Department of Mathematics and Computer Science
University of Southern Denmark, Odense

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Marco Chiarandini

DM545/DM871 – Linear and integer programming

Sheet 1, Spring 2021

Exercise 1

Consider the matrices:

$$A = \begin{bmatrix} 2 & 0 \\ -4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -7 & 2 \\ 5 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 9 \\ -3 & 0 \\ 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 1 & 8 \\ 3 & 0 & 2 \\ 4 & -6 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 0 & 3 & 0 \\ -5 & 1 & 1 \\ 7 & 6 & 2 \end{bmatrix}$$

In each part compute the given expression. Where the computation is not possible explain why.

1. $D + E$
2. $D - E$
3. $5A$
4. $2B - C$
5. $2(D + 5E)$
6. $(C^T B)A^T$
7. $2\text{tr}(AB)$
8. $\det(E)$

Exercise 2

Consider the following system of linear equations in the variables $x, y, z \in \mathbb{R}$.

$$\begin{aligned} -2y + 3z &= 3 \\ 3x + 6y - 3z &= -2 \\ -3x - 8y + 6z &= 5 \end{aligned}$$

1. Write the augmented matrix of this system.
2. Reduce this matrix to row echelon form by performing a sequence of elementary row operations.
3. Solve the system and write its general solution in parametric form.

Exercise 3

Consider the following matrix

$$M = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}.$$

1. Find M^{-1} by performing row operations on the matrix $[M \mid I]$.

2. Is it possible to express M as a product of elementary matrices? Explain why or why not.

Exercise 4

1. Given the point $[3, 2]$ and the vector $[-1, 0]$ find the vector and parametric (Cartesian) equation of the line containing the point and parallel to the vector.
2. Find the vector and parametric (Cartesian) equations of the plane in $\mathbb{R}^3$ that passes through the origin and is orthogonal to $\mathbf{v} = [3, -1, -6]$.