

DM561 – Linear Algebra and Applications

DM562 – Scientific Programming

Obligatory Assignment 4, Autumn 2018

Deadline: Friday, December 14 at noon

This document is associated with the files `asg4.py` and `draw.py` and data files, which are available in the git repository. The file `asg4.py` is the only one that need to be edited and submitted.

In the Introduction to Python – Part 3, on slide 9, we compared three important models for growth functions in computer science applications:

exponential model	$y = ae^{bx}$
power function model	$y = ax^b$
logarithmic model	$y = a + b \ln x$

where a and b are to be determined to fit experimental data as closely as possible. In this exercise you will work with a procedure called *linearization*, by which the data are transformed to a form in which a least squares straight line fit can be used to approximate the constants. Some calculus is required to carry out the task.

Let x denote the different size of two square matrices that we multiply with each other and y the computation time registered by executing, for example, the script `benchmark` from Assignment 1.

We will assume to have collected the following data (also available in the associated python script:

x	2	3	4	5	6	7	8	9
y	1.75	1.91	2.03	2.13	2.22	2.30	2.37	2.43

We will fit a linear model and the three models above using least squares and decide which is the best model.

Linear function Implement the function `least_squares(A,b)` that takes an appropriate matrix A and vector \mathbf{b} and returns the least square solution \mathbf{z} of the system $A\mathbf{z} = \mathbf{b}$. In the implementation, you are not allowed to use the Numpy and Scipy function `linalg.lstsq(A,b)`. Using the function `least_squares(A,b)` determine the linear function $y = ax + b$ that best fit the given data (x_j, y_j) .

Exponential function Making the substitution

$$Y = \ln y$$

in the equation $y = ae^x$ produces the equation $Y = bx + \ln a$, whose graph in the xY -plane is a line of slope b and Y -intercept $\ln a$. Verify this fact!

Hence, a curve of the form $y = ae^x$ can be fitted to the given n data points (x_j, y_j) by letting $Y_j = \ln y_j$, then fitting a straight line to the transformed data points (x_j, Y_j) by least squares to find b and $\ln a$, and then computing a from $\ln a$. Implement this method to fit an exponential model in the function `exponential_fit` that uses your `linalg.lstsq(A,b)`. (You may need to use the functions `np.log` and `np.exp` for computing the natural logarithm). You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted exponential curve.

Power function Making the substitutions

$$X = \ln x \qquad Y = \ln y$$

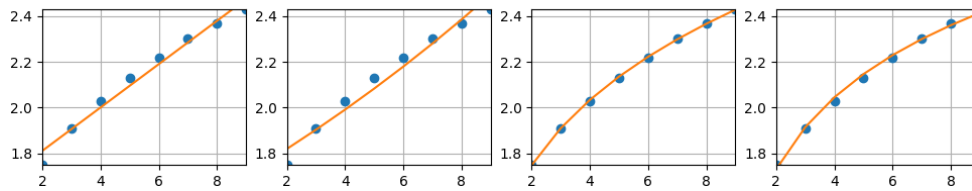


Figure 1: The result of Task 1.

in the equation $y = ax^b$ produces the equation $Y = bX + \ln a$, whose graph in the XY -plane is a line of slope b and Y -intercept $\ln a$. Verify this fact!

Hence, a curve of the form $y = ae^x$ can be fitted to the given n data points (x_j, y_j) by letting $X_j = \ln x_j$ and $Y_j = \ln y_j$, then fitting a straight line to the transformed data points (X_j, Y_j) by least squares to find b and $\ln a$, and then computing a from $\ln a$. Implement this method to fit an power model in the function `power_fit`. You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted curve.

Logarithmic function Making the substitution

$$X = \ln x$$

in the equation $y = a + b \ln x^b$ produces the equation $Y = a + bX$, whose graph in the XY -plane is a line of slope b and y -intercept a . Verify this fact!

Hence, a curve of the form $y = a + b \ln x$ can be fitted to the given n data points (x_j, y_j) by letting $X_j = \ln x_j$, then fitting a straight line to the transformed data points (X_j, y_j) by least squares to find b and a , and then computing a from $\ln a$. Implement this method to fit a logarithmic model in the function `logarithmic_fit`. You can visualize the situation executing the file `draw.py` that uses the function that you implement to draw a plot with the points and the fitted curve.

Training error Implement the function `training_error(f,x,y)` that returns the sum of squared errors for the model `f`. Using this function compare the training error of the four models and determine which model has the best training error.