

DM561
Linear Algebra with Applications

Intoduction to Python - Part 2

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[Based on booklet Python Essentials]

Outline

Exception Handling
File Input and Output
Numpy

1. Exception Handling

2. File Input and Output

3. Numpy

Data Access

Numerical Computing with NumPy

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An **exception** formally indicates an error and terminates the program early.

```
>>> print(x)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'x' is not defined

>>> [1, 2, 3].fly()
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
AttributeError: 'list' object has no attribute 'fly'
```

Built-in Exceptions

Exception	Indication
AttributeError	An attribute reference or assignment failed.
ImportError	An import statement failed.
IndexError	A sequence subscript was out of range.
NameError	A local or global name was not found.
TypeError	An operation or function was applied to an object of inappropriate type.
ValueError	An operation or function received an argument that had the right type but an inappropriate value.
ZeroDivisionError	The second argument of a division or modulo operation was zero.

See <https://docs.python.org/3/library/exceptions.html> for the complete list of built-in exception

Raising Exceptions

```
>>> if 7 is not 7.0:                # Raise an exception with an error message.  
...     raise Exception("ints and floats are different!")  
...
```

Traceback (most recent call last):

File "<stdin>", line 2, in <module>

Exception: ints and floats are different!

```
>>> for x in range(10):  
...     if x > 5:                    # Raise a specific kind of exception.  
...         raise ValueError("'x' should not exceed 5.")  
...     print(x, end=' ')  
...
```

0 1 2 3 4 5

Traceback (most recent call last):

File "<stdin>", line 3, in <module>

ValueError: 'x' should not exceed 5.

Handling Exceptions

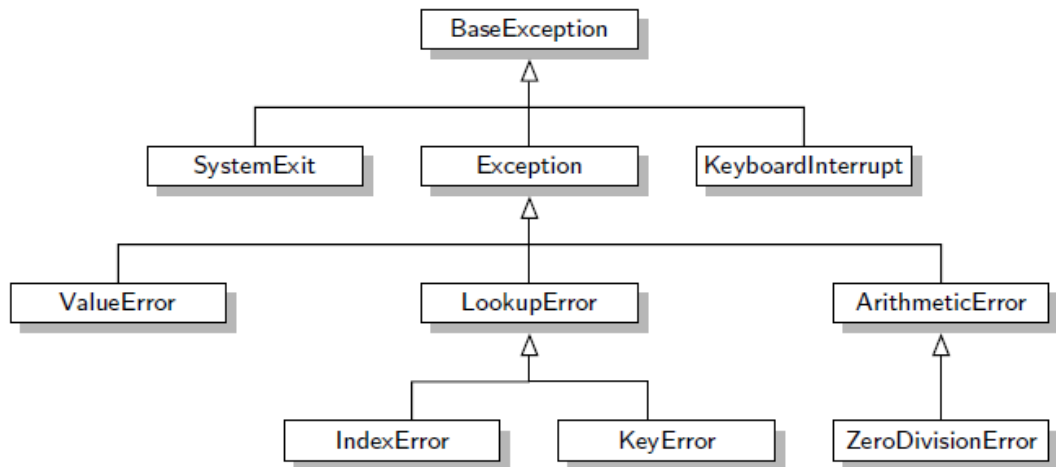
To prevent an exception from halting the program, it must be handled by placing the problematic lines of code in a `try` block.

```
>>> try:
...     print("Entering try block...", end='')
...     house_on_fire = False
... except ValueError as e: # Skipped because there was no exception.
...     print("caught a ValueError.")
...     house_on_fire = True
... except TypeError as e: # Also skipped (if just ``except:'' then always caught)
...     print("caught a TypeError.")
...     house_on_fire = True
... else:
...     print("no exceptions raised.")
... finally: # always executed, even if a return statement or an uncaught ←
...     exception occurs
...     print("The house is on fire:", house_on_fire)
... 
```

Entering try block...no exceptions raised.

The house is on fire: False

Exception Hierarchy



Working with Exception Objects

```
>>> try:
...     raise Exception('spam', 'eggs')
... except Exception as inst:
...     print(type(inst))      # the exception instance
...     print(inst.args)      # arguments stored in .args
...     print(inst)           # __str__ allows args to be printed directly,
...                             # but may be overridden in exception subclasses
...     x, y = inst.args      # unpack args
...     print('x =', x)
...     print('y =', y)
...
<class 'Exception'>
('spam', 'eggs')
('spam', 'eggs')
x = spam
y = eggs
```

Custom Exception Classes

```
>>> class TooHardError(Exception):  
...     pass  
...  
>>> raise TooHardError("This lab is impossible!")  
Traceback (most recent call last):  
  File "<stdin>", line 1, in <module>  
__main__.TooHardError: This lab is impossible!
```

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File Reading

```
>>> myfile = open("hello_world.txt", 'r')    # Open a file for reading.
>>> print(myfile.read())                    # Print the contents of the file.
Hello,                                       # (it's a really small file.)
World!

>>> myfile.close()                          # Close the file connection.
```

the 'mode' can be 'r', 'w', 'x', 'a'

A More Secure Way

```
>>> myfile = open("hello_world.txt", 'r')    # Open a file for reading.
>>> try:
...     contents = myfile.readlines()        # Read in the content by line.
... finally:
...     myfile.close()                      # Explicitly close the file.

# Equivalently, use a 'with' statement to take care of errors.
>>> with open("hello_world.txt", 'r') as myfile:
...     contents = myfile.readlines()
...                                     # The file is closed automatically.
```

Reading and Writing

Attribute	Description
<code>closed</code>	True if the object is closed.
<code>mode</code>	The access mode used to open the file object.
<code>name</code>	The name of the file.
Method	Description
<code>close()</code>	Close the connection to the file.
<code>read()</code>	Read a given number of bytes; with no input, read the entire file.
<code>readline()</code>	Read a line of the file, including the newline character at the end.
<code>readlines()</code>	Call <code>readline()</code> repeatedly and return a list of the resulting lines.
<code>seek()</code>	Move the cursor to a new position.
<code>tell()</code>	Report the current position of the cursor.
<code>write()</code>	Write a single string to the file (spaces are not added).
<code>writelines()</code>	Write a list of strings to the file (newline characters are not added).

```
>>> with open("out.txt", 'w') as outfile:    # Open 'out.txt' for writing.
...     for i in range(10):
...         outfile.write(str(i**2)+' ')    # Write some strings (and spaces).
...
>>> outfile.closed                          # The file is closed automatically.
True
```

String Methods

Method	Returns
<code>count()</code>	The number of times a given substring occurs within the string.
<code>find()</code>	The lowest index where a given substring is found.
<code>isalpha()</code>	True if all characters in the string are alphabetic (a, b, c, ...).
<code>isdigit()</code>	True if all characters in the string are digits (0, 1, 2, ...).
<code>isspace()</code>	True if all characters in the string are whitespace (" ", '\t', '\n').
<code>join()</code>	The concatenation of the strings in a given iterable with a specified separator between entries.
<code>lower()</code>	A copy of the string converted to lowercase.
<code>upper()</code>	A copy of the string converted to uppercase.
<code>replace()</code>	A copy of the string with occurrences of a given substring replaced by a different specified substring.
<code>split()</code>	A list of segments of the string, using a given character or string as a delimiter.
<code>strip()</code>	A copy of the string with leading and trailing whitespace removed.

String Methods

```
# str.join() puts the string between the entries of a list.
```

```
>>> words = ["state", "of", "the", "art"]
```

```
>>> "-".join(words)
```

```
'state-of-the-art'
```

```
# str.split() creates a list out of a string, given a delimiter.
```

```
>>> "One fish\nTwo fish\nRed fish\nBlue fish\n".split('\n')
```

```
['One fish', 'Two fish', 'Red fish', 'Blue fish', '']
```

```
# If no delimiter is provided, the string is split by whitespace characters.
```

```
>>> "One fish\nTwo fish\nRed fish\nBlue fish\n".split()
```

```
['One', 'fish', 'Two', 'fish', 'Red', 'fish', 'Blue', 'fish']
```

Format

```
# Join the data using string concatenation.
>>> day, month, year = 10, "June", 2017
>>> print("Is today", day, str(month) + ', ', str(year) + "?")
Is today 10 June, 2017?
```

```
# Join the data using str.format().
>>> print("Is today {} {}, {}?".format(day, month, year))
Is today 10 June, 2017?
```

```
>>> iters = int(1e7)
>>> chunk = iters // 20
>>> for i in range(iters):
...     print("\r[{:<20}] i = {}".format('='*((i//chunk)+1), i), end='', flush=True)
```

```
# From version 3.6
>>> print(f'Is today {day} {month}, {year}? The vlaue of pi is: {math.pi:.3f}')
Is today 10 June, 2017? The value of pi is: 3.142.
```

JSON To dict

Reading a JSON (JavaScript Object Notation) file, content example:

```
{
  "first_name": "Søren",
  "last_name": "Sørensen",
  "age": 25,
  "address": {
    "street_address": "Rodvej 45",
    "city": "Odense",
    "postal_code": "5000"
  },
  "phone_numbers": [
    {
      "type": "home",
      "number": "+45 7575757"
    },
    {
      "type": "mobile",
      "number": "+45 99999999"
    }
  ],
  "gender": {
    "type": "male"
  }
}
```

```
import json

with open('example.json') as json_file:
    data = json.load(json_file)

print(json.dumps(data, indent=4))

with open('example2.json', 'w') as outfile:
    json.dump(data, outfile)
```

Add the `ensure_ascii=False` argument to the `json.dump` in order to print correctly the 'ø' character.

Unicode

Python's string type uses the [Unicode Standard](#) for representing characters

code point | glyph | name

```
0061    'a'; LATIN SMALL LETTER A
0062    'b'; LATIN SMALL LETTER B
0063    'c'; LATIN SMALL LETTER C
...
007B    '{'; LEFT CURLY BRACKET
...
2167    'VIII'; ROMAN NUMERAL EIGHT
2168    'IX'; ROMAN NUMERAL NINE
...
265E    '♞'; BLACK CHESS KNIGHT
265F    '♟'; BLACK CHESS PAWN
...
1F600   '😄'; GRINNING FACE
1F609   '😏'; WINKING FACE
...
```

code point values integer in the range
0 to 0x10FFFF (about 1.1 million values)

notation U + 265E \rightsquigarrow 0x265e (9822 in decimal).

Unicode code points are **encoded** in binary format, below shown in exadecimal system:

P				y				t				h				o				n			
0x50	00	00	00	79	00	00	00	74	00	00	00	68	00	00	00	6f	00	00	00	6e	00	00	00
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

(little endian order)

The standard encoding is UTF-8 that uses a variable number of bytes, 1, 2, 3 or 4.

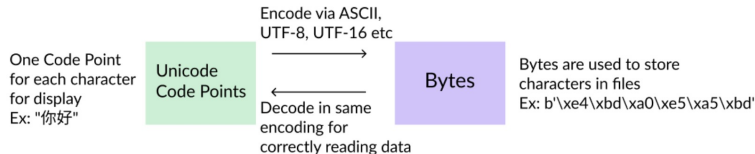
Displaying the correct glyph from unicode is generally the job of a GUI toolkit or a terminal's font renderer.

Python and Unicode

Encode: from unicode code points to Bytes

Decode: from Bytes to unicode code points

```
>>> "\u0394" # or \U00000394
Δ # if GUI cannot visualize it then \u0394
>>> "\N{GREEK CAPITAL LETTER DELTA}"
Δ
>>> "\N{GREEK CAPITAL LETTER DELTA}".encode("utf-8") # str.encode()
b'\xce\x94'
>>> b'\xce\x94'.decode("utf-8") # bytes.decode()
Δ
```



By default Python uses UTF-8 encoding, so this works:

```
>>> øv = 3  
... øv
```

In Python 3, `str` internal representation for strings is unicode code points

```
>>> word = "øvelse" # u"øvelse" is obsolete
```

`ord` integer representing the Unicode code point of that character. `chr` the inverse.

```
>>> s = "a\xac\u1234\u20ac\u00008000"
... #      ^^^^ two-digit hex escape
... #      ^^^^^ four-digit Unicode escape
... #      ^^^^^^^^^ eight-digit Unicode escape
>>> [ord(c) for c in s]
[97, 172, 4660, 8364, 32768] # the code point
```

```
>>> u = chr(40960) + 'abcd' + chr(1972)
>>> u.encode('utf-8')
b'\xea\x80\x80abcd\xde\xb4'
```


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Numpy

Module implementing multi-dimensional vectors useful for applied and computational mathematics.

```
>>> import numpy as np

# Create a 1-D array by passing a list into NumPy's array() function.
>>> np.array([8, 4, 6, 0, 2])
array([8, 4, 6, 0, 2])
```

ndarray object. Each dimension is called an **axis**: For a 2-D array, the 0-axis indexes the rows and the 1-axis indexes the columns.

```
# Create a 2-D array by passing a list of ←
# lists into array().
>>> A = np.array( [ [1, 2, 3], [4, 5, 6] ] )
>>> print(A)
[[1 2 3]
 [4 5 6]]
```

```
# Access to elements:
>>> print(A[0, 1], A[1, 2])
2 6

# Elements of a 2D array are 1D ←
# arrays.
>>> A[0]
array([1, 2, 3])
```

Basic Array Operations

Operators + and * for built-in lists (and strings too) :

```
# Addition concatenates lists together.
```

```
>>> [1, 2, 3] + [4, 5, 6]
```

```
[1, 2, 3, 4, 5, 6]
```

```
# Multiplication concatenates a list with itself a given number of times.
```

```
>>> [1, 2, 3] * 4
```

```
[1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3]
```

Basic Array Operations

```
>>> x, y = np.array([1, 2, 3]), np.array([4, 5, 6])

# Addition or multiplication by a scalar acts on each element of the array.
>>> x + 10                                # Add 10 to each entry of x.
array([11, 12, 13])

>>> x * 4                                # Multiply each entry of x by 4.
array([ 4,  8, 12])

# Add two arrays together (component-wise).
>>> x + y
array([5, 7, 9])

# Multiply two arrays together (component-wise).
>>> x * y
array([ 4, 10, 18])
```

Array Attributes

An ndarray object has several attributes, some of which are listed below.

Attribute	Description
<code>dtype</code>	The type of the elements in the array.
<code>ndim</code>	The number of axes (dimensions) of the array.
<code>shape</code>	A tuple of integers indicating the size in each dimension.
<code>size</code>	The total number of elements in the array.

Data Types

All elements of a NumPy array must have the same data type!!

Data type	Description
bool_	Boolean
int8	8-bit integer
int16	16-bit integer
int32	32-bit integer
int64	64-bit integer
uint8	Unsigned 8-bit integer
uint16	Unsigned 16-bit integer
uint32	Unsigned 32-bit integer
uint64	Unsigned 64-bit integer
float16	Half-precision float
float32	Single-precision float
float64	Double-precision float (default type for most computations)
complex64	Complex number represented by two single-precision floats
complex128	Complex number represented by two double-precision floats

Change Data Types

To change an existing array's type, use the array's `astype()` method.

```
# A list of integers becomes an array of integers.
>>> x = np.array([0, 1, 2, 3, 4])
>>> print(x)
[0 1 2 3 4]
>>> x.dtype
dtype('int64')
```



```
# Change the data type to one of NumPy's float types.
>>> x = x.astype(np.float64)
>>> print(x)
[ 0.  1.  2.  3.  4.]
>>> x.dtype
dtype('float64')
```

Array Creation Routines

Function	Returns
<code>arange()</code>	Array of sequential integers (like <code>list(range())</code>).
<code>linspace()</code>	Array of given shape and type of evenly spaced numbers over a specified interval.
<code>empty()</code>	Array of given shape and type, without initializing entries.
<code>identity()</code>	The square identity array.
<code>eye()</code>	2-D array with ones on the diagonal and zeros elsewhere.
<code>ones()</code>	Array of given shape and type, filled with ones.
<code>ones_like()</code>	Array of ones with the same shape and type as a given array.
<code>zeros()</code>	Array of given shape and type, filled with zeros.
<code>zeros_like()</code>	Array of zeros with the same shape and type as a given array.
<code>full()</code>	Array of given shape and type, filled with a specified value.
<code>full_like()</code>	Full array with the same shape and type as a given array.

Each of these functions accepts the keyword argument `dtype` to specify the data type. Common types include `np.bool_`, `np.int64`, `np.float64`, and `np.complex128`.

Array Creation Routines

```
>>> np.arange(3,7,2) # np.arange(start, stop, step)
array([3, 5])
```

```
>>> np.linspace(2.0, 3.0, num=5) # np.linspace(start, stop, num)
array([2.  , 2.25, 2.5  , 2.75, 3.  ])
```

Array Creation Routines

```
# A 1-D array of 5 zeros.
```

```
>>> np.zeros(5)
array([ 0.,  0.,  0.,  0.,  0.] )
```

```
# A 2x5 matrix (2-D array) of integer ones.
```

```
>>> np.ones((2,5), dtype=np.int)      # The shape is specified as a tuple.
array([[1, 1, 1, 1, 1],
       [1, 1, 1, 1, 1]])
```

```
# The 2x2 identity matrix.
```

```
>>> I = np.eye(2)
>>> print(I)
[[ 1.  0.]
 [ 0.  1.]]
```

```
# Array of 3s the same size as 'I'.
```

```
>>> np.full_like(I, 3)                # Equivalent to np.full(I.shape, 3).
array([[ 3.,  3.],
       [ 3.,  3.]])
```

Array Creation Routines

Function	Description
<code>diag()</code>	Extract a diagonal or construct a diagonal array.
<code>tril()</code>	Get the lower-triangular portion of an array by replacing entries above the diagonal with zeros.
<code>triu()</code>	Get the upper-triangular portion of an array by replacing entries below the diagonal with zeros.

```
>>> A = np.array([[1, 2, 3],  
                  [4, 5, 6],  
                  [7, 8, 9]])  
  
# Get only the upper triangular ←  
  entries of 'A'.  
>>> np.triu(A)  
array([[1, 2, 3],  
       [0, 5, 6],  
       [0, 0, 9]])
```

```
# Get the diagonal entries of 'A' as a ←  
  1-D array.  
>>> np.diag(A)  
array([1, 5, 9])  
  
# diag() can also create a diagonal ←  
  matrix from a 1-D array.  
>>> np.diag([1, 11, 111])  
array([[ 1,  0,  0],  
       [ 0, 11,  0],  
       [ 0,  0, 111]])
```

Random Sampling

Similar to standard library's 'Random' module but more efficient

Function	Description
<code>choice()</code>	Take random samples from a 1-D array.
<code>random()</code>	Uniformly distributed floats over $[0, 1)$.
<code>randint()</code>	Random integers over a half-open interval.
<code>random_integers()</code>	Random integers over a closed interval.
<code>randn()</code>	Sample from the standard normal distribution.
<code>permutation()</code>	Randomly permute a sequence / generate a random sequence.
Function	Distribution
<code>beta()</code>	Beta distribution over $[0, 1]$.
<code>binomial()</code>	Binomial distribution.
<code>exponential()</code>	Exponential distribution.
<code>gamma()</code>	Gamma distribution.
<code>geometric()</code>	Geometric distribution.
<code>multinomial()</code>	Multivariate generalization of the binomial distribution.
<code>multivariate_normal()</code>	Multivariate generalization of the normal distribution.
<code>normal()</code>	Normal / Gaussian distribution.
<code>poisson()</code>	Poisson distribution.
<code>uniform()</code>	Uniform distribution.

Random Sampling

```
# 5 uniformly distributed values in the interval [0, 1).  
>>> np.random.random(5)  
array([ 0.21845499,  0.73352537,  0.28064456,  0.66878454,  0.44138609])  
  
# A 2x5 matrix (2-D array) of integers in the interval [10, 20).  
>>> np.random.randint(10, 20, (2,5))  
array([[17, 12, 13, 13, 18],  
       [16, 10, 12, 18, 12]])
```

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Array Slicing

$$A[0] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

$$A[2,1] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

Array Slicing

$$A[1] = A[1,:] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

$$A[:,2] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

$$A[1:,:2] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

$$A[1:-1,1:-1] = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}$$

Array Slicing

```
>>> x = np.arange(10); x
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
>>> x[3]                                     # The element at index 3.
3
>>> x[:3]                                   # Everything up to index 3 (exclusive).
array([0, 1, 2])
>>> x[3:]                                   # Everything from index 3 on.
array([3, 4, 5, 6, 7, 8, 9])
>>> x[3:8]                                  # The elements from index 3 to 8.
array([3, 4, 5, 6, 7])
>>> A = np.array([[0,1,2,3,4],[5,6,7,8,9]])
>>> A
array([[0, 1, 2, 3, 4],
       [5, 6, 7, 8, 9]])
>>> A[1, 2]                                 # The element at row 1, column 2.
7
>>> A[:, 2:]                                # All of the rows, from column 2 on.
array([[2, 3, 4],
       [7, 8, 9]])
```

Array Slicing

- Indexing and slicing operations return a **view** of the array.
- Changing a view of an array also changes the original array.
- That is, **arrays are mutable**.
- To create a copy of an array, use `np.copy()` or the array's `copy()` method.
- Changes to a copy of an array does not affect the original array
- Copying is less efficient than getting a view.

Fancy Indexing

Via either an array of indices or an array of boolean values (**mask**) to extract specific elements.

```
>>> x = np.arange(0, 50, 10)          # The integers from 0 to 50 by tens.
>>> x
array([ 0, 10, 20, 30, 40])

# An array of integers extracts the entries of 'x' at the given indices.
>>> index = np.array([3, 1, 4])       # Get the 3rd, 1st, and 4th elements.
>>> x[index]                          # Same as np.array([x[i] for i in index]).
array([30, 10, 40])

# A boolean array extracts the elements of 'x' at the same places as 'True'.
>>> mask = np.array([True, False, False, True, False])
>>> x[mask]                           # Get the 0th and 3rd entries.
array([ 0, 30])
```

Fancy Indexing

```
>>> y = np.arange(10, 20, 2)          # Every other integers from 10 to 20.
>>> y
array([10, 12, 14, 16, 18])

# Extract the values of 'y' larger than 15.
>>> mask = y > 15                     # Same as np.array([i > 15 for i in y]).
>>> mask
array([False, False, False,  True,  True], dtype=bool)
>>> y[mask]                           # Same as y[y > 15]
array([16, 18])

# Change the values of 'y' that are larger than 15 to 100.
>>> y[mask] = 100
>>> print(y)
[10 12 14 100 100]
```

Shaping

```
>>> A = np.arange(12)                # The integers from 0 to 12 (exclusive).
>>> print(A)
[ 0  1  2  3  4  5  6  7  8  9 10 11]

# 'A' has 12 entries, so it can be reshaped into a 3x4 matrix.
>>> A.reshape((3,4))                # The new shape is specified as a tuple.
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])

# Reshape 'A' into an array with 2 rows and the appropriate number of columns.
>>> A.reshape((2,-1))
array([[ 0,  1,  2,  3,  4,  5],
       [ 6,  7,  8,  9, 10, 11]])
```

Shaping

```
>>> A = np.arange(12).reshape((3,4))
>>> A
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11]])

# Flatten 'A' into a one-dimensional array.
>>> np.ravel(A)                                # Equivalent to A.reshape(A.size)
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11])

# Transpose the matrix 'A'.
>>> A.T                                         # Equivalent to np.transpose(A).
array([[ 0,  4,  8],
       [ 1,  5,  9],
       [ 2,  6, 10],
       [ 3,  7, 11]])
```

- Caution: reshape is just a view on the array, it does not copy the data. Beware: the following does [shallow copy](#):

```
>>> N=A
>>> N[0,0] = 0 # now A[0,0] has become 0
>>> M=A.reshape(1,9)
>>> M[0,7]=10 # now A[2,1] has become 10
>>> A
array([[ 0.,  2.,  3.],
       [ 4.,  5.,  6.],
       [ 7., 10.,  9.]])
```

- We can see the base object by:

```
>>> print(A.base)
None
>>> print(M.base) # the base object of M is the matrix A
[[ 0.  2.  3.]
 [ 4.  5.  6.]
 [ 7. 10.  9.]])
```

- To implement a [deep copy](#) you must use `np.copy(A)`

Note on Vector Shape

- By default, all NumPy 1D arrays (including column slices) are automatically reshaped into “flat” (ie, row) 1D arrays.
- This contrasts with mathematical notation where vectors are represented vertically
- NumPy methods such as `dot()` are implemented to purposefully work well with 1D “row arrays”.
- `np.transpose()` does not alter 1D arrays.
- Do not force a 1D vector to be a column vector unless necessary.
- To force a “column array” use `np.reshape()`, `np.vstack()`

Function	Description
<code>concatenate()</code>	Join a sequence of arrays along an existing axis
<code>hstack()</code>	Stack arrays in sequence horizontally (column wise).
<code>vstack()</code>	Stack arrays in sequence vertically (row wise).
<code>column_stack()</code>	Stack 1-D arrays as columns into a 2-D array.
<code>row_stack()</code>	Stack 1-D arrays as rows into a 2-D array.

Stacking

```
>>> A = np.arange(6).reshape((2,3))
>>> B = np.zeros((4,3))

>>> np.vstack((A,B,A)) # same as np.concatenate([A,B,A],axis=0) and row_stack()
array([[ 0.,  1.,  2.],           # A
       [ 3.,  4.,  5.],
       [ 0.,  0.,  0.],           # B
       [ 0.,  0.,  0.],
       [ 0.,  0.,  0.],
       [ 0.,  0.,  0.],
       [ 0.,  1.,  2.],           # A
       [ 3.,  4.,  5.]])
```

```
>>> A = A.T
>>> B = np.ones((3,4))

# hstack() # same as np.concatenate([A,B,A],axis=1) and column_stack()
>>> np.hstack((A,B,A))
array([[ 0.,  3.,  1.,  1.,  1.,  1.,  0.,  3.],
       [ 1.,  4.,  1.,  1.,  1.,  1.,  1.,  4.],
       [ 2.,  5.,  1.,  1.,  1.,  1.,  2.,  5.]])
```

You need to use `column_stack()` to stack arrays horizontally 1-D arrays.

```
>>> np.column_stack((A, np.zeros(3), np.ones(3), np.full(3, 2)))
array([[ 0.,  3.,  0.,  1.,  2.],
       [ 1.,  4.,  0.,  1.,  2.],
       [ 2.,  5.,  0.,  1.,  2.]])
```

<http://docs.scipy.org/doc/numpy-1.10.1/reference/routines.array-manipulation.html>

Broadcasting

NumPy tries to automatically align arrays for component-wise operations whenever possible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

$$x = [10 \ 20 \ 30]$$

$$A + x = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 10 & 20 & 30 \end{bmatrix} = \begin{bmatrix} 11 & 22 & 33 \\ 11 & 22 & 33 \\ 11 & 22 & 33 \end{bmatrix}$$

$$A + x.\text{reshape}((3,-1)) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = \begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}$$

Outline

Exception Handling
File Input and Output
Numpy

1. Exception Handling

2. File Input and Output

3. Numpy

Data Access

Numerical Computing with NumPy

Universal Functions

Universal function: operates on an entire array element-wise. More efficient than looping.

Function	Description
<code>«abs()»</code> or <code>absolute()</code>	Calculate the absolute value element-wise.
<code>exp()</code> / <code>log()</code>	Exponential (e^x) / natural log element-wise.
<code>maximum()</code> / <code>minimum()</code>	Element-wise maximum / minimum of two arrays.
<code>sqrt()</code>	The positive square-root, element-wise.
<code>sin()</code> , <code>cos()</code> , <code>tan()</code> , etc.	Element-wise trigonometric operations.

```
>>> x = np.arange(-2,3)
>>> print(x, np.abs(x))           # Like np.array([abs(i) for i in x]).
[-2 -1  0  1  2] [2 1 0 1 2]

>>> np.sin(x)                    # Like np.array([math.sin(i) for i in x]).
array([-0.90929743, -0.84147098,  0.          ,  0.84147098,  0.90929743])
```

Universal Functions

- Some functions from the module `math` do not work with array.
- It is possible to make them element-wise via `vectorize`.

```
>>> def f(x):  
    return 0 if x<=5 else 1  
# f(A) # error  
>>> np.vectorize(f)(A)  
array([[0, 0, 0],  
       [0, 0, 1],  
       [1, 1, 1]])
```

Operations along an Axis

Most array methods have an `axis` argument that allows an operation to be done along a given axis.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$A.\text{sum}(\text{axis}=0) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} = [4 \ 8 \ 12 \ 16]$$

$$A.\text{sum}(\text{axis}=1) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} = [10 \ 10 \ 10 \ 10]$$

Summary

1. Exception Handling

2. File Input and Output

3. Numpy

Data Access

Numerical Computing with NumPy