Primal heuristics

$$\min_{S \subseteq N} \{ c(S) : v(S) \geq K \}$$

Knapsack
facility loc

Greedy heuristic

1. $S^0 = \emptyset$
2. $$j_t = \arg\min \frac{c(S^{t-1} \cup \{j_t\}) - c(S^{t-1})}{v(\quad) - v(\quad)}$$

   $$\min \frac{\text{additional cost}}{\text{unit of resource}}$$
3. if S^{t-1} is feasible and costs less than $S^{t-1} \cup \{j_t\}$ then stop with S^{t-1}
4. else $S^t = S^{t-1} \cup \{j_t\}$
5. if $t = n$ stop with S^t
6. else $t = t+1$ goto 2

Local Search

init S
while $\exists\, S'$ in $N(S)$: $f(S') < f(S)$:
    $S = S'$

Tabu search   SA   Genetic Alg

Tabu search, sim ann, genetic alg.

MIP heuristics

$$\max\{cx : x \in X \subseteq \mathbb{Z}^n_+\} = z_{IP}$$

Let $x \in LP(IP)$:

Rounding : nearest integer $\lceil x_j \rfloor$

Shift : shift x_j to $x_j + a$ if decreases infeasibility

Fix : set x_j to some value $x_j \in \mathbb{Z}$
tighten bounds and resolve

Solve with fixed vars and rand.

Solve a mixed ILP.

Repeat

- Dive and Fix

0–1 problem, x^* LP sol

$F = \{j : x^*_j \notin \{0,1\}\}$

Let $i = \arg\min \{\min[x^*_j, 1-x^*_j]\}$ closest to

$j \in F$ ... int

$x_i = \lceil x_i \rfloor$

Solve LP with x_i fixed

Repeat until feasible

- Neighborhood rounding

$$\max\{cx : x \in X,\ \lfloor x_j^* \rfloor \le x_j \le \lceil x_j^* \rceil\ \ j \in N\}$$

- Feasibility Pump

$$\max\{cx : Ax \le b,\ l \le x \le u,\ x \in \mathbb{Z}^n\}$$

$x^* = x^{LP}$

$\hat{x} = \lceil x^* \rfloor$

if $\hat{x} \notin P$:

$$\min \sum_{j \in B : \hat{x}_j = l_j} (x_j - l_j) + \sum_{j \in B : \hat{x}_j = u_j} (u_j - x_j) + \sum_{j : l_j < \hat{x}_j < u_j} \delta_j$$

$$Ax \le b$$

$$\delta_j = |x_j - \hat{x}_j|$$

$$l \le x \le u$$

$x^* = ?$

Repeat

Improvement heuristics

Given feasible sol x^*

- Local Branching

$$\max\left\{cx : x \in X \cap \left\{x : \sum_{j: x_j^*=0} x_j + \sum_{j: x_j^*=1} (1-x_j) \le k\right\}\right\}$$

- Proximity search

$x^* \in X \subset \{0,1\}^n$

find a sol that improves by δ and is as close to x^* as poss.

$$\min\left(\sum_{j: x_j^*=0} x_j + \sum_{j: x_j^*=1} (1-x_j)\right)$$

$$\sum_{j=1}^{n} c_j x_j \le cx^* - \delta$$

$$x \in X$$

- Relaxation Induced Neigh. Search. (RINS)

$x^* \in X$ and x^{LP}

$F = \{ j \in N : x_j^* = x_j^{LP} \}$

$$\max\left\{cx : x \in X \cap \{x : x_j = x_j^*, j \in F\}\right\}$$

• Polishing

K best sol is stored

Select 2 or more sol $\{x^1 \dots x^r\}$

Fix $F = \{ j \in [1:n] : x^1_j = x^t_j \text{ for } t = 1 \dots r \}$

$\max \{ cx : x \in X \wedge \{ x : x_j = x^*_j \ \forall j \in F \}$

User defined MIP heur.

Relax and Fix

$$z = \max \{ c^1x^1 + c^2x^2 ,\ A^1x^1 + A^2x^2 = b,$$
$$x^1 \in \mathbb{Z}^{n_1}_+$$
$$x^2 \in \mathbb{Z}^{n_2}_+$$

1 Relax

$$\bar{z} = \max\ c^1x^1 + c^2x^2$$
$$A^1x^1 + A^2x^2 = b$$
$$x^1 \in \mathbb{Z}^{n_1}_+ \quad \Rightarrow (\bar{x}^1, \bar{x}^2)$$
$$x^2 \in \mathbb{R}^{n_2}_+$$

2 Fix

$$\underline{z} = \max\ c^1x^1 + c^2x^2$$
$$A^1x^1 + A^2x^2 = b$$
$$x^1 = \bar{x}^1 \quad \Rightarrow (\bar{x}_1, x^{\sim 2})$$
$$x^2 \in \mathbb{Z}^{n_2}$$

3 Heuristic sol

$x^H = (\bar{x}^1, \bar{x}^2)$ w $\underline{z} = cx^H \leq z \leq \bar{z}$

Use of the multiperiod:

$[t_k \ \sigma_k \ \tau_k]$ $\quad t_k \leq \sigma_k \leq \tau_k$

$t_k = \sigma_{k-1} + 1$

kth step

$$\max \ cx$$
$$x \in X$$
$$x_j = x_j^F \quad j = 1 \ldots \sigma_{k-1}$$
$$x_j \in \mathbb{Z}_+^1 \quad j = t_k \ldots \tau_k$$
$$x_j \in \mathbb{R}_+^1 \quad j > \tau_k$$

$x_j^F = x_j^*$ for $j = t_k \ldots \sigma_k$

30 periods

10 vars int.

6 vars fix

$[1 \ 6 \ 10]$ $[7 \ 12 \ 16]$ $[13 \ 18 \ 22]$

$[19 \ 24 \ 28]$ $[25 \ 30 \ 30]$

Large Neighb.. Search

$\max \{cx : x \in X, x_j = x_j^* \ j \in N \setminus U\}$

eg $V = [7, 12]$ to improve
between 7 and 12

- Extended Formulation