 & 1 & 3 \\ 0 & 2 & 6 \\ 0 & 1 & 3 \end{vmatrix} = 0$; $\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 0$

$\Rightarrow \dim V_{\lambda_1} = 3 - 1 = 2 = m_{\lambda_1}$ $\begin{cases} x_2 + 3x_3 = 0 \Rightarrow 3x_3 = -x_2 \Rightarrow x_2 = -3x_3 \\ 2x_2 + 6x_3 = 0 \Rightarrow 2x_2 - 2x_2 = 0 \end{cases}$

$V_{\lambda_1} = \{(0, -3x_3, x_3) / x_3 \in \mathbb{R}\} = \{x_3(0,-3,1) / x_3 \in \mathbb{R}\} \Rightarrow \mathcal{R}_1 = \{(0,-3,1)\}$

$V_{\lambda_2} = \{x \in \mathbb{R}^3 / f(x) = 10x\} \Rightarrow \begin{cases} -5x_1 + x_2 + 3x_3 = 0 \\ -3x_2 + 6x_3 = 0 \\ x_2 - 2x_3 = 0 \end{cases} \Rightarrow \begin{vmatrix} -5 & 1 & 3 \\ 0 & -3 & 6 \\ 0 & 1 & -2 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow \dim V_{\lambda_2} = 3 - 2 = 1 = m_{\lambda_2}$ $\begin{cases} -5x_1 + x_2 = -3x_3 \Rightarrow -5x_1 = -5x_3 \Rightarrow x_1 = x_3 \\ x_2 = 2x_3 \end{cases} \Rightarrow V_{\lambda_2} = \{x_3(1,2,1) / x_3 \in \mathbb{R}\}$

$\Rightarrow \mathcal{R}_2 = \{(1,2,1)\} \Rightarrow \mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$