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DM545/DM871 – Linear and integer programming

Sheet 4, Spring 2023

Exercises with the symbol $^+$ are to be done at home before the class. Exercises with the symbol * will be tackled in class and should be at least read at home. The remaining exercises are left for self training after the exercise class.

Exercise 1* Sensitivity Analysis and Revised Simplex

A furniture-manufacturing company can produce four types of product using three resources.

- A bookcase requires three hours of work, one unit of metal, and four units of wood and it brings in a net profit of 19 Euro.
- A desk requires two hours of work, one unit of metal and three units of wood, and it brings in a net profit of 13 Euro.
- A chair requires one hour of work, one unit of metal and three units of wood and it brings in a net profit of 12 Euro.
- A bedframe requires two hours of work, one unit of metal, and four units of wood and it brings in a net profit of 17 Euro.
- Only 225 hours of labor, 117 units of metal and 420 units of wood are available per day.

In order to decide how much to make of each product so as to maximize the total profit, the managers solve the following LP problem

$$
\begin{aligned}
\max \quad & 19x_1 + 13x_2 + 12x_3 + 17x_4 \\
& 3x_1 + 2x_2 + x_3 + 2x_4 \leq 225 \\
& x_1 + x_2 + x_3 + x_4 \leq 117 \\
& 4x_1 + 3x_2 + 3x_3 + 4x_4 \leq 420 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

The final tableau has x_1, x_3 and x_4 in basis. With the help of a computational environment such as Python for carrying out linear algebra operations, address the following points:

a) Write A_B, A_N, $A_B^{-1}A_N$, the final simplex tableau and verify that the solution is indeed optimal.

b) What is the increase in price (reduced cost) that would make product x_2 worth to be produced?

c) What is the marginal value (shadow price) of an extra hour of work or amount of metal and wood?

d) Are all resources totally utilized, i.e. are all constraints "binding", or is there slack capacity in some of them? Answer this question in the light of the complementary slackness theorem.

e) From the economical interpretation of the dual why product x_2 is not worth producing? What is its imputed cost?

Solve the following variations:

1. The net profit brought in by each desk increases from 13 Euro to 15 Euro.
2. The availability of metal increases from 117 to 125 units per day
3. The company may also produce coffee tables, each of which requires three hours of work, one unit of metal, two units of wood and bring in a net profit of 14 Euro.
4. The number of chairs produced must be at most five times the numbers of desks

Exercise 2+

Solve the systems $\mathbf{y}^T E_1 E_2 E_3 E_4 = [1\ 2\ 3]$ and $E_1 E_2 E_3 E_4 \mathbf{d} = [1\ 2\ 3]^T$ with

$$E_1 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0.5 & 0 \\ 0 & 4 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_4 = \begin{bmatrix} -0.5 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Exercise 3* Factory Planning and Machine Maintenance

A firm makes seven products $1, \ldots, 7$ on the following machines: 4 grinders, 2 vertical drills, 3 horizontal drills, 1 borer, and 1 planer.
Each product yields a certain contribution to the profit (defined as selling price minus cost of raw materials expressed in Euro/unit). These quantities (in Euro/unit) together with the production times (hours/unit) required on each process are given below.

product	1	2	3	4	5	6	7
profit	10	6	8	4	11	9	3
grinding	0.5	0.7	0	0	0.3	0.2	0.5
vdrill	0.1	0.2	0	0.3	0	0.6	0
hdrill	0.2	0	0.8	0	0	0	0.6
boring	0.05	0.03	0	0.07	0.1	0	0.08
planning	0	0	0.01	0	0.05	0	0.05

In the first month (January) and the five subsequent months certain machines will be down for maintenance. These machines will be:

January	1 grinder
February	2 hdrill
March	1 borer
April	1 vdrill
May	1 grinder
May	1 vdrill
June	1 planer
June	1 hdrill

There are marketing limitations on each product in each month. That is, in each month the amount sold for each product cannot exceed these values:

product	1	2	3	4	5	6	7
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

It is possible to store products in a warehouse. The capacity of the storage is 100 units per product type per month. The cost is 0.5 Euro per unit of product per months. There are no stocks in the first month but it is desired to have a stock of 50 of each product type at the end of June.
The factory works 6 days a week with two shifts of 8 hours each day. (It can be assumed that each month consists of 24 working days.)
The factory wants to determine a production plan, that is, the quantity to produce, sell and store in each month for each product, that maximizes the total profit.

Task 1 Model the factory planning problem for the month of January as an LP problem.

Task 2 Model the multi-period (from January to June) factory planning problem as an LP problem. Use mathematical notation and indicate in general terms how many variables and how many constraints your model has.