[redacted] LARISA
GRUPA: 141

20.03.2019
(Curs)

Tema 2

1) $(\mathbb{R}^4, +, \cdot)|_{\mathbb{R}}$; $U = \{x \in \mathbb{R}^4 / x_2 - x_3 = 0;\ x_1 + x_4 = 0\}$

a) REPER ÎN U
b) $W \subseteq \mathbb{R}^4$ o.î. $\mathbb{R}^4 = U \oplus W$
c) $\rho: U \oplus W \to U \oplus W$; $\rho(0,1,2,-1) = ?$
S - SIMETRIE ; $S(0,1,2,-1) = ?$

a) $U: \begin{cases} x_2 - x_3 = 0 \Rightarrow x_2 = x_3 \\ x_1 + x_4 = 0 \Rightarrow x_1 = -x_4 \end{cases} \Rightarrow U = \{(x_1, x_2, x_2, -x_1) / x_1, x_2 \in \mathbb{R}\} =$
$= \{x_1(1,0,0,-1) + x_2(0,1,1,0) / x_1, x_2 \in \mathbb{R}\}$

$A = \left(\begin{array}{cc|cc} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \operatorname{rang}(A) = 2 \Rightarrow \dim_{\mathbb{R}} U = 4 - 2 = 2$
$U = S(A)$

$\mathcal{R} = \{(1,0,0,-1); (0,1,1,0)\}$ SG. PT. U $\Big\} \Rightarrow \mathcal{R}$ REPER ÎN U
$|\mathcal{R}| = \dim_{\mathbb{R}} U = 2 \Rightarrow \mathcal{R}$ SLI

b) EXTINDEM $\mathcal{R}$ LA UN REPER ÎN $\mathbb{R}^4$

$$\begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{4+1+1} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \neq 0$$

$W = \langle (0,1,1,0); (0,1,0,0) \rangle \Rightarrow \mathbb{R}^4 = U \oplus W$

c) $X = (0,1,2,-1)$. DESCOMPUN X ÎN RAPORT CU $U \oplus W$.
$\mathcal{R} = \{(1,0,0,1); (0,1,1,0); (1,0,0,0); (0,1,0,0)\}$
$X = (0,1,2,-1) = a(1,0,0,-1) + b(0,1,1,0) + c(1,0,0,0) + d(0,1,0,0)$
$\qquad\qquad\qquad\qquad\quad u \in U \qquad\qquad\qquad\quad w \in W$

$(0,1,2,-1) = (a+c, b+d, b, -a)$

$\begin{cases} a + c = 0 \Rightarrow c = -1 \\ b + d = 1 \Rightarrow d = -1 \\ b = 2 \\ -a = -1 \Rightarrow a = 1 \end{cases} \Rightarrow (1,2,-1,-1)$ COORD. X ÎN RAPORT CU $\mathcal{R}$

$u = 1(1,0,0,-1) + 2(0,1,1,0) = (1,2,2,-1)$
$w = -1(1,0,0,0) + (-1)(0,1,0,0) = (-1,-1,0,0)$

$X = (0,1,2,-1) = (1,2,2,-1) + (-1,-1,0,0)$

$\rho(X) = \rho(u+w) = u = (1,2,2,-1)$

$\Delta = 2\rho - id_V = 2(1,2,2,-1) - (0,1,2,-1) = (2,3,2,-1)$

2) $f: \mathbb{R}^3 \to \mathbb{R}^3$; $f(X) = (x_1 + x_2 - x_3 ; 2x_1 + x_2 + x_3 ; x_1)$

a) f LINIARĂ c) $\ker f = ?$ $\operatorname{Im} f = ?$

b) MATRICE f cu $\mathcal{R}_0$ d) REPER ÎN $\ker$ ȘI Im

a) $f(ax+by) = af(x) + bf(y)$ → TREBUIE SĂ DEMONSTRĂM

$ax + by = (ax_1 + by_1 ; ax_2 + by_2 ; ax_3 + by_3)$

$f(ax+by) = (ax_1 + by_1 + ax_2 + by_2 - ax_3 - by_3 ; 2ax_1 + 2by_1 + ax_2 + by_2 + ax_3 + by_3 ; ax_1 + by_1) = a(x_1 + x_2 - x_3 ; 2x_1 + x_2 + x_3 ; x_1) + b(y_1 + y_2 - y_3 ; 2y_1 + y_2 + y_3 ; y_1) = af(x) + bf(y)$

b) $Y = AX$

$$\begin{pmatrix} x_1 + x_2 - x_3 \\ 2x_1 + x_2 + x_3 \\ x_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \det(A) = 1 + 1 = 2 \neq 0 \Rightarrow \operatorname{rang}(A) = 3$$

d+c) $\ker f = \{X \in \mathbb{R}^3 \mid f(X) = {}^{A}0_{\mathbb{R}^3}\}$

$$\left.\begin{cases} x_1 + x_2 - x_3 = 0 \\ 2x_1 + x_2 + x_3 = 0 \\ x_1 = 0 \end{cases}\right\} \Rightarrow x_2 = x_3 \ (= 0)$$

$\ker f = S(A) \Rightarrow \dim_{\mathbb{R}} \ker f = 3 - 3 = 0$

$\ker f = \{(0,0,0)\} \Rightarrow \mathcal{R}_1 = \{(0,0,0)\}$

• $\operatorname{Im} f = \{y \in \mathbb{R}^3 \mid (\exists) x \in \mathbb{R}^3 \text{ a.î. } f(x) = y\}$

(*) $\begin{cases} x_1 + x_2 - x_3 = y_1 \\ 2x_1 + x_2 + x_3 = y_2 \\ x_1 = y_3 \end{cases}$ ESTE COMPATIBIL ($\Delta_A = 2 \neq 0$); $\Delta_C = \begin{vmatrix} 1 & 1 & y_1 \\ 2 & 1 & y_2 \\ 1 & 0 & y_3 \end{vmatrix} = 0 \Rightarrow$

$\Rightarrow y_3 + y_2 - y_1 - 2y_3 = 0 \Rightarrow y_2 - y_1 - y_3 = 0 \Rightarrow y_2 = y_1 + y_3$

$\operatorname{Im} f = \{(y_1, y_2, y_3) \in \mathbb{R}^3 \mid y_2 = y_1 + y_3\} = \{(y_1, y_1 + y_3, y_3) \mid y_1, y_3 \in \mathbb{R}\}$

$y_1(1,1,0) + y_3(0,1,1)$

$\mathcal{R}_2 = \{(1,1,0); (0,1,1)\}$ SG. PT. $\operatorname{Im} f$ $\Rightarrow \mathcal{R}_2$ REPER ÎN $\operatorname{Im} f$

$\operatorname{rang}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = 2$ (MAX) $\Rightarrow \mathcal{R}_2$ = SU

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Tema 2

1) $(\mathbb{R}^4, +, \cdot)/\mathbb{R}$; $V_1 = \{(a,b,c,0) / a,b,c \in \mathbb{R}\}$; $V_2 = \{(0,0,d,e) / d,e \in \mathbb{R}\}$

a) $\mathbb{R}^4 = V_1 + V_2$ b) SUMA NU E DIRECTĂ

a) $V_1 = \{a(1,0,0,0) + b(0,1,0,0) + c(0,0,1,0) / a,b,c \in \mathbb{R}\}$

$V_2 = \{d(0,0,1,0) + e(0,0,0,1) / d,e \in \mathbb{R}\}$

$\mathcal{R}_0 = \{e_1 = (1,0,0,0); e_2 = (0,1,0,0); e_3 = (0,0,1,0); e_4 = (0,0,0,1)\}$ (REPER CANONIC ÎN $\mathbb{R}^4$)

$\operatorname{rang}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = 3 \Rightarrow \left.\begin{array}{l} \{e_1, e_2, e_3\} = SLI \\ \langle \{e_1, e_2, e_3\} \rangle = V_1 \end{array}\right\} \Rightarrow \mathcal{R}_1 = \{e_1, e_2, e_3\}$ REPER ÎN V_1

$\operatorname{rang}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = 2 \Rightarrow \left.\begin{array}{l} \{e_3, e_4\} = SLI \\ \langle \{e_3, e_4\} \rangle = V_2 \end{array}\right\} \Rightarrow \mathcal{R}_2 = \{e_3, e_4\}$ REPER ÎN V_2

$\mathcal{R}_0 = \mathcal{R}_1 \cup \mathcal{R}_2$

Dar $\mathcal{R}_1 \cap \mathcal{R}_2 = \{e_3\} \neq \emptyset$ $\Rightarrow \mathbb{R}^4 = V_1 + V_2$

$\dim_{\mathbb{R}}(V_1 + V_2) = \dim_{\mathbb{R}} V_1 + \dim_{\mathbb{R}} V_2 - \dim_{\mathbb{R}}(V_1 \cap V_2) =$

$= 3 + 2 - 1 = 4 = \dim_{\mathbb{R}} \mathbb{R}^4$

b) SUMA NU E DIRECTĂ PT. CĂ $V_1 \cap V_2 \neq \emptyset$, $\dim_{\mathbb{R}}(V_1 \cap V_2) = 1$ (de la punctul a)

2) $(\mathbb{R}^4, +, \cdot)/\mathbb{R}$; $U_1 = \{x \in \mathbb{R}^4 / x_1 + x_2 + x_3 + x_4 = 0\}$

$U_2 = \{x \in \mathbb{R}^4 / x_1 = x_2 = x_3 = 0\}$

a) $\mathbb{R}^4 = U_1 \oplus V_2$

b) SĂ SE DESCOMPUNĂ $x = (1,2,0,1)$ ÎN RAPORT CU SUMA DIRECTĂ

a) $U_1 : x_1 + x_2 + x_3 + x_4 = 0$

$A_1 = (1\ 1\ 1\ 1) \Rightarrow \operatorname{rang}(A_1) = 1$

$U_1 = S(A_1) \Rightarrow \dim_{\mathbb{R}} U_1 = 4 - 1 = 3$

$U_2: \begin{cases} X_1 = 0 \\ X_2 = 0 \\ X_3 = 0 \end{cases}$ $A_2 = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \operatorname{rang}(A_2) = 3 \; ; \; U_2 = S(A_2)$

$\dim_{\mathbb{R}} U_2 = 4 - 3 = 1$

$U: \begin{cases} X_1 + X_2 + X_3 + X_4 = 0 \Rightarrow X_4 = 0 \\ X_1 = 0 \\ X_2 = 0 \\ X_3 = 0 \end{cases} \Rightarrow \quad U = \{0_{\mathbb{R}^4}\} \Rightarrow$

$U = U_1 \cap U_2$

$\Rightarrow \dim_{\mathbb{R}}(U_1 + U_2) = \dim_{\mathbb{R}} U_1 + \dim_{\mathbb{R}} U_2 + \dim_{\mathbb{R}} U = 3 + 1 = 4 = \dim_{\mathbb{R}} \mathbb{R}^4 \Rightarrow$

$\Rightarrow \mathbb{R}^4 = U_1 \oplus U_2$

b) $U_1 = \{(-X_2 - X_3 - X_4, X_2, X_3, X_4) \mid X_2, X_3, X_4 \in \mathbb{R}\} = \{X_2(-1,1,0,0) + X_3(-1,0,1,0) +$

$+ X_4(-1,0,0,1) \mid X_2, X_3, X_4 \in \mathbb{R}\}$

$\mathcal{R}_1 = \{(-1,1,0,0); (-1,0,1,0); (-1,0,0,1)\}$ SG PT. $U_1 \} \Rightarrow \mathcal{R}_1$ REPER

$|\mathcal{R}_1| = \dim_{\mathbb{R}} U_1 = 3 \Rightarrow \mathcal{R}_1 = SU'$

$U_2 = \{(0,0,0,X_4) \mid X_4 \in \mathbb{R}\} = \{X_4(0,0,0,1) \mid X_4 \in \mathbb{R}\}$

$\mathcal{R}_2 = \{(0,0,0,1)\}$ SG PT U_2 $\} \Rightarrow \mathcal{R}_2$ REPER

$|\mathcal{R}_2| = \dim_{\mathbb{R}} U_2 = 1 \Rightarrow \mathcal{R}_2 = SU'$

$X = (1,2,0,1) = \overbrace{a(-1,1,0,0) + b(-1,0,1,0)}^{u \in U_1} + \overbrace{c(-1,0,0,1) + d(0,0,0,1)}^{v \in U_2} =$

$= (-a-b-c, a, b, c+d)$

$\begin{cases} -a-b-c = 1 \Rightarrow c = -3 \\ a = 2 \\ b = 0 \\ c+d = 1 \Rightarrow d = 4 \end{cases} \Rightarrow (2,0,-3,4)$ COORD. LUI x ÎN RAPORT CU $\mathcal{R}_1 \cup \mathcal{R}_2$

$u = 2(-1,1,0,0) + 0(-1,0,1,0) = (-2,2,0,0) \in U_1$

$v = -3(-1,0,0,1) + 4(0,0,0,1) = (3,0,0,1) \in U_2$

$(1,2,0,1) = (-2,2,0,0) + (3,0,0,1)$