Course Timetabling

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University Course Timetabling Research

- Course Timetabling [ITC-2002, ITC-2007], [Di Gaspero et al. (2007)], [Bettinelli et al. (2015)]
  Curriculum-Based (CB-CTT) $\rightsquigarrow$ Post-Enrolment-Based (PE-CTT)

- Solution Approaches: Modeling approach + solving algorithms:
  - Direct representation
    branch and bound, construction heuristics, metaheuristic methods
  - Mixed Integer Linear Programming (MILP)
    branch-and-cut
  - Satisfiability problems (SAT)
    backtracking-based algorithms
  - Constraint Satisfaction Problems (CSP)
    constraint propagation + backtracking-based algorithms

University Course Timetabling Research

Common assumptions of CB-CTT and PE-CTT: weekly periodicity + classes of equal length

Inadequate in many institutions (SDU included)

- different requirements for the number of classes in each week of the semester
- different duration for each single class
- precedence constraints among classes, eg, introduction classes preceed exercise classes

ITC2019: Course Timetabling Competition 2019 (`www.itc2019.org`) including these features and student sectioning.

In the next slides we focus on the problem at SDU.

Open question: is there a possible reduction between the SDU problem and the problem at ITC2019?

Outline

Timetabling at SDU

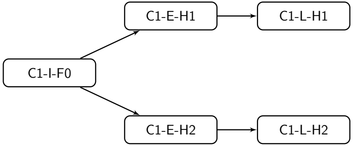
The timeline of timetabling activities for the Spring semester

week 37	39	40-52	49-50	52-2	6-23
Room negotiation	Teachers communicate organization of classes	Timetabling mandatory courses (CB-CTT)	Students register to courses	Timetabling elective courses (PE-CTT)	Semester

Problem Description: Input

- A set of periods (timeslots) $P = \{(h, d, w) \mid h \in \text{Hours}, d \in \text{Days}, w \in Weeks\}$
- A set of events E each event with a duration $\ell(e)$, $e \in E$ + precedence digraph $D = (V, A)$

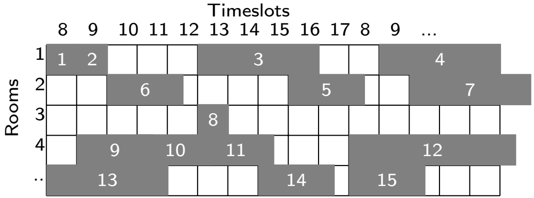
Course	Session	Section	Event
C1	Intro	F	C1-I-F
	Exercises	H1	C1-E-H1
		H2	C1-E-H2
	CompLab	H1	C1-L-H1
		H2	C1-L-H2

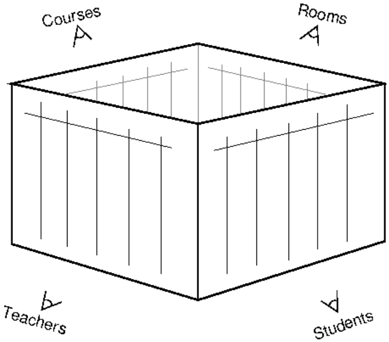


- A set of rooms R (seminar room + dummy)
- People: a set of students S, a set of teachers T
  - A collection of enrollments $\mathcal{Q} = \{E_s \subset E | s \in S\}$ that are events a student has subscribed to (post enrollment model)
  - A collection of teaching duties $\mathcal{D} = \{D_t \subset E | t \in T\}$
  - Teacher unavailabilites $\mathcal{U} = \{U_t \subset P | t \in T\}$
- Schedule of mandatory courses $M = \{(e, r, p) \mid e \in E, r \in R, p \in P\} \equiv$ preassignments

Problem Description: Task

Schedule events in the semester such that the timetable is feasible and appealing from different perspectives (resources)





Problem Description

Constraints (hard constraints)

Enforce All Events Scheduled	Enforce Fixed Preassignments	Enforce Precedences
Prevent Room Conflicts	Enforce Fixed Rooms	Enforce Banned Slots
Prevent Staff Conflicts	Enforce Max One Event x Day x Crs	Enforce Pairings

Objective(s) (soft constraints):

Weekly Stability	Student/Instructor Conflicts	Bad Slots
Usage of seminarrum	Events x Day x Tch	

The classical approach:
Violations of these criteria are penalized with *appropriately* chosen weights in a (single) objective function to minimize

Collective welfare approach?

Outline

1. Problem Description

2. MILP Approach

Mixed Integer Linear Programming Formulations (1/2)

- Compact MILP formulation (aka Monolithic) [Burke et al. (2008, 2010a)]
  variables: $x_{erp} \in \{0, 1\}$ whether event $e \in E$ placed in room $r \in R$ in timetslot $p \in P$

- Two-stage formulation [Lach and Lübbecke (2012)]
  1. Variables: $x_{ep} \in \{0, 1\}$ whether event $e \in E$ placed in timeslot $p \in P$
  2. One-sided perfect matching in a bipartite graph with additional constraints

- Divide-and-conquer approach [Hao and Benlic (2011)]
  1. Generate a partition E_i of the set of events.
  2. Solve each subproblem $P(E_i), i = 1, ..., k$, with a MILP solver to compute lower bound LB_i.
  3. Sum up the k values LB_i to achieve the final lower bound to CB-CTT.

- Column generation approach [Cacchiani et al. (2013)]: Two Weekly Schedule Types
  1. A vector $\mathbf{x} \in B^{|E|\times|R|\times|P|}$ made of components $x_{erp} \in \mathbb{B}$ representing the assignment of event $e \in E$ to room $r \in R$ at timeslot $p \in P$.
     Used to consider penalties for room capacity and room stability.
  2. A vector $\boldsymbol{\theta} \in B^{|E|\times|P|}$ made up of components $\theta_{ep} \in \mathbb{B}$ that indicates if an event $e \in E$ is scheduled at timeslot $p \in P$.
     Used to consider penalties for curriculum compactness and minimum working days.

Mixed Integer Linear Programming Formulations (2/2)

- A Resource Constrained Project Scheduling Model
  1. Variables: Time indexed variables of starting time of an event

$$x_{erp} \in \{0,1\} \qquad \forall e \in E,\ r \in R,\ p = (h,d,w) \in P$$

  denote whether an event $e \in E$ is located in room $r \in R$ and starts at timeslot $p = (h,d,w) \in P$

- Patterns (or classes in extended sense) [Muüller, Rudová, Muüllerová, 2019]
  1. Patterns are generated: In a pattern all meetings start at the same time, run for the same number of slots and are placed in the same room.
  2. Variables $x_{ci} \in \{0,1\}$ selects a pattern for a class, y_{cr} selects a room for a class.

```
<class id="40" limit="34" parent="39">
    <room id="16" penalty="4"/>
    <room id="21" penalty="0"/>
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    <room id="3" penalty="0"/>
    <room id="13" penalty="0"/>
    <room id="25" penalty="4"/>
    <room id="27" penalty="0"/>
    <room id="7" penalty="0"/>
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    <time days="0001000" start="96" length="22" weeks="011111111111110" penalty="0"/>
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    <time days="0001000" start="120" length="22" weeks="011111111111110" penalty="6"/>
    <time days="0000100" start="120" length="22" weeks="011110111111110" penalty="0"/>
    <time days="0001000" start="144" length="22" weeks="011111111111110" penalty="6"/>
    <time days="0000100" start="144" length="22" weeks="011110111111110" penalty="0"/>
    <time days="0001000" start="168" length="22" weeks="011111111111110" penalty="6"/>
    <time days="0000100" start="168" length="22" weeks="011110111111110" penalty="2"/>
    <time days="0001000" start="192" length="22" weeks="011111111111110" penalty="0"/>
    <time days="0000100" start="192" length="22" weeks="011110111111110" penalty="8"/>
```
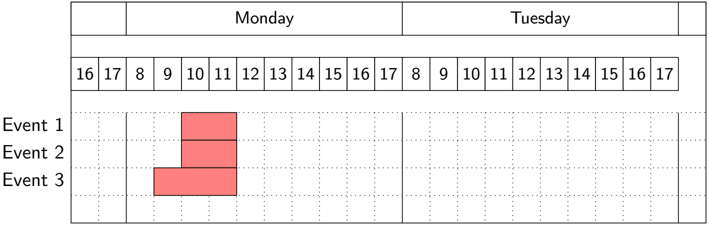

The Resource Constrained Project Scheduling Model
Rooms

Hard constraints:

- Prevent Room Conflicts + Room Availability + Suitable Events

$$\sum_{e \in E} \sum_{h=\max\{8, s-\ell(e)\}}^{s} x_{erhdw} \leq a_{rhdw} \qquad \forall r \in R, s = 8, \ldots, 17$$



$$\begin{gathered} x_{1,\cdot,1,10,w} + x_{1,\cdot,1,11,w} + \\ x_{2,\cdot,1,10,w} + x_{2,\cdot,1,11,w} + \\ x_{3,\cdot,1,9,w} + x_{3,\cdot,1,10,w} + x_{3,\cdot,1,11,w} \leq 1 \end{gathered}$$

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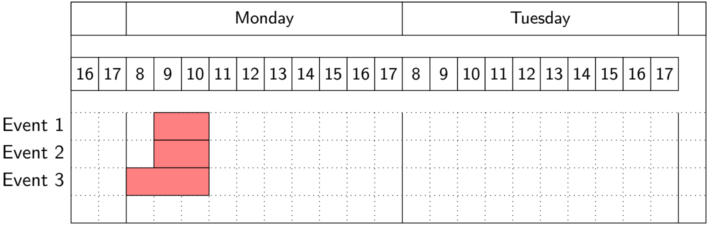

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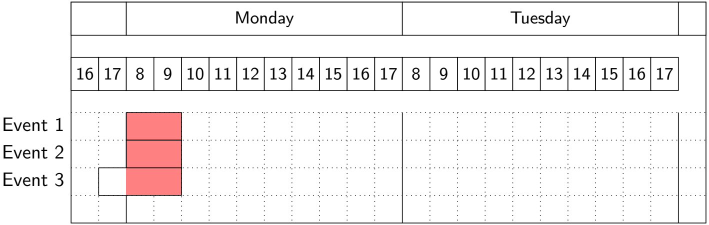

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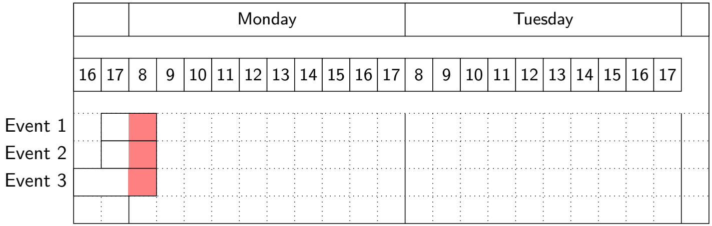

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