DM872
Mathematical Optimization at Work

Introduction

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Outline

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Who is here?

31 registered in BlackBoard

Prerequisites

- Programming
- Linear Algebra
- Linear and Integer Programming (DM545/DM871)

from DM545 (5 ECTS)
who??

- Math-economy
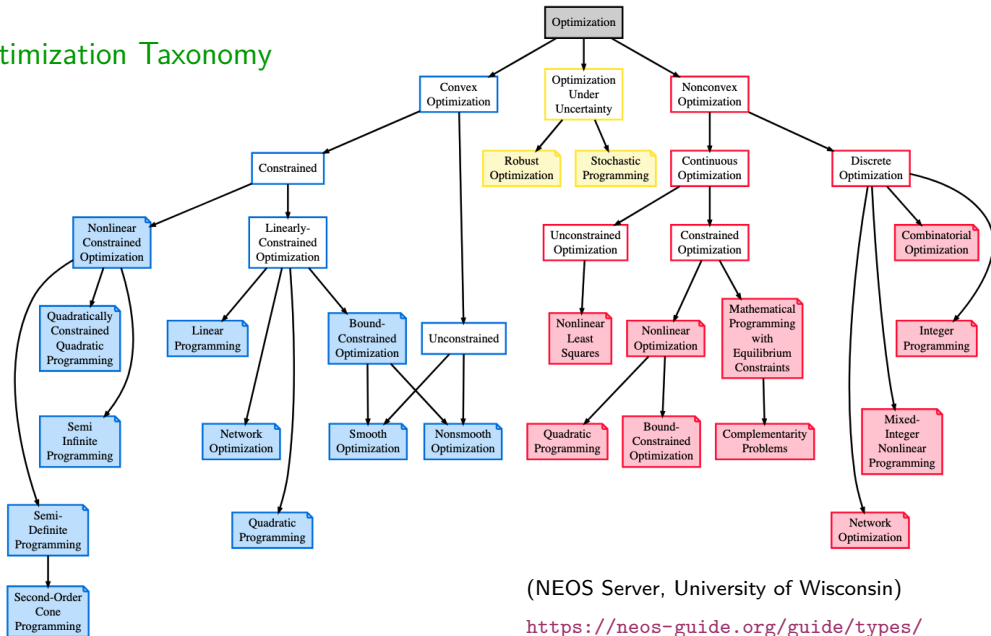- Others?

from DM871 (5 ECTS)
who??

- Computer Science (Master)
- Applied Mathematics
- Others?

Aims of the course

Learn about solving large scale, real-life problems with mixed integer linear programming:

- **advanced techniques** for integer linear programming

- applications

- implementations

$\rightsquigarrow$ You will see the theory and apply the tools learned to solve real life problems using computer software

Optimization Taxonomy

Optimization

- Convex Optimization
- Optimization Under Uncertainty
  - Robust Optimization
  - Stochastic Programming
- Nonconvex Optimization

Convex Optimization
- Constrained
  - Nonlinear Constrained Optimization
    - Quadratically Constrained Quadratic Programming
    - Semi Infinite Programming
    - Semi-Definite Programming
      - Second-Order Cone Programming
  - Linearly-Constrained Optimization
    - Linear Programming
      - Network Optimization
      - Quadratic Programming
    - Bound-Constrained Optimization
      - Smooth Optimization
      - Nonsmooth Optimization
  - Unconstrained

Nonconvex Optimization
- Continuous Optimization
  - Unconstrained Optimization
    - Nonlinear Least Squares
  - Constrained Optimization
    - Nonlinear Optimization
      - Quadratic Programming
      - Bound-Constrained Optimization
    - Mathematical Programming with Equilibrium Constraints
      - Complementarity Problems
- Discrete Optimization
  - Combinatorial Optimization
    - Integer Programming
  - Mixed-Integer Nonlinear Programming
  - Network Optimization

(NEOS Server, University of Wisconsin)

https://neos-guide.org/guide/types/
https://docs.mosek.com/modeling-cookbook/index.html

Contents of the Course (aka Pensum)

Advanced mixed integer linear programming techniques

0	LP and ILP Recap
1	More on Modeling
2	Cutting Planes
3	Dantzig-Wolfe decomposition
4	(Delayed) Column generation
5	Branch and price
6	Lagrangian relaxation
7	Matheuristics

Applications

7	TSP
8	Vehicle Routing
9	Vehicle Scheduling
10	Crew Scheduling
11	Machine Learning
12	Educational Timetabling

Practical Information

Teachers: Marco Chiarandini (imada.sdu.dk/u/march/)
     Konstantin Pavlikov (https://portal.findresearcher.sdu.dk/en/persons/kop/)
Instructor: None

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Alternative views of the schedule:

- mitsdu.sdu.dk, SDU Mobile
- Official course description (skemaplan)
- https://github.com/DM872/Resources/
- https://imada.sdu.dk/u/march/Timetables/Semesters/F26/html/DM872.html

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Schedule:
- Scheduled $16 \times 4 = 64$ hours ($\sim$ 32 classes)
- Officially joint classes: 48 hours ($\sim$ 24 classes)
- We try to finish at the beginning of May, 3,4 weeks earlier than scheduled.

No distinction between classes, they are all common and with possible hands on activities.

Communication Means

- ItsLearning (LMS) ⇔ Git Repository
  (link https://github.com/DM872/Resources/)

- Announcements + Discussion Board + Submissions in ItsLearning

- Ask peers

- Write to Marco (marco@imada.sdu.dk)

- Visit Marco in his office in working hours (9-17)

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Course Material

Git Repository is the main reference for list of contents

It contains:

- list of topics and references

- slides

- tutorials

- reading materials

- exercise sheets

- assignments

Updated daily.

Classes

- Goal: Moderate course/track material and guide learning
  - main material is the literature and exercises
  - no lecture notes beyond the slides

- Questions and discussions welcome and encouraged at any time
  - if you have questions, ideas, or disagreement: interrupt and ask!
  - discussing and questioning what you read/hear is part of the lecture

Exercises

- We start them in class, in groups of two

- You continue and finish them as self study

- Try to occasionally gather in different groups

- Prepared tasks are starting point for engaged and self-responsible learning

- We discuss any question or idea in plenum in the class after or you ask for personal feedback in the classes

Assessment

- Two obligatory medium size projects, evaluation by external censor

- First is individual and second is in group of at most two persons

- Learning goals:
  - first, understand and apply knowledge gained in a similar context, narrow scope
  - second, analyze, select and apply an appropriate method studied to a new context, open scope. Discuss and reflect.

- (language: Danish and/or English)

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- Gen AI: we follow SDU regulations https://mitsdu.dk/en/aiatsdu.
  Report in Appendix if you have used and if so share the entire chat. If the Appendix is missing the submission is not valid and you fail the exam.

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- Final grade: overall evaluation but as starting point the average grade rounded up

Implementations

- Python 3.10+ (check pyenv)

- MILP specific:
  - gurobipy, Gurobi 13+ (commercial 100 000 DKK, alternatives: Cplex, Xpress)
  - SCIP Optimization Suite + PyScipOpt (Commercial alternative Gurobi or Cplex $\approx$ 100 000 Dkk)
  - Python-MIP + CBC or Gurobi 13+
  - Pyomo + Ipopt, CBC, HiGHS, Gurobi 13+, Cplex

- jupyter, jupyterLab (= interactive python) or Google CoLab

- VS Code

Average Working Load

Expected 135 working hourse for 5 ECTS

- Preparations (before/after lecture): [24 classes $\times 4h$]
  Read up on non-optional material (at least the slides)
  Use optional material or own research to close gaps in understanding

- Exercises: [$10 \times 1h$]
  Go through the weekly sheets (github)
  Solve tasks left from classes
  Discuss with peers

- Project work: [$24h + 24h$]
  Research literature, Implement, Analyse

150 hours in total. 8.2 hours per week.