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February 14, 2023
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DM545/DM871 – Linear and integer programming

Sheet 0, Spring 2023

Review of elements from Linear Algebra that are used in DM545/DM871.

Exercise 1

Consider the matrices:

$$A = \begin{bmatrix} 2 & 0 \\ -4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -7 & 2 \\ 5 & 3 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 9 \\ -3 & 0 \\ 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 1 & 8 \\ 3 & 0 & 2 \\ 4 & -6 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 0 & 3 & 0 \\ -5 & 1 & 1 \\ 7 & 6 & 2 \end{bmatrix}$$

In each part compute the given expression. Where the computation is not possible explain why.

1. $D + E$
2. $D - E$
3. $5A$
4. $2B - C$
5. $2(D + 5E)$
6. $(C^T B)A^T$
7. $2\text{tr}(AB)$
8. $\det(E)$

Exercise 2

Consider the following system of linear equations in the variables $x, y, z \in \mathbb{R}$.

$$\begin{aligned} -2y + 3z &= 3 \\ 3x + 6y - 3z &= -2 \\ -3x - 8y + 6z &= 5 \end{aligned}$$

1. Write the augmented matrix of this system.
2. Reduce this matrix to row echelon form by performing a sequence of elementary row operations.
3. Solve the system and write its general solution in parametric form.

Exercise 3

Consider the following matrix

$$M = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}.$$

1. Find M^{-1} by performing row operations on the matrix $[M \mid I]$.
2. Is it possible to express M as a product of elementary matrices? Explain why or why not.

Exercise 4

1. Given the point $[3, 2]$ and the vector $[-1, 0]$ find the vector and parametric (Cartesian) equation of the line containing the point and parallel to the vector.
2. Find the vector and parametric (Cartesian) equations of the plane in $\mathbb{R}^3$ that passes through the origin and is orthogonal to $\mathbf{v} = [3, -1, -6]$.

Exercise 5

Write a one line description of the methods you know to compute the inverse of a square matrix.

Exercise 6

Calculate by hand the inverse of

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

and check your result with the function `numpy.linalg.inv`.

Exercise 7

Use Cramer's rule to express the solution of the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

Exercise 8

Given two points in the Cartesian plane $\mathbb{R}^2$, $A = (1, 2)$ and $B = (3, 4)$ write the vector parametric equation and the Cartesian equation of the line that passes through them. Express the vector equation as an affine combination of the two points.

Exercise 9

Express the segment in $\mathbb{R}^2$ between the points $A = (1, 2)$ and $B = (3, 4)$ as a convex combination of its extremes.

Exercise 10

Write a generic vector parametric equation and a generic Cartesian equation of a plane in $\mathbb{R}^3$.

Exercise 11

Write a generic Cartesian equation of an hyperplane in $\mathbb{R}^n$ that does not pass through the origin.

Exercise 12

Prove that the following vectors in $\mathbb{R}^3$ linearly independent?
- $[6, 9, 5]^T$
- $[5, 5, 7]^T$
- $[2, 0, 7]^T$