DM872
Mathematical Optimization at Work

Cut and solve

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Outline

1. Primal Heuristics

2. Cut and Solve

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Primal Heuristics

Independent on MIP solvers

- Greedy heuristics
- Local search
- Metaheuristics

Inside MIP solvers

- Construction heuristics
    - Rounding, Shift, Fix
    - Dive and Fix
    - Neighborhood Rounding
    - Feasibility pump
- Improvement heuristics
    - Local Branching
    - Proximity Search
    - Relaxation Induced Neighborhood Search (RINS)
    - Polishing Heuristic

User defined MIP heuristics

- Relax and Fix
- Large neighborhood search
- Extended formulations

Outline

1. Primal Heuristics

2. Cut and Solve

Cut and Solve

- Iteration $\equiv$ node in search path
- piercing cut a cut that removes at least one feasible solution from the original (unrelaxed) problem solution space.

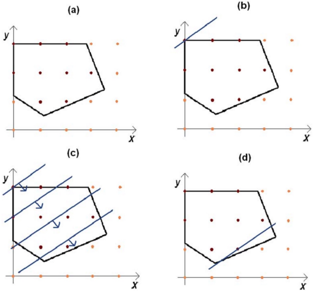
```
algorithm cut_and_solve (IP)
    select cut
    find optimal feasible solution in space removed by cut
       update best if necessary
    add cut to problem
    find lower bound
    if (lower bound >= best) return best
    otherwise, repeat
```

Example

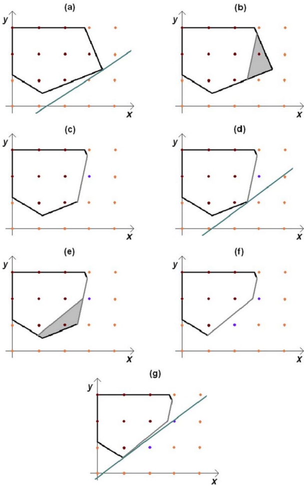
$$\min Z = y - \frac{4}{5}x$$

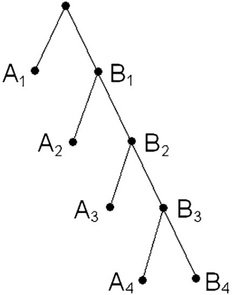
subject to:

$$x \geqslant 0$$
$$y \leqslant 3$$
$$y + \frac{3}{5}x \geqslant \frac{6}{5}$$
$$y + \frac{13}{6}x \leqslant 9$$
$$y - \frac{5}{13}x \geqslant \frac{1}{14}$$
$$x \in I$$
$$y \in I$$



Example





Generic piercing cut procedure

- Partition binary variables in a small set S and a large set L.
  - sparse problem solved on the set S while setting variables in L to zero.

$$\sum_{x_i \in L} x_i = 0$$

  - piercing cut

$$\sum_{x_i \in L} x_i \geq 1$$

- the assumption is that being sparse in feasible (integer) solutions, this problem should be easier to solve.
- general guidelines to select S:
  - Each piercing cut should remove the solution to the current relaxed problem so as to prevent this solution from being found in subsequent iterations.
  - The space that is removed by the piercing cut should be adequately sparse, so that the optimal solution can be found relatively easily.
  - The piercing cuts should attempt to capture an optimal solution for the original problem. The algorithm will not terminate until an optimal solution has been cut away and consequently made the incumbent.
  - In order to guarantee termination, each piercing cut should contain at least one feasible solution for the original, unrelaxed, problem.

Generic Cut and Solve

```
algorithm generic_cut_and_solve (BIP)
    relax integrality and solve LP
    if (LP solution >= best) return best
    let S = {variables with reduced costs <= alpha}
    find optimal feasible solution in S
       update best if necessary
       if (LP solution >= best) return best
    add (sum of variables not in S >= 1) to BIP
    repeat
```

- Rationale: Reduce cost is a lower bound on the increase of the LP solution cost if the value of the variables is increase by one unit. Hence variables with reduced costs of low absolute value are likely to disrupt the least the objective function value.
- α should be small enough to leave the sparse problem easy to solve and large enough to admit a feasible an possibly optimal solution
- note that since all variables currently in basis have reduced cost of null then the current optimal solution will be part of the sparse problem and cut away from the rest.