

DM865 – Spring 2018
Heuristics and Approximation Algorithms

Experimental Analysis

Marco Chiarandini

Department of Mathematics & Computer Science
University of Southern Denmark

Outline

1. Experimental Analysis

- Motivations and Goals

- Descriptive Statistics

 - Performance Measures

 - Sample Statistics

- Scenarios of Analysis

 - A. Single-pass heuristics

 - B. Asymptotic heuristics

- Guidelines for Presenting Data

Outline

1. Experimental Analysis

- Motivations and Goals

- Descriptive Statistics

 - Performance Measures

 - Sample Statistics

- Scenarios of Analysis

 - A. Single-pass heuristics

 - B. Asymptotic heuristics

- Guidelines for Presenting Data

Contents and Goals

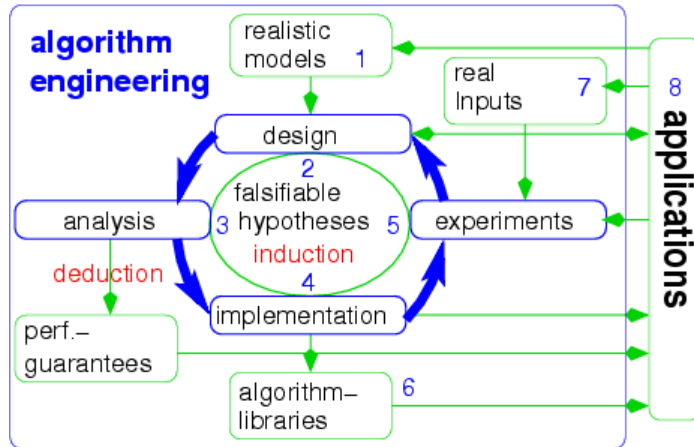
Provide a view of issues in [Experimental Algorithmics](#)

- [Exploratory data analysis](#)
- Presenting results in a concise way with graphs and tables
- Organizational issues and Experimental Design
- Basics of [inferential statistics](#)
- Sequential statistical testing: race, a methodology for tuning

The goal of Experimental Algorithmics is not only producing a sound [analysis](#) but also adding an important tool to the development of a good solver for a given problem.

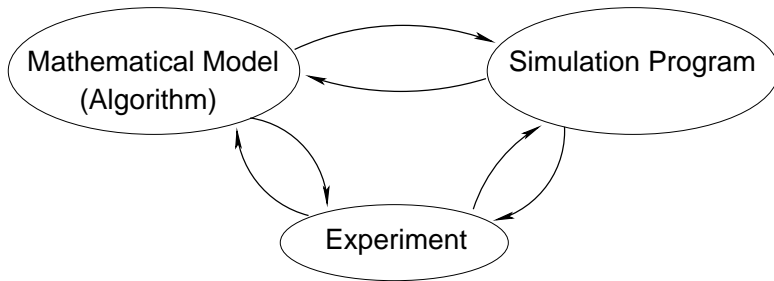
Experimental Algorithmics is an important part in the algorithm production cycle, which is referred to as [Algorithm Engineering](#)

The Engineering Cycle



from <http://www.algorithm-engineering.de/>

Experimental Algorithmics



In empirical studies we consider simulation programs which are the implementation of a mathematical model (the algorithm)

[McGeoch, 1996]

Experimental Algorithmics

Goals

- Defining standard **methodologies**
- Comparing **relative performance** of algorithms so as to identify the best ones for a given application
- **Characterizing** the behavior of algorithms
- Identifying **algorithm separators**, *i.e.*, families of problem instances for which the performance differ
- Providing new insights in **algorithm design**

Fairness Principle

Fairness principle: being completely fair is perhaps impossible but try to remove any possible bias:

- possibly all algorithms must be implemented with the **same style**, with the **same language** and **sharing common subprocedures and data structures**
- the code must be **optimized**, e.g., using the best possible data structures
- running times must be comparable, e.g., by running experiments on the **same computational environment** (or redistributing them randomly)

Definitions

The most typical scenario considered in analysis of search heuristics

Asymptotic heuristics with time/quality limit decided *a priori*

The algorithm \mathcal{A}^∞ is halted when time expires or a solution of a given quality is found.

Deterministic case: \mathcal{A}^∞ on π returns a solution of cost x .

The performance of \mathcal{A}^∞ on π is a scalar $y = x$.

Randomized case: \mathcal{A}^∞ on π returns a solution of cost X , where X is a random variable.

The performance of \mathcal{A}^∞ on π is the univariate $Y = X$.

[This is not the only relevant scenario: to be refined later]

Random Variables and Probability

Statistics deals with random (or stochastic) variables.

A variable is called random if, prior to observation, its outcome cannot be predicted with certainty.

The uncertainty is described by a probability distribution.

Discrete variables

Probability distribution:

$$p_i = P[x = v_i]$$

Cumulative Distribution Function (CDF)

$$F(v) = P[x \leq v] = \sum_i p_i$$

Mean

$$\mu = E[X] = \sum x_i p_i$$

Variance

$$\sigma^2 = E[(X - \mu)^2] = \sum (x_i - \mu)^2 p_i$$

Continuous variables

Probability density function (pdf):

$$f(v) = \frac{dF(v)}{dv}$$

Cumulative Distribution Function (CDF):

$$F(v) = \int_{-\infty}^v f(v) dv$$

Mean

$$\mu = E[X] = \int x f(x) dx$$

Variance

$$\sigma^2 = E[(X - \mu)^2] = \int (x - \mu)^2 f(x) dx$$

Generalization

For each general problem \mathcal{P} (e.g., TSP, GCP) we denote by Π a set (or class) of instances and by $\pi \in \Pi$ a single instance.

On a specific instance, the random variable Y that defines the performance measure of an algorithm is described by its probability distribution/density function

$$Pr(Y = y \mid \pi)$$

It is often more interesting to generalize the performance on a class of instances Π , that is,

$$Pr(Y = y, \Pi) = \sum_{\pi \in \Pi} Pr(Y = y \mid \pi) Pr(\pi)$$

Sampling

In experiments,

1. we sample the population of instances and
2. we sample the performance of the algorithm on each sampled instance

If on an instance π we run the algorithm r times then we have r replicates of the performance measure Y , denoted Y_1, \dots, Y_r , which are independent and identically distributed (i.i.d.), i.e.

$$Pr(y_1, \dots, y_r | \pi) = \prod_{j=1}^r Pr(y_j | \pi)$$

$$Pr(y_1, \dots, y_r) = \sum_{\pi \in \Pi} Pr(y_1, \dots, y_r | \pi) Pr(\pi).$$

Instance Selection

In **real-life applications** a simulation of $p(\pi)$ can be obtained by historical data.

In **simulation studies** instances may be:

- real world instances
- random variants of real world-instances
- online libraries
- randomly generated instances

They may be grouped in classes according to some features whose impact may be worth studying:

- type (for features that might impact performance)
- size (for scaling studies)
- hardness (focus on hard instances)
- application (e.g., CSP encodings of scheduling problems), ...

Within the class, instances are drawn with uniform probability $p(\pi) = c$

Statistical Methods

The analysis of performance is based on finite-size sampled data. Statistics provides the methods and the mathematical basis to

- describe, summarizing, the data (descriptive statistics)
- make inference on those data (inferential statistics)

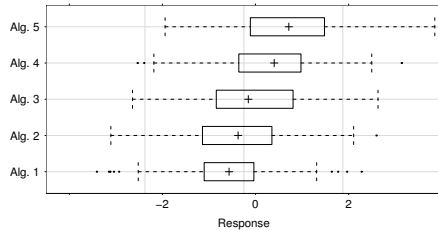
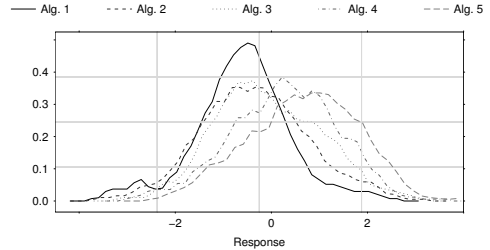
Statistics helps to

- guarantee reproducibility
- make results reliable
(are the observed results enough to justify the claims?)
- extract relevant results from large amount of data

In the **practical context** of heuristic design and implementation (i.e., **engineering**), statistics helps to take correct design decisions with the **least amount of experimentation**

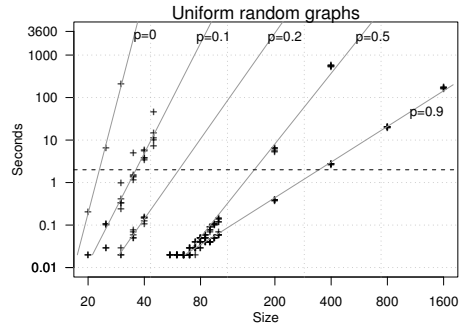
Objectives of the Experiments

- **Comparison:**
bigger/smaller, same/different, Algorithm Configuration, Component-Based Analysis
 - Standard statistical methods: *experimental designs, test hypothesis and estimation*



Objectives of the Experiments

- **Comparison:**
bigger/smaller, same/different, Algorithm Configuration, Component-Based Analysis
 - Standard statistical methods: *experimental designs, test hypothesis and estimation*
- **Characterization:**
Interpolation: fitting models to data
Extrapolation: building models of data, explaining phenomena
 - Standard statistical methods: *linear and non linear regression*
model fitting



Measures and Transformations

On a single instance

Design: Several runs on an instance

	Algorithm 1	Algorithm 2	...	Algorithm k
Instance 1	x_{11}	x_{21}		x_{k1}
\vdots	\vdots	\vdots		\vdots
Instance r	x_{1r}	x_{2r}		x_{kr}

Measures and Transformations

On a single instance

Computational effort indicators

- number of elementary operations/algorithmic iterations
(e.g., search steps, objective function evaluations, number of visited nodes in the search tree, consistency checks, etc.)
- total CPU time consumed by the process
(sum of *user* and *system* times returned by `getrusage`)

Solution quality indicators

- value returned by the cost function
- error from optimum/reference value
- (optimality) gap $\frac{UB-LB}{LB+\epsilon}$ (if $\max \frac{UB-LB}{UB+\epsilon}$)
 ϵ is an infinitesimal for the case $LB = 0$ but $UB - LB \neq 0$
- ranks

Measures and Transformations

On a class of instances

Design A: One run on various instances

	Algorithm 1	Algorithm 2	...	Algorithm k
Instance 1	X_{11}	X_{12}		X_{1k}
\vdots	\vdots	\vdots		\vdots
Instance b	X_{b1}	X_{b2}		X_{bk}

Design B: Several runs on various instances

	Algorithm 1	Algorithm 2	...	Algorithm k
Instance 1	X_{111}, \dots, X_{11r}	X_{121}, \dots, X_{12r}		X_{1k1}, \dots, X_{1kr}
Instance 2	X_{211}, \dots, X_{21r}	X_{221}, \dots, X_{22r}		X_{2k1}, \dots, X_{2kr}
\vdots	\vdots	\vdots		\vdots
Instance b	X_{b11}, \dots, X_{b1r}	X_{b21}, \dots, X_{b2r}		X_{bk1}, \dots, X_{bkr}

Measures and Transformations

On a class of instances

Computational effort indicators

- no transformation if the interest is in studying scaling
- standardization if a fixed time limit is used
- geometric mean (used for a set of numbers whose values are meant to be multiplied together or are exponential in nature),
- otherwise, better to group homogeneously the instances

Solution quality indicators

Different instances imply different scales \Rightarrow need for an invariant measure

(However, many other measures can be taken both on the algorithms and on the instances
[McGeoch, 1996])

On a class of instances (cont.)

Solution quality indicators

- Distance or error from a reference value (assume minimization case):

$$e_1(x, \pi) = \frac{x(\pi) - \bar{x}(\pi)}{\hat{\sigma}(\pi)} \quad \text{standard score}$$

$$e_2(x, \pi) = \frac{x(\pi) - x^{opt}(\pi)}{x^{opt}(\pi)} \quad \text{relative error}$$

$$e_3(x, \pi) = \frac{x(\pi) - x^{opt}(\pi)}{x^{worst}(\pi) - x^{opt}(\pi)} \quad \text{invariant [Zemel, 1981]}$$

- optimal value computed exactly or known by construction
- surrogate value such bounds or best known values
- Rank (no need for standardization but loss of information)

Sampling

- We work with samples (instances, solution quality) drawn from populations



Summary Measures

Measures to describe or characterize a population

- Measure of central tendency, location
- Measure of dispersion

One such a quantity is

- a **parameter** if it refers to the population (Greek letters)
- a **statistics** if it is an *estimation* of a population parameter from the sample (Latin letters)

Measures of central tendency

- Arithmetic Average (Sample mean)

$$\bar{X} = \frac{\sum x_i}{n}$$

- *Quantile*: value above or below which lie a fractional part of the data (used in nonparametric statistics)
 - Median

$$\mathcal{M} = x_{(n+1)/2}$$

- Quartile

$$Q_1 = x_{(n+1)/4} \quad Q_3 = x_{3(n+1)/4}$$

- q -quantile

q of data lies below and $1 - q$ lies above

- Mode

value of relatively great concentration of data
(*Unimodal* vs *Multimodal* distributions)

Measure of dispersion

- Sample range

$$R = x_{(n)} - x_{(1)}$$

- Sample variance

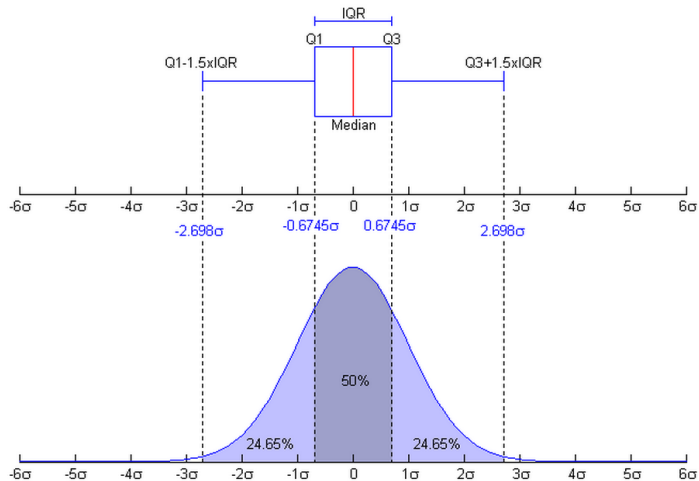
$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{X})^2$$

- Standard deviation

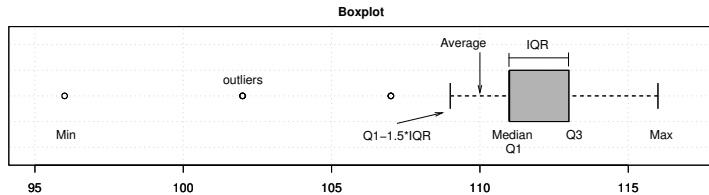
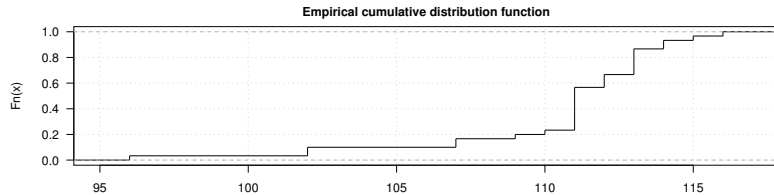
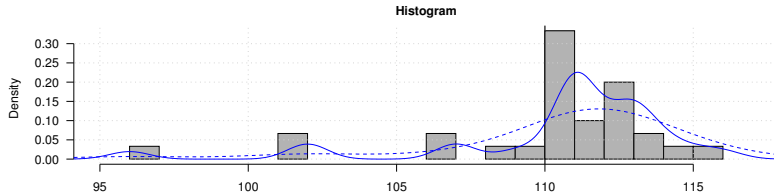
$$s = \sqrt{s^2}$$

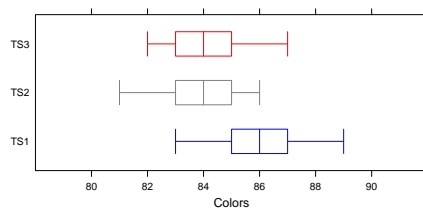
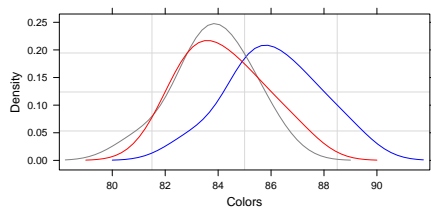
- Inter-quartile range

$$IQR = Q_3 - Q_1$$



Boxplot and a probability density function (pdf) of a Normal $N(0,1)$ Population. (source: Wikipedia)
[see also: <http://informationandvisualization.de/blog/box-plot>]





In R

```
> x<-runif(10,0,1)
  mean(x), median(x), quantile(x), quantile(x,0.25)
  range(x), var(x), sd(x), IQR(x)
> fivenum(x)
#(minimum, lower-hinge, median, upper-hinge, maximum)
[1] 0.18672 0.26682 0.28927 0.69359 0.92343
> summary(x)
> aggregate(x,list(factors),median)
> boxplot(x)
```

Scenarios

- A. Single-pass heuristics
- B. Asymptotic heuristics
(can be run indefinitely with a chance of continuing to make progress):

Two approaches:

- 1. Univariate
 - 1.a Time as an external parameter decided *a priori*
 - 1.b Solution quality as an external parameter decided *a priori*
- 2. Cost dependent on running time:

Scenario A

Single-pass heuristics

Deterministic case: \mathcal{A}^\dagger on class Π returns a solution of cost x with computational effort t (e.g., running time).

The performance of \mathcal{A}^\dagger on class Π is the vector $\vec{y} = (x, t)$.

Randomized case: \mathcal{A}^\dagger on class Π returns a solution of cost X with computational effort T , where X and T are random variables.

The performance of \mathcal{A}^\dagger on class Π is the bivariate $\vec{Y} = (X, T)$.

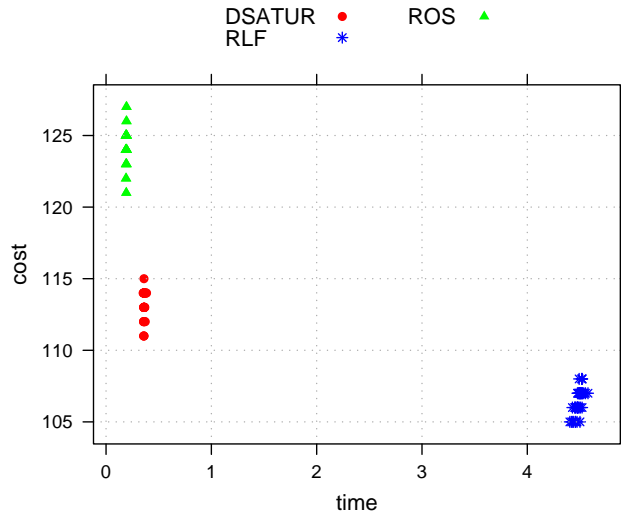
Example

Scenario:

- ▷ 3 heuristics \mathcal{A}_1^+ , \mathcal{A}_2^+ , \mathcal{A}_3^+ on class Π .
- ▷ homogeneous instances or need for data transformation.
- ▷ 1 or r runs per instance
- ▶ **Interest:** inspecting solution cost and running time to observe and compare the level of approximation and the speed.

Tools:

- Scatter plots of solution-cost and run-time



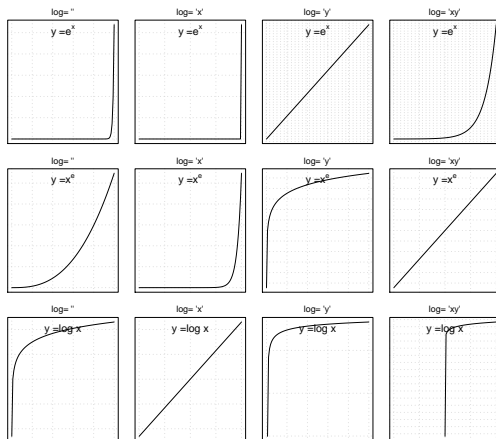
Multi-Criteria Decision Making

Needed some definitions on **dominance relations**

In **Pareto sense**, for points in \mathbf{R}^2

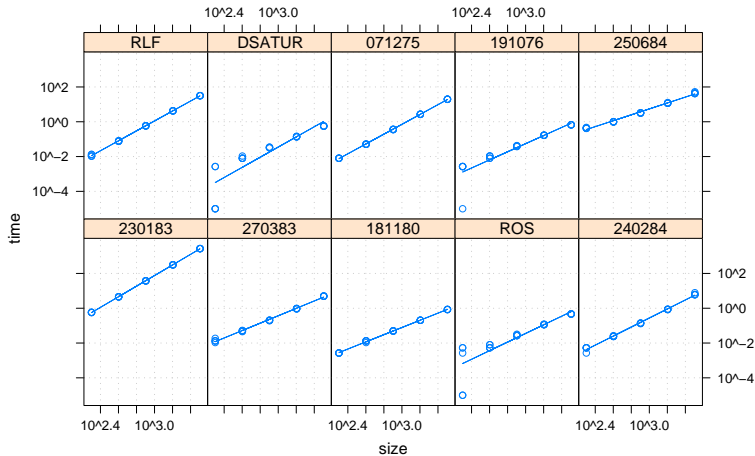
$\vec{x}^1 \preceq \vec{x}^2$	weakly dominates	$x_i^1 \leq x_i^2$ for all $i = 1, \dots, n$
$\vec{x}^1 \parallel \vec{x}^2$	incomparable	neither $\vec{x}^1 \preceq \vec{x}^2$ nor $\vec{x}^2 \preceq \vec{x}^1$

Scaling Analysis

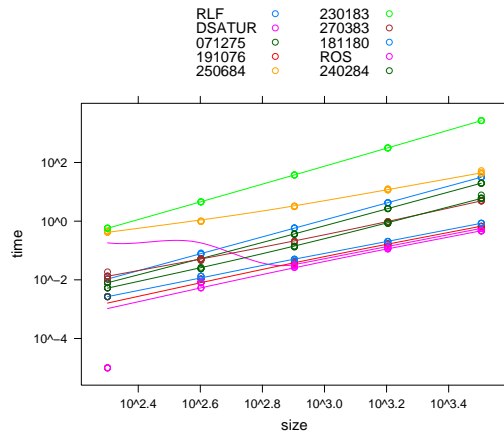


Linear regression in log-log plots \Rightarrow polynomial growth

Linear regression in log-log plots \Rightarrow polynomial growth



Comparative visualization



Scenarios

- A. Single-pass heuristics
- B. Asymptotic heuristics
(can be run indefinitely with a chance of continuing to make progress):

Two approaches:

- 1. Univariate
 - 1.a Time as an external parameter decided *a priori*
 - 1.b Solution quality as an external parameter decided *a priori*
- 2. Cost dependent on running time:

Scenario B

Asymptotic heuristics

There are two approaches:

- 1.a. Time as an external parameter decided *a priori*.

The algorithm is halted when time expires.

Deterministic case: \mathcal{A}^∞ on class Π returns a solution of cost x .

The performance of \mathcal{A}^∞ on class Π is the scalar $y = x$.

Randomized case: \mathcal{A}^∞ on class Π returns a solution of cost X , where X is a random variable.

The performance of \mathcal{A}^∞ on class Π is the univariate $Y = X$.

Example

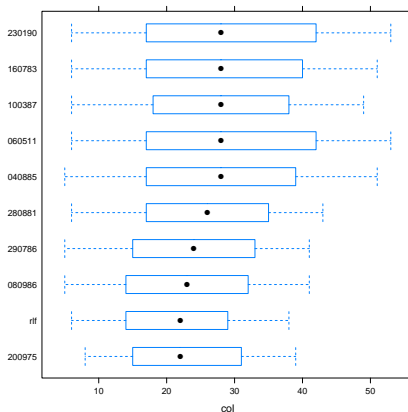
Scenario:

- ▷ 3 heuristics \mathcal{A}_1^∞ , \mathcal{A}_2^∞ , \mathcal{A}_3^∞ on class Π .
(Or 3 heuristics \mathcal{A}_1^∞ , \mathcal{A}_2^∞ , \mathcal{A}_3^∞ on class Π without interest in computation time because negligible or comparable)
- ▷ homogeneous instances (no data transformation) or heterogeneous (data transformation)
- ▷ 1 or r runs per instance
- ▷ a priori time limit imposed
- ▶ **Interest:** inspecting solution cost

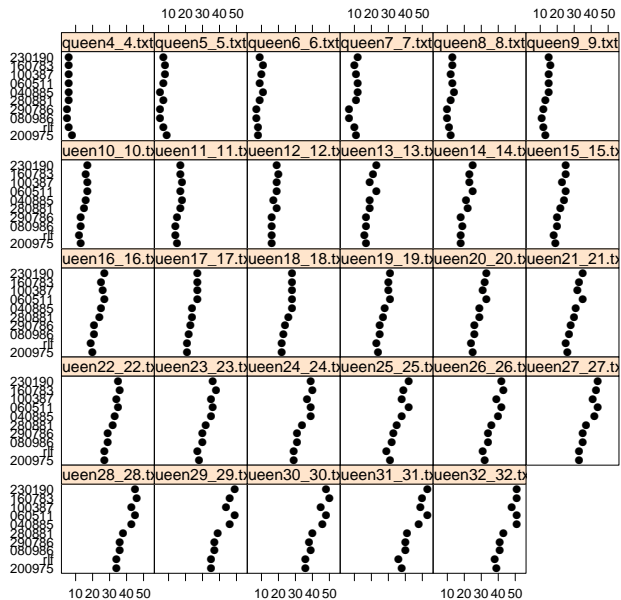
Tools:

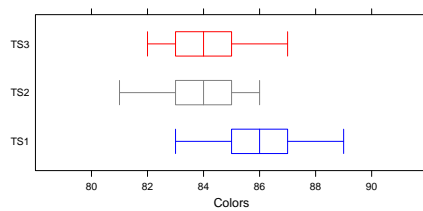
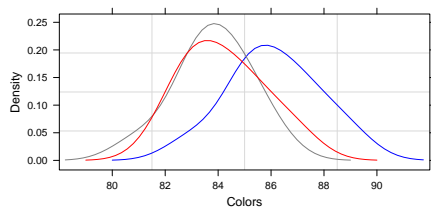
- Histograms (summary measures: mean or median or mode?)
- Boxplots
- Empirical cumulative distribution functions (ECDFs)


```
## load the data
> load("results.rda")
> levels(DATAS$instance)
[1] "queen4_4.txt" "queen5_5.txt" "queen6_6.txt" "queen7_7.txt"
[5] "queen8_8.txt" "queen9_9.txt" "queen10_10.txt" "queen11_11.txt"
[9] "queen12_12.txt" "queen13_13.txt" "queen14_14.txt" "queen15_15.txt"
[13] "queen16_16.txt" "queen17_17.txt" "queen18_18.txt" "queen19_19.txt"
[17] "queen20_20.txt" "queen21_21.txt" "queen22_22.txt" "queen23_23.txt"
[21] "queen24_24.txt" "queen25_25.txt" "queen26_26.txt" "queen27_27.txt"
[25] "queen28_28.txt" "queen29_29.txt" "queen30_30.txt" "queen31_31.txt"
[29] "queen32_32.txt"
> bwplot(reorder(alg, col, median)~col, data=DATA)
```



```
> bwplot(reorder(alg, col, median) ~ col | instance, data=DATA, as.table=TRUE)
```

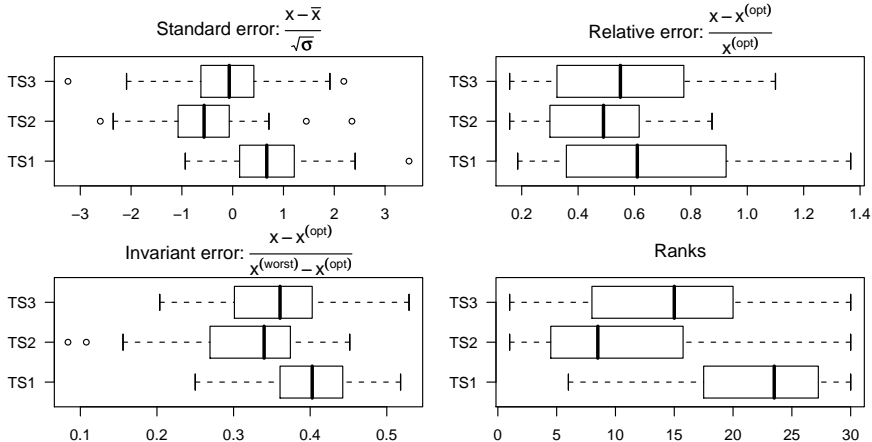




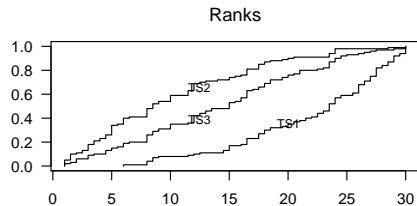
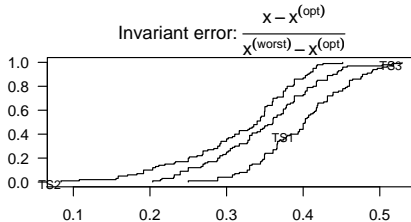
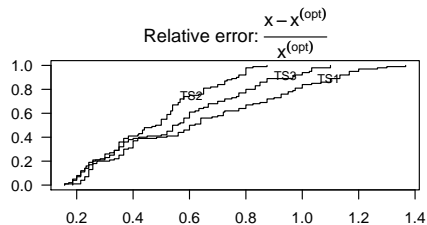
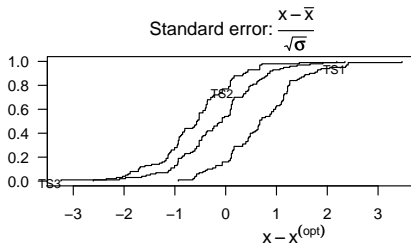
R Pointers

- rstudio cheatsheets <https://www.rstudio.com/resources/cheatsheets/>
 - Data Import Cheat Sheet
 - Data Transformation Cheat Sheet
 - Data Wrangling https://rpubs.com/bradleyboehmke/data_wrangling <https://www.rstudio.com/wp-content/uploads/2015/02/data-wrangling-cheatsheet.pdf>
 - Data Visualization Cheat Sheet
- ggplot2: Elegant Graphics for Data Analysis. Wickham, Hadley
<https://www.springer.com/gp/book/9783319242750> <http://ggplot2.org/book/>
based on grammar:
 - Wickham H (2010) A layered grammar of graphics. J Comput Graph Stat 19(1):3–28
 - Wilkinson L (2005) The grammar of graphics. Statistics and computing, 2nd edn. Springer, New York

On a class of instances



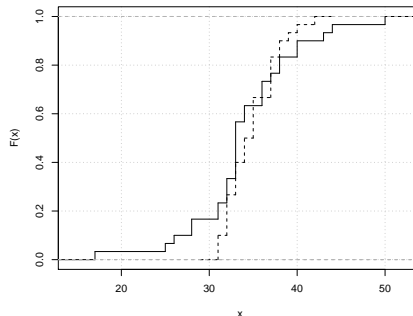
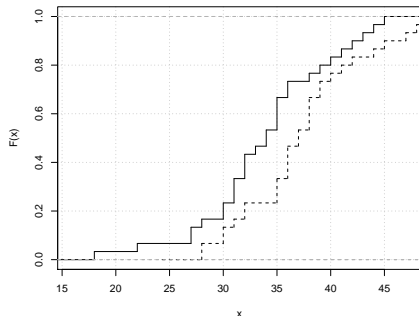
On a class of instances



Stochastic Dominance

Definition: Algorithm \mathcal{A}_1 probabilistically dominates algorithm \mathcal{A}_2 on a problem instance, iff its CDF is always "below" that of \mathcal{A}_2 , i.e.:

$$F_1(x) \leq F_2(x), \quad \forall x \in X$$



R code behind the previous plots

We load the data and plot the comparative boxplot for each instance.

```
> load("TS.class-G.dataR")
> G[1:5,]
  alg          inst run sol time.last.imp tot.it parz.it exit.it exit.time opt
1 TS1 G-1000-0.5-30-1  1  59      9.900619   5955     442     5955  10.02463  30
2 TS1 G-1000-0.5-30-1  2  64      9.736608   3880     130     3958  10.00062  30
3 TS1 G-1000-0.5-30-1  3  64      9.908618   4877      49     4877  10.03263  30
4 TS1 G-1000-0.5-30-1  4  68      9.948622   6996     409     6996  10.07663  30
5 TS1 G-1000-0.5-30-1  5  63      9.912620   3986      52     3986  10.04063  30
>
> library(lattice)
> bwplot(alg ~ sol | inst, data=G)
```

If we want to make an aggregate analysis we have the following choices:

- maintain the raw data,
- transform data in standard error,
- transform the data in relative error,
- transform the data in an invariant error,
- transform the data in ranks.

Maintain the raw data

```
> par(mfrow=c(3,2),las=1,font.main=1,mar=c(2,3,3,1))  
> #original data  
> boxplot(sol~alg,data=G,horizontal=TRUE,main="Original data")
```

Transform data in standard error

```
> #standard error
> T1 <- split(G$sol,list(G$inst))
> T2 <- lapply(T1,scale,center=TRUE,scale=TRUE)
> T3 <- unsplit(T2,list(G$inst))
> T4 <- split(T3,list(G$alg))
> T5 <- stack(T4)
> boxplot(values~ind,data=T5,horizontal=TRUE,main=expression(paste("Standard error: "
> library(latticeExtra)
> ecdfplot(~values,group=ind,data=T5,main=expression(paste("Standard error:
",frac(x-bar(x),sqrt(sigma))))))

> #standard error
> G$scale <- 0
> split(G$scale, G$inst) <- lapply(split(G$sol, G$inst), scale,center=TRUE,scale=TRUE)
```

Transform the data in relative error

```
> #relative error
> G$err2 <- (G$sol-G$opt)/G$opt
> boxplot(err2~alg,data=G,horizontal=TRUE,main=expression(paste("Relative error: ",frac(x-x^(opt),x-x^(opt))))
> ecdfplot(G$err2,group=G$alg,main=expression(paste("Relative error: ",frac(x-x^(opt),x-x^(opt))))
```

Transform the data in an invariant error

We use as surrogate of x^{worst} the median solution returned by the simplest algorithm for the graph coloring, that is, the ROS heuristic.

```
> #error 3
> load("ROS.class-G.dataR")
> F1 <- aggregate(F$sol,list(inst=F$inst),median)
> F2 <- split(F1$x,list(F1$inst))
> G$ref <- sapply(G$inst,function(x) F2[[x]])
> G$err3 <- (G$sol-G$opt)/(G$ref-G$opt)
> boxplot(err3~alg,data=G,horizontal=TRUE,main=expression(paste("Invariant error: ",frac(x-x^(opt),x-x^(ref))))
> ecdfplot(G$err3,group=G$alg,main=expression(paste("Invariant error: ",frac(x-x^(opt),x-x^(ref))))
```

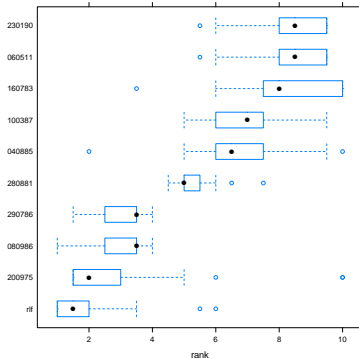
Transform the data in ranks

```
> #rank  
> G$rank <- G$sol  
> split(G$rank, G$inst) <- lapply(split(G$sol, D$inst), rank)  
> bwplot(rank~reorder(alg,rank,median),data=G,horizontal=TRUE,main="Ranks")  
> ecdfplot(rank,group=alg,data=G,main="Ranks")
```

```

> ## Let's make the ranks of the colors
> T1 <- split(DATA["col"], DATA["instance"])
> T2 <- lapply(T1, rank, na.last = "keep")
> T3 <- unsplit(T2, DATA["instance"])
> DATA$rank <- T3
>
> ## we plot the ranks for an aggregate analysis
> ## reorder sort the factor algorithm by median values
> bwplot(reorder(alg, rank, median) ~ rank, data = DATA)

```



Scenarios

A. Single-pass heuristics

B. Asymptotic heuristics

(can be run indefinitely with a chance of continuing to make progress):

Two approaches:

1. Univariate

- 1.a Time as an external parameter decided *a priori*

- 1.b Solution quality as an external parameter decided *a priori*

2. Cost dependent on running time:

Scenario B

Asymptotic heuristics

There are two approaches:

- 1.b. **Solution quality** as an external parameter decided *a priori*. The algorithm is halted when quality is reached.

Deterministic case: \mathcal{A}^∞ on class Π finds a solution in running time t .

The performance of \mathcal{A}^∞ on class Π is the scalar $y = t$.

Randomized case: \mathcal{A}^∞ on class Π finds a solution in running time T , where T is a random variable.

The performance of \mathcal{A}^∞ on class Π is the univariate $Y = T$.

Dealing with Censored Data

Asymptotic heuristics, Approach 1.b

- ▶ Heuristic \mathcal{A}^+ stopped before completion or \mathcal{A}^∞ truncated (always the case)
- ▶ **Interest:** determining whether a prefixed goal (optimal/feasible) has been reached

The computational effort to attain the goal can be specified by a cumulative distribution function $F(t) = P(T < t)$ with T in $[0, \infty)$.

If in a run i we stop the algorithm at time L_i then we have a **Type I right censoring**, that is, we know either

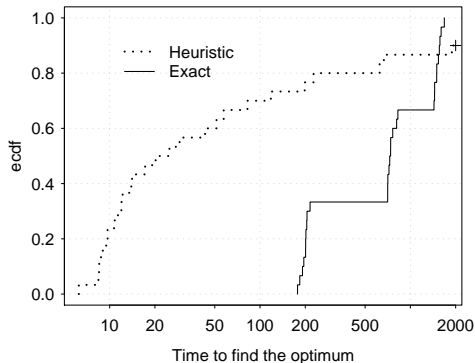
- T_i if $T_i \leq L_i$
- or $T_i \geq L_i$.

Hence, for each run i we need to record $\min(T_i, L_i)$ and the indicator variable for observed optimal/feasible solution attainment, $\delta_i = I(T_i \leq L_i)$.

Example

Asymptotic heuristics, Approach 1.b: Example

- ▶ An exact *vs* an heuristic algorithm for the *2-edge-connectivity augmentation problem*.
- ▶ **Interest:** time to find the optimum on different instances.



Uncensored:

$$F(t) = \frac{\# \text{ runs} < t}{n}$$

Censored:

$$F(t) = \frac{\# \text{ runs} < t}{n}$$

Scenarios

- A. Single-pass heuristics
- B. Asymptotic heuristics
(can be run indefinitely with a chance of continuing to make progress):

Two approaches:

- 1. Univariate
 - 1.a Time as an external parameter decided *a priori*
 - 1.b Solution quality as an external parameter decided *a priori*
- 2. Cost dependent on running time:

Scenario B

Asymptotic heuristics

There are two approaches:

2. Cost dependent on running time:

Deterministic case: \mathcal{A}^∞ on π returns a current best solution x at each observation in t_1, \dots, t_k .

The performance of \mathcal{A}^∞ on π is the **profile** indicated by the vector $\vec{y} = \{x(t_1), \dots, x(t_k)\}$.

Randomized case: \mathcal{A}^∞ on π produces a monotone stochastic process in solution cost $X(\tau)$ with any element dependent on the predecessors.

The performance of \mathcal{A}^∞ on π is the **multivariate** $\vec{Y} = (X(t_1), X(t_2), \dots, X(t_k))$.

Example

Scenario:

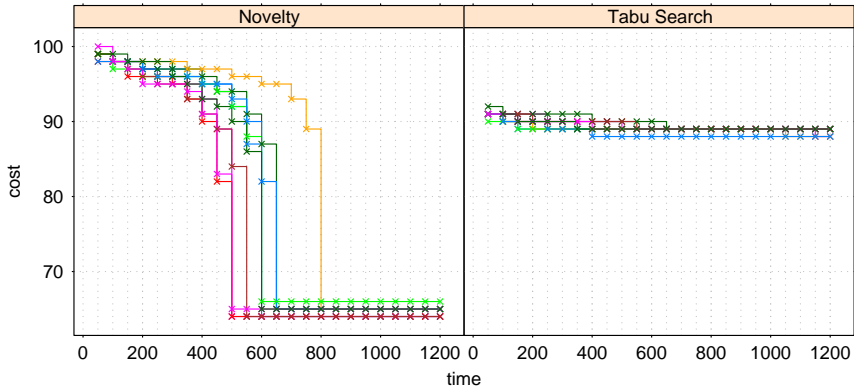
- ▷ 3 heuristics \mathcal{A}_1^∞ , \mathcal{A}_2^∞ , \mathcal{A}_3^∞ on instance π .
- ▷ single instance hence no data transformation.
- ▷ r runs
- ▶ **Interest:** inspecting solution cost over running time to determine whether the comparison varies over time intervals

Tools:

- Quality profiles

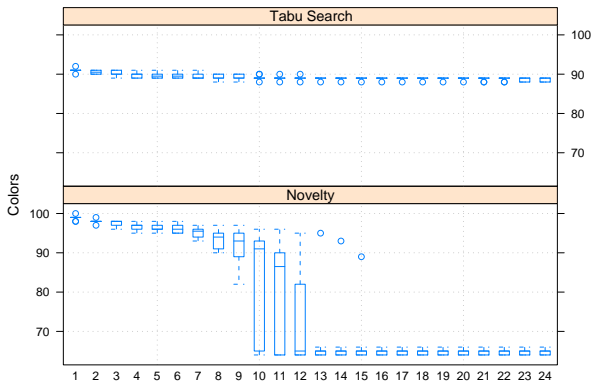
The performance is described by **multivariate random variables** of the kind $\vec{Y} = \{Y(t_1), Y(t_2), \dots, Y(t_k)\}$.

Sampled data are of the form $\vec{Y}_i = \{Y_i(t_1), Y_i(t_2), \dots, Y_i(t_k)\}$, $i = 1, \dots, 10$ (10 runs per algorithm on one instance)



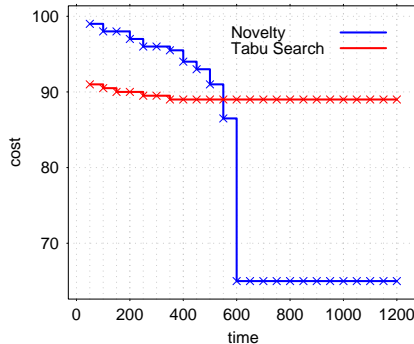
The performance is described by **multivariate random variables** of the kind $\vec{Y} = \{Y(t_1), Y(t_2), \dots, Y(t_k)\}$.

Sampled data are of the form $\vec{Y}_i = \{Y_i(t_1), Y_i(t_2), \dots, Y_i(t_k)\}$, $i = 1, \dots, 10$ (10 runs per algorithm on one instance)



The performance is described by **multivariate random variables** of the kind $\vec{Y} = \{Y(t_1), Y(t_2), \dots, Y(t_k)\}$.

Sampled data are of the form $\vec{Y}_i = \{Y_i(t_1), Y_i(t_2), \dots, Y_i(t_k)\}$, $i = 1, \dots, 10$ (10 runs per algorithm on one instance)



The median behavior of the two algorithms

Summary

Visualize your data for your **analysis** and for **communication** to others

Explore your data:

- make plots: histograms, boxplots, empirical cumulative distribution functions, correlation/scatter plots
- look at the numerical data and interpret them in practical terms: computation times, distance from optimum
- look for patterns

All the above both at a single instance level and at an aggregate level.

Making Plots

<http://algo2.iti.uni-karlsruhe.de/sanders/courses/bergen/bergenPresenting.pdf>

[Sanders, 2002]

- Should the experimental setup from the exploratory phase be redesigned to increase conciseness or accuracy?
- What parameters should be varied? What variables should be measured?
- How are parameters chosen that cannot be varied?
- Can tables be converted into curves, bar charts, scatter plots or any other useful graphics?
- Should tables be added in an appendix?
- Should a 3D-plot be replaced by collections of 2D-curves?
- Can we reduce the number of curves to be displayed?
- How many figures are needed?
- Should the x-axis be transformed to magnify interesting subranges?

- Should the x-axis have a logarithmic scale? If so, do the x-values used for measuring have the same basis as the tick marks?
- Is the range of x-values adequate?
- Do we have measurements for the right x-values, i.e., nowhere too dense or too sparse?
- Should the y-axis be transformed to make the interesting part of the data more visible?
- Should the y-axis have a logarithmic scale?
- Is it misleading to start the y-range at the smallest measured value?
(if not too much space wasted start from 0)
- Clip the range of y-values to exclude useless parts of curves?
- Can we use banking to 45°?
- Are all curves sufficiently well separated?
- Can noise be reduced using more accurate measurements?
- Are error bars needed? If so, what should they indicate? Remember that measurement errors are usually not random variables.

- Connect points belonging to the same curve.
- Only use splines for connecting points if interpolation is sensible.
- Do not connect points belonging to unrelated problem instances.
- Use different point and line styles for different curves.
- Use the same styles for corresponding curves in different graphs.
- Place labels defining point and line styles in the right order and without concealing the curves.
- Give axis units
- Captions should make figures self contained.
- Give enough information to make experiments reproducible.
- Golden ratio rule: make the graph wider than higher [Tufte 1983].
- Rule of 7: show at most 7 curves (omit those clearly irrelevant).
- Avoid: explaining axes, connecting unrelated points by lines, cryptic abbreviations, microscopic lettering, pie charts

References

- Birattari M., Stützle T., Paquete L., and Varrentrapp K. (2002). **A racing algorithm for configuring metaheuristics**. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2002)*, edited by L. et al., pp. 11–18. Morgan Kaufmann Publishers, New York.
- Chiarandini M. (2009). **Experimental analysis of optimization heuristics using R**. Lecture notes available at <http://www.imada.sdu.dk/~marco/Teaching/Files/Rnotes.pdf>.
- Sanders P. (2002). **Presenting data from experiments in algorithmics**. In *Experimental Algorithmics – From Algorithm Design to Robust and Efficient Software*, vol. 2547 of **LNCS**, pp. 181–196. Springer.