

レポートタイトル

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1 はじめに

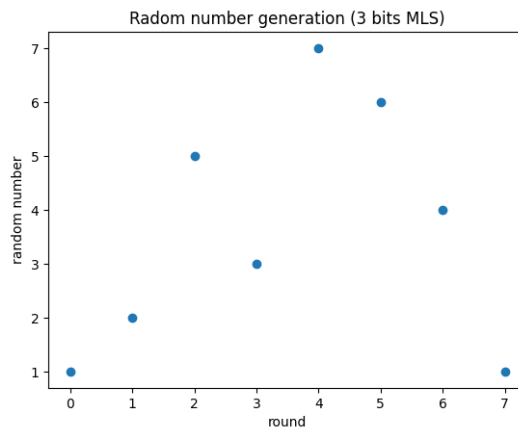


図1 図の貼り方

2 実験 1-5

2.1 目的

これからランダムウォーク実験を行うにあたって、ランダムな数を生成することが必要であるが、パソコンはランダムな数を生成できない。そこでコンピューター上では数式を用いて擬似乱数列—乱数のように思えるが、初期状態が決定すれば未来の数列が決定してしまうので真にランダムではない数列—を生成するのだが、このとき C 言語における `srand(time(NULL))` や、python における `rand()` のようないわゆる組み込み関数を用いずに擬似乱数列を生成するのが今回の実験の目的である。

2.2 理論

擬似乱数列の生成を行う。このとき LFSR 法を用いた。LFSR とは次の数を直前の数に基づきシフト演算を使って漸化式的に決定した数列 (次の数が直前の数の線形写像になっているシフトレジスタ) のことで、このとき直前の数から次の数を求める漸化式をうまく設定することで、全ビットが 0 という状態以外のすべての

取る整数列を作ることができることが分かっており、これを最長 LFSR と呼ぶ。今回はこの最長 LFSR を作
 ることで擬似乱数列を生成した。

ここで、最長 LFSR を生成できる漸化式を次に示す。このとき、下に示す漸化式は 2 進数である

$$a_{n+1} = b_{n+1} + c_{n+1} \& 1$$

但し

$$b_{n+1} = (a_n << 1) \& (10^{bit} - 1)$$

(しつこいようだが、10 はいわゆる 2 進数の 10 であり、つまり 10 進数における 2 である)

$$c_{n+1} = \sum_i (a_n >> k_i)$$

このとき、 c_{n+1} を以下のように設定することで、最長 LFSR を計算することができる：

n	XNOR from	n	XNOR from	n	XNOR from	n	XNOR from
3	3,2	45	45,44,42,41	87	87,74	129	129,124
4	4,3	46	46,45,26,25	88	88,87,17,16	130	130,127
5	5,3	47	47,42	89	89,51	131	131,130,84,83
6	6,5	48	48,47,21,20	90	90,89,72,71	132	132,103
7	7,6	49	49,40	91	91,90,8,7	133	133,132,82,81
8	8,6,5,4	50	50,49,24,23	92	92,91,80,79	134	134,77
9	9,5	51	51,50,36,35	93	93,91	135	135,124
10	10,7	52	52,49	94	94,73	136	136,135,11,10
11	11,9	53	53,52,38,37	95	95,84	137	137,116
12	12,6,4,1	54	54,53,18,17	96	96,94,49,47	138	138,137,131,130
13	13,4,3,1	55	55,31	97	97,91	139	139,136,134,131
14	14,5,3,1	56	56,55,35,34	98	98,87	140	140,111
15	15,14	57	57,50	99	99,97,54,52	141	141,140,110,109
16	16,15,13,4	58	58,39	100	100,63	142	142,121
17	17,14	59	59,58,38,37	101	101,100,95,94	143	143,142,123,122
18	18,11	60	60,59	102	102,101,36,35	144	144,143,75,74
19	19,6,2,1	61	61,60,46,45	103	103,94	145	145,93
20	20,17	62	62,61,6,5	104	104,103,94,93	146	146,145,87,86
21	21,19	63	63,62	105	105,89	147	147,146,110,109
22	22,21	64	64,63,61,60	106	106,91	148	148,121
23	23,18	65	65,47	107	107,105,44,42	149	149,148,40,39
24	24,23,22,17	66	66,65,57,56	108	108,77	150	150,97
25	25,22	67	67,66,58,57	109	109,108,103,102	151	151,148
26	26,6,2,1	68	68,59	110	110,109,98,97	152	152,151,87,86
27	27,5,2,1	69	69,67,42,40	111	111,101	153	153,152
28	28,25	70	70,69,55,54	112	112,110,69,67	154	154,152,27,25
29	29,27	71	71,65	113	113,104	155	155,154,124,123
30	30,6,4,1	72	72,66,25,19	114	114,113,33,32	156	156,155,41,40
31	31,28	73	73,48	115	115,114,101,100	157	157,156,131,130
32	32,22,2,1	74	74,73,59,58	116	116,115,46,45	158	158,157,132,131
33	33,20	75	75,74,65,64	117	117,115,99,97	159	159,128
34	34,27,2,1	76	76,75,41,40	118	118,85	160	160,159,142,141
35	35,33	77	77,76,47,46	119	119,111	161	161,143
36	36,25	78	78,77,59,58	120	120,113,9,2	162	162,161,75,74
37	37,5,4,3,2,1	79	79,70	121	121,103	163	163,162,104,103
38	38,6,5,1	80	80,79,43,42	122	122,121,63,62	164	164,163,151,150
39	39,35	81	81,77	123	123,121	165	165,164,135,134
40	40,38,21,19	82	82,79,47,44	124	124,87	166	166,165,128,127
41	41,38	83	83,82,38,37	125	125,124,18,17	167	167,161
42	42,41,20,19	84	84,71	126	126,125,90,89	168	168,166,153,151
43	43,42,38,37	85	85,84,58,57	127	127,126		
44	44,43,18,17	86	86,85,74,73	128	128,126,101,99		

但し、漸化式中の bit 、 k_i は、表にある表現を用いて

$$bit = n$$

$$k_i = XNOR from$$

と表せる

2.3 実験方法

実験に用いた python プログラムを次に示す

Listing 1 キャプション 2

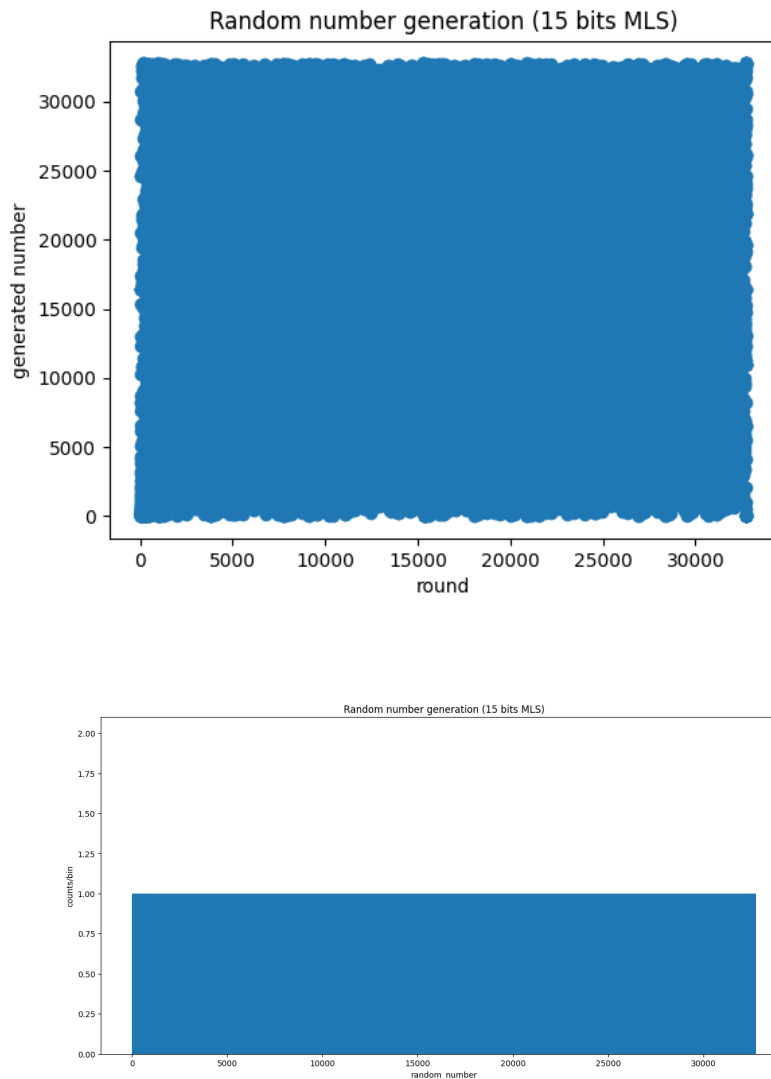
```

1 import matplotlib.pyplot as plt
2
3 feedbacks = [[],[[3,2],[4,3],
4               [5,3],[6,5],[7,6],[8,6,5,4],[9,5],
5               [10,7],[11,9],[12,11,10,4],[13,12,11,8],[14,13,12,2],
6               [15,14],[16,14,13,11],[17,14],[18,11],[19,18,17,14],
7               [20,17],[21,19],[22,21],[23,18],[24,23,22,17]
8               ]
9
10 def calculate_lfsr(initNumber, bits):
11     num = initNumber # initial number
12     maxrounds = 2**bits
13     formater = "0" + str(bits) + "b"
14
15     rands = []
16
17     for i in range(maxrounds):
18         # print(num, "(" , format(num, formater), ")")
19         rands.append(num)
20
21         a = (num << 1) & (maxrounds-1)
22
23         #seems both acceptable, but b=1 would become more complex and difficult to estimate
24         # b = 1
25         b = 0
26
27         for j in range(len(feedbacks[bits])):
28             target = feedbacks[bits][j] - 1
29             b = ((b & 1) ^ (num >> target) & 1) & 1
30
31         num = a+(b&1)
32
33     return rands
34
35 def plot_results_a(rands, bits):
36     nums = range(2**bits)
37     title_str = "Random number generation ({:d} bits MLS)".format(bits)
38     plt.scatter(nums, rands)
39     plt.title(title_str)
40     plt.xlabel("round")
41     plt.ylabel("generated number")
42     plt.show()
43
44 def plot_results_b(rands, bits):
45     title_str = "Random number generation ({:d} bits MLS)".format(bits)
46     plt.figure()
47     plt.hist(rands, bins=2**bits, range=[0,2**bits])
48     plt.title(title_str)
49     plt.xlabel("random_number")
50     plt.ylabel("counts/bin")
51     plt.show()
52
53 def main():
54     init = 3 # change variants here
55     bits = 9 # change variants here
56     # this program can calculate from 3 bits to 24 bits, but calculating 24 bits never finishes ! (due to
57     # the amount of calculation)
58     local_rands = calculate_lfsr(init, bits)
59     plot_results_a(local_rands, bits)
60     plot_results_b(local_rands, bits)
61
62 if __name__ == "__main__":
63     main()

```

2.4 実験結果

このプログラムを 15bit で実行した結果を次に示す

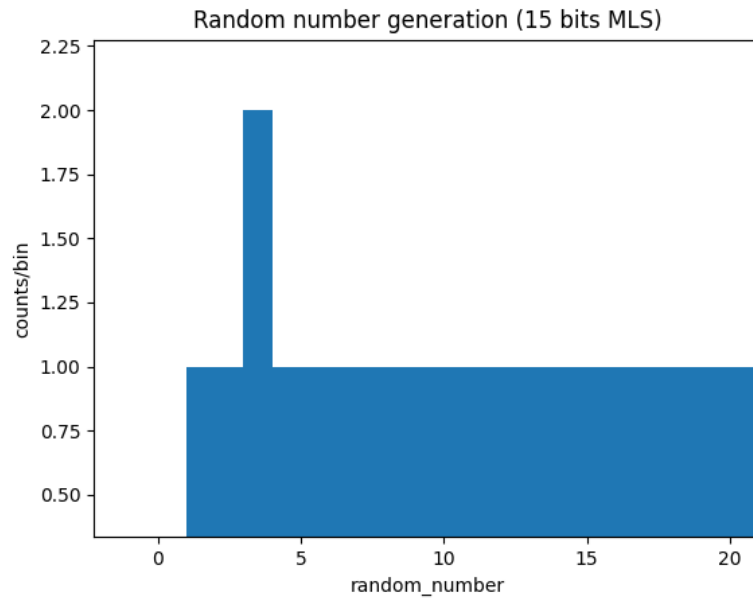


ここに示したヒストグラムのよう、初期値である 3 (ソースコード 1.py の 54 行目で指定した) を除いた全ての数が 1 から 2^{15} まで 1 回ずつのみ現れ、3 のみ 2 回現れている。

3 実験 2

3.1 目的

$Z^d(\in j = (j_1, j_2, \dots, j_d))$ と表現される d 次元格子において、毎回 $2d$ 個の隣接点から 1 点を等確率で選んで進んでいく運動を d 次元単純ランダムウォークと呼ぶ。これはブラウン運動などと共に、統計力学・量子力



学・数理ファイナンスのようなランダムな運動を数学的に記述するモデルのなかで、もっとも基本的なものの一つとして知られている。今回はそのなかでもっとも基礎的な1次元単純ランダムウォークを実行し、粒子の動きを調べることで、確率過程モデルの基礎を確認する。

3.2 理論

t 回目の試行における粒子の座標を x_t とおくと、

$$P(x_{t+1} - x_t = 1) = 1/2$$

$$P(x_{t+1} - x_t = -1) = 1/2$$

を満たすとする (単純ランダムウォーク)

3.3 実験方法

3.3.1 実験 2-4

実験に用いた Python プログラムを次に示す

Listing 2 キャプション 2

```

1 import matplotlib.pyplot as plt
2 import time
3
4
5 feedbacks = [[], [], [], [3, 2], [4, 3],
6              [5, 3], [6, 5], [7, 6], [8, 6, 5, 4], [9, 5],
7              [10, 7], [11, 9], [12, 11, 10, 4], [13, 12, 11, 8], [14, 13, 12, 2],
8              [15, 14], [16, 14, 13, 11], [17, 14], [18, 11], [19, 18, 17, 14],
9              [20, 17], [21, 19], [22, 21], [23, 18], [24, 23, 22, 17]
10            ]
11
12
13 def calculate_lfsr(initNumber, bits):

```

```

14     num = initNumber # initial number
15     maxrounds = 2**bits
16     formater = "0" + str(bits) + "b"
17
18     rands = []
19
20     for i in range(maxrounds):
21         # print(num, "(" , format(num, formater), ")")
22         rands.append(num)
23
24         a = (num << 1) & (maxrounds-1)
25
26         #seems both acceptable, but b=1 would become more complex and difficult to estimate
27         # b = 1
28         b = 0
29
30         for j in range(len(feedbacks[bits])):
31             target = feedbacks[bits][j] - 1
32             b = ((b & 1) ^ (num >> target) & 1) & 1
33
34         num = a+(b&1)
35
36     return rands
37
38
39 def plot_results_a(rands, bits):
40     nums = range(2**bits)
41     title_str = "Random number generation ({:d} bits MLS)".format(bits)
42     plt.scatter(nums, rands)
43     plt.title(title_str)
44     plt.xlabel("round")
45     plt.ylabel("generated number")
46     plt.show()
47
48
49 def plot_results_b(rands, bits):
50     title_str = "Random number generation ({:d} bits MLS)".format(bits)
51     plt.figure()
52     plt.hist(rands, bins=2**bits, range=[0,2**bits])
53     plt.title(title_str)
54     plt.xlabel("random_number")
55     plt.ylabel("counts/bin")
56     plt.show()
57
58
59 def random_walk(rands, bits, checkpoints):
60     x=0
61     pos_at_checkpoints = [0]
62
63     for t in range(1, max(checkpoints)+1):
64         if rands[(t-1)%(2**bits)] & 1 == 0: # this is the equivalent of "rands[] % 2 == 0"
65             x = x+1
66         else:
67             x = x-1
68
69         if t in checkpoints:
70             pos_at_checkpoints.append(x)
71
72     # print(checkpoints)
73     # print(pos_at_checkpoints)
74
75     return pos_at_checkpoints
76
77
78 def main():
79     # this program can calculate from 3 bits to 24 bits, but calculating 24 bits never finishes ! (due to
80     # the amount of calculation)
81
82     # change variants under here
83     bits = 12
84     cycle = 2**bits # but DO NOT CHANGE HERE!
85     # trials = 5
86     trials = 1
87     checkpoints = range(cycle)
88     # checkpoints = [0,200,350,int(0.3*cycle),int(0.5*cycle),int(0.7*cycle),cycle]
89     # checkpoints.sort()
90     markers = ['+', 'x', 'D', 'd']
91     # change variants above here

```

```

92     pos_at_checkpoints = []
93
94     for i in range(trials):
95         init = int(time.time()*100)**2 % (2**bits)
96         rands = calculate_lfsr(init, bits)
97         pos_at_checkpoints = random_walk(rands, bits, checkpoints)
98         plt.plot(checkpoints, pos_at_checkpoints, label = "initial number = {:d}".format(init))
99         # plt.scatter(checkpoints, pos_at_checkpoints, alpha=0.6, marker = markers[i%len(markers)], label =
          "initial number = {:d}".format(init))
100
101     title_str = "Random number generation ({:d} bits MLS)".format(bits)
102     plt.title(title_str)
103     plt.xlabel("time")
104     plt.ylabel("x position")
105     plt.legend()
106     plt.show()
107
108     # plot_results_a(local_rands, bits)
109     # plot_results_b(local_rands, bits)
110
111
112 if __name__ == "__main__":
113     main()

```

3.3.2 実験 2-5

実験に用いた Python プログラムを次に示す

Listing 3 キャプション 2

```

1  import matplotlib.pyplot as plt
2  import time
3
4
5  feedbacks = [[],[],[],[3,2],[4,3],
6               [5,3],[6,5],[7,6],[8,6,5,4],[9,5],
7               [10,7],[11,9],[12,11,10,4],[13,12,11,8],[14,13,12,2],
8               [15,14],[16,14,13,11],[17,14],[18,11],[19,18,17,14],
9               [20,17],[21,19],[22,21],[23,18],[24,23,22,17]
10             ]
11
12
13 def calculate_lfsr(initNumber, bits):
14     num = initNumber # initial number
15     maxrounds = 2**bits
16     formater = "0" + str(bits) + "b"
17
18     rands = []
19
20     for i in range(maxrounds):
21         # print(num, "(" , format(num, formater), ")")
22         rands.append(num)
23
24         a = (num << 1) & (maxrounds-1)
25
26         #seems both acceptable, but b=1 would become more complex and difficult to estimate
27         # b = 1
28         b = 0
29
30         for j in range(len(feedbacks[bits])):
31             target = feedbacks[bits][j] - 1
32             b = ((b & 1) ^ (num >> target) & 1) & 1
33
34         num = a+(b&1)
35
36     return rands
37
38
39 def plot_results_a(rands, bits):
40     nums = range(2**bits)
41     title_str = "Random number generation ({:d} bits MLS)".format(bits)
42     plt.scatter(nums, rands)
43     plt.title(title_str)
44     plt.xlabel("round")
45     plt.ylabel("generated number")
46     plt.show()

```

```

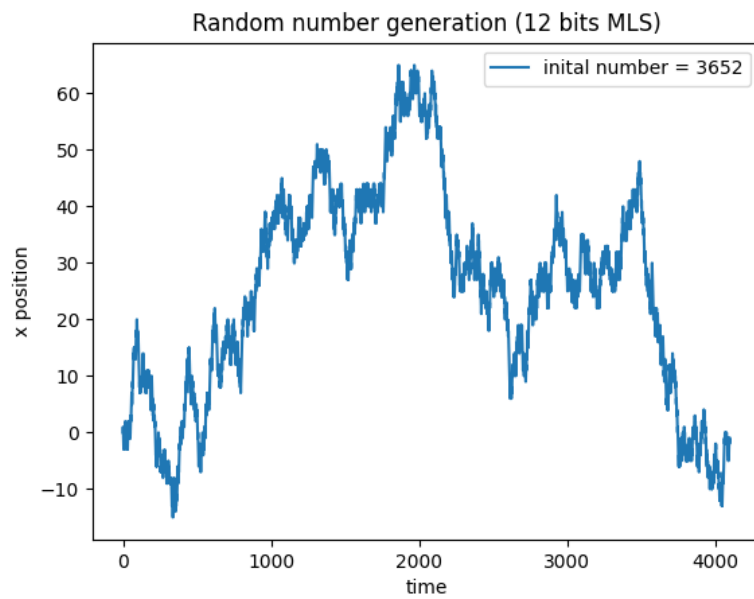
47
48
49 def plot_results_b(rands, bits):
50     title_str = "Random number generation ({:d} bits MLS)".format(bits)
51     plt.figure()
52     plt.hist(rands, bins=2**bits, range=[0,2**bits])
53     plt.title(title_str)
54     plt.xlabel("random_number")
55     plt.ylabel("counts/bin")
56     plt.show()
57
58
59 def random_walk(rands, bits, checkpoints):
60     x=0
61     pos_at_checkpoints = [0]
62
63     for t in range(1, max(checkpoints)+1):
64         if rands[(t-1)%(2**bits)] & 1 == 0: # this is the equivalent of "rands[] % 2 == 0"
65             x = x+1
66         else:
67             x = x-1
68
69         if t in checkpoints:
70             pos_at_checkpoints.append(x)
71
72     # print(checkpoints)
73     # print(pos_at_checkpoints)
74
75     return pos_at_checkpoints
76
77
78 def main():
79     # this program can calculate from 3 bits to 24 bits, but calculating 24 bits never finishes ! (due to
80     # the amount of calculation)
81
82     # change variants under here
83     bits = 12
84     cycle = 2**bits # but DO NOT CHANGE HERE!
85     trials = 5
86     # trials = 1
87     # checkpoints = range(cycle)
88     checkpoints = [0,200,350,int(0.3*cycle),int(0.5*cycle),int(0.7*cycle),cycle]
89     checkpoints.sort()
90     markers = ['+', 'x', 'D', 'd']
91     # change variants above here
92
93     pos_at_checkpoints = []
94
95     for i in range(trials):
96         init = int(time.time()*100)**2 % (2**bits)
97         rands = calculate_lfsr(init, bits)
98         pos_at_checkpoints = random_walk(rands, bits, checkpoints)
99         # plt.plot(checkpoints, pos_at_checkpoints, label = "initial number = {:d}".format(init))
100        plt.scatter(checkpoints, pos_at_checkpoints, alpha=0.6, marker = markers[i%len(markers)], label = "
            initial number = {:d}".format(init))
101        title_str = "Random number generation ({:d} bits MLS)".format(bits)
102        plt.title(title_str)
103        plt.xlabel("time")
104        plt.ylabel("x position")
105        plt.legend()
106        plt.show()
107
108    # plot_results_a(local_rands, bits)
109    # plot_results_b(local_rands, bits)
110
111 if __name__ == "__main__":
112     main()

```

3.4 実験結果

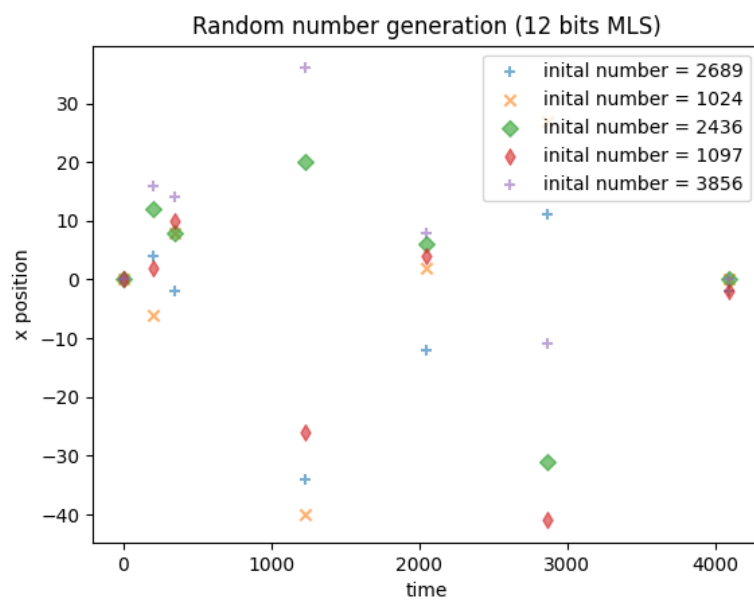
3.4.1 実験 2-4

実験の結果を次に示す



3.4.2 実験 2-5

実験の結果を次に示す



4 実験 3

4.1 目的

4.2 理論

4.3 実験方法

4.3.1 実験 3-1

実験に用いた Python プログラムを次に示す

Listing 4 キャプション 2

```
1 import matplotlib.pyplot as plt
2 import time
3 import numpy as np
4
5
6 feedbacks = [[],[],[],[3,2],[4,3],
7              [5,3],[6,5],[7,6],[8,6,5,4],[9,5],
8              [10,7],[11,9],[12,11,10,4],[13,12,11,8],[14,13,12,2],
9              [15,14],[16,14,13,11],[17,14],[18,11],[19,18,17,14],
10             [20,17],[21,19],[22,21],[23,18],[24,23,22,17]
11             ]
12
13
14 def calculate_lfsr(initNumber, bits):
15     num = initNumber # initial number
16     maxrounds = 2**bits
17     formater = "0" + str(bits) + "b"
18
19     rands = []
20
21     for i in range(maxrounds):
22         # print(num, "(" , format(num, formater), ")")
23         rands.append(num)
24
25         a = (num << 1) & (maxrounds-1)
26
27         #seems both acceptable, but b=1 would become more complex and difficult to estimate
28         b = 1
29         # b = 0
30
31         for j in range(len(feedbacks[bits])):
32             target = feedbacks[bits][j] - 1
33             b = ((b & 1) ^ (num >> target) & 1) & 1
34
35         num = a+(b&1)
36
37     return rands
38
39
40 def plot_results_a(rands, bits):
41     nums = range(2**bits)
42     title_str = "Random number generation ({:d} bits MLS)".format(bits)
43     plt.scatter(nums, rands)
44     plt.title(title_str)
45     plt.xlabel("round")
46     plt.ylabel("generated number")
47     plt.show()
48
49
50 def plot_results_b(rands, bits):
51     title_str = "Random number generation ({:d} bits MLS)".format(bits)
52     plt.figure()
53     plt.hist(rands, bins=2**bits, range=[0,2**bits])
54     plt.title(title_str)
55     plt.xlabel("random_number")
56     plt.ylabel("counts/bin")
57     plt.show()
58
```

```

59
60 def random_walk(rands, bits, checkpoints):
61     x=0
62     pos_at_checkpoints = [0]
63
64     for t in range(1, max(checkpoints)+1):
65         if rands[(t-1)*(2**bits)] & 1 == 0: # this is the equivalent of "rands[] % 2 == 0"
66             x = x+1
67         else:
68             x = x-1
69
70         if t in checkpoints:
71             pos_at_checkpoints.append(x)
72
73     # print(checkpoints)
74
75     return pos_at_checkpoints
76
77
78 def main():
79     # this program can calculate from 3 bits to 24 bits, but calculating 24 bits never finishes ! (due to
80     # the amount of calculation)
81
82     # change variants under here
83     bits = 12
84     cycle = 2**bits # but DO NOT CHANGE HERE!
85     # trials = int(cycle*0.7)
86     trials = 300
87     # checkpoints = range(cycle)
88     checkpoints = [0,20,35,int(0.3*cycle),int(0.5*cycle),int(0.7*cycle),cycle]
89     checkpoints.sort()
90     checkpoint = 2
91     markers = ['+', 'x', 'D', 'd']
92     # change variants above here
93     # initialize the variants
94     all_pos_at_checkpoints = []
95     inits = []
96     # initializing ends here
97
98
99     for i in range(trials):
100         init = int(time.time()*10000)**2 % (2**bits)
101         rands = calculate_lfsr(init, bits)
102         pos_at_checkpoints = random_walk(rands, bits, checkpoints)
103         all_pos_at_checkpoints = np.append(all_pos_at_checkpoints, pos_at_checkpoints, axis=0)
104
105     all_pos_at_checkpoints = np.reshape(all_pos_at_checkpoints, [trials, len(checkpoints)]).astype(np.int64)
106
107     calculating_set = all_pos_at_checkpoints[:, checkpoint]
108     plt.scatter(list(range(trials)), calculating_set)
109     plt.xlabel("trial id")
110     plt.ylabel("x position (@ t = {:d})".format(checkpoints[checkpoint]))
111     plt.title("x position vs. trials as of {:d} bits".format(bits))
112     plt.show()
113     # plot_results_a(local_rands, bits)
114     # plot_results_b(local_rands, bits)
115
116
117 if __name__ == "__main__":
118     main()

```

4.3.2 実験 3-2,3-3

実験に用いた Python プログラムを次に示す

Listing 5 キャプション 2

```

1 import matplotlib.pyplot as plt
2 import time
3 import numpy as np
4
5
6 feedbacks = [[],[],[],[3,2],[4,3],
7              [5,3],[6,5],[7,6],[8,6,5,4],[9,5],

```

```

8         [10,7],[11,9],[12,11,10,4],[13,12,11,8],[14,13,12,2],
9         [15,14],[16,14,13,11],[17,14],[18,11],[19,18,17,14],
10        [20,17],[21,19],[22,21],[23,18],[24,23,22,17]
11    ]
12
13
14    def calculate_lfsr(initNumber, bits):
15        num = initNumber # initial number
16        maxrounds = 2**bits
17        formater = "0" + str(bits) + "b"
18
19        rands = []
20
21        for i in range(maxrounds):
22            # print(num, "(" , format(num, formater), ")")
23            rands.append(num)
24
25            a = (num << 1) & (maxrounds-1)
26
27            #seems both acceptable, but b=1 would become more complex and difficult to estimate
28            b = 1
29            # b = 0
30
31            for j in range(len(feedbacks[bits])):
32                target = feedbacks[bits][j] - 1
33                b = ((b & 1) ^ (num >> target) & 1) & 1
34
35            num = a+(b&1)
36
37        return rands
38
39
40    def plot_results_a(rands, bits):
41        nums = range(2**bits)
42        title_str = "Random number generation ({:d} bits MLS)".format(bits)
43        plt.scatter(nums, rands)
44        plt.title(title_str)
45        plt.xlabel("round")
46        plt.ylabel("generated number")
47        plt.show()
48
49
50    def plot_results_b(rands, bits):
51        title_str = "Random number generation ({:d} bits MLS)".format(bits)
52        plt.figure()
53        plt.hist(rands, bins=2**bits, range=[0,2**bits])
54        plt.title(title_str)
55        plt.xlabel("random_number")
56        plt.ylabel("counts/bin")
57        plt.show()
58
59
60    def random_walk(rands, bits, checkpoints):
61        x=0
62        pos_at_checkpoints = [0]
63
64        for t in range(1, max(checkpoints)+1):
65            if rands[(t-1)%(2**bits)] & 1 == 0: # this is the equivalent of "rands[] % 2 == 0"
66                x = x+1
67            else:
68                x = x-1
69
70            if t in checkpoints:
71                pos_at_checkpoints.append(x)
72
73            # print(checkpoints)
74
75        return pos_at_checkpoints
76
77
78    def main():
79        # this program can calculate from 3 bits to 24 bits, but calculating 24 bits never finishes ! (due to
80        # the amount of calculation)
81
82        # change variants under here
83        bits = 12
84        cycle = 2**bits # but DO NOT CHANGE HERE!
85        trials = int(cycle*0.7)
86        # trials = 1

```

```

86 # checkpoints = range(cycle)
87 checkpoints = [0,int(0.0375*cycle),int(0.075*cycle),int(0.15*cycle),int(0.3*cycle),int(0.5*cycle),int
      (0.7*cycle),int(0.85*cycle),int(0.925*cycle),int(0.9625*cycle),cycle]
88 checkpoints.sort()
89 markers = ['+', 'x', 'D', 'd']
90 # change variants above here
91
92 all_pos_at_checkpoints = []
93 means = []
94 stds = []
95
96 for i in range(trials):
97     init = int(time.time()*1000000) % (2**bits)
98     rand = calculate_lfsr(init, bits)
99     pos_at_checkpoints = random_walk(rand, bits, checkpoints)
100     all_pos_at_checkpoints = np.append(all_pos_at_checkpoints, pos_at_checkpoints, axis=0)
101
102 all_pos_at_checkpoints = np.reshape(all_pos_at_checkpoints, [trials, len(checkpoints)]).astype(np.int64
      )
103
104 for i in range(len(checkpoints)):
105     calculating_set = all_pos_at_checkpoints[:, i]
106     plt.hist(calculating_set, histtype = "step", label = "steps = {:d}".format(checkpoints[i]))
107
108     mean = np.mean(calculating_set)
109     means = np.append(means, mean)
110     std = np.std(calculating_set)
111     stds = np.append(stds, std)
112
113     print(means, stds)
114
115 plt.title("x position counts vs. x position")
116 plt.xlabel("x positions at certain times")
117 plt.ylabel("counts/bin")
118 plt.yscale("log")
119 plt.legend()
120 plt.show()
121
122 plt.clf()
123 plt.close()
124
125 plt.title("times vs. means at that times")
126 plt.xlabel("times")
127 plt.ylabel("means of x positions")
128 plt.scatter(checkpoints, means)
129 plt.show()
130
131 plt.clf()
132 plt.close()
133
134 plt.title("times vs. standard deviations at that times")
135 plt.xlabel("times")
136 plt.ylabel("standard deviations of x positions")
137 plt.scatter(checkpoints, stds)
138 plt.show()
139
140
141 if __name__ == "__main__":
142     main()

```

4.4 実験結果

4.4.1 実験 3-1

実験の結果を次に示す

4.4.2 実験 3-2,3-3

実験の結果を次に示す

