DM865 – Spring 2020

Heuristics and Approximation Algorithms

Local Search for Traveling Salesman Problem

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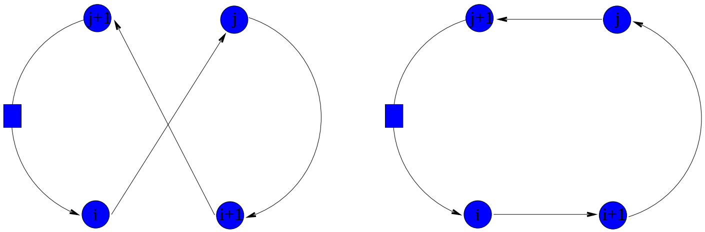
Intra-route Neighborhoods

2-opt

$$\{i, i+1\}\{j, j+1\} \longrightarrow \{i, j\}\{i+1, j+1\}$$

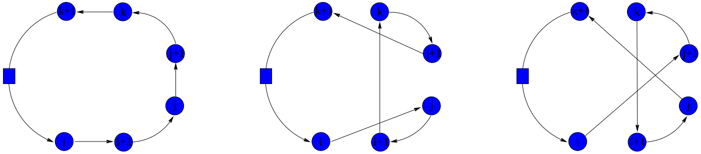


$O(n^2)$ possible exchanges
One path is reversed

Intra-route Neighborhoods

3-opt

$$\{i, i+1\}\{j, j+1\}\{k, k+1\} \longrightarrow \dots$$



$O(n^3)$ possible exchanges

Paths can be reversed

Possible 3-Exchanges

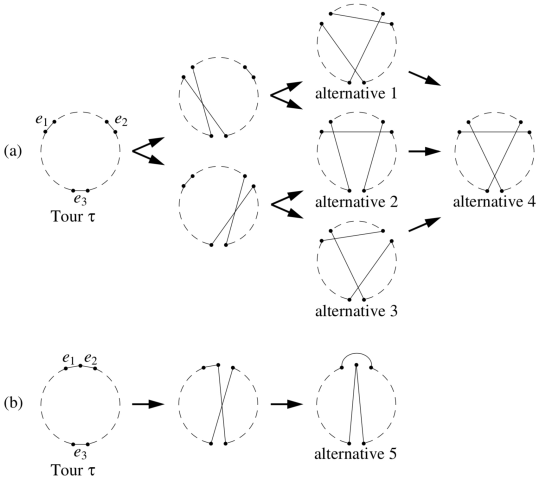
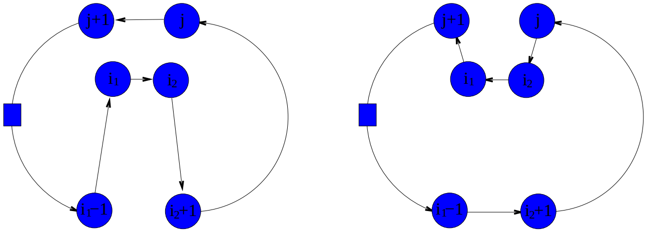


Figure from [MAK]

Intra-route Neighborhoods

Or-opt [Or (1976)]

$$\{i_1-1, i_1\}\{i_2, i_2+1\}\{j, j+1\} \longrightarrow \{i_1-1, i_2+1\}\{j, i_1\}\{i_2, j+1\}$$



sequences of one, two, three consecutive vertices relocated

$O(n^2)$ possible exchanges — No paths reversed

Table 17.1 Cases for k-opt moves.

k	No. of Cases
2	1
3	4
4	20
5	148
6	1,358
7	15,104
8	198,144
9	2,998,656
10	51,290,496

[Appelgate Bixby, Chvátal, Cook, 2006]

Local Search Example

Random-order first improvement for the TSP

- **Given:** TSP instance G with vertices $v_1, v_2, \dots, v_n$.
- **Search space:** Hamiltonian cycles in G;
- **Neighborhood relation** N**:** standard 2-exchange neighborhood
- **Initialization:**

  search position := fixed canonical tour $< v_1, v_2, \dots, v_n, v_1 >$
  "mask" P := random permutation of $\{1, 2, \dots, n\}$

- **Search steps:** examine neighbors in order of P (does not change throughout search)
  evaluate neighbors w.r.t. cost of tour $f(s)$
  accept the first improvement
- **Termination:** when no improving search step possible
  (local minimum)

Local Search Example

Iterative Improvement for TSP

TSP-2opt-first(s)
input: *an initial candidate tour* $s \in S(\in)$
output: *a local optimum* $s \in S_\pi$

for $i = 1$ to $n - 1$ **do**
    for $j = i + 1$ to n **do**
        if $P[i] + 1 \geq n$ or $P[j] + 1 \geq n$ **then** *continue* ;
        if $P[i] + 1 = P[j]$ or $P[j] + 1 = P[i]$ **then** *continue* ;

$$\begin{aligned}\Delta_{ij} \quad = \quad & d(\pi_{P[i]}, \pi_{P[j]}) + d(\pi_{P[i]+1}, \pi_{P[j]+1}) + \\ & -d(\pi_{P[i]}, \pi_{P[i]+1}) - d(\pi_{P[j]}, \pi_{P[j]+1})\end{aligned}$$

        if $\Delta_{ij} < 0$ **then**
            `UpdateTour(s,P[i],P[j])`

is it really?

Local Search Example

Iterative Improvement for TSP

TSP-2opt-first(s)
input: *an initial candidate tour* $s \in S(\in)$
output: *a local optimum* $s \in S_\pi$

FoundImprovement:=TRUE;
while FoundImprovement **do**
  FoundImprovement:=FALSE;
  for $i = 1$ to $n - 1$ **do**
    for $j = i + 1$ to n **do**
      if $P[i] + 1 \geq n$ or $P[j] + 1 \geq n$ **then** *continue* ;
      if $P[i] + 1 = P[j]$ or $P[j] + 1 = P[i]$ **then** *continue* ;
      $$\begin{aligned}\Delta_{ij} \quad = \quad & d(\pi_{P[i]}, \pi_{P[j]}) + d(\pi_{P[i]+1}, \pi_{P[j]+1}) + \\ & -d(\pi_{P[i]}, \pi_{P[i]+1}) - d(\pi_{P[j]}, \pi_{P[j]+1})\end{aligned}$$
      if $\Delta_{ij} < 0$ **then**
        `UpdateTour(s,P[i],P[j])`
        FoundImprovement=TRUE

Local Search Example

Efficient implementations of 2-opt, 2H-opt and 3-opt local search.

A. Neighborhood pruning (exact or heuristic)
   Fixed radius search + Candidate lists + DLB

B. Delta evaluation (already in $O(1)$)

C. Data structures

Details at black board and references [Bentley 92,Johnson McGeoch 2002, Appelgate Bixby, Chvátal, Cook, 2006]

Local Search for TSP

- k-exchange heuristics
  - 2-opt
  - 2.5-opt
  - Or-opt
  - 3-opt
- complex neighborhoods
  - Lin-Kernighan
  - Helsgaun's Lin-Kernighan
  - Dynasearch
  - ejection chains approach

Implementations exploit speed-up techniques

A. neighborhood pruning:
   - fixed radius nearest neighborhood search
   - neighborhood lists: restrict exchanges to most interesting candidates
   - don't look bits: focus local search to "interesting" part
B. delta evaluation
C. sophisticated data structures

Implementation examples by Stützle: `http://www.sls-book.net/implementations.html`

TSP data structures

Tour representation:

- determine pos of v in π
- determine succ and prec
- check whether u_k is visited between u_i and u_j
- execute a k-exchange (reversal)

Possible choices:

- $|V| < 1.000$ array for π and π^{-1}
- $|V| < 1.000.000$ two level tree
- $|V| > 1.000.000$ splay tree

Moreover static data structure:

- priority lists
- k-d trees

Look at implementation of local search for TSP by T. Stützle:

File: http://www.imada.sdu.dk/~marco/DM811/Resources/ls.c

```
two_opt_b(tour); % best improvement, no speedup
two_opt_f(tour); % first improvement, no speedup
two_opt_best(tour); % first improvement with dlbs,
                   % fixed radius near neighbour searches, neughbourhood lists
two_opt_first(tour); % best improvement with dlbs,
                   % fixed radius near neighbour searches, neughbourhood lists
three_opt_first(tour); % first improvement
```

Table 17.2 Computer-generated source code for k-opt moves.

k	No. of Lines
6	120,228
7	1,259,863
8	17,919,296

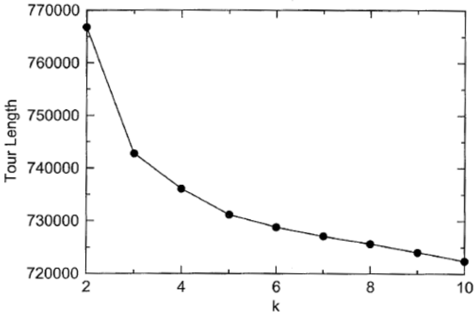


Figure 17.1 k-opt on a 10,000-city Euclidean TSP.