Part 1: Simulation Exercise

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Part 1: Simulation Exercise Instructions In this project you will investigate the *exponential distribution* in R and compare it with the Central Limit Theorem.

The exponential distribution can be simulated in R with **rexp(n, lambda)** where:

- 1. **lambda** is the rate parameter.
- 2. The *mean* of exponential distribution is 1/lambda.
- 3. the standard deviation is also 1/lambda.

Set lambda = 0.2 for all of the simulations.

You will investigate the distribution of averages of 40 exponentials.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponentials.

You should:

- 1. Show the sample mean and compare it to the theoretical mean of the distribution.
- 2. Show the variance and compare it to the theoretical variance of the distribution.
- 3. Show that the distribution is approximately normal.

Exponential distribution: The exponential distribution is defined by its cumulative distribution function

$$F(x) = 1 - e^{-\lambda x}$$

The R function rexp generates random variables with an exponential distribution.

Question 1: Show the sample mean and compare it to the theoretical mean distribution

```
set.seed(777)
n <- 40
Lambda <- 0.2
Simulations <- 1000</pre>
```

Theoretical values:

- 1. Theoretical Mean is equal to: 5
- 2. Theoretical Standard Deviation is equal to: 0.7905694
- 3. Theoretical Variance is equal to: 0.625

```
SampleMean <- 0
#For, this will create our n means from the rexp funtion.

for(i in 1:Simulations) {
   SampleMean <- c(SampleMean, mean(rexp(n, Lambda)))
}
mean(SampleMean)</pre>
```

[1] 4.964824

The sample mean is very close to the theoretical mean distribution. The difference is = 0.030211

Question 2: Show the sample variance and compare it to the thoretical variance of the distribution.

```
Variance <- var(SampleMean)</pre>
```

[1] 0.6625188

The sample variance is very close to the theoretical variance.

The difference is = -0.0134832

Show that the distribution is approximately normal

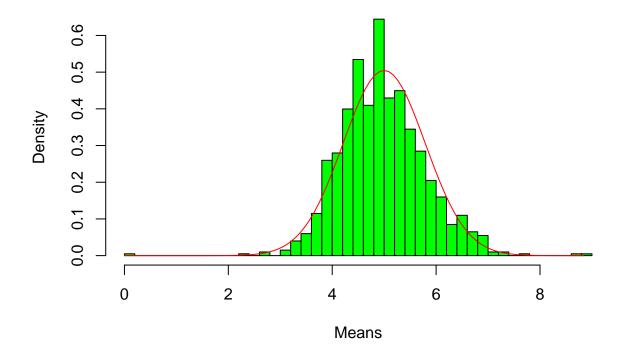
This can be shown with 2 graph tools:

- 1. Histogram of the n means
- 2. The normal probability plot

First we examine the Histogram,

```
hist(SampleMean, breaks = n, prob = T, col = "green", xlab = "Means")
x <- seq(min(SampleMean), max(SampleMean), length = 100)
lines(x, dnorm(x, mean = 1/Lambda, sd = (1/Lambda/sqrt(n))), pch = 25, col = "Red")</pre>
```

Histogram of SampleMean

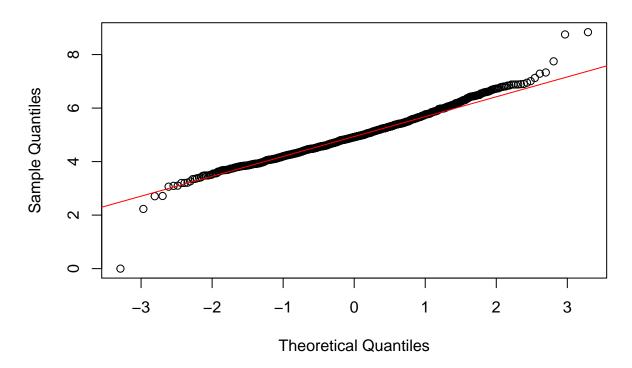


We conclude that the histogram appears to be bell shaped, and the mean is very close to the theoretical mean.

Then, for our Normal probability plot:

```
qqnorm(SampleMean)
qqline(SampleMean, col = "Red")
```

Normal Q-Q Plot



The data should form an approximate straight line, in this case it appears almost linear, we can assume the data set is approximately normally distributed