

# DM561 – Linear Algebra and Applications

## DM562 – Scientific Programming

### Obligatory Assignment 2, Autumn 2019

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**Deadline: Monday, November 23 at noon**

This document is associated with the files `asg2.py` and `draw.py` and data files, which are available in the git repository. The file `asg2.py` is the only one that needs to be edited and submitted.

In the Introduction to Python - Part 3, on slide 12, we compared three important models for growth functions in computer science applications:

|                             |                   |
|-----------------------------|-------------------|
| <b>exponential model</b>    | $y = ae^{bx}$     |
| <b>power function model</b> | $y = ax^b$        |
| <b>logarithmic model</b>    | $y = a + b \ln x$ |

where  $a$  and  $b$  are to be determined to fit experimental data as closely as possible. In this exercise you will work with a procedure called *linearization*, by which the data are transformed to a form in which a least squares straight line fit can be used to approximate the constants.

Let  $x$  denote the different size of two square matrices that we multiply with each other and  $y$  the computation time registered by executing, for example, the script benchmark from Assignment 1.

We will assume to have collected the following data  $D = \{(x_i, y_i)\}$  (also available in the associated python script):

|     |      |      |      |      |      |      |      |      |
|-----|------|------|------|------|------|------|------|------|
| $x$ | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| $y$ | 1.75 | 1.91 | 2.03 | 2.13 | 2.22 | 2.30 | 2.37 | 2.43 |

We will fit a linear model and the three models above using least squares and decide which is the best model.

**Linear function** Implement the function `least_squares(A,b)` that takes an appropriate matrix  $A$  and vector  $\mathbf{b}$  and returns the least square solution  $\mathbf{z}$  of the system  $A\mathbf{z} = \mathbf{b}$ . In the implementation, you are not allowed to use the function `linalg.lstsq(A,b)` from Numpy and Scipy. Using the function `least_squares(A,b)` determine the linear function  $y = ax + b$  that best fits the given data  $(x_i, y_i)$ . You can visualize the situation executing the file `draw.py` that uses the code of the function `linear_model(x,y)` that you implement to draw a plot with the points and the fitted linear regression.

**Exponential function** Making the substitution

$$Y = \ln y$$

in the equation  $y = ae^{bx}$  produces the equation  $Y = bx + \ln a$ , whose graph in the  $xY$ -plane is a line of slope  $b$  and  $Y$ -intercept  $\ln a$ . Verify this fact!

Hence, a curve of the form  $y = ae^{bx}$  can be fitted to the given  $n$  data points  $(x_i, y_i)$  by letting  $Y_i = \ln y_i$ , then fitting a straight line to the transformed data points  $(x_i, Y_i)$  by least squares to find  $b$  and  $\ln a$ , and then computing  $a$  from  $\ln a$ . Implement this method to fit an exponential model in the function `exponential_fit` that uses your `least_squares(A,b)`. (You may need to use the functions `np.log` and `np.exp` for computing the natural logarithm). You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted exponential curve.

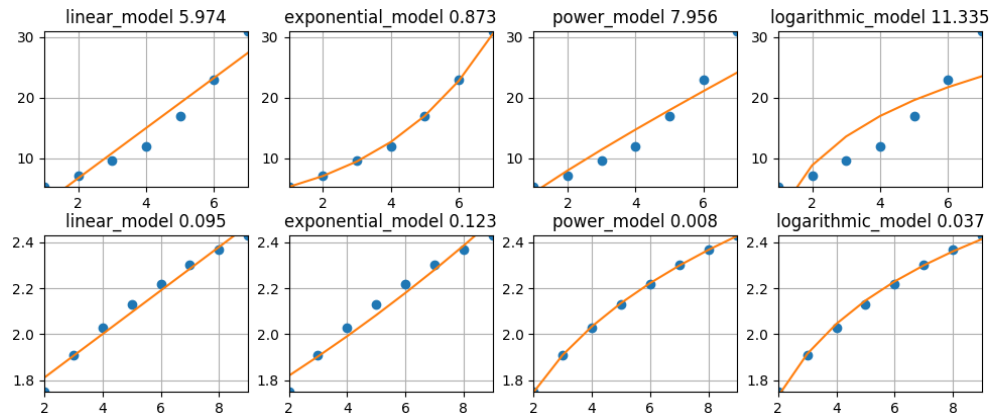


Figure 1: The result of this assignment with training error on two different data sets. Which curve fits best the data in the two data sets?

### Power function Making the substitutions

$$X = \ln x \quad Y = \ln y$$

in the equation  $y = ax^b$  produces the equation  $Y = bX + \ln a$ , whose graph in the  $XY$ -plane is a line of slope  $b$  and  $Y$ -intercept  $\ln a$ . Verify this fact!

Hence, a curve of the form  $y = ax^b$  can be fitted to the given  $n$  data points  $(x_i, y_i)$  by letting  $X_i = \ln x_i$  and  $Y_i = \ln y_i$ , then fitting a straight line to the transformed data points  $(X_i, Y_i)$  by least squares to find  $b$  and  $\ln a$ , and finally computing  $a$  from  $\ln a$ . Implement this method to fit a power model in the function `power_fit`. You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted curve.

### Logarithmic function Making the substitution

$$X = \ln x$$

in the equation  $y = a + b \ln x$  produces the equation  $Y = a + bX$ , whose graph in the  $XY$ -plane is a line of slope  $b$  and  $y$ -intercept  $a$ . Verify this fact!

Hence, a curve of the form  $y = a + b \ln x$  can be fitted to the given  $n$  data points  $(x_i, y_i)$  by letting  $X_i = \ln x_i$  and then fitting a straight line to the transformed data points  $(X_i, Y_i)$  by least squares to find  $b$  and  $a$ . Implement this method to fit a logarithmic model in the function `logarithmic_fit`. You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted curve.

**Training error** Implement the function `training_error(f,x,y)` that returns the **root** sum of squared errors for the model  $f$ . Using this function compare the training error of the four models and determine which model has the best training error.