Tehnici de Optimizare
- Seminar 5 -
Aplicaţii.

1 Support Vector Machine

$$\pi_H(x_a) = x_a - \frac{w^T x_a + b}{\|w\|^2} w$$

$$\underbrace{\frac{w^T x_a + b}{\|w\|}}_{r} \frac{w}{\|w\|} = x_a - \pi_H(x_a) \Rightarrow \mathrm{dist}_H(x_a) = r = \frac{|w^T x_a + b|}{\|w\|}$$

$$w^T x_a + b = 1 \Rightarrow r = \frac{1}{\|w\|}$$

$$\begin{aligned} \min_{w,b} \quad & \frac{1}{2}\|w\|^2 \\ \text{s.l.} \quad & y_i(w^T x^i + b) \geq 1, \quad \forall i = 1, \cdots, N \end{aligned}$$

Conditia Slater este valabila. Presupunem ca exista $(w, b) \in Q$.

$$L(w, b, \mu) = \frac{1}{2}\|w\|^2 + \sum_i \mu_i [y_i(w^T x^i + b) - 1]$$

c Sistem KT:

$$\begin{aligned} & \nabla_w L(w, b, \mu) = 0, \;\; \nabla_b L(w, b, \mu) = 0 \\ & 1 - y_i(w^T x^i + b) \leq 0, \forall i, \;\; \mu \geq 0 \\ & \mu_i [y_i(w^T x^i + b) - 1] = 0, \forall i \end{aligned}$$

Complementaritatea implica $\mu_i > 0$ atunci $y_i(w^T x^i + b) = 1 \Leftrightarrow x^i$ apartine hiperplanului auxiliar.

$$w(\mu) = -\sum_i \mu_i y_i x^i, \;\; \sum_i \mu_i y_i = 0$$

Functia duala:

$$\begin{aligned}\phi(\mu) &= -\frac{1}{2}\|\sum_i \mu_i y_i x^i\|^2 + b\sum_i \mu_i y_i - \sum_i \mu_i \\ &= -\frac{1}{2}\|\sum_i \mu_i y_i x^i\|^2 - \sum_i \mu_i \\ &= -\frac{1}{2}\mu^T H \mu - \sum_i \mu_i,\end{aligned}$$

unde $H_{ij} = y_i y_j (x^i)^T x^j$. Problema duala:

$$\begin{aligned}\min_{\mu} \ & \frac{1}{2}\mu^T H \mu + \sum_i \mu_i \\ \text{s.l.} \ & \mu \geq 0, \sum_i \mu_i y_i = 0\end{aligned}$$

Observatii:

- $I^* = \{i : \mu_i^* > 0\}$ atunci $w^* = -\sum_{i \in I^*} \mu_i^* y_i x^i$
- $y_i((w^*)^T x^i + b) = 1$, oricare $i \in I^*$; aleg oricare $i \in I^*$, calculam $b = \frac{1}{y_i} - (w^*)^T x^i$