Intermediates-Specific Technical Change, Structural Transformation, and Growth

Davide M. Difino*

Goethe University Frankfurt

October 25, 2025

Job Market Paper – Latest version here

As economies develop, they increasingly rely on services intermediates in production. While recent research highlights the importance of these shifts for aggregate dynamics, the mechanisms underlying them remain understudied. This paper quantifies the role of biased, intermediates-specific technical change as a key mechanism behind the rise of services intermediates – and its implications for broader structural transformation and aggregate growth – using a two-sector model with intermediates-specific technical change and a full input-output structure. Calibrated with U.S. data, the model indicates that input-specific technical change has been driving the majority of the rise in services intermediates in the services-producing sector, but not in the goods-producing sector. This heterogeneity accounts for both the stagnation of value-added productivity in services and several aggregate trends: almost half of the increase in the services' share of intermediates and employment, roughly one-fifth of the rise in final expenditure shares, and approximately a 25% reduction in aggregate real GDP growth relative to an unbiased counterfactual. These findings establish biased intermediates-specific technical change as a central driver of the evolving production structure, the aggregate productivity slowdown, and structural transformation.

Keywords: Biased Technical Change, Structural transformation, Growth

JEL classification: E13, O41, O33

*Email: difino@wiwi.uni-frankfurt.de. I thank my advisors, Georg Duernecker and Leo Kaas, for their invaluable guidance. I am also grateful to Alessio Moro and Ákos Valentinyi for their support and advices during my visits to the University of Cagliari and Manchester University. I thank Julieta Caunedo, Nick Pretnar, Markus Trunschke, and seminar participants at Goethe University Frankfurt and the University of Cagliari for helpful comments. All errors are my own.

1 Introduction

A well-established empirical regularity is that, as economies develop, economic activity shifts toward services. This phenomenon, commonly referred to as "structural change," has important implications – for example, for aggregate growth (Duernecker et al., 2024), wage stagnation (Ngai and Sevinc, 2025), and the transmission of monetary policy (Galesi and Rachedi, 2019) – and thus spurred a lively literature in the last decade. This literature on structural change focuses primarily on the rising service shares in value-added, consumption, employment, and investment. On the other hand, the observed increase in the use of services as intermediate inputs is relatively understudied. Recently, it started receiving attention for its relevance to aggregate dynamics. For example, Sposi (2019) shows how differences in intermediate service intensity drive differences in comparative advantage and terms of trade between advanced and developing economies. In contrast, Baqaee and Farhi (2019) argues that intermediate demand shapes aggregate growth and amplifies the effects of sectoral productivity shocks. Furthermore, Rubbo (2023) shows that the composition of intermediate demand plays a key role in the transmission of monetary policy. However, while the implications of changes in the structure of intermediate demand are well studied, the factors driving the rise in service intermediates are less well understood. This paper contributes to the latter question by asking: what drives the observed expansion of intermediate services?

The standard mechanism in the literature links the rise of services to development through the growing relative price gap between goods and services (e.g., Ngai and Pissarides, 2007; Herrendorf et al., 2014; Swikecki, 2017 for broader structural change, Gaggl et al., 2023 for structural change in intermediates). This paper argues that substitution alone cannot account for the observed increase in the service share of intermediate inputs and proposes an additional mechanism: biased intermediates-specific technical change. The argument is straightforward: technological progress not only raises efficiency but also reshapes the mix of intermediates on which production relies (as studied, for example, by Acemoglu and Azar, 2020). Innovations often require new or enhanced service intermediates alongside existing goods intermediates. For instance, advances in connected vehicle technologies increase the use of external software development and cybersecurity services in automotive manufacturing; the growth of e-commerce and digital platforms drives retail firms to expand expenditures on cloud hosting, data analytics, and logistics services; and precision agriculture technologies complement greater reliance on data, external verification, and farm management services[1]. These cases illustrate that technological progress can generate rising demand for service intermediates, raising the share of services in production. It also motivates a central question: has technological change driven

[1] For example, Getahun et al. (2024)

production toward greater demand for service intermediates? In the terminology of Acemoglu (2002), is technical change services intermediates-biased?

This paper proceeds in three main steps. The first establishes empirical regularities regarding the rise in service intermediates. The second develops a structural framework that rationalizes these facts in the light of biased intermediates-specific technical change. Finally, the model, calibrated with U.S. data, is used to run a counterfactual to quantify the role of biased intermediates-specific technical change upon structural change and aggregate economic growth.

The empirical analysis begins by documenting the evolution of the service share of total intermediates using data from the World Input-Output Database (WIOD), covering 40 countries, 11 industries, and 50 years. The objective is to characterize how the structure of intermediate demand has changed over time and to isolate the mechanisms behind the observed rise in service intermediates. First, I ask whether the increase in the service share of total intermediates arises from industries becoming more service-intensive or from a reallocation of aggregate intermediate demand toward service-intensive industries. A shift-share decomposition separates these two channels. The results show that both within-industry increases in service intensity and between-industry reallocations contribute to the aggregate rise: a result in line with Gaggl et al. (2023). However, when decomposition is performed separately for the goods-producing and services-producing sectors, the reallocation component within each broad sector is negligible. The aggregate composition effect, therefore, primarily reflects reallocation between the goods-producing and services-producing sectors rather than reallocation between sub-sectors. Secondly, the empirical analysis highlights systematic heterogeneity between the two sectors. On average, the services-producing sector exhibits higher service-intermediate intensity than the goods-producing sector. Moreover, the elasticity of the service share of intermediates with respect to sectoral labor productivity is larger in the services-producing sector, suggesting that productivity increases are associated with larger growth in service intermediates use among service producers. Finally, I test whether relative prices – the standard mechanism emphasized in models of structural change – can explain the observed correlation between productivity and service-intermediate intensity. Panel regressions controlling for relative prices show that the log ratio of service-to-goods intermediates remains positively and significantly correlated with sectoral value added per worker. Hence, relative prices alone cannot account for the correlation between productivity and service intensity. Taken together, these results yield three stylized facts. First, the aggregate composition effect largely reflects reallocation between broad sectors rather than within them. Second, the goods-producing and services-producing sectors differ systematically in service-intermediate intensity and in their responsiveness to productivity. Third, relative prices cannot fully explain the link between productivity and service intensity. These findings motivate a parsimonious two-sector framework that captures the key mechanisms driving the rise in service intermediates –within-sector intensification and cross-sector reallocation – while allowing for heterogeneity across sectors. The residual correlation between productivity and service intensity, unexplained by prices, points to an additional mechanism: intermediates-specific technical change.

To interpret the empirical evidence, the paper develops a two-sector general-equilibrium model where each sector produces its output – either goods or services – by combining labor with a CES aggregate of goods and services intermediates. Technical change is intermediates-specific, allowing efficiency to evolve differently for goods and service intermediates, and may also differ across sectors. This structure captures both substitution between intermediates driven by relative prices and shifts in the underlying technology that make certain intermediates more productive inputs in production. The model delivers a closed-form expression for the log service share of intermediates in each sector, which can be decomposed into two additive components: the standard substitution effect and a novel ”bias” term. The substitution effect reflects the standard price-based mechanism highlighted in the structural-change literature: as the relative price of services changes, firms adjust their intermediate mix according to intermediate complementarity. The second term captures how differential changes in intermediates-specific productivities (”bias” in Acemoglu, 2002's terminology) affect intermediate composition. This decomposition breaks down the observed changes in service intensity into two forces: changes in relative prices and evolving intermediates-specific technical bias. The model's input-output structure allows sector-specific technical bias to affect not only the aggregate intermediate demand but also relative prices. Via changes in relative prices, biased intermediates-specific technical change affects the service shares in value added, employment, and final expenditure shares.

The model is estimated using data for the United States. Production parameters are identified through structural production-function estimation following the approach of León-Ledesma et al. (2010). Monte Carlo simulations confirm the reliability of the estimation procedure in recovering underlying parameters. Estimation results show that technical change is service-biased in the services-producing sector and approximately neutral in the goods-producing sector. In other words, technological progress in services has been directed toward raising the efficiency of service intermediates relative to goods intermediates. Growth-accounting exercises further indicate that this bias is a major source of the observed divergence in sectoral (value-added) productivity growth: service-biased technical change in services slows measured labor-productivity growth in that sector. Consequently, biased technical change amplifies the productivity gap between the goods- and services-producing sectors. When the estimated parameters are used to simulate the model, it closely replicates the historical evolution of service intensity within both sectors and the observed rise in the service share of final expenditures.

To quantify the contribution of intermediates-specific technical change, I run a counterfactual experiment in which technical change is held constant across intermediates. The counterfactual paths of service intensity, employment, and final expenditures are compared to their fitted counterparts from the calibrated model. The results show that, relative to this no-bias baseline, service-biased technical change accounts for over half of the observed increase in the service share of aggregate intermediates in the United States between 1965 and 2014. It also explains more than half of the rise in the service share of employment and roughly 20 percent of the increase in final expenditures and value-added shares. These magnitudes underscore the quantitative relevance of intermediates-specific technical change as a driver of structural transformation. At the same time, the model implies that biased intermediates-specific technical change slows down aggregate economic growth: service-biased technical change in the services-producing sector accounts for about one-quarter of the observed shortfall in aggregate GDP relative to the unbiased counterfactual. The intuition is straightforward: services-biased technical change reorients total output toward the service-producing sector, which is characterized by lower productivity growth.

This paper contributes to three strains of literature. First, it contributes to the literature on input-output composition and the rise of service intermediates. Berlingieri (2013) documents the growing role of professional and business services in U.S. production. In contrast, Sposi (2019) shows how cross-country differences in intermediates can explain the hump-shaped structure of manufacturing's share of value added. By showing that within-sector reallocations dominate in both goods and services, this paper provides new evidence on the microstructure of intermediates shifts. Second, it contributes to the structural transformation literature. Whereas most studies emphasize changes in value-added, consumption, or employment shares (e.g., Kongsamut et al., 2001; Herrendorf et al., 2014; Herrendorf et al., 2021), this paper focuses on structural change in intermediates, complementing recent work by Valentinyi (2021) and Gaggl et al. (2023). It introduces biased technical change as a new channel linking intermediate use to sectoral transformation–distinct from the relative-price and preference mechanisms emphasized in prior work (e.g., Ngai and Pissarides, 2007; Boppart, 2014; Comin et al., 2021). Third, it contributes to the production-network literature, which studies how shocks and productivity changes propagate through input-output linkages (e.g., Acemoglu et al., 2016; Baqaee and Farhi, 2018 and 2019; Miranda-Pinto, 2021) and affect aggregate growth (Acemoglu and Azar, 2020). There, the role of intermediate and sector-specific technical change within networks remains understudied. This work demonstrates how incorporating intermediates-specific productivity growth into a comprehensive input-output framework reveals the impact of biased intermediates-specific technical change on broader structural transformation and aggregate growth.

The remainder of the paper proceeds as follows. Section 2 presents the data, decomposition, and price regressions. Section 3 develops the theoretical framework and characterizes equilibrium. Section 4 details the estimation and calibration. Section 5 reports the quantitative and counterfactual results. Section 6 discusses two extensions to the baseline model: non-homothetic preferences and capital accumulation. Finally, section 7 concludes.

2 Cross-country empirical evidence

2.1 Data

The dataset used in this section is constructed from the harmonized World Input-Output Database (WIOD). The data cover 40 countries and 22 or 54 industries (depending on vintage). While the WIOD tables also provide linkages between countries, we abstract from international trade by focusing on total intermediate inputs used by each country-sector-year tuple, irrespective of their provenance. Industry-level ”use” and ”make” tables are joined across the two WIOD vintages (long run, covering 1965-2000, and short run, covering 1999-2014), with industry definitions harmonized to match 12 consistent industries, which can be associated with a broad two-sector classification (goods and services), as shown in Table 1, using cumulative chain-linking. Nominal values are deflated using sectoral price indices, which are themselves obtained through cumulative chain-linking and normalized to 1 in the base year of 1965. From these, we compute real input–output quantities and corresponding price series for goods, services, and final demand.

Isic3 code	Broad sector	Description
AtB	Goods	Agriculture, Hunting, Forestry, and Fishing
C	Goods	Mining and Quarrying
D	Goods	Total Manufacturing
E	Goods	Electricity, Gas and Water Supply
F	Goods	Construction
G	Services	Wholesale and Retail Trade
H	Services	Hotels and Restaurants
I	Services	Transport, Storage, Post and Telecommunications
J	Services	Financial Intermediation
K	Services	Real Estate, Renting and Business Activities
LtQ	Services	Community Social and Personal Services

Table 1: WIOD Sector classification

2.2 Which sub-sectors drive intermediate service demand?

Is the demand for service intermediates driven by the growth of some service-intensive industry? Or is it a manifestation of a broad increase in service intensity across all industries? Wedecompose changes in the log service share to goods shares into ”within” and ”between” components. Specifically, let’s define the aggregate services input share as

$$S_{ct} = \frac{\sum_j x^s_{ict}}{\sum_j x^s_{ict} + x^g_{ict}} = \sum_i \underbrace{\frac{x^s_{ict}}{x^s_{ict} + x^g_{ict}}}_{\chi_{ict}} \cdot \underbrace{\frac{x^s_{ict} + x^g_{ict}}{\sum_j x^s_{ict} + x^g_{ict}}}_{\omega_{ict}}$$

where ω_{ict} represents sector i’s share of total intermediate input and χ_{ict} denotes the services input intensity of sector j. A (mid-point) approximation of the chain rule allows us to decompose the change in service share of aggregate inputs as

$$\Delta \ln S_{ct} \approx \underbrace{\sum_j \bar{\omega}_{jct} \Delta \chi_{jct}}_{\text{"Within"}} + \underbrace{\sum_j \bar{\chi}_{jct} \Delta \omega_{jct}}_{\text{"Between"}}$$

The decomposition allows us to disentangle whether the rise in the services’ share of total intermediate demand stems from individual industries becoming more services-intensive (”within”) or from structural shifts toward inherently services-intensive sectors (”between”, or, equivalently, composition effect). Table 6 reports a shift– share decomposition of the rise in the services share of intermediate demand. When all industries are considered – that is, when we examine the role of all industries in determining total intermediate demand – both the ”between” and ”within” components are substantial, accounting for approximately 40% and 60% respectively of the average rise in the services’ share of total intermediates across countries, as illustrated in the leftmost boxplot in figure 1. That is, even if most of the increase in the services’ share of total intermediates is driven by industries becoming more service-intensive, there is a sizable reallocation toward more service-intensive sectors. For example, in the United States (see appendix B for country-level results), roughly 0.43 of the increase in demand for services is captured by the between component and about 0.57 by the within component, with a negligibly small residual. This result aligns with the empirical findings of Gaggl et al. (2023).

Literature on structural transformation often focuses on the reallocation between broad sectors, such as goods and services. Indeed, this paper begins with an examination of the rise of service intermediaries. Along with the well-known reallocation toward services in final demand and value-added, it follows that total output moves toward services. A question follows: how much of the

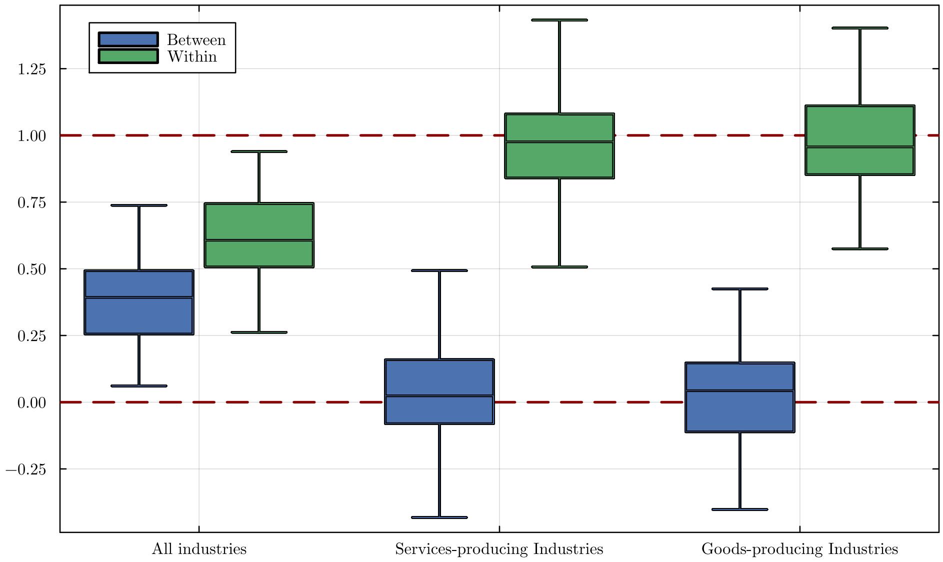

Figure 1: Within-Between Decomposition (all country average, 1965-2014)

reallocation highlighted in the previous paragraph is due to structural reallocation between broad sectors? As is shown in the two remaining boxplots in figure 1, (almost) all of it. In fact, replicating the analysis above by broad sector, that is equiring on the industries' role upon the service share of **sectoral** total intermediates, shows that the overwhelming majority of the reallocation is driven by increases in industries' service intensity, and reallocation within industries is negligible or, sometimes, negative (that is, reallocation moves toward good-intensive industries). As shown in figure 1, this holds for both goods and the services-producing sectors. For example, in the U.S., reallocation between goods-producing industries *reduced* the services' share of sectoral intermediate by 25%, implying that the observed change (approximately seven percentage points) is entirely (and more) driven by industries becoming more services-intensive. The same goes for the services-producing sector, where the rise in the service share of sectoral intermediates (approximately 17 percentage points) is driven for a very large majority ($\sim 95\%$) by the within effect, reallocation accounting for only the residual 5%.

From the data, we can derive two key conclusions. First, the growth in the services share of intermediate inputs is *not* primarily the artifact of a small number of service-intensive industries expanding at the expense of all others. Rather, almost every industry within each broad sector has become more service-intensive over time. Furthermore, while both sectors become more service-intensive (that is, the service share in the total intermediates used by the sector rose), structural transformation between broad sectors accounts for the large majority of the "between" reallocation between industries.

Secondly, both broad sectors - as shown in figure 2 - show a positive correlation between the services'

Figure 2: Structural change in intermediates by broad sector

share of sectoral intermediates and *sectoral* value-added per worker. That is, as sectors become more productive (net of intermediate use, as per the definition of value-added), they use more services in their productive process. In other words, there is a structural transformation toward services in the demand for intermediates from each sector. However, the correlation is heterogeneous between sectors, showing a stronger correlation (0.07 vs 0.03, both highly statistically significant) and a smaller variance in the services-producing sector compared to the goods-producing one. Furthermore, from the y-axis of figure 2 we can see that the services-producing sector is, generally speaking, more service-intensive compared to the goods-producing one. In fact, while in the least productive country-year pair, we have that in the services-producing sector $\sim$ 40% of all intermediates are services, in the goods-producing sector the share is approximately 10%.

These facts together imply that a two-sector model with a goods-producing sector and a services-producing one parsimoniously captures the key facts of structural transformation in intermediates: 1. structural change in intermediates happens at the sector level, 2. sectoral reallocation happens at the sector level, 3. Sectors exhibit heterogeneity in service intensity levels and correlations with productivity.

2.3 Can relative prices explain the rise of service intermediates?

The literature on structural transformation provides a key mechanism to explain why demand moves across categories: the substitution effect. The idea is that, as relative prices change (which, due to supply-side mechanisms, such as heterogeneous productivity growth, tend to be correlated with development), so does demand, due to complementarity. How agents react to price changes depends on their willingness to substitute the relatively more expensive good with the now relatively cheaper one. Works such as Ngai and Pissarides (2007) identify the substitution effect as one of the two mechanisms driving the rising share of services in final demand. In the literature that tackles the demand of intermediates from the producing sector, such as Valentinyi (2021) and Gaggl et al. (2023), it is the only mechanism explored. We ask: how well can relative prices explain the observed service share in (sectoral) intermediate demand?

Sector:	Goods			Services		
	Services share of sectoral inputs (log)					
	(1)	(2)	(3)	(4)	(5)	(6)
Relative Prices (log, Services/ Goods)	0.213***	0.216***	0.191***	0.138***	0.112***	0.100***
	(0.020)	(0.022)	(0.023)	(0.012)	(0.013)	(0.012)
GDP per worker (log)		-0.004			0.022***	
		(0.008)			(0.005)	
Sect. value added per worker (log)			0.025*			0.072***
			(0.011)			(0.006)
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
N	1,181	1,181	1,181	1,181	1,181	1,181
R^2	0.924	0.924	0.924	0.925	0.926	0.933
Within-R^2	0.091	0.091	0.095	0.112	0.127	0.215

Table 2: Panel regression of services' share of total sectoral intermediates

Table 2 reports panel regressions of the log ratio of service to goods inputs on relative prices, controlling for country and year fixed effects. Columns (1) and (4) include only relative prices, estimated separately for the goods- and services-producing sectors. In both cases, the coefficients are positive, precisely estimated, and highly significant: higher relative service prices are associated with a larger service share of sectoral intermediates. This positive correlation is consistent with the stylized fact of structural transformation and, as shown in Section 3, implies that intermediate inputs are gross complements. Adding GDP per worker as a control in Columns (2) and (5) leaves the coefficient on relative prices positive and significant. Even after accounting for relative prices, richer countries use a higher share of services in production–particularly in the services-producing sector. Columns (3) and (6) replace aggregate income with sectoral productivity, measured by value added per worker. The pattern remains: the coefficients on relative prices and productivity are positive and statistically significant in both sectors, and model fit improves notably for the services-producing sector, as indicated by the increase in the within-R^2. In summary, more productive sector-country pairs use relatively more services as intermediate inputs even after controlling for relative prices. This residual correlation suggests that this mechnanism alone cannot account for the observed rise in service intermediates – pointing to an additional mechanism beyond relative price effects.

To provide additional evidence, let's estimate residualized measures of sectoral outcomes in the style of Comin et al. (2021). Specifically, for each sector $s \in \{\text{Goods}, \text{Services}\}$ and country c, we regress the log of sectoral intermediate intensity – defined as the share ratio between services and goods intermediate, in nominal terms – on relative input prices and country fixed effects:

$$\ln \frac{\text{Input}_{cst}^{Services}}{\text{Input}_{cst}^{Goods}} = \beta_s \text{RelPrice}_{cst} + \gamma_{cs} + \varepsilon_{cst}, \qquad \forall s \in [Goods, Services],$$

where $\text{RelPrice}_{ct} = \ln p_{ct}^{Services} - \ln p_{ct}^{Goods}$ is the relative price of service to goods inputs, and γ_c denotes country fixed effects. We obtain fitted residuals $\hat{\varepsilon}_{cst}$ for each outcome $X \in \{\text{VA}, \text{Emp}\}$ using:

$$\ln X_{cst} = \beta_s^X \text{RelPrice}_{cst} + \gamma_{cs}^X + \varepsilon_{cst}^X, \qquad \forall s \in [Goods, Services].$$

We then construct residualized service input shares and value-added per worker, defined as the residual component of the dependent variable after controlling for relative prices and country effects. Finally, we plot the residualized input shares against the residualized sectoral outcomes. The results, shown in figure 3, show a statistically significant residual correlation of service intensity and value-added for the services-producing sectors. In the goods-producing sector, the residual correlation remains statistically significant, albeit small.

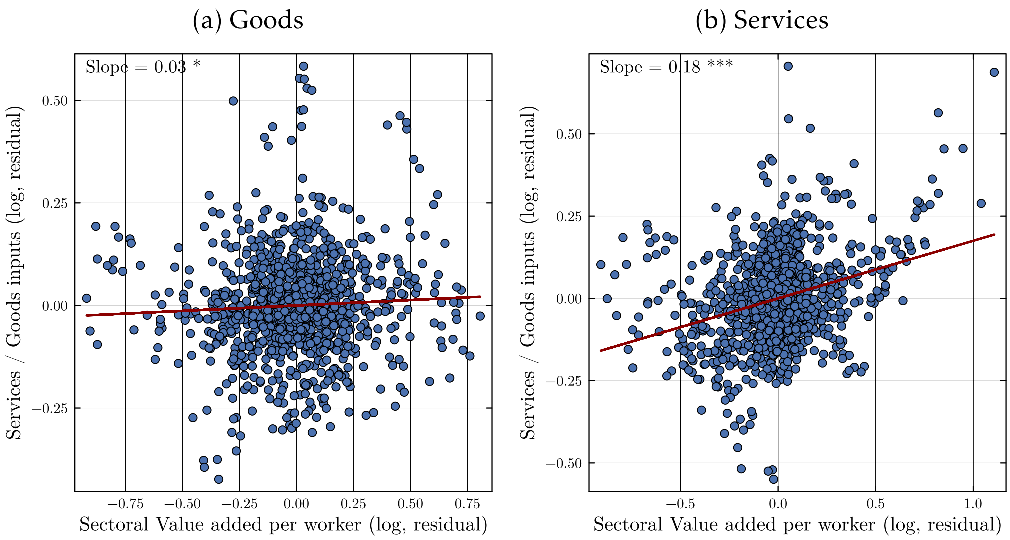

Figure 3: Residual correlation between services intermediates and sectoral productivity

Overall, the analysis in this section provides tentative evidence that relative prices, although important, do not fully account for the rise of intermediate services. In the regression, the explanatory power of the model increases once income and productivity are taken into account, and the within-R^2 values indicate that a substantial share of the variation remains unaccounted for. The partialing-out procedure shown in Figure 3 confirms the hypothesis. These findings suggest that factors beyond relative prices may play a role in the observed rise in the importance of services as intermediate inputs; in particular, there is a residual correlation between service intensity and sectoral productivity.

How can productivity affect intermediate composition? In the next section, we will explore theoretically how technical change might drive the observed rise in the service share of intermediates.

3 Theoretical Model

3.1 Technology

In the economy, there are two competitive sectors, "Goods" (g) and "Services" ("s"), each producing a single differentiated product. Each sector solves a standard cost minimization problem, where the cost function is:

$$S_{i,t}(y_{i,t}; A^{go}_{i,t}, \mathbf{a}_{i,t}, \mathbf{p}_t, \mathbf{r}_t) = \left\{ \arg\min_{\mathbf{x}_{i,t}, \boldsymbol{k}_{i,t}} \mathbf{p}^T_{i,t}\mathbf{x}_{i,t} + \mathbf{r}^T_t \boldsymbol{k}_{i,t} : \quad y_{i,t} = A^{go}_{i,t} f\left(\mathbf{a}_{i,t} \circ \mathbf{x}_{i,t}, \boldsymbol{k}_{i,t}\right) \right\} \tag{1}$$

where $A^{go}_{i,t}$ is (gross output's) total factor productivity (labor productivity) for sector i, while $\mathbf{a}_{i,t}$ is a vector of input-augmenting productivities a la Acemoglu (2002). Finally, $\mathbf{k}$ is a vector of perfectly mobile factors. Notice that due to perfect mobility of factors and intermediates, input prices ($\mathbf{p}$ and $\mathbf{r}$) equalize across sectors. All productivity terms ($\mathbf{A}^{go}_i$ and $\mathbf{a}_i$) are assumed to be exogenous. As standard in the macroeconomics literature working with intermediates (e.g. Moro, 2012, and Gaggl et al., 2023, for some closely related work), we assume the sector-specific production function features a composite input structure, where intermediate goods are aggregated into a single aggregate intermediate input M, which then enters the production function alongside factors –

$$y_{i,t} = A^{go}_{i,t} f_i[M_i(\mathbf{a}_{i,t} \circ \mathbf{x}_{i,t}), \mathbf{k}_{i,t}]$$

– via a well-behaved function $f_i(\cdot)$. The intermediates aggregator M takes a CES form:

$$M\left(\mathbf{a}_{i,t} \circ \mathbf{x}_{i,t}\right) = C_i \cdot \left[\gamma_i (a_{ig,t} x_{ig,t})^{\frac{\sigma_i - 1}{\sigma_{i,t}}} + (1-\gamma_i)(a_{is,t} x_{is,t})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}.$$

Where the notation goes as follows: $x_{\text{"use sector" "producer sector",time}}$.: For example, $x_{sg,t}$ represents the quantity of output of the sector "g" used in the productive process of sector "s" at time t. Solving the optimization problem in equation 1 yields the standard CES demand function. The implied relative demand of intermediates is summarized in proposition 1.

Proposition 1. *The (log) sectoral intermediate services intensity can be written as*

$$\ln \frac{p_{is,t}x_{is,t}}{p_{ig,t}x_{ig,t}} = \sigma_i \ln\left(\frac{1-\gamma_i}{\gamma_i}\right) + \underbrace{(1-\sigma_i)(\ln a_{ig,t} - \ln a_{is,t})}_{\text{bias in technical change } \equiv \ln \phi_{i,t}} + \underbrace{(1-\sigma_i)(\ln p_{is,t} - \ln p_{ig,t})}_{\text{substitution effect}}, \quad \forall i \in [g,s]. \tag{2}$$

Proposition 1 states that the (log relative) demand for services intermediates can be decomposed into two main drivers: the standard substitution effect studied by the literature and bias in technical change. As in Acemoglu (2002) (which, however, dealt with factor-specific productivities), the mechanism behind the two drivers is the same, namely input complementarity, as shown by the dependency of both upon the magnitude of the substitution parameter σ_i: if intermediates are "gross-complements" ($\sigma < 1$), then intermediates demand is larger for the more expensive - via substitution effect - and the *least* productive intermediate. If intermediates are "gross-substitutes" ($\sigma > 1$), then the opposite is the case: demand is larger for the cheapest and most productive sector.

In a certain way, bias in technical change and the substitution effect are two sides of the same coin. In the standard multisector model (e.g. Ngai and Pissarides, 2007), relative prices are inversely proportional to sectoral productivity. That is, prices will be lower in the more productive sectors. Thus, via substitution, demand is larger for the least productive sector if varieties are gross complements: substitution compares the "external" productivity (TFP), while technical bias compares the "internal" productivity of the intermediates.

Henceforth, we will assume that intermediates are gross complements in all sectors, formalized in assumption 3.1.

Assumption 3.1. *Goods and services intermediates are gross complements in all sectors:*

$$\sigma_i < 1 \qquad \forall i \in [g,s] \tag{3}$$

Later, in section 4, it will be shown that this is indeed the case in the U.S.: in fact, the elasticity of substitution between intermediates is close to 0 in both the goods-producing and the services-producing sectors. That is, the intermediates aggregator M takes a Leontief production function. However, we will abstain from imposing a Leontief aggregator. While it would simplify the math, it would also obfuscate the role of the elasticity of substitution in driving the demand for intermediates. Therefore, for generality's sake, we will maintain the CES functional form for M. For the remainder of this work, it will be convenient to reparametrize the model to highlight the role of technical bias. Therefore, without loss of generality, as in Lashkari et al. (2024), let's re-parameterize the productivity terms as:

$$\phi_{i,t} = \left(\frac{a_{is,t}}{a_{ig,t}}\right)^{1-\sigma} \qquad \text{and} \qquad \theta_{i,t} = a_{ig,t} \cdot C_i, \qquad \forall i \in [g,s].$$

That is, $\phi_{i,t}$ represents how much services-biases is technology in sector i: from equation 2 notice that $\phi > 1$ implies that technical change is **services-biased**, if $\phi = 1$ then technical change is said to be **input-neutral** (or "unbiased") and, finally, if $\phi < 1$, then technical change is **goods-biased**. Again, notice that, as in equation 2, ϕ depends upon the value of the elasticity of substitution between intermediates (σ). If intermediates are gross complements ($\sigma < 1$), then an increase in the goods-specific productivity compared to the service-specific one will increase the service share of intermediate inputs for sector i. On the other hand, if intermediates are gross substitutes ($\sigma > 1$), then an increase in the productivity ratio will lead to a decrease in the service share. Finally, θ can be interpreted as the "total intermediate" productivity level.

Here we will make the following simplifying assumption:

Assumption 3.2. *The economy features only one factor, labor (ℓ).*

In the section 6, we expand the model to account for capital accumulation and its use in the productive process. As shown there, capital accumulation does not affect the key dynamics. Therefore, for simplicity's sake, here we will focus on the 1-factor case. As standard in the literature (see, for example Moro, 2012 and Gaggl et al., 2023), let's assume that the sectoral production functions take a Cobb-Douglas form:

$$y_{i,t} = A^{go}_{i,t} \cdot M_i \left(\mathbf{x}_{i,t}; \phi_{i,t}\right)^{\alpha_i} \left(\ell_{i,t}\right)^{1-\alpha_i}. \tag{4}$$

The parameter α represents the intermediate intensity of the i sector and, equivalently, the intermediate' share of total cost. Indeed, the intermediate' share of total cost has remained relatively stable in the WIOD data. After the reparametrization, the intermediate aggregator reads

$$M\left(\mathbf{x}_{i,t}; \theta_{i,t}, \phi_{i,t}\right) = \theta_{i,t} \left[\gamma_i (x_{ig,t})^{\frac{\sigma_i - 1}{\sigma_{i,t}}} + (1-\gamma_i)(\phi_{i,t})^{\frac{1}{\sigma}} (x_{is,t})^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}. \tag{5}$$

while the associated ideal price index is

$$\mathcal{P}^M_{i,t} = \frac{1}{\theta_{i,t}} \left[\gamma_i^{\sigma_i} \left(p_{ig,t}\right)^{1-\sigma_i} + (1-\gamma_i)^{\sigma_i} \phi_{i,t} \left(p_{is,t}\right)^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}, \qquad \forall i \in [g,s]. \tag{6}$$

The demand for intermediates and labor follows the standard CES function forms:

$$\ell_{i,t} = (1-\alpha_i)\cdot\frac{p_{i,t}y_{i,t}}{w_t} \qquad \forall i \in [g,s] \tag{7}$$

$$M_{i,t} = \alpha_i\cdot\frac{p_{i,t}y_{i,t}}{\mathcal{P}^M_{i,t}} \qquad \forall i \in [g,s] \tag{8}$$

Combining the demand functions, we obtain the following condition

$$\frac{A^{go}_{i,t}p_{i,t}}{(\mathcal{P}^M_{i,t})^{\alpha_i}} = \frac{w_t}{(1-\alpha_i)} \tag{9}$$

Which implies that, via perfect mobility of labor, we have that real wage prices are such that:

$$\frac{p_{s,t}}{p_{g,t}} = \frac{1-\alpha_g}{1-\alpha_s}\cdot\frac{A^{go}_{g,t}}{A^{go}_{s,t}}\cdot\frac{(\mathcal{P}^M_{s,t})^{\alpha_s}}{(\mathcal{P}^M_{g,t})^{\alpha_g}} \tag{10}$$

Therefore, as standard, relative pricers are inversely proportional to relative productivity: prices are relatively higher for the relatively *less* productive sector. However, in the presence of intermediate networks, relative prices are directly proportional to intermediate prices; therefore, via the intermediate price index, bias in technical change affects equilibrium prices, as will be made explicit later in proposition 4.

3.2 Final demand

In the economy, there is a representative consumer, supplying a single unit of labor in exchange for a wage w_t. The consumer's preferences across goods take a CES functional form, where the utility level $u(\mathbf{c}_t)$ is implicitly defined as:

$$u(\mathbf{c}_t) = \sum_{i\in[g,s]} \eta_{c,i}\cdot(c_{i,t})^{\frac{\sigma_c-1}{\sigma_c}}.$$

In the absence of capital accumulation, the consumer problem can be collapsed into a period-over-period allocation problem, where the agent, given wages – and thus its budget constraint due to the labor normalization – and prices, decides how to allocate consumption between goods and services to maximize profits. The standard intratemporal optimization problem reads

$$u^*_t = \left\{\arg\max_{\mathbf{c}_{i,t}} u(\mathbf{c}_t): \quad w_t = \mathbf{p}^T_{i,t}\mathbf{c}_t\right\}$$

While analytically convenient, the standard CES demand has proved to be counterfactual in the face of the empirical evidence of structural transformation in final demand. In particular, the literature stresses the role of income effects stemming from non-homothetic preferences (for example, Boppart, 2014, Comin et al., 2021, and Matsuyama, 2022). While relevant to structural change, in the main body of this work, we will work with homothetic preferences for clarity's sake. The role of non-homotheticity will be discussed with an extended model in section 6. Solving the optimization problem yields the standard CES demand function, where the relative demand for services and goods is a function of relative prices only.

$$log\left(\frac{p_{s,t}c_{s,t}}{p_{g,t}c_{g,t}}\right) = \ln\left(\frac{1-\eta}{\eta}\right) + \underbrace{(1-\sigma_c)\cdot(\ln p_{s,t} - \ln p_{g,t})}_{\text{substitution effect}} \tag{11}$$

The associated price index – that is, $\mathcal{P}_t^C(\mathbf{p})$ such that $\mathcal{P}_t^C(\mathbf{p})\cdot u_t^* = w_t$ – reads:

$$\mathcal{P}^C(\mathbf{p}_t) = \left[\eta^{\sigma_c}\cdot\left(p_{g,t}\right)^{1-\sigma_c} + (1-\eta)^{\sigma_c}\cdot(p_{s,t})^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}} \tag{12}$$

Without loss of generality, let us set the consumer price index as the numeraire. That is, let's assume that the consumer price index (CPI), a standard empirical measure of inflation, is constant. Setting the consumer price index as the numeraire allows a simple intuition for aggregate value-added in the upcoming section, as it will broadly match the empirical computation of real GDP. However, this would not be the case with non-homothetic preferences, as will be discussed in section 6.

3.3 Equilibrium

Having outlined the technology of production at the sectoral level and the behavior of the representative household. It is now useful to place the model within a general equilibrium framework. In this setting, both the goods-producing and the services-producing sectors interact through their demand and supply of intermediate inputs, as well as through the allocation of the unique primary factor, labor.

Definition 3.1 (Competitive equilibrium). *The equilibrium is a collection of prices $(\boldsymbol{p}, w)$ and allocations $(\boldsymbol{c}, \boldsymbol{x}_g, \boldsymbol{x}_s)$ such that*

1. *Each sector solves its cost minimization problem given prices and productivities $(A_i^{go}, \phi_{i,t})$.*

2. *The representative household solves its cost minimization problem.*

3. *Factor and intermediate input markets clear:*

$$y_{i,t} = c_{i,t} + x_{gi,t} + x_{si,t} \qquad \forall i \in [g,s] \tag{13}$$

$$1 = \ell_{g,t} + \ell_{s,t} \tag{14}$$

More concretely, market clearing requires that the output of each sector be fully absorbed as an intermediate input in production or as final demand. Since labor is the only factor of production under Assumption 3.2, labor market clearing reduces to the condition that the total supply of labor equals the sum of labor demanded across sectors. Similarly, equilibrium in the markets for intermediates implies that the demand for sectoral output, whether used in the production of goods or services, or for final demand, must equal the supply $y_{i,t}$. The equilibrium of the model, alongside its input–output connection, is shown in figure 4.

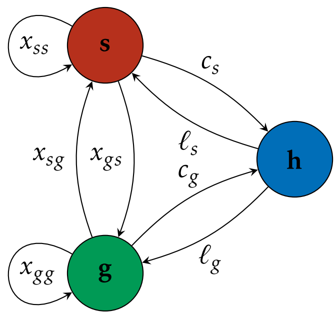

Figure 4: Graph of the input–output structure

3.4 Aggregation

This section explores the aggregate behavior of the economy. Let's start by defining sectoral value-added as the difference between nominal gross output and the nominal value of intermediate inputs:

$$\mathcal{P}^{Y}_{i,t} Y_{i,t} = p_{i,t} y_{i,t} - \mathcal{P}^{M}_{i,t} M_{i,t} \qquad \forall i \in [g, s],$$

where $p_{i,t} y_{i,t}$ is nominal gross output, $\mathcal{P}^{M}_{i,t} M_{i,t}$ denotes the nominal cost of intermediates, and $\mathcal{P}^{Y}_{i,t}$ is the price index associated with real GDP, $Y_{i,t}$. To compute real value-added, following the empirical state-of-the-art, we need to compute changes in nominal value-added keeping prices constant: ($\Delta p_{i,t} = \Delta \mathcal{P}^{M}_{i,t} = 0$). It is equivalent to the base-period price chain-index used by statistical agencies when computing real value-added. Taking a log difference at constant prices yields the approximation:

$$\mathcal{P}^{Y}_{i,t} Y_{i,t} \Delta \ln Y_{i,t} \approx p_{i,t} y_{i,t} \Delta \ln y_{i,t} - \mathcal{P}^{M}_{i,t} M_{i,t} \Delta \ln M_{i,t}|_{\mathcal{P}^{M}_{i}=const.}, \qquad \forall i \in [g, s],$$

where we are using the short-hand

$$\Delta \ln X_t \equiv \ln\left(\frac{X_{t+1}}{X_t}\right)$$

for a generic variable X. Notice that from equation 6, it must be that $\Delta p_{i,t} = \Delta \mathcal{P}^M_{i,t} = 0 \implies \Delta\theta_{i,t} = \Delta\phi_{i,t} = 0$. That is, keeping intermediate prices constant implies keeping intermediate productivities constant. Therefore, from equation 5 we have that:

$$\Delta \ln M_{i,t}|_{\mathcal{P}^M_i = const.} = \Delta \ln M_i - \Delta \ln \theta_{i,t} + \chi_{is,t} \cdot \Delta \ln \phi_{i,t}.$$

Replacing and using the first order conditions yields:

Lemma 1 (Evolution of sectoral value-added)**.**

$$\Delta \ln Y_{i,t} = \underbrace{\frac{\Delta \ln A^{go}_{i,t} + \alpha_i \left(\Delta \ln \theta_{i,t} - \frac{\chi_{is,t}}{1-\sigma_i} \Delta \ln \phi_{i,t}\right)}{1-\alpha_i}}_{\equiv \Delta \ln A^{va}} + \Delta \ln \ell_{i,t}, \tag{15}$$

where $\chi_{is,t}$ is the service share of total intermediates for sector i:

$$\chi_{ij,t} = \frac{p_{j,t} x_{ij,t}}{\sum_j p_{j,t} x_{ij,t}}.$$

This equation allows us to express sectoral real value-added explicitly as a function of the primitives. In particular, notice that by "chain-linking" the growth rates of Y – that is, defining Δ with respect to a reference point $t = 0$ (that is, $\Delta \ln X_t = \ln X_t - \ln X_0$, $\forall X$) and taking the exponential – gives us the *real value-added at t = 0 prices*:

$$\bar{Y}_{i,t} = \underbrace{\left\{ \bar{A}^{go}_{i,t} \cdot \left[\bar{\theta}_{i,t} \cdot \left(\frac{\phi^{\chi_{is,t}}_{i,t}}{\phi^{\chi_{is,0}}_{i,0}} \right)^{-1} \right]^{\alpha_i} \right\}^{\frac{1}{1-\alpha_i}}}_{\bar{A}^{va}_{i,t}} \bar{\ell}_{i,t}, \tag{16}$$

where $\bar{X}_{i,t} = Y_{i,t}/Y_{i,0}$ represents the normalized variable with respect to a generic reference point $t = 0$. This approach is consistent with how real value-added is computed empirically: all commonly used measures of real value-added use chain-linking and thus provide value not in levels, but relative to a reference point. Equation 16 represents the closed-form production function for value-added, which takes a convenient "AL" functional form, and provides the basic intuition for the term $A^{va}_{i,t}$: it represents the value-added labor productivity of the economy[2].

In the absence of intermediates-specific productivities, sectoral gross-output (A^{go}) and values-added TFP (A^{va}) are log-proportional, the difference driven by the size of intermediate intensity, α_i. More interesting is the case in which there is intermediates-specific technical change. First, "total intermediate productivity" (θ) enters A^{va} log-linearly, again weighted by intermediate intensity: a hardly surprising result, since the productions functions (equations 4) can be rewritten, without loss of generality, with $A^{go}_{i,t} \times \theta^{\alpha_i}_{i,t}$, $\forall i \in [g,s]$ as the new TFP terms exploiting the constant-return-to-scale property of the Cobb-Douglas production funtion. Secondly, and more interestingly, technical bias, θ_i, enters the growth rate of value-added TFP *negatively* and is weighted by the service's share of total intermediates: given θ, a larger ϕ implies a reduction in the relative productivity of service intermediates and its impact upon aggregate value-added productivity is driven by the quantity of services used in the productive process. Equation 15 implies that, even if fundamental productivities (A^{Go},θ and ϕ) were to grow at constant (non-zero) rates, then the value-added TFP A^{va}_i would not grow at a steady rate: with non zero $\Delta \ln \phi_i$, the intermediate shares $\chi_{is,t}$ does change over time. Furthermore, the dependence of value-added TFP upon service shares implies that general equilibrium effects enter value-added TFP via relative prices.

To simplify aggregation, let's make the following assumption:

Assumption 3.3. *Intermediate intensity parameters are symmetric between the two sectors:*

$$\alpha = \alpha_g = \alpha_s$$

This assumption will be relaxed in the quantitative exercise.

Proposition 2 (Aggregate Value-Added)**.** *Under assumption 3.3, aggregate value-added per worker in terms of the numeraire - consumer price index - is*

$$Y_t = \mathcal{A}_t \mathcal{L}_t,$$

[2] In the model with capital (in section 6), the chain-linked value-added will take a form:

$$\bar{Y}_{i,t} = \underbrace{\left\{ \bar{A}^{go}_{i,t} \cdot \left[\bar{\theta}_{i,t} \cdot \left(\frac{\phi^{\chi_{is,t}}_{i,t}}{\phi^{\chi_{is,0}}_{i,0}} \right)^{-1} \right]^{\alpha_i} \right\}^{\frac{1}{1-\alpha_i}}}_{\bar{A}^{va}_{i,t}} \left(\bar{k}^{1-\beta}_{i,t} \bar{\ell}^{\beta}_{i,t} \right)$$

That is, in the 2-factor model, a functionally equivalent $A^{va}_{i,t}$ will represent value-added TFP. However, in the two models (with and without capital), A^{go} is not the same measure: in the model without capital, TFP includes the capital contribution.

where $\mathcal{L}_t$ (normalized to 1) is the total labor endowment. Aggregate TFP ($\mathcal{A}$) is such that

$$\mathcal{A}_t = \mathcal{C}^Y \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{(\mathcal{P}_{jt}^M)^{\alpha}}{A_{jt}^{go}} \right)^{1-\sigma_c} \right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}, \tag{17}$$

where $\mathcal{C}^Y$ is a constant term. The intermediate price indexes solve

$$\mathcal{P}_{i,t}^M = \frac{1}{(1-\alpha)\theta_{i,t}} \left[\gamma_i^{\sigma_i} \left(\frac{(\mathcal{P}_{g,t}^M)^{\alpha}}{A_{g,t}^{go}} \right)^{1-\sigma_i} + (1-\gamma_i)^{\sigma_i} \phi_{i,t} \left(\frac{(\mathcal{P}_{s,t}^M)^{\alpha}}{A_{s,t}^{go}} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \qquad \forall i \in [g,s]. \tag{18}$$

Proof. See appendix A.1 ∎

Proposition 2 states that the model allows for aggregation of sectoral value-added into an aggregate production function whose total-factor productivity, $\mathcal{A}$, is a function of fundamental productivities. Equation (17) highlights the fundamental non-linearity of aggregate TFP in this economy. The structure of production generates non-linearity through the interaction between input–output linkages and biased technical change. To isolate this channel, consider the following case:

Example. *Assume that $\alpha = 0$ for all i, so that production uses only labor. In this case,* (18) *simplifies to*

$$\mathcal{A}_t = \mathcal{C}^Y \left[\sum_{j \in g,s} \eta_j^{\sigma_c} \left(A_{jt}^{go} \right)^{\sigma_c - 1} \right]^{\frac{1}{\sigma_c - 1}}. \tag{19}$$

That is, a standard CES aggregator of sectoral gross-output productivities.

Once intermediates are reintroduced ($\alpha > 0$), the sectoral price indices in (18) generally do not equalize, reflecting heterogeneity in input-augmenting productivity parameters ($\theta_i, \phi_{i,t}$). This heterogeneity prevents the aggregation of sectoral labor productivities into a simple CES form. Instead, it embeds a non-linear dependence of aggregate TFP on the differences between intermediate-enhancing productivities across sectors. To observe the role of *differences* in intermediates-specific productivity across sectors, let's proceed with the following example:

Example. *Assume that input productivities are identical across sectors, with $\phi_{i,t} = \phi$ and $\theta_i = \theta$ for all $i \in g,s$. Furthermore, assume $\sigma_i = \sigma$ for all $i \in [g,s]$. In this special case, aggregate TFP takes the form*

$$\mathcal{A}_t = \mathcal{C}^Y \left[\sum_{j \in [g,s]} \eta_j \left(\frac{A_{jt}^{go}}{\bar{A}_t^{\frac{\alpha}{1-\alpha}}} \right)^{\sigma_c - 1} \right]^{\frac{1}{\sigma_c - 1}}, \tag{20}$$

where $\bar{A}_t$ is the (biased) mean of sectoral productivities:

$$\bar{A}_t \equiv \frac{1}{(1-\alpha)\theta}\left[\gamma^{\sigma}\left(A^{go}_{g,t}\right)^{\sigma-1} + (1-\gamma)^{\sigma}\phi\cdot\left(A^{go}_{s,t}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

This example makes transparent how sectoral asymmetries in input bias feed into aggregate TFP: even when intermediate intensities are symmetric, differences in ϕ (relative productivity of services intermediates) and θ (level shifters) induce a distortionary aggregation of sectoral labor productivities into the economy-wide index.

3.5 Model Dynamics

This section explores the evolution over time of key aggregate variables.

Proposition 3 (Aggregate value-added growth with biased technical change)**.** *The evolution over time of aggregate value-added reads:*

$$\Delta \ln Y_t \approx \sum_{i\in[g,s]} \frac{P^Y_{j,t}Y_{j,t}}{\sum_{j\in[g,s]}P^Y_{j,t}Y_{j,t}}\Delta\ln A^{va}_{i,t} \tag{21}$$

$$\approx \underbrace{\sum_{i\in[g,s]}\lambda_{i,t}\left(\Delta\ln A^{go}_{i,t}+\alpha\Delta\ln\theta_{i,t}\right)}_{\textit{TFP component}} - \underbrace{\sum_{i\in[g,s]}s_{i,t}\Delta\ln\phi_{i,t}}_{\textit{Bias component}}, \tag{22}$$

where $\lambda_{i,t}$ are the sectoral total output shares of aggregate value-added share (a.k.a. Domar weights),

$$\lambda_{i,t} = \frac{p_{i,t}y_{i,t}}{\sum_{j\in[g,s]}P^Y_{j,t}Y_{j,t}},$$

and $s_{i,t}$ is the sectoral intermediate services share of the aggregate value-added share,

$$s_{i,t} = \frac{p_{s,t}x_{is,t}}{\sum_{j\in[g,s]}P^Y_{j,t}Y_{j,t}}.$$

Proof. See appendix A.2 ■

Proposition 3 is a generalization of the well-known Hulten's Theorem (Hulten, 1978, with recent contributions by Baqaee and Farhi, 2019) in the presence of intermediates-specific productivities. It shows the first-order (or instantaneous) impact of changes in sectoral productivity upon aggregate growth. Notice, in fact, that it ignores the effect that changes in productivity have upon the weights $\lambda_{i,t}$

and $s_{i,t}$. It is thus an approximation of the more complex dynamics that arise due to the presence of the production network. However, notice that the results in this section are a natural consequence of the results from the previous section: for example, Proposition 3 follows naturally from an approximation of the chain rule of equation 20.

Hulten's theorem suggests that production networks play a role in the dynamics of aggregate value-added. To see that, notice that equation 21 states that aggregate value-added growth is the weighted average of sectoral (gross-output) TFP. However, the weights are NOT the sectoral shares of aggregate value-added for each subsector: they are the *total output* shares of aggregate value-added, often referred to as Domar weights (Domar, 1961). That is, the sectoral share of aggregate value-added, plus the intermediates produced by that sector as a share of aggregate value-added. Notice that the sum of the Domar weights is, in the presence of intermediates, larger than one by definition: this represents the intrinsic "multiplier effect" of input–output linkages. Furthermore, note that what matters is the growth of sectoral gross-output TFP, not value-added TFP. In fact, from Equation 15, we know that even with neutral technical change, sectoral gross output and value-added TFP grow at different rates. Proposition 3 provides an explicit generalization in the presence of intermediates-specific productivities. First, compared to the standard Hulten's theorem, we can now split the growth rate in aggregate value-added growth into two different channels: a TFP component, akin to the standard Hulten Theorem, and a Bias component. While the TFP component aggregates sectoral TFP growth via Domar weights, the Bias term aggregates sectoral bias in technical change using the sectoral service share of aggregate value-added as weights. Intuitively, the more a sector uses services as intermediates, the more a shock in ϕ (that is, a change in the relative productivity of different intermediates) affects aggregate real value-added growth.

Proposition 4 (Prices evolution)**.** *The evolution over time of prices is such that*

$$\Delta \ln p_{i,t} = \overbrace{\sum_{j\in[g,s]} \hat{\Omega}_{ij,t}\Omega_{is,t}\Delta \ln \phi_{j,t}}^{\text{Bias component}} - \overbrace{\sum_{j\in[g,s]} \hat{\Omega}_{ij,t}(\Delta \ln A^{go}_{j,t} + \alpha_i \Delta \ln \theta_{i,t})}^{\text{TFP component}} - \overbrace{\Delta \ln Y_t}^{\text{Numeraire adjustment}}, \tag{23}$$

where $\hat{\Omega}_{ij,t}$ are elements of the Leontief matrix inverse $\hat{\Omega}_{ij,t} = (1-\Omega_t)^{-1}$, and

$$\Omega_t = \begin{bmatrix} \Omega_{gg,t} & \Omega_{gs,t} \\ \Omega_{sg,t} & \Omega_{ss,t} \end{bmatrix}$$

is the (sales-based) input–output matrix.

Proof. See Appendix A.3 ■

Proposition 4 shows the evolution over time of prices. Equation 23, called the "forward price equation" in Baqaee and Rubbo (2023), shows the first-order impact of productivity changes upon prices. Compared to the aggregation in Proposition 3, now the input–output structure is explicitly represented by the Leontief inverse, the inverse of the complement of the input–output matrix. Again, we can decompose the change in prices into a TFP effect and a bias term. As expected, prices decline with sectoral gross-output productivities (as made explicit by equation 10). However, bias in technical changes does *increase* relative prices. It follows trivially from the fact that ϕ is, given θ, a *decline* in service-intermediate productivity. Therefore, the more a sector is dependent upon service intermediates, the more its total productivity declines as ϕ grows, ceteris paribus.

Jointly, Proposition 3 and 4 provide a clear intuition of the role of input–output networks in aggregate dynamics. To see it, let's start from the case with unbiased technical change: without intermediates-specific productivities, equations 21 and 23 become

$$\Delta \ln Y_t \approx \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A^{go}_{i,t} \quad \text{and} \quad \Delta \ln p_{i,t} \approx - \sum_{j \in [g,s]} \hat{\Omega}_{ij,t} \Delta \ln A^{go}_{j,t}. \tag{24}$$

Assume there is an idiosyncratic TPF shock in the services-producing sector. As standard in multi-sector models, this will result in a decline in the relative price of services. However, since all sectors use services as intermediates, this means that production becomes cheaper for all sectors. Again, this will lower equilibrium prices, leading to a further reduction in production costs and, again, lower equilibrium prices. The resulting sequence converges toward the new equilibrium, characterized by a lower relative price for services compared to an economy without intermediates.

Let's now move to the characterization of the asymptotic behavior of the economy. Without loss of generality, let's define

$$\tilde{A}^{go}_{i,t} = A^{go}_{i,t} \cdot \theta^{\alpha}_{i,t} \qquad \forall i \in [g,s].$$

That is, TFP after the "total-intermediate" productivity term is moved outside of the intermediate aggregator, exploiting the constant-return-to-scale property of the Cobb-Douglas production functions. We will make the following simplifying assumption, common in the structural transformation literature.

Assumption 3.4. *Sectoral productivities grow at constant, and (potentially) heterogeneous rates:*

$$\Delta \ln \tilde{A}^{go}_{i,t} = g(\tilde{A}^{go}_{i}) \quad \textit{and} \quad \ln \phi_{i,t} = g(\phi_i) \qquad \forall i \in [g,s] \textit{ and } \forall t.$$

As standard in the structural transformation literature, let's assume that the growth rate of TFP is higher in the goods-producing sector relative to the services-producing sector. This assumption is consistent with the stylized facts of economic development. The second assumption is that technical change is increasingly more service-biased in the services-producing sector compared to the goods-producing sector. Formally:

Assumption 3.5. *Technical change is such that:*

1. $g(\tilde{A}_g^{go}) > g(\tilde{A}_s^{go}) \geq 0$
2. $g(\phi_s) > g(\phi_g) \geq 0.$

Assumption 3.5 aligns with the empirical evidence for the US from 1965 to 2014: in fact, section 4 will demonstrate that technical change has been unbiased in the goods-producing sector and services-biased in the services-producing sector. The stylized facts of economic growth (also known as Kaldor's facts) posit that aggregate quantities, such as wages and aggregate value-added, ought to grow at a constant rate, that is, that the economy moves along an Aggregate Balanced Growth Path (AGBP). Due to the use of a single production factor and the lack of an intertemporal allocation problem from the representative household, the model is trivially consistent with an AGBP.

Proposition 5. *Under assumptions 3.4 and 3.5, there is an aggregate balanced growth path (ABGP) where all aggregate variables grow at a constant and symmetric rate:*

$$\Delta \ln W = \Delta \ln Y = \Delta \ln C = \Delta \ln \mathcal{A}$$

Proposition 5 follows trivially from our definition of the numeraire and market-clearing conditions. In section 6, we characterized the existence of a non-trivial ABGP in the case with capital accumulation. More interesting is the asymptotic behavior of the economy outside of the ABGP, shown in proposition 6.

Proposition 6 (Asymptotic Aggregate Balanced Growth Path)**.** *Under Assumptions 3.1, 3.3, 3.4, and 3.5 - outside the ABGP - the economy converges to an asymptotic ABGP in which aggregate value-added grows at a constant rate -*

$$\lim_{t\to\infty} \Delta \ln Y_t = \lim_{t\to\infty} \Delta \ln A_{s,t}^{va} \tag{25}$$

$$= \frac{1}{1-\alpha} \cdot \left(\Delta \ln A_{s,t}^{go} + \alpha \Delta \ln \theta_{s,t}\right) - \underbrace{\frac{\alpha}{(1-\sigma_s)(1-\alpha)}}_{>1} \cdot \Delta \ln \phi_{s,t}, \tag{26}$$

Proof. See appendix A.4 ∎

The intuition is straightforward: as the economy expands – due to the non-negativity assumption in the growth rates of productivity, assumption 3.5 – the services-producing sector dominates both the intermediate use and final consumption roles in nominal terms. In fact, as the economy grows, the relative price of services increases relative to goods (equation 23). Consequently, the nominal share of services in both sectors (equation 2) and in final demand (equation 11) grows steadily. Asymptotically, services would be the only sector operating in the economy: the growth rate of aggregate value-added will then be the same as the growth rate of the value-added of the services-producing sector. The bias terms enter equation 25 with a negative sign: services-biased technical change slows down economic growth in the long run. This is a consequence of the definition of ϕ: since it is defined as $(a_{gs}/a_{ss})^{(1-\sigma)}$, with $\sigma < 1$, an increase in ϕ implies a *decline* in the relative productivity of services. Therefore, for any fixed θ (which, remember, is the goods-specific productivity), an increasing ϕ means that the productivity of services intermediates is actually declining.

Equation 25 gives us a theoretical prediction: if the services-producing sector is increasingly services-biased, this will hurt long-run economic performance. This prediction will be proved correct using US data in the next section.

4 Calibration

4.1 Measuring Technical Change

Let's start by quantifying the direction and magnitude of bias in technical change. No work has yet attempted to estimate technical change at the intermediate level: the abundant literature on directed technical change primarily focuses on the allocation of factors (namely, capital and labor) rather than intermediates. Therefore, this section introduces a novel approach, based upon León-Ledesma et al. (2010) and Lashkari et al. (2024), to estimating technical change in intermediates. Due to its novelty, I will test the approach's ability to recover key parameters via a Monte Carlo simulation.

A well-known issue that plagues the estimation of biased technical change is transparent in equation 2: technical bias is a function of the elasticity of substitution. Therefore, it is impossible to estimate the elasticity of substitution between intermediates and the bias in technical change thereof separately. In fact, the ex-ante choice of the shape of technical change affects the estimation of the elasticity of substitution: for example Antras (2004) argues that, assuming Hicks-neutral technical change biases the estimated elasticity of substitution (between factors) toward unity – that is, the Cobb-Douglas case. Let's start from the generic case with an arbitrary number of factors: the sectoral production function reads

$$y_{i,t} = f(M_{i,t}, k_{i,t}, \mathbf{k}_{i,t}),$$

where M has the same CES functional form as in the previous sections. As in equation 2, the relative demand for intermediates is:

$$\frac{\chi_{is,t}}{\chi_{ig,t}} = \ln\frac{p_{is,t}x_{is,t}}{p_{ig,t}x_{ig,t}} = \sigma_i \ln\left(\frac{1-\gamma_i}{\gamma_i}\right) + (1-\sigma_i)(\ln p_{is,t} - \ln p_{ig,t}) + \ln\phi_{i,t}, \quad \forall i \in [g,s].$$

Notice that the optimal relative demand for intermediates does not depend on the use of factors. That is, the equation above holds for any well-behaved $f(\cdot)$. Therefore, the calibration shown here is independent of assumption 3.2 maintained in the theoretical model. Furthermore, it follows that knowledge of the CES aggregator M is sufficient to determine the demand for intermediates. Following Klump et al. (2012), let's normalize the intermediate aggregator with respect to a reference period $t = 0$[3]. That is

$$\bar{M}_{i,t} = \theta_{i,t} \cdot \left[\Gamma_i \cdot \bar{x}_{ig,t}^{\frac{\sigma_i-1}{\sigma_i}} + (1-\Gamma_i)(\bar{\phi}_{i,t})^{\frac{1}{\sigma_i}}(\bar{x}_{is,t})^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}}, \tag{28}$$

where $\bar{x}_{i,t} \equiv x_{i,t}/x_{i,0}$ and

$$\Gamma_i \equiv \frac{p_{g,0}x_{ig,0}}{p_{g,0}x_{ig,0} + p_{s,0}x_{is,0}}$$

That is, the goods-intensity at the reference point t = 0. The normalization of the CES aggregator is a necessary step, which can be either explicit, as in this case, or implicit. In fact, CES production functions are necessarily defined and derived relative to a benchmark period, as argued by León-Ledesma et al. (2010). Secondly, normalization simplifies the estimation by removing the parameter γ, which is replaced with a constant observable value. The new (log) relative input intensity reads:

$$\ln\bar{\phi}_{i,t} = \ln\frac{\bar{\chi}_{is,t}}{\bar{\chi}_{ig,t}} - \underbrace{(1-\sigma_i)(\ln\bar{p}_{is,t} - \ln\bar{p}_{ig,t})}_{h(\bar{\mathbf{p}};\sigma_i)} \tag{29}$$

where we "inverted" the bias term ϕ to define it as a function of parameters and observables. Notice

[3]The normalization can be quickly derived by noticing that

$$x_{ij0} = \gamma_{ij}^{\frac{1}{\sigma_i}} \cdot a_{ij}^{\sigma_i-1}\left(\frac{p_{g0}}{\mathcal{P}_{i,0}^M}\right)^{-\sigma} M_{i,0} \quad \forall i,j \in \{g,s\} \quad \Longrightarrow \quad \gamma_{ij} = \Gamma_j^i\left(\alpha_{ij0}\cdot x_{ij0}\right)^{\frac{1-\sigma_i}{\sigma_i}} \cdot M_{i,0}^{\frac{\sigma_i-1}{\sigma_i}}. \tag{27}$$

Solving for γ and replacing it into the CES aggregator and the equation in proposition 1 yields equations 28 and 29, respectively.

how the Γ term disappears due to normalization. Substituting into 28 yields:

$$\ln(\bar{M}_{i,t}) = \ln\left\{\left[\Gamma_i \bar{x}_{ig,t}^{\frac{\sigma_i-1}{\sigma_i}} + (1-\Gamma_i)(\bar{\phi}_{i,t}(\bar{\mathbf{p}}_i,\bar{\mathbf{x}}_i))^{\frac{1}{\sigma_i}} \cdot \bar{x}_{is,t}^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}}\right\} + \bar{\theta}_{i,t}$$

$$\equiv f_i(\bar{\mathbf{p}}_i,\bar{\mathbf{x}}_i;\Gamma_i,\sigma_i) + \bar{\theta}_{i,t}$$

A system of two equations can summarise the sectoral technology and optimality behavior:

$$\begin{cases} \ln\bar{\psi}_{i,t} = \frac{h_i(\bar{\mathbf{p}}_i;\sigma_i)}{1-\sigma_i} - \frac{\ln\bar{\chi}_{i,t}}{1-\sigma_i} \\ \ln\bar{\theta}_{i,t} = \ln\bar{M}_i - f_i(\bar{\mathbf{p}}_i,\bar{\mathbf{x}}_i;\Gamma_i,\sigma_i) \end{cases} \tag{30}$$

where we redefine $\ln\bar{\psi}_{i,t} = -(\ln\bar{\phi}_{i,t})/(1-\sigma_i) = \ln(\bar{a}_{is}/\bar{a}_{ig})$, that is, the relative productivity of services intermediate in sector i to make the dependency of ϕ upon σ explicit. The methodology in this section consists of the joint estimation of the system above, where both sectoral technology and the optimization behavior are taken into account. As argued by Diamond et al. (1978), it is impossible to jointly estimate biased technical change and (constant) elasticity of substitution in the non-Cobb-Douglas case unless assumptions are made regarding the functional form for technological progress. Here, following Lashkari et al. (2024), we set that productivities evolve according to a random walk with drift.

Assumption 4.1. *(log) productivities evolve according to a random walk with drift:*

$$\ln\tilde{\psi}_{i,t} = \ln\tilde{\psi}_{i,t-1} + \mu_i^{\psi} + \epsilon_{i,t}^{\psi} \qquad i \in [g,s] \tag{31}$$

$$\ln\theta_{i,t} = \ln\theta_{i,t-1} + \mu_i^{\theta} + \epsilon_{i,t}^{\theta} \qquad i \in [g,s] \tag{32}$$

Here, μ_i^{ϕ} and μ_i^{θ} can be interpreted as the *average* growth rate over the sample period. To disentangle the issue of price endogeneity – sectoral decision making depends on prices that are endogenous to the firm optimization problem – let's make this standard assumption:

Assumption 4.2. *Lagged sectoral choices, $\bar{\mathbf{x}}_{i,t-1}$, are orthogonal to the current productivity innovations $\epsilon_{ij,t}$. That is,* $\mathbb{E}\left[\epsilon_{ij,t} \mid \bar{\mathbf{x}}_{i,t-1}\right] = 0$

Under these assumptions, the three parameters in system 30, namely the elasticity of substitution σ_i and the average growth rates of bias in technical change μ_{ij}^{ϕ} and good-enhancing productivity μ_{ij}^{θ}, can be estimated using GMM, as in Wooldridge (2009). The joint estimation of the input choice and the normalized production function proper via GMM is equivalent to the two-step estimation in

Olley and Pakes (1996) and Petrin et al. (2004). Since the problem has three unknowns (σ_i, μ^ϕ, μ^ϕ) in two moments, we need to exploit the time dimension to ensure identification. Thus, following Lashkari et al. (2024) and Trunschke and Judd (2024), we use lagged values of intermediate inputs as instruments. That gives four moments for three parameters, ensuring over-identification.

4.1.1 Monte-Carlo Simulation

Before bringing this approach to the data, let's test its capability to recover the parameters using a Monte Carlo simulation. Following León-Ledesma et al. (2010), we generate synthetic data for a single sector via the following steps:

1. **Technology processes –**

$$
\begin{aligned}
\ln \tilde{\psi}_t &= \ln \tilde{\psi}_{t-1} + \mu^\psi + \epsilon_{\psi,t} && \epsilon_{\psi,t} \sim \mathcal{N}(0, \sigma_\psi) \\
\ln \theta_t &= \ln \theta_{t-1} + \mu^\theta + \epsilon_{\theta,t} && \epsilon_{\theta,t} \sim \mathcal{N}(0, \sigma_\theta)
\end{aligned}
$$

.

2. **Input series –** $z_{i,t} = z_{i,t-1} \exp\Big(\mu^z + \epsilon_{z,t}\Big), \quad z \in \{x_g, x_s\}$, with $\epsilon_{z,t} \sim \mathcal{N}(0, \sigma_z)$.
3. **Equilibrium output and prices** are solved simultaneously from the normalized production function and first-order conditions.

Given the synthetic dataset of output and input quantities, and prices, the Monte Carlo econometrician applies the algoritm above (equation 17) to estimate the parameters σ, μ^ϕ, and μ^θ over 5,000 replications of the synthetic datasets with 50 observations each, where each sample is computed over different draws of the errors $\epsilon_{\phi,t}$, $\epsilon_{\theta,t}$, $\epsilon_{x_g,t}$, and $\epsilon_{x_s,t}$. We set $[\sigma_\phi, \sigma_g, \sigma_{x_g}, \sigma_{x_s}] = [0.025, 0.015, 0.1, 0.2]$. Finite-sample performance is assessed by bias, root mean squared error (RMSE), and convergence frequencies.

Parameter	True	Mean	Std	Bias	RMSE	Conv Rate
σ	0.600	0.597	0.038	-0.003	0.038	1.00
μ_ϕ	0.020	0.020	0.008	-0.000	0.008	1.00
μ_θ	0.010	0.010	0.003	0.000	0.003	1.00

Table 3: Monte Carlo Results (N=50, Sims=5000)

The Monte Carlo results in Table 3 confirm that the estimator performs well in finite samples, with negligible bias and unitary convergence rates.

4.1.2 Estimation with U.S. data

Applying the procedure to U.S. data from 1965–2014 yields sector-specific estimates reported in Table 4. In both sectors, the estimated elasticity of substitution is very close to zero, suggesting perfect complementarity between intermediates, that is, the M aggregate takes a Leontief functional form. Therefore, the only source of differences in intermediate *quantity* over time is given by evolving bias in technical change. This result is expected: while recent estimates (such as Gaggl et al. (2023)) observe non-zero elasticity of substitution between services and goods intermediates in the U.S. over a similar time interval, their result is likely biased toward unity due to their assumption that technical change is neutral toward intermediate demand (Antras, 2004).

	σ	μ^{ψ}	μ^{θ}
Goods	0.001 (0.026)	0.001 (0.005)	-0.001 (0.001)
Services	0.002 (0.002)	-0.012** (0.004)	0.008** (0.003)

Table 4: estimation of production parameters by sector

Bias in technical change shows sharp heterogeneity between sectors: in the good-producing sector, the drift of bias in technical change μ^{ψ} is indistinguishable from zero. By contrast, in the services-producing sector, the growth rate of ψ is negative. Remeber that, due to $\sigma_i \approx 0$, we have that bias in technical change (ψ) is equal to $-\psi_i \ \forall i \in [g, a]$. That is, services-bias technical change is the opposite of relative change in service-specific intermediate productivity, as transparent in equation 2. Therefore, the negative growth rate of ψ in the services-producing sector implies that technical change is increasingly services-biased in the services-producing sector. On the other hand, since μ^{ψ} is close to zero, we conclude that technical change is neutral (or at least, stable) in the good-producing sector. Similarly, μ^{θ} is close to zero (and slightly negative) for the good-producing sector: the total intermediates productivity has roughly remained constant, while total intermediate productivity has been growing steadily in the services-producing sector.

Figure 5a illustrates the resulting time path of residual ϕ_t and θ_t, computed as

$$\ln \bar{\phi}_{i,t} = \ln \bar{\chi}_{i,t} - h_i(\bar{\mathbf{p}}_{i,t}, \bar{\mathbf{x}}_{i,t}; \hat{\sigma}_i) \qquad \forall i \in [g, a]$$

$$\ln \bar{\theta}_{i,t} = \ln \bar{M}_{i,t} - f_i(\bar{\mathbf{p}}_{i,t}, \bar{\mathbf{x}}_{i,t}; \Gamma_i, \hat{\sigma}_i) \qquad \forall i \in [g, a]$$

where $\bar{\chi}_{i,t}$, $\bar{M}_{i,t}$, $\bar{p}_{i,t}$, and $\bar{x}_{i,t}$ are the empirically observed values, while $\hat{\sigma}_i$ is the estimated value of sectoral elasticity of substitution. In the first panel (5a), the residual ϕ by sector exhibits a roughly similar pattern across the two sectors until the mid-1980s. Subsequently, a sharp heterogeneity emerges: the services-producing sector becomes increasingly services-biased, whereas the goods-

(a) Evolution of residual ϕ by sector (b) Evolution of residual θ by sector

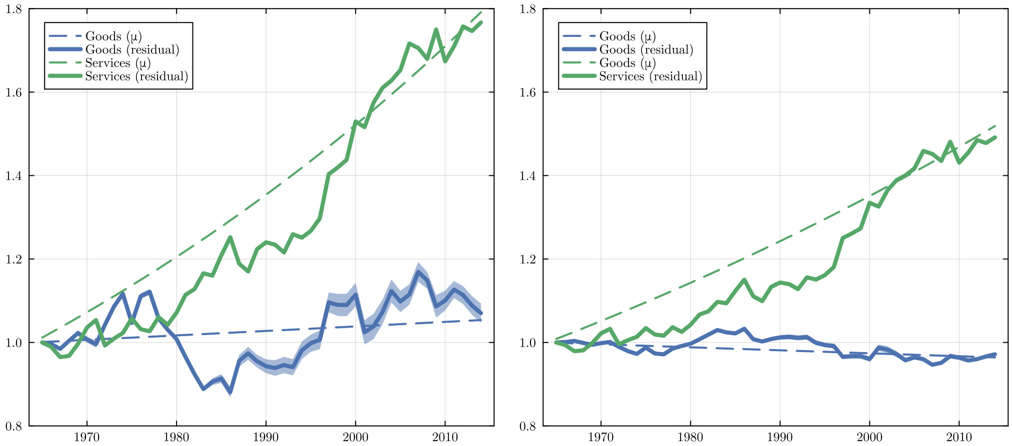

Figure 5: Time series of intermediates-specific productivities

producing sector experiences technical change that remains roughly neutral.

4.2 Sectoral Productivities

Sectoral (gross-output) labor productivity (A^{go}) is derived as a residual of the (normalized) productive process:

$$\ln \bar{A}_{i,t} = \ln Y_{i,t} - \alpha_i \cdot \ln \bar{M}_{i,t}(\mathbf{x}_{i,t}; \sigma_i, \phi_{i,t}, \Gamma_i) + (1-\alpha_i) \cdot \ln \bar{\ell}_{i,t} \qquad i \in [g, s].$$

The parameter α_i is the average sectoral intermediate share of nominal total output. Sectoral employment data (ℓ_i) come from BEA's NIPA tables. Figure 5 shows the time series for the sectoral productivity between 1965-2014: as expected, the goods-producing sector experienced a sizeable growth of productivity over the last 50 years. On the other hand, the services-producing sector is characterized by a stagnating productivity level.

However, notice that for many results (such as proposition 6) what matters for aggregate dynamics is not $A^{go}_{i,t}$ in isolation, but actually $A^{go}_{i,t} \times \theta^{\alpha_i}_{i,t}$, that is, the sectoral labor productivity after moving the total-intermediate productivity (θ) out of the intermediate aggregator M. The resulting TFP term is shown in the second panel of figure 6. Notice that, due to the sharp rise in total-intermediate productivity (θ) in the services-producing sector, the gap in this adjusted TFP term between goods and services is significantly smaller. It suggests some tentative pessimism regarding proposition 6: the asymptotic growth rate of the economy is bound to the (so far) negative growth rate of the value-added labor productivity of the services-producing sector. This prediction must, however, be taken carefully, as argued by Duernecker et al. (2024), where the author observes that, due to gross-substitability within

(a) Gross-output productivity ($\ln A^{go}$) (b) Value-added productivity ($\ln A^{va}$)

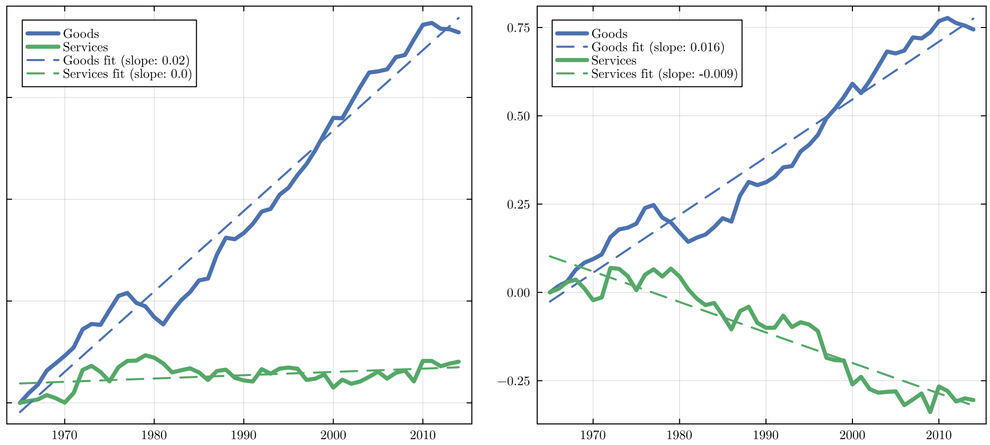

Figure 6: Time series of sectoral (log) productivity by sector

services-producing industries, final demand will move toward the more productive services-producing industries, thus ensuring a higher growth rate of the value-added of the aggregate services-producing sector. If the same mechanism applies to the demand for service intermediates, it is a matter for future research.

4.3 Decomposition of sectoral labor productivity

In equation 15, we derive a decomposition of sectoral labor productivity into three different components:

$$\Delta \ln A^{va} = \underbrace{\frac{\Delta \ln A^{go}_{i,t}}{1-\alpha_i}}_{\text{Gross-output TFP}} + \underbrace{\frac{\alpha_i \Delta \ln \theta_{i,t}}{1-\alpha_i}}_{\text{Total intermediate}} - \underbrace{\frac{\alpha_i \chi_{is,t} \Delta \ln \phi_{i,t}}{1-\alpha_i}}_{\text{Bias}}.$$

Given our knowledge of $A^{go}_{i,t}$, $\theta_{i,t}$, $\phi_{i,t}$, and intermediate shares, we can now decompose the three different mechanisms: the result is shown in figure 7. It suggests drastically different compositions of labor productivity growth between sectors. In the first panel, we observe that the large majority of the changes in labor productivity in the good-producing sector between 1965 and 2014 are driven by gross-output TFP. Bias in Technical change and intermediate total productivity play a very marginal role. The opposite is the case for the services-producing sector: here, bias plays a key role, slowing down aggregate labor productivity to stagnation. It suggests a key fact: service-biased technical change is the key driver of the slowdown of value-added labor productivity in the services-producing sector.

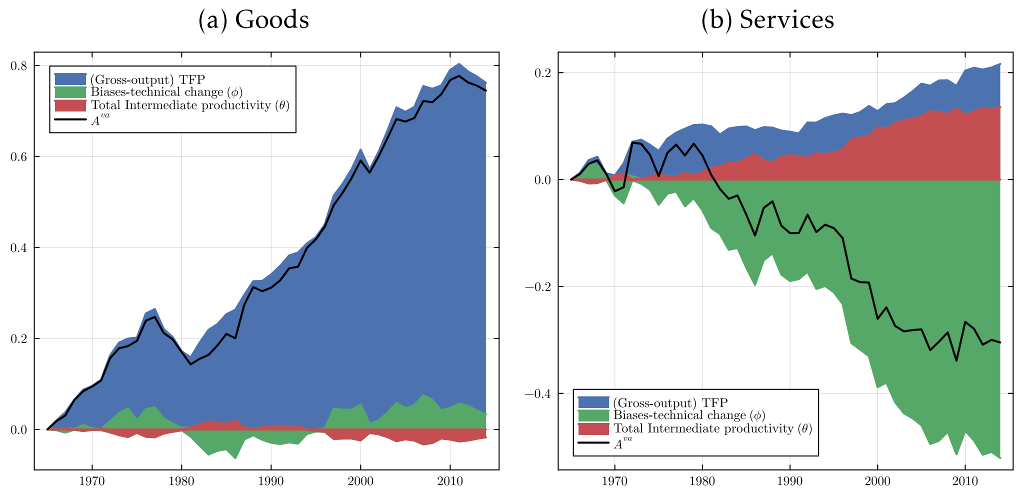

Figure 7: Decomposition of (log, value-added) labor productivity by sector

5 Quantitative Exercise

The quantitative exercise consists of solving the static competitive equilibrium described in Section 3 repeatedly along the historical sequence of estimated state variables (A^{go}, θ, and ϕ), given the parameters summarized in Table 5.

Parameter	Value	Sorce
α_G	0.52	Average Intermediate Share of nominal Output, (1965-2014, WIOD)
α_S	0.27	Average Intermediate Share of nominal Output, (1965-2014, WIOD)
Γ_G	0.77	Goods Share of nominal Output, (1965, WIOD)
Γ_S	0.41	Goods Share of nominal Output, (1965, WIOD)
Γ_C	0.79	Goods Share of nominal Final Household expenditure (1965, WIOD)
σ_G	0.00	Estimated via GMM (section 4.1)
σ_S	0.00	Estimated via GMM (section 4.1)
σ_C	0.17	Estimated via OLS (Equation 11)

Table 5: Calibrated parameters

Given the period-t primitives (parameters, labor productivity, and Technical bias), the equilibrium is obtained by solving simultaneously the log form of the wage FOCs, firms' input-demand first-order conditions, the household FOC, and the market-clearing constraints, that is, the system that characterizes competitive equilibrium in Section 3. The numeraire is chosen so that the wage equals the representative consumer's expenditure index, consistent with the normalization in the theoretical model.

When calibrated with estimated parameters and the empirically recovered sequences of ϕ_t and A_t, for both aggregate intermediates and final demand, the model reproduces the main features of the U.S. structural transformation, as shown in figure 8. In particular, the model generates a pronounced

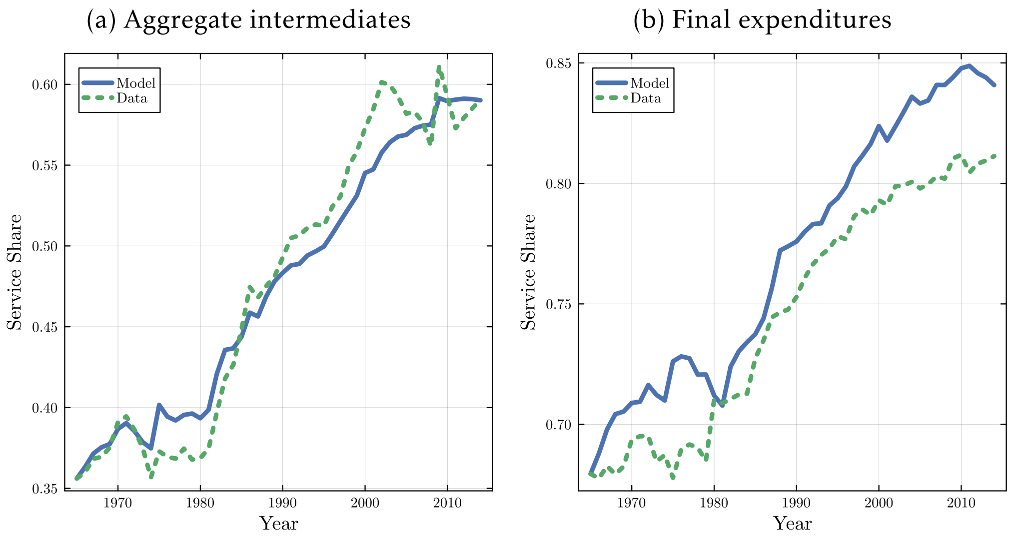

Figure 8: Service share by measure (nominal, Fitted vs Data)

rise in service intensity within both the goods and the services-producing sectors. Final expenditure shares also closely mirror the historical co-movements. The remarkable fit of the change in the service share of nominal aggregate intermediates suggests that the model captures well the general equilibrium mechanism of the model. Notice that, due to the Leontief functional form, *real* demand for intermediates is a function of technical bias only: that is, the relative quantity of services intermediates is an exogenous state. However, the fact that *nominal* values match suggests that the model successfully captures the evolution of relative prices.

5.1 Counterfactual exercise: structural transformation

To explore the role of technical change upon key measures of structural change, let's run a counterfactual exercise where we estimate the model after setting a symmetric and null bias in technical change (that is, $\phi_i = 1, \forall i \in [g,s]$). Figure 9 shows the result, where we plot the four key different measures of structural change: the service share of total intermediates, employment, value-added, and total expenditures. Across all measures, "shutting down" bias in technical change led to a decrease in the service share. Therefore, broadly speaking, we can state that bias in technical change is a relevant mechanism driving all relevant measures of structural change. The mechanism, common to all measures, is that bias in technical change affects relative prices, as we know from equation 23. As is clear there, services-biased technical change increases the relative price of the affected sector. As we have seen in the previous section, technical change is biased only in the services-producing sector. Therefore, the prices of services grow faster compared to the prices of goods. This change in relative

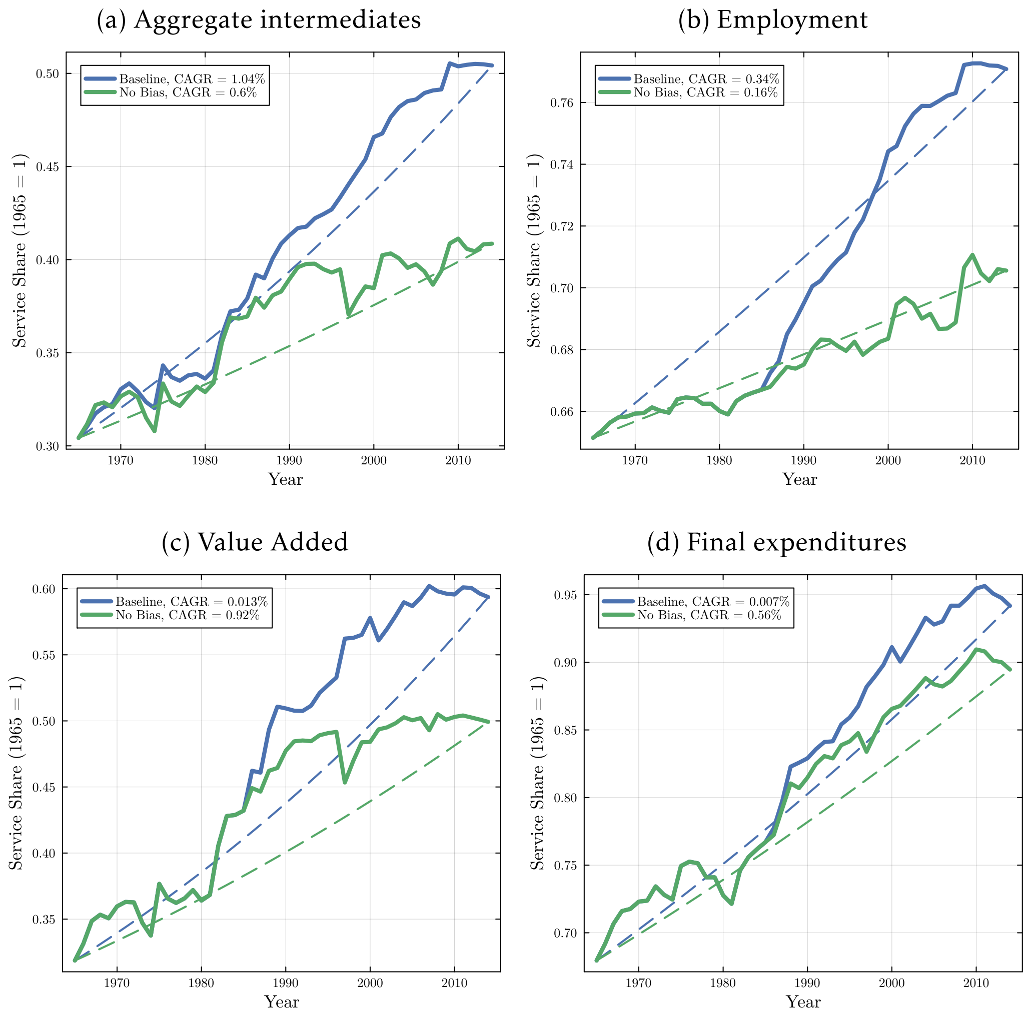

Figure 9: Service shares by measure (1965 = 1)

prices is shown in figure 10. As expected, in the counterfactual exercise, the relative price of services is lower. As predicted, the service-biased technical change in the services-producing sector increases the relative price of services.

This change in relative price has a cascade effect on all measures of structural change: first, it leads to an increase in the service share of aggregate intermediates (figure 9a), in addition to the direct effect from bias in technical change, as highlighted in equation 2. In fact, service-biased technical change in the services-producing sector is responsible for $\sim 50\%$ of the observed increase in the service share of total intermediates between 1965 and 2014. Secondly, it leads to a rise in nominal final expenditure as shown in figure 9d: since the demand for services depends only on relative prices (equation 11), the difference between the baseline model and the countrerfactual exercise is proportional to the change in price in figure 10, accounting for $\sim 25\%$ of the change in the service share over the relevant time period. Finally, since service-biased technical change in the services-producing sector leads to an

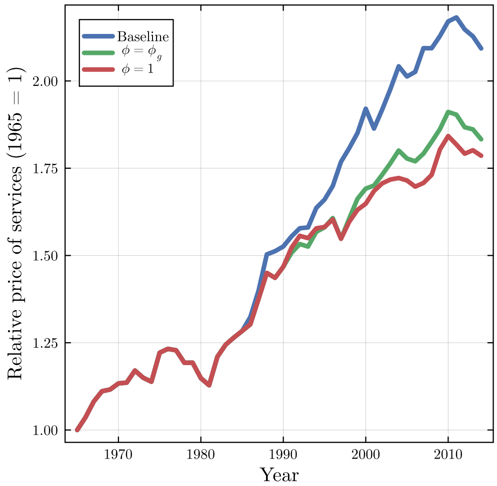

Figure 10: evolution of relative prices (Services / Goods)

increase in the service share of both intermediates and final demand, it cumulates into an increase in the service share of total output. It is represented by the rise in the service share of total employment (figure 9b). That is, heterogeneous bias in technical change is a key mechanism in broader structural change. In the United States, from 1965 to 2014, it accounts for more than half of the rise in the services share of aggregate intermediate inputs and employment share, and roughly 20% of the growth in the services share of final expenditures.

5.2 Counterfactual exercise: aggregate growth

From proposition 3, we know that service-biased intermediate technical change has a first-order negative effect upon aggregate growth. However, it also has a second-order impact: it increases the aggregate demand for services in the economy, as shown in the previous section, thus increasing the service (Domar) weight in equation 21. That is, as the services-producing sector becomes increasingly service-intensive in its productive process, it represents an ever-increasing share of the nominal output, making the growth rate of aggregate value-added converge toward the growth of its productivity (as in equation 25). Since the growth rate of the services-producing sector's labor productivity is stagnant (see figure 7) compared to the one for the goods-producing sector, it slows down aggregate value-added.

Figure 11 shows the role of bias in technical change upon aggregate value-added. As expected, service-biased technical change slowed down aggregate economic growth in the U.S. between 1965 and 2014. The counterfactual growth path (with intermediate-neutral technical change) diverged from the baseline model in the mid-1980s, as the technical bias component in the services-producing sector

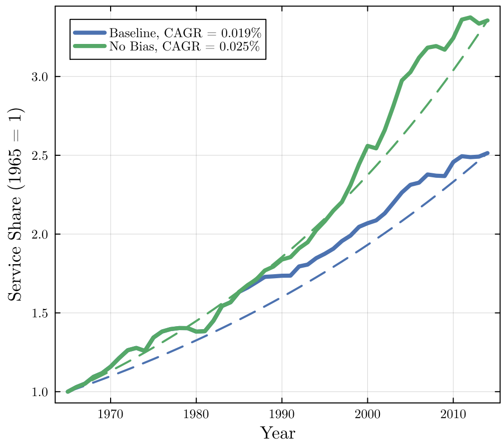

Figure 11: Counterfactual Exercise (Economic Growth)

began to diverge from that in the goods-producing sector. In total, services-biased technical change leads to a decline of aggregate GDP of more than 25% compared to the counterfactual simulation.

6 Extensions

6.1 Non-homothetic preferences

In section 3, we assumed the representative household's preferences to be homothetic. Less formally, the allocation of final demand between sectoral outputs (goods and services) is to be unaffected by income level: a richer household will consume the same composition of final expenditures (arbeit in larger quantities), ceteris paribus (e.g., when facing the same prices). While this is a commonly held assumption in a large swath of macroeconomics academic literature – often considered a desirable property of the demand system – recent research focusing on sectoral reallocation argues that income drives reallocation in both cases and empirically is non-negligible.

In this subsection, the assumption of homotheticity in final demand will be relaxed. Compared to the theoretical model in section 3, let's assume that the consumer's preferences across goods take a non-homothetic CES functional form (Comin et al., 2021 and Matsuyama, 2022), where the utility level $u(\mathbf{c}_t)$ is implicitly defined as:

$$\sum_{i\in[g,s]} \eta_{c,i} \cdot \left(\frac{c_{i,t}}{u(\mathbf{c}_t)^{\epsilon_i}} \right)^{\frac{\sigma_c - 1}{\sigma_c}} = 1.$$

where the parameters $\boldsymbol{\epsilon}$ drive non-homotheticity: notice that if we set $\epsilon_i = 1 \ \forall i \in [g,s]$ equation 3.2

collapses to the standard CES aggregator in section 3. The relative shares in final consumption can be written as

$$log\left(\frac{p_{s,t}c_{s,t}}{p_{g,t}c_{g,t}}\right) = \ln\left(\frac{1-\eta}{\eta}\right) + \underbrace{(1-\sigma_c)\cdot(\ln p_{s,t} - \ln p_{g,t})}_{\text{substitution effect}} + \underbrace{(1-\sigma_c)\cdot(\epsilon_s - \epsilon_g)\cdot \ln u_t}_{\text{income effect}} \tag{33}$$

Equation 11 highlights the role of the non-homotheticity parameters $\boldsymbol{\epsilon}$: the consumption good characterized by a higher ϵ will see its expenditure share rise with welfare (or, approximately, income). Therefore, we can label the good with a higher ϵ as the **luxury** good. The associated (ideal) price index – that is, $\mathcal{P}_t^C(U_t,\mathbf{p})$ such that $\mathcal{P}_t^C(U_t,\mathbf{p})\cdot U_t = E(U_t,\mathbf{p})$ – reads:

$$\mathcal{P}^C(U_t;\mathbf{p}_t) = \left[\eta^{\sigma_c}\cdot\left(\frac{p_{g,t}}{U_t^{1-\epsilon_g}}\right)^{1-\sigma_c} + (1-\eta)^{\sigma_c}\cdot\left(\frac{p_{s,t}}{U_t^{1-\epsilon_s}}\right)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}$$

Notice that $\mathcal{P}^C(U_t;\mathbf{p}_t)$ differs from the standard price aggregator in the baseline model (equation 12) in that the price index is affected by the utility level via the non-homothetic parameters. It follows that the price index inherits the non-homothetic properties of the consumer's preferences: facing the same prices, the ideal price index will be different for individuals of varying levels of utility U (and therefore, by duality, of expenditure level). The non-homotheticity of the consumer's Ideal Price Index has a significant implication. While in the baseline model setting, the price index as the numeraire makes the definition of aggregate value-added equivalent to the empirical best practice, this is no longer the case with non-homothetic preferences. It follows from the fact that, empirically, the computation of real value-added relies on the assumption that the underlying aggregator (that need not be defined) is homothetic: a requirement for chain indexing using the standard indexes (Törnqvist, Fischer, Paasche, Laspereys, etc.) as done by statistical agencies[4]. Therefore, in the model with non-homothetic preferences, aggregate value-added takes a different interpretation: it represents the welfare-relevant aggregate value-added, or, equivalently, nominal value-added deflated by a welfare-relevant price index that does not match empirical practice.

Figure 12 shows a replication of the quantitative exercises in section 5 with and without non-homothetic preferences, the former being the same as in section 5 and shown here for reference's sake. The non-homotheticity parameters for goods (ϵ_G) and services (ϵ_S) are set to 1.0 and 1.4, respectively. While this value matches the empirical value for the U.S. from Comin et al. (2021), it is chosen here for illustrative purposes. The key takeaway is that, as expected, non-homotheticity does not affect the demand for intermediates (sub-figure 12a) in any appreciable manner. It is not, however, the case for

[4]The BEA's NIPA tables, for example, compute real value-added by chain indexing using a Fisher Index (Whelan, 2002).

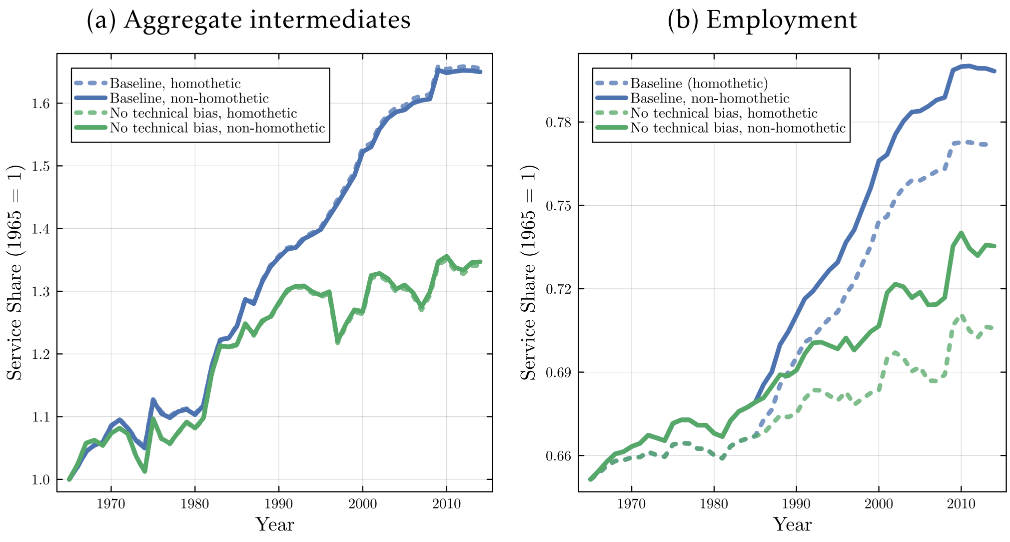

Figure 12: Numerical exercise with non-homothetic preferences

the service share of employment (sub-figure!12b): as the economy grows, the demand for services in final consumption grows faster compared to the model with non-homothetic preferences. It is a natural consequence of services being a ”luxury” expenditure item: the richer the consumer is, the more services she buys. However, notice that the counterfactual exercises show a proportional decline in both the non-homothetic and homothetic cases: non-homothetic preferences have a marginal impact on the impact of biased technical change upon aggregate dynamics, the main mechanism being via relative prices.

6.2 Capital

In the theoretical model in section 3, we assumed that there was a single factor, labor. However, the standard Neoclassical framework maintains the presence of both labor and capital. In this section, we will show, theoretically, that investment dynamics have no direct effect on the mechanism explored in this work. In contrast, investment into capital is both a key driver of aggregate growth (as in the canonical Solow (1962) framework). A relevant environment for structural change (Herrendorf et al., 2021 and Gaggl et al., 2023), it mostly works in parallel to technical bias. Here, we will present the key results from extending the model in section 3 with capital accumulation. A complete description of the model can be found in appendix C.

Let's start by noticing that the relative demand of intermediates, equation 2, is unaffected by the functional form of the production function f and the number of intermediates: the presence of capital in the production function does not affect intermediate composition. Following the same approach as the main body, we have:

Lemma 2 (Evolution of sectoral value-added (with capital))**.**

$$\Delta \ln Y_{i,t} = \underbrace{\frac{\Delta \ln A^{go}_{i,t} + \alpha_i(\Delta \ln \theta_{i,t} - \chi_{is,t}\Delta \ln \phi_{i,t})}{1-\alpha_i}}_{\equiv \Delta \ln A^{va}} + \beta \Delta \ln \ell_{i,t} + (1-\beta)\Delta \ln \kappa_{i,t}, \tag{34}$$

Proof. See appendix C

It is immediate to see that in equation 34, the growth rate of sectoral value-added TFP is equivalent to the one in the main model (equation 15): the use of capital does not affect the role of biased technical change upon sectoral value-added TFP. The same goes for aggregate TFP, as shown by proposition 3.

Lemma 3 (Aggregate value-added (with capital))**.** *Aggregate value-added in terms of the numeraire reads:*

$$Y_t = \underbrace{\mathcal{C}^Y \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}^M_{j,t}}{A^{go}_{j,t}} \right)^{\sigma_c - 1} \right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}}_{\mathcal{A}_t} \cdot K_t^{1-\beta} \underbrace{\mathcal{L}^\beta}_{=1}$$

Furthermore,

$$r_t = (1-\beta)\mathcal{A}_t K_t^{-\beta} \qquad \textit{and} \qquad w_t = \beta \mathcal{A}_t K_t^{1-\beta}$$

Proof. See appendix C

Furthermore, proposition 3 states the existence of an aggregate production function, alongside the aggregate factor compensation r_t and w_t as functions of aggregate quantities. Therefore, we have the standard Houlten theorem:

Proposition 7. *The growth rate of aggregate value-added reads:*

$$\Delta \ln Y_t = \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A^{va}_{i,t} + (1-\beta)\Delta \ln K_t,$$

where $\mathcal{I}_t$ is aggregate investment at time t.

Proof. See appendix C

Again, the (first-order) evolution of aggregate value-added is the weighted average of sectoral value-added productivities. However, we know that the latter is not affected by capital accumulation (equation 34). Due to the existence of an aggregate production function, the model with capital accumulation is compatible with a balanced growth path, as shown in proposition 8.

Proposition 8. *An aggregate balanced growth path (ABGP) exists where all aggregate variables grow at a constant and symmetric rate:*

$$\Delta \ln K = \Delta \ln W = \Delta \ln Y = \Delta \ln C = \Delta \ln K = \frac{\Delta \ln \mathcal{A}}{\beta} = \frac{\sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A_{i,t}^{va}}{\beta}$$

Proof. See appendix C

Proposition 8 states that the role of biased technical change upon aggregate growth in a balanced growth path is directly unaffected by the capital accumulation process.

7 Conclusion

This paper studies the role of biased technical change in intermediate demand on the process of structural transformation. The empirical evidence presented here shows that most of the reallocation toward services intermediates occurs within sub-sectors but aggregates into large between-sector shifts, and that relative prices alone cannot account for the observed dynamics. A two-sector model with service-biased sectoral technical change provides a parsimonious framework to interpret these facts, linking biased innovation to the composition of intermediates, sectoral outcomes, and aggregate growth. Quantitative results suggest that service-biased technical change was a central force behind the rise of service intermediates in the United States between 1965 and 2014, accounting for a substantial share of sectoral reallocation and exerting a nontrivial drag on real aggregate value-added. More broadly, the findings underscore that the bias in technical change is a first-order determinant of structural transformation and aggregate performance.

References

Daron Acemoglu. Directed technical change. *The review of economic studies*, 69(4):781–809, 2002.

Daron Acemoglu and Pablo D Azar. Endogenous production networks. *Econometrica*, 88(1):33–82, 2020.

Daron Acemoglu, Ufuk Akcigit, and William Kerr. Networks and the macroeconomy: An empirical exploration. *Nber macroeconomics annual*, 30(1):273–335, 2016.

Pol Antras. Is the us aggregate production function cobb-douglas? new estimates of the elasticity of substitution. *Contributions in Macroeconomics*, 4(1):20121005, 2004.

David Baqaee and Elisa Rubbo. Micro propagation and macro aggregation. *Annual Review of Economics*, 15(1):91–123, 2023.

David Rezza Baqaee and Emmanuel Farhi. Macroeconomics with heterogeneous agents and input-output networks. Technical report, National Bureau of Economic Research, 2018.

David Rezza Baqaee and Emmanuel Farhi. The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019.

Giuseppe Berlingieri. Outsourcing and the rise in services. 2013.

Timo Boppart. Structural change and the kaldor facts in a growth model with relative price effects and non-gorman preferences. *Econometrica*, 82(6):2167–2196, 2014.

Diego Comin, Danial Lashkari, and Martí Mestieri. Structural change with long-run income and price effects. *Econometrica*, 89(1):311–374, 2021.

Peter Diamond, Daniel McFadden, and Miguel Rodriguez. Measurement of the elasticity of factor substitution and bias of technical change. In *Contributions to economic analysis*, volume 2, pages 125–147. Elsevier, 1978.

Evsey D Domar. On the measurement of technological change. *The Economic Journal*, 71(284):709–729, 1961.

Georg Duernecker, Berthold Herrendorf, and Akos Valentinyi. Structural change within the services sector and the future of cost disease. *Journal of the European Economic Association*, 22(1):428–473, 2024.

Paul Gaggl, Aspen Gorry, and Christian Vom Lehn. Structural change in production networks and economic growth. 2023.

Alessandro Galesi and Omar Rachedi. Services deepening and the transmission of monetary policy. *Journal of the European Economic Association*, 17(4):1261–1293, 2019.

Sewnet Getahun, Habtamu Kefale, and Yohannes Gelaye. Application of precision agriculture technologies for sustainable crop production and environmental sustainability: A systematic review. *The Scientific World Journal*, 2024(1):2126734, 2024.

Berthold Herrendorf, Richard Rogerson, and Akos Valentinyi. Growth and structural transformation. *Handbook of economic growth*, 2:855–941, 2014.

Berthold Herrendorf, Richard Rogerson, and Akos Valentinyi. Structural change in investment and consumptiona unified analysis. *The Review of Economic Studies*, 88(3):1311–1346, 2021.

Charles R Hulten. Growth accounting with intermediate inputs. *The Review of Economic Studies*, 45 (3):511–518, 1978.

Rainer Klump, Peter McAdam, and Alpo Willman. The normalized ces production function: theory and empirics. *Journal of Economic surveys*, 26(5):769–799, 2012.

Piyabha Kongsamut, Sergio Rebelo, and Danyang Xie. Beyond balanced growth. *The Review of Economic Studies*, 68(4):869–882, 2001.

Danial Lashkari, Arthur Bauer, and Jocelyn Boussard. Information technology and returns to scale. *American Economic Review*, 114(6):1769–1815, 2024.

Miguel A León-Ledesma, Peter McAdam, and Alpo Willman. Identifying the elasticity of substitution with biased technical change. *American Economic Review*, 100(4):1330–1357, 2010.

Kiminori Matsuyama. Non-ces aggregators: A guided tour. *Buffett Institute Global Poverty Research Lab Working Paper Forthcoming*, 2022.

Jorge Miranda-Pinto. Production network structure, service share, and aggregate volatility. *Review of Economic Dynamics*, 39:146–173, 2021.

Alessio Moro. Biased technical change, intermediate goods, and total factor productivity. *Macroeconomic Dynamics*, 16(2):184–203, 2012.

L Rachel Ngai and Christopher A Pissarides. Structural change in a multisector model of growth. *American economic review*, 97(1):429–443, 2007.

L Rachel Ngai and Orhun Sevinc. A multisector perspective on wage stagnation. *Review of Economic Dynamics*, 56:101269, 2025.

Steven Olley and Ariel Pakes. The dynamics of productivity in the telecommunications equipment industry, 1996.

Amil Petrin, Brian P Poi, and James Levinsohn. Production function estimation in stata using inputs to control for unobservables. *The Stata Journal*, 4(2):113–123, 2004.

Elisa Rubbo. Networks, phillips curves, and monetary policy. *Econometrica*, 91(4):1417–1455, 2023.

Robert M Solow. Technical progress, capital formation, and economic growth. *The American Economic Review*, 52(2):76–86, 1962.

Michael Sposi. Evolving comparative advantage, sectoral linkages, and structural change. *Journal of Monetary Economics*, 103:75–87, 2019.

Tomasz Swikecki. Determinants of structural change. *Review of economic dynamics*, 24:95–131, 2017.

Markus Trunschke and Kenneth L Judd. Estimating gross output production functions. Technical report, National Bureau of Economic Research, 2024.

Akos Valentinyi. Structural transformation, input-output networks, and productivity growth. *Structural Transformation and Economic Growth (STEG) Pathfinding paper*, 2021.

Karl Whelan. A guide to us chain aggregated nipa data. *Review of income and wealth*, 48(2):217–233, 2002.

Jeffrey M Wooldridge. On estimating firm-level production functions using proxy variables to control for unobservables. *Economics letters*, 104(3):112–114, 2009.

A Proofs and Derivations

A.1 Proof of proposition 2

This proof start from noticing that

$$y_{i,t} = A^{go}_{i,t} M^{\alpha}_{i,t} \left(\frac{Y_{i,t}}{A^{va}_{i,t}} \right)^{1-\alpha}$$

and therefore, replacing into the sectoral FOC yields:

$$M_{i,t} = \alpha \frac{p_{i,t}}{\mathcal{P}^M_{i,t}} A^{go}_{i,t} M^{\alpha}_{i,t} \left(\frac{Y_{i,t}}{A^{va}_{i,t}} \right)^{1-\alpha} = \left(\frac{\alpha p_{i,t}}{\mathcal{P}^M_{i,t}} \right)^{\frac{1}{1-\alpha}} \frac{(A^{go}_{i,t})^{\frac{1}{1-\alpha}}}{A^{va}_{i,t}} Y_{i,t}.$$

Furthermore we also have that

$$y_{i,t} = A_{i,t}^{go} M_{i,t}^{\alpha} \left(\frac{Y_{i,t}}{A_{i,t}^{va}} \right)^{1-\alpha} = \left(\frac{\alpha p_{i,t}}{\mathcal{P}_{i,t}^{M}} \right)^{\frac{\alpha}{1-\alpha}} \frac{(A_{i,t}^{go})^{\frac{1}{1-\alpha}}}{A_{i,t}^{va}} Y_{i,t}$$

Notice that with no technical bias, we have that $(A_{i,t}^{go})^{\frac{1}{1-\alpha}} = A_{i,t}^{va}$. The value-added price is such that

$$\mathcal{P}_{i,t}^{Y} = \frac{p_{i,t} y_{i,t} - \mathcal{P}_{i,t}^{M} M_{i,t}}{Y_{i,t}} = \frac{\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}}}{A_{i,t}^{va}} \cdot \left(\frac{A_{i,t}^{go} p_{i,t}}{(P_{i,t}^{M})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \tag{35}$$

Aggregate value-added is therefore,

$$Y_t = \sum_{i \in [g,s]} \mathcal{P}_{i,t}^{Y} Y_{i,t} = \left(\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} \right) \sum_{i \in [g,s]} \left(\frac{A_{i,t}^{go} p_{i,t}}{(P_{i,t}^{M})^{\alpha}} \right)^{\frac{1}{1-\alpha}} \ell_{i,t}$$

Notice that from equation 9, we have that

$$\frac{A_{i,t}^{go} p_{i,t}}{(\mathcal{P}_{i,t}^{M})^{\alpha}} = \frac{w_t}{(1-\alpha)} \equiv B_t \qquad \forall i \in [g,s]$$

Therefore,

$$Y_t = \left(\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} \right) B_t^{\frac{1}{1-\alpha}} \underbrace{\sum_{i \in [g,s]} \ell_{i,t}}_{=1}$$

From our choice of numeraire we have that

$$\mathcal{P}_{it}^{C} = \left[\eta^{\sigma_c} \cdot \left(p_{g,t} \right)^{1-\sigma_c} + (1-\eta)^{\sigma_c} \cdot \left(p_{s,t} \right)^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}} = 1$$

Rearranging:

$$p_{i,t} = \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{p_{j,t}}{p_{i,t}} \right)^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}}$$

Notice that from equation 10, we also obtain that

$$p_{i,t} = \left\{ \sum_{j \in [g,s]} \eta_j^{\sigma_c} \left[\frac{A_{i,t}^{go}}{A_{j,t}^{go}} \left(\frac{\mathcal{P}_{j,t}^{M}}{\mathcal{P}_{i,t}^{M}} \right) \right]^{1-\sigma_c} \right\}^{\frac{1}{1-\sigma_c}}.$$

Rearranging:

$$B_t = \frac{A_{i,t}^{go} p_{i,t}}{(\mathcal{P}_{i,t}^{M})^{\alpha}} = \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}_{j,t}^{M}}{A_{j,t}^{go}} \right)^{\sigma_c - 1} \right]^{\frac{1}{1-\sigma_c}}.$$

Therefore

$$Y_t = \underbrace{\mathcal{C}^Y \left[\sum_{j\in[g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}^M_{j,t}}{A^{go}_{j,t}} \right)^{\sigma_c - 1} \right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}}_{\mathcal{A}_t}.$$

Finally, the equation ruling $\mathcal{P}^M_{i,t}$ is quickly obtained by replacing condition 9 into the definition of $\mathcal{P}^M_{i,t}$. That is, replace

$$p_{i,t} = \frac{(\mathcal{P}^M_{i,t})^\alpha}{A^{go}_{i,t}} \cdot \frac{w_t}{1-\alpha} \qquad \forall i \in [g,s]$$

into

$$\mathcal{P}^M_{i,t} = \frac{1}{\theta_{i,t}} \left[\gamma_i^{\sigma_i} \left(p_{ig,t} \right)^{1-\sigma_i} + (1-\gamma_i)^{\sigma_i} \phi_{s,t} \left(p_{is,t} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}}, \qquad \forall i \in [g,s].$$

Therefore

$$\mathcal{P}^M_{i,t} = \frac{1}{(1-\alpha)\theta_{i,t}} \left[\gamma_i^{\sigma_i} \left(\frac{(\mathcal{P}^M_{g,t})^\alpha}{A^{go}_{g,t}} \right)^{1-\sigma_i} + (1-\gamma_i)^{\sigma_i} \phi_{i,t} \left(\frac{(\mathcal{P}^M_{s,t})^\alpha}{A^{go}_{s,t}} \right)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \qquad \forall i \in [g,s]$$

A.2 Proof of proposition 3

Proof. Aggregate value-added is a Laspereys index of sectoral value-added:

$$\Delta \ln Y_t = \sum_{i\in[g,s]} \frac{P^Y_{i,t} Y_{i,t}}{\sum_{j\in[g,s]} P^Y_{j,t} Y_{j,t}} \Delta \ln Y_{i,t}$$

Since $\mathcal{P}^Y_i Y_i = w_t \ell_{i,t}$, then

$$\Delta \ln Y_t = \sum_{i\in[g,s]} \ell_{i,t} \cdot \Delta \ln Y_{i,t}$$

notice that

$$\ell_i = (1-\alpha_i) \frac{p_{i,t} y_{i,t}}{w_t} \equiv (1-\alpha_i)\lambda_{i,t}$$

where $\lambda_{i,t}$ are the Domar weights. Therefore

$$\Delta \ln Y_t = \sum_{i\in[g,s]} (1-\alpha_i)\lambda_{i,t} \cdot \Delta \ln Y_{i,t} \tag{36}$$

$$= \sum_{i\in[g,s]} (1-\alpha_i)\lambda_{i,t} \cdot \Delta \ln A^{va}_{i,t} + \sum_{i\in[g,s]} \underbrace{(1-\alpha_i)\lambda_{i,t}}_{=\ell_{i,y}} \cdot \Delta \ln \ell_{i,t} \tag{37}$$

$$= \sum_{i\in[g,s]} \lambda_{i,t}\left(\Delta \ln A^{go}_{i,t} + \alpha_i \Delta \ln \theta_{i,t}\right) - \sum_{i\in[g,s]} s_{i,t}\Delta \ln \theta_{i,t} + \underbrace{\sum_{i\in[g,s]} \Delta \ell_{i,t}}_{=0}, \tag{38}$$

where we exploited the fact that

$$\alpha_i \chi_{si,t} \lambda_{i,t} = \alpha_i \frac{p_{s,t} x_{is,t}}{P^M_{i,t} M_{i,t}} \frac{p_{i,t} y_{i,t}}{\sum_{j\in[g,s]} P^Y_{j,t} Y_{j,t}} = \alpha_i \underbrace{\frac{p_{i,t} y_{i,t}}{P^M_{i,t} M_{i,t}}}_{\equiv 1/\alpha_i} \underbrace{\frac{p_{s,t} x_{is,t}}{\sum_{j\in[g,s]} P^Y_{j,t} Y_{j,t}}}_{\equiv s_{i,t}} = s_{i,t}$$

■

A.3 Proof of proposition 4

From the sectoral F.O.C. zero-profit condition we can derive:

$$\Delta \ln p_i = \alpha_i \Delta \ln P^M_{i,t} + (1-\alpha_i)\Delta \ln w_t - \Delta \ln A^{go}_{i,t}$$

Since

$$\Delta \ln P^M_{i,t} \approx \chi_{ig,t}\Delta \ln p_{g,t} + \chi_{is,t}\Delta \ln p_{s,t} + \chi_{is,t}\Delta \ln \phi_{i,t} - \Delta \ln \theta_{i,t}$$

Notice that

$$\alpha_i \chi_{ij,t} = \frac{P^M_{i,t} M_{i,t}}{p_{i,t} y_{i,t}} \frac{p_{j,t} x_{ij,t}}{P^M_{i,t} M_{i,t}} = \frac{p_{j,t} x_{ij,t}}{p_{i,t} y_{i,t}} \equiv \Omega_{ij,t}$$

is the intermediate j' share of total output for sector i. Therefore

$$\Delta \ln p_i = \Omega_{ig,t}\Delta \ln p_{g,t} + \Omega_{is,t}\Delta \ln p_{s,t} + (1-\alpha_i)\Delta \ln w_t - \Delta \ln A^{go}_{i,t} - \alpha_i \Delta \ln \theta_{i,t} + \underbrace{\alpha_i \chi_{is,t}}_{\equiv \Omega_{is,t}} \Delta \ln \phi_{i,t}$$

In matrix form:

$$\Delta \ln \mathbf{p}_t = \Omega \Delta \ln \mathbf{p}_t - \Delta \ln \mathbf{A}^{go}_t + \Delta \ln \tilde{\boldsymbol{\theta}}_t + \Omega_s \Delta \ln \boldsymbol{\phi}_t - \boldsymbol{\alpha}_t \Delta \ln w_t$$

where

$$\mathbf{p}_t = \begin{bmatrix} p_{g,t} \\ p_{s,t} \end{bmatrix}, \quad \mathbf{A}_t = \begin{bmatrix} A_{g,t} \\ A_{s,t} \end{bmatrix}, \quad \tilde{\boldsymbol{\theta}}_t = \begin{bmatrix} \alpha_g \theta_{g,t} \\ \alpha_s \theta_{s,t} \end{bmatrix}, \quad \boldsymbol{\phi}_t = \begin{bmatrix} \phi_{g,t} \\ \phi_{s,t} \end{bmatrix},$$

$$\Omega_t = \begin{bmatrix} \Omega_{gg,t} & \Omega_{gs,t} \\ \Omega_{sg,t} & \Omega_{ss,t} \end{bmatrix}, \quad \Omega_{s,t} = \begin{bmatrix} \Omega_{gs,t} \\ \Omega_{ss,t} \end{bmatrix} \qquad \boldsymbol{\alpha}_t = \begin{bmatrix} 1-\alpha_{g,t} \\ 1-\alpha_{s,t} \end{bmatrix}$$

Notice that the sectoral budget constraint implies that

$$\frac{p_{g,t}x_{ig,t}}{p_{i,t}y_{i,t}}+\frac{p_{s,t}x_{is,t}}{p_{i,t}y_{i,t}}+\underbrace{\frac{w_t\ell_{i,t}}{p_{i,t}y_{i,t}}}_{1-\alpha_i}=1.$$

Solving for $\mathbf{p}_t$ yields

$$\Delta\ln\mathbf{p}_t=(\mathbf{1}-\Omega)^{-1}\left(\Delta\ln\mathbf{A}_t^{go}-\Delta\ln\tilde{\boldsymbol{\theta}}_t+\Omega_s\Delta\ln\boldsymbol{\phi}_t\right)-\Delta\ln w_i$$

where we exploited the fact that the $\Delta\ln w_i$ term cancels out as in Baqaee and Rubbo (2023) and that $w=Y$ due to market clearing and the numeraire choice. Define the Leontief inverse $(\mathbf{1}-\Omega)^{-1}$ as $\hat{\Omega}$. The system above can be rewritten as

$$\Delta\ln p_{i,t}=\overbrace{\sum_{j\in[g,s]}\hat{\Omega}_{ij,t}\Omega_{is,t}\Delta\ln\phi_{j,t}}^{\text{Bias component}}-\overbrace{\sum_{j\in[g,s]}\hat{\Omega}_{ij,t}(\Delta\ln A_{j,t}^{go}-\alpha_i\Delta\ln\theta_{i,t})}^{\text{TFP component}}-\overbrace{\Delta\ln Y_t}^{\text{Numeraire adjustment}}$$

A.4 Proof of proposition 6

Redefine $\tilde{\mathcal{P}}^M=\mathcal{P}_i^M\cdot\theta_i$. Therefore

$$\tilde{\mathcal{P}}_{i,t}^M=\left[\frac{\gamma_i^{\sigma_i}}{\left(A_{g,t}^{go}\theta_g^{\alpha_g}\right)^{1-\sigma_i}}\left(\tilde{\mathcal{P}}_{g,t}^M\right)^{\alpha_i(1-\sigma_i)}+\phi_{i,t}\frac{(1-\gamma_i)^{\sigma_i}}{\left(A_{s,t}^{go}\theta_s^{\alpha_s}\right)^{1-\sigma_i}}\left(\tilde{\mathcal{P}}_{s,t}^M\right)^{\alpha_i(1-\sigma_i)}\right]^{\frac{1}{1-\sigma_i}}\cdot Y_t^{1-\alpha_i}\qquad\forall i\in[g,s]$$

Due do the exponential nature of the productivity terms, we have that, asymptotically, the service term dominates. Therefore, we have that

$$\lim_{t\to\infty}\tilde{\mathcal{P}}_{i,t}^M=\lim_{t\to\infty}\left[\phi_{i,t}\frac{(1-\gamma_i)^{\sigma_i}}{\left(\tilde{A}_{s,t}^{go}\right)^{(1-\sigma_i)}}\left(\tilde{\mathcal{P}}_{s,t}^M\right)^{\alpha_i(1-\sigma_i)}\right]^{\frac{1}{1-\sigma_i}}\cdot Y_t^{1-\alpha_i}\qquad\forall i\in[g,s].$$

Rearranging

$$\lim_{t\to\infty}\tilde{\mathcal{P}}_{i,t}^M=B_i\cdot\phi_{i,t}^{\frac{1}{1-\sigma_i}}\cdot\frac{\left(\tilde{\mathcal{P}}_{s,t}^M\right)^{\alpha_i}}{\tilde{A}_{s,t}^{go}}\cdot Y^{1-\alpha_i}\tag{39}$$

Under the simplifying assumption that $\alpha_i=\alpha\ \forall i\in[g,s]$,

$$\lim_{t\to\infty}\frac{\left(\tilde{\mathcal{P}}_{st}^M\right)^{\alpha}}{\left(\tilde{\mathcal{P}}_{gt}^M\right)^{\alpha}}=const.\cdot\left(\frac{\phi_s^{\frac{1}{1-\sigma_s}}}{\phi_g^{\frac{1}{1-\sigma_g}}}\right)^{\alpha}$$

and thus

$$\lim_{t\to\infty}\frac{\frac{(\tilde{\mathcal{P}}^M_{st})^\alpha}{\tilde{A}^{go}_{s,t}}\cdot Y^{\epsilon_s-\alpha}}{\frac{(\tilde{\mathcal{P}}_{gt})^\alpha}{\tilde{A}^{go}_{g,t}}\cdot Y^{\epsilon_g-\alpha}}=\lim_{t\to\infty} const.\cdot\left(\frac{\tilde{A}^{go}_{g,t}}{\tilde{A}^{go}_{s,t}}\right)\left(\frac{\phi_s^{\frac{1}{1-\sigma_s}}}{\phi_g^{\frac{1}{1-\sigma_g}}}\right)^\alpha\cdot Y^{\epsilon_s-\epsilon_g}$$

Since $\Delta\ln\tilde{A}^{go}_{g,t}>\Delta\ln\tilde{A}^{go}_{s,t}$, $\Delta\ln\phi_{s,t}>\Delta\ln\phi_{s,t}$, and $\epsilon_s-\epsilon_g>0$ then

$$\lim_{t\to\infty}\frac{\left(\tilde{\mathcal{P}}^M_{st}\right)^\alpha}{\tilde{A}^{go}_{s,t}}\cdot Y^{\epsilon_s-\alpha}>>\lim_{t\to\infty}\frac{\left(\tilde{\mathcal{P}}^M_{gt}\right)^\alpha}{\tilde{A}^{go}_{g,t}}\cdot Y^{\epsilon_s-\alpha}$$

Therefore, asymptotically, in

$$\left[\sum_{j\in\{g,s\}}\eta_j^{\sigma_c}\left(\frac{(\tilde{\mathcal{P}}^M_{jt})^\alpha}{A^{go}_{j,t}\theta_j^{\alpha_j}}\right)^{1-\sigma_c}Y_t^{(\epsilon_j-\alpha_j)(1-\sigma_c)}\right]^{\frac{1}{1-\sigma_c}}=1$$

the service term dominates and thus

$$\lim_{t\to\infty}\eta_s^{\frac{\sigma_s}{1-\sigma_s}}\frac{\left(\tilde{\mathcal{P}}^M_{st}\right)^\alpha}{\tilde{A}^{go}_{s,t}}\cdot Y^{\epsilon_s-\alpha}=1$$

Notice that from equation 39 we have that

$$\lim_{t\to\infty}\tilde{\mathcal{P}}^M_{s,t}=B_s\frac{Y_t\cdot\phi_{s,t}^{\frac{1}{(1-\sigma_s)(1-\alpha)}}}{\left(\tilde{A}^{go}_{s,t}\right)^{\frac{1}{1-\alpha}}}$$

Thus

$$\lim_{t\to\infty}\frac{\eta_s^{\frac{\sigma_s}{1-\sigma_s}}}{\tilde{A}^{go}_{s,t}}\cdot\left(B_s^\alpha\frac{Y_t^\alpha\cdot\phi_{s,t}^{\frac{\alpha}{(1-\sigma_s)(1-\alpha)}}}{\left(\tilde{A}^{go}_{s,t}\right)^{\frac{\alpha}{1-\alpha}}}\right)\cdot Y^{\epsilon_s-\alpha}=\lim_{t\to\infty}\left(\eta_s^{\frac{\sigma_s}{1-\sigma_s}}B_s^\alpha\frac{\cdot\phi_{s,t}^{\frac{\alpha}{(1-\sigma_s)(1-\alpha)}}}{\left(\tilde{A}^{go}_{s,t}\right)^{\frac{1}{1-\alpha}}}\right)\cdot Y^{\epsilon_s}=1$$

rearranging

$$\lim_{t\to\infty}Y_t^{\epsilon_s}=\lim_{t\to\infty}const.\cdot\tilde{A}^{go}_{s,t}\phi_{s,t}^{-\frac{\alpha}{(1-\sigma_s)(1-\alpha)}}$$

which implies that

$$\lim_{t\to\infty}\Delta\ln Y_t=\frac{1}{\epsilon_s(1-\alpha)}\cdot\Delta\ln\tilde{A}^{go}_{s,t}-\frac{\alpha}{\epsilon_s(1-\sigma_s)(1-\alpha)}\cdot\Delta\ln\phi_{s,t}$$

B Additional empirical results

B.1 Country-level Shift-Share decomposition

Country	Delta	Between	Within	Residual
AUS	0.27	0.300	0.700	0.000
AUT	0.23	0.252	0.748	0.000
BEL	0.26	0.231	0.769	-0.000
BRA	0.07	0.320	0.680	-0.000
CAN	0.12	0.687	0.313	-0.000
CHN	-0.00	6.746	-5.746	-0.000
DEU	0.28	0.274	0.726	0.000
DNK	0.31	0.294	0.706	0.000
ESP	0.22	0.261	0.739	-0.000
FIN	0.21	0.257	0.743	-0.000
FRA	0.18	0.459	0.541	0.000
GBR	0.29	0.478	0.522	-0.000
GRC	0.12	0.556	0.444	-0.000
HKG	0.24	0.380	0.620	-0.000
IRL	0.32	0.588	0.412	-0.000
ITA	0.22	0.310	0.690	0.000
JPN	0.17	0.428	0.572	0.000
KOR	0.04	0.422	0.578	0.000
MEX	0.09	0.469	0.531	-0.000
NLD	0.18	0.424	0.576	-0.000
PRT	0.10	0.620	0.380	0.000
SWE	0.12	0.332	0.668	-0.000

Country	Delta	Between	Within	Residual
TWN	0.12	0.061	0.939	-0.000
USA	0.22	0.426	0.574	0.000
BGR	0.01	1.429	-0.429	-0.000
CHE	0.03	-0.263	1.263	-0.000
CYP	0.02	0.404	0.596	-0.000
CZE	0.03	-0.437	1.437	-0.000
EST	-0.02	1.940	-0.940	-0.000
HRV	0.04	-0.106	1.106	0.000
HUN	0.03	0.182	0.818	0.000
IDN	-0.04	-0.121	1.121	0.000
LTU	-0.02	-1.111	2.111	0.000
LUX	0.08	0.695	0.305	0.000
LVA	-0.05	0.661	0.339	0.000
MLT	0.18	0.958	0.020	0.022
NOR	-0.00	2.588	-1.588	0.000
POL	-0.08	0.234	0.766	0.000
ROU	0.10	0.382	0.618	0.000
ROW	-0.08	0.216	0.784	0.000
RUS	0.06	0.430	0.570	-0.000
SVK	-0.06	0.738	0.262	-0.000
SVN	0.05	0.537	0.463	-0.000
TUR	0.04	0.412	0.588	-0.000

Table 6: Within–Between decomposition (demand for services)

Country	Delta	Between	Within	Residual
AUS	0.21	0.124	0.876	-0.000
AUT	0.12	-0.110	1.110	-0.000
BEL	0.12	-0.116	1.116	-0.000
BRA	0.05	-0.049	1.049	-0.000
CAN	0.05	0.270	0.730	0.000
CHN	0.02	0.312	0.688	-0.000
DEU	0.22	-0.003	1.003	0.000
DNK	0.25	-0.005	1.005	0.000
ESP	0.11	-0.171	1.171	-0.000
FIN	0.12	-0.137	1.137	-0.000
FRA	0.11	-0.091	1.091	0.000
GBR	0.10	0.009	0.991	-0.000
GRC	0.05	0.416	0.584	0.000
HKG	0.21	0.025	0.975	0.000
IRL	0.34	0.396	0.604	0.000
ITA	0.15	0.063	0.937	0.000
JPN	0.09	0.023	0.977	-0.000
KOR	0.06	0.398	0.602	-0.000
MEX	0.08	0.057	0.943	-0.000
NLD	0.10	0.104	0.896	0.000
PRT	0.04	-0.347	1.347	0.000
SWE	0.09	-0.008	1.008	-0.000

Country	Delta	Between	Within	Residual
TWN	0.10	-0.098	1.098	0.000
USA	0.07	-0.248	1.248	0.000
BGR	-0.01	-0.503	1.503	-0.000
CHE	0.02	-0.678	1.678	0.000
CYP	0.02	-0.122	1.122	0.000
CZE	0.03	-0.432	1.432	-0.000
EST	-0.01	6.649	-5.649	0.000
HRV	0.02	-1.169	2.169	-0.000
HUN	0.02	0.207	0.793	-0.000
IDN	-0.01	0.039	0.961	-0.000
LTU	-0.03	-0.962	1.962	-0.000
LUX	0.07	0.359	0.641	0.000
LVA	-0.02	0.104	0.896	-0.000
MLT	0.01	1.200	-0.200	-0.000
NOR	0.00	3.372	-2.372	0.000
POL	-0.08	0.055	0.945	-0.000
ROU	0.07	0.111	0.889	0.000
ROW	-0.09	0.005	0.995	-0.000
RUS	0.04	0.171	0.829	0.000
SVK	-0.07	0.493	0.507	-0.000
SVN	0.02	-0.013	1.013	0.000
TUR	0.01	-0.003	1.003	0.000

Table 7: Within–Between decomposition (demand for services, only Goods industries)

Country	Delta	Between	Within	Residual
AUS	0.13	-0.546	1.546	0.000
AUT	0.27	0.118	0.882	-0.000
BEL	0.31	0.078	0.922	-0.000
BRA	0.03	-0.353	1.353	-0.000
CAN	0.04	0.344	0.656	-0.000
CHN	0.07	-0.228	1.228	-0.000
DEU	0.21	0.100	0.900	0.000
DNK	0.19	-0.005	1.005	-0.000
ESP	0.22	0.094	0.906	0.000
FIN	0.16	-0.112	1.112	-0.000
FRA	0.06	0.149	0.851	0.000
GBR	0.25	0.047	0.953	-0.000
GRC	0.07	0.074	0.926	0.000
HKG	0.06	0.228	0.772	0.000
IRL	0.11	0.206	0.794	0.000
ITA	0.15	-0.108	1.108	-0.000
JPN	0.06	-0.874	1.874	-0.000
KOR	0.00	2.454	-1.454	0.000
MEX	-0.03	-0.225	1.225	-0.000
NLD	0.11	-0.012	1.012	-0.000
PRT	0.03	0.770	0.230	-0.000
SWE	0.06	-0.257	1.257	-0.000

Country	Delta	Between	Within	Residual
TWN	0.14	0.039	0.961	-0.000
USA	0.17	0.053	0.947	-0.000
BGR	0.04	0.425	0.575	-0.000
CHE	0.03	-0.071	1.071	0.000
CYP	-0.02	1.075	-0.075	0.000
CZE	0.03	-0.052	1.052	-0.000
EST	-0.00	5.120	-4.120	0.000
HRV	0.06	-0.062	1.062	-0.000
HUN	0.05	0.011	0.989	0.000
IDN	-0.07	-0.402	1.402	-0.000
LTU	-0.01	2.394	-1.394	-0.000
LUX	0.02	-0.149	1.149	0.000
LVA	-0.02	0.086	0.914	0.000
MLT	0.04	0.601	0.249	0.150
NOR	0.01	-0.363	1.363	0.000
POL	-0.03	-0.355	1.355	0.000
ROU	0.07	0.158	0.842	-0.000
ROW	0.02	-0.140	1.140	-0.000
RUS	0.04	0.039	0.961	0.000
SVK	0.03	0.141	0.859	0.000
SVN	0.03	0.312	0.688	-0.000
TUR	0.02	-1.058	2.058	-0.000

Table 8: Within–Between decomposition (demand for services, only Services industries)

C Model with capital

C.1 Technology

In the economy, there are two competitive sectors, "Goods" (g) and "Services" ("s"), each producing a single differentiated product. As standard in the macroeconomics literature (e.g. Moro, 2012, and Gaggl et al., 2023), let's assume that the sector-specific production function features a composite input structure, where intermediate goods are aggregated into a single aggregate intermediate input M, which then enters the production function alongside capital and labor:

$$y_i = A^{go}_{i,t} M_i\left(\mathbf{x}_i; \phi_{i,t}\right)^{\alpha_i} \left(\ell_i^{\beta} \kappa_i^{1-\beta}\right)^{1-\alpha_i}.$$

ϕ and A^{go} being technical bias a la Acemoglu, 2002 and sectoral TFP respectively. As before, we assume that M takes a CES form:

$$M\left(\mathbf{x}_{i,t}, \phi_{i,t}\right) = \theta_{i,t}\left[\gamma_i (x_{ig,t})^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\gamma_i)\phi_{i,t}^{\frac{1}{\sigma_i}} (x_{is,t})^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}.$$

The standard cost minimization problem yields the following demand for intermediates:

$$x_{ig,t} = \gamma_i^{\sigma_i}\left(\frac{p_{gt}}{\mathcal{P}^M_{i,t}}\right)^{-\sigma_i} \cdot M_{i,t} \qquad \forall i \in [g,s] \tag{40}$$

$$x_{is,t} = \phi_{i,t}(1-\gamma_i)^{\sigma_i}\left(\frac{p_{st}}{\mathcal{P}^M_{i,t}}\right)^{-\sigma_i} \cdot M_{i,t} \qquad \forall i \in [g,s]. \tag{41}$$

where $\mathcal{P}_i$ is the CES ideal intermediate price index:

$$\mathcal{P}_{i,t}^M \equiv \left[\gamma_i^{\sigma_i} p_{gt}^{1-\sigma_i} + \frac{(1-\gamma_i)^{\sigma_i}}{\phi_{i,t}} \cdot p_{st}^{1-\sigma_i}\right]^{\frac{1}{1-\sigma_i}}.$$

such that $\mathcal{P}_{i,t}^M \cdot M_{i,t} = p_{ig}x_{ig} + p_{is}x_{is}$. The demand for labor, capital, and intermediate goods is such that

$$\ell_{i,t} = (1-\alpha_i)\beta \cdot \frac{p_{i,t}y_{i,t}}{w_t} \qquad \forall i \in [g,s] \tag{42}$$

$$\kappa_{i,t} = (1-\alpha_i)(1-\beta) \cdot \frac{p_{i,t}y_{i,t}}{r_t} \qquad \forall i \in [g,s] \tag{43}$$

$$M_{i,t} = \alpha_i \cdot \frac{p_{i,t}y_{i,t}}{\mathcal{P}_{i,t}^M} \qquad \forall i \in [g,s] \tag{44}$$

Combining the demand functions, we obtain the following condition

$$\frac{A_{i,t}^{go} p_{i,t}}{(\mathcal{P}_{i,t}^M)_i^{\alpha}} = \left(\frac{w_t}{\beta(1-\alpha_i)}\right)^{\beta} \left(\frac{r_t}{(1-\beta)(1-\alpha_i)}\right)^{1-\beta} \tag{45}$$

C.2 Household

There is an infinitely lived representative household whose lifetime utility is:

$$U = \sum_{t=0}^{\infty} \beta^t \ln u(\mathbf{c}_t)$$

where $0 < \beta < 1$ is the discount rate and $\mathbf{c} \equiv [c_g, c_s]$ is consumption bundle. Due to time separability, the household problem can be split into an intratemporal problem where the agent maximizes u given per-period expenditures, and an intertemporal problem, where the consumer chooses how much to spend in each period, given the results of the intratemporal problem. The utility function $u(\cdot)$ is the same as that used in the main body of this work, and so is the intratemporal optimization problem: the optimal per-period allocation of expenditures between goods and services is the same as in equation 11. As before, consumers supply inelastically a unit of labor, rewarded with a wage w_t, but now also can save, facing an interest rate r_t. The intertemporal problem reads.

$$V = \left\{ \max_{\{u,\mathcal{I}\}_0^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln u(\mathbf{c}_t): \quad E(u_t;\mathbf{p}) + \mathcal{I}_{t+1} \leq w_t + (1+r_t)\mathcal{I}_t \right\}$$

Which yields the standard Euler Equation

$$\frac{E_{t+1}}{E_t} = \beta(1 + r_{t+1})$$

and the transversality condition

$$\lim_{t\to\infty} \beta^t (1+r_t) \cdot \frac{\mathcal{I}_t}{E_t} = 0$$

C.3 Investment

Aggregate capital evolves according to:

$$K_{t+1} = (1-\delta)K_t + \mathcal{I}_t$$

It is fully mobile across sectors. The market-clearing condition reads:

$$K_t = K_{gt} + K_{st}$$

A competitive sector creates new capital (Investment) by bundling both goods and services:

$$\mathcal{I}_t = \left[\gamma_{\mathcal{I}}(x_{\mathcal{I}g,t})^{\frac{\sigma_{\mathcal{I}}-1}{\sigma_{\mathcal{I}}}} + (1-\gamma_{\mathcal{I}})(x_{\mathcal{I}s,t})^{\frac{\sigma_{\mathcal{I}}-1}{\sigma_{\mathcal{I}}}}\right]^{\frac{\sigma_{\mathcal{I}}}{\sigma_{\mathcal{I}}-1}}$$

Implying the following demand for capital intermediates:

$$x_{\mathcal{I}g,t} = \gamma_{\mathcal{I}} \cdot \left(p_{gt}\right)^{-\sigma_{\mathcal{I}}} \cdot \mathcal{I}_t$$
$$x_{\mathcal{I}s,t} = (1-\gamma_{\mathcal{I}}) \cdot (p_{st})^{-\sigma_{\mathcal{I}}} \cdot \mathcal{I}_t$$

The price of aggregate investment reads:

$$\mathcal{P}_{\mathcal{I}t} \equiv \frac{1}{-}\left[\gamma_{\mathcal{I}}^{\sigma_I} \cdot p_{gt}^{1-\sigma_{\mathcal{I}}} + (1-\gamma_{\mathcal{I}})^{\sigma_I} \cdot p_{st}^{1-\sigma_{\mathcal{I}}}\right]^{\frac{1}{1-\sigma_{\mathcal{I}}}} = 1$$

C.4 Equilibrium and aggregate dynamics

Definition C.1 (Competitive equilibrium)**.** *The equilibrium is a collection of prices $(\boldsymbol{p}, w)$ and allocations $(\boldsymbol{c}, \boldsymbol{x}_g, \boldsymbol{x}_s)$ such that*

1. *Each sector solves its cost minimization problem given prices and productivities $(A_i^{go}, \phi_{i,t})$.*

2. *The representative household has its intertemporal and intratemporal problems*

3. *Factor and intermediate input markets clear:*

$$y_{i,t} = c_{i,t} + x_{gi,t} + x_{si,t} + x_{\mathcal{I}i,t} \qquad \forall i \in [g,s] \tag{46}$$

$$1 = \ell_{g,t} + \ell_{s,t} \tag{47}$$

$$K_t = \kappa_{g,t} + \kappa_{s,t} \tag{48}$$

C.4.1 Static Aggregation

Following the same approach as the main body, we have:

Proposition 9 (Evolution of sectoral value-added)**.**

$$\Delta \ln Y_{i,t} = \underbrace{\frac{\Delta \ln A^{go}_{i,t} + \alpha_i(\Delta \ln \theta_{i,t} - \chi_{is,t}\Delta \ln \phi_{i,t})}{1-\alpha_i}}_{\equiv \Delta \ln A^{va}} + \beta \Delta \ln \ell_{i,t} + (1-\beta)\Delta \ln \kappa_{i,t}, \tag{49}$$

where, we assumed that $\alpha = \alpha_i$ and $\beta = \beta_i$ $\forall i \in [g,s]$. From chain-linking:

$$\bar{Y}_{i,t} = \underbrace{\left\{ \bar{A}^{go}_{i,t} \cdot \left[\bar{\theta}_{i,t} \cdot \left(\frac{\phi^{\chi_{is,t}}_{i,t}}{\phi^{\chi_{is,0}}_{i,0}} \right)^{-1} \right]^{\alpha_i} \right\}^{\frac{1}{1-\alpha_i}}}_{\bar{A}^{va}_{i,t}} \bar{\ell}^{\beta}_{i,t} \bar{\kappa}^{1-\beta}_{i,t}. \tag{50}$$

Proposition 10. *Aggregate value-added in term of the numeraire reads:*

$$Y_t = \underbrace{\mathcal{C}^Y \left[\sum_{j\in[g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}^M_{j,t}}{A^{go}_{j,t}} \right)^{\sigma_c - 1} \right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}}_{\mathcal{A}_t} \cdot K_t^{1-\beta}$$

Furthermore,

$$r_t = (1-\beta)\mathcal{A}_t K_t^{-\beta} \qquad \textit{and} \qquad w_t = \beta \mathcal{A}_t K_t^{1-\beta}$$

Proof. Notice that

$$y_{i,t} = A^{go}_{i,t} M^{\alpha}_{i,t} \left(\frac{Y_{i,t}}{A^{va}_{i,t}} \right)^{1-\alpha}$$

And, replacing into the sectoral FOC yields:

$$M_{i,t} = \alpha \frac{p_{i,t}}{\mathcal{P}^M_{i,t}} A^{go}_{i,t} M^{\alpha}_{i,t} \left(\frac{Y_{i,t}}{A^{va}_{i,t}}\right)^{1-\alpha} = \left(\frac{\alpha p_{i,t}}{\mathcal{P}^M_{i,t}}\right)^{\frac{1}{1-\alpha}} \frac{(A^{go}_{i,t})^{\frac{1}{1-\alpha}}}{A^{va}_{i,t}} Y_{i,t}$$

Therefore, we also have that

$$y_{i,t} = A^{go}_{i,t} M^{\alpha}_{i,t} \left(\frac{Y_{i,t}}{A^{va}_{i,t}}\right)^{1-\alpha} = \left(\frac{\alpha p_{i,t}}{\mathcal{P}^M_{i,t}}\right)^{\frac{\alpha}{1-\alpha}} \frac{(A^{go}_{i,t})^{\frac{1}{1-\alpha}}}{A^{va}_{i,t}} Y_{i,t}.$$

Notice that with no technical bias, we have that $(A^{go}_{i,t})^{\frac{1}{1-\alpha}} = A^{va}_{i,t}$. The value-added price is such that

$$\mathcal{P}^Y_{i,t} = \frac{p_{i,t} y_{i,t} - \mathcal{P}^M_{i,t} M_{i,t}}{Y_{i,t}} = \frac{\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}}}{A^{va}_{i,t}} \cdot \left(\frac{A^{go}_{i,t} p_{i,t}}{(P^M_{i,t})^{\alpha}}\right)^{\frac{1}{1-\alpha}}. \tag{51}$$

Aggregate value-added is therefore,

$$Y_t = \sum_{i\in[g,s]} \mathcal{P}^Y_{i,t} Y_{i,t} = \left(\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}}\right) \sum_{i\in[g,s]} \left(\frac{A^{go}_{i,t} p_{i,t}}{(P^M_{i,t})^{\alpha}}\right)^{\frac{1}{1-\alpha}} \ell^{\beta}_{i,t} \kappa^{(1-\beta)}_{i,t}.$$

Notice that we have that.

$$\frac{A^{go}_{i,t} p_{i,t}}{(\mathcal{P}^M_{i,t})^{\alpha}} = \left(\frac{w_t}{\beta(1-\alpha)}\right)^{\beta} \left(\frac{r_t}{(1-\beta)(1-\alpha)}\right)^{1-\beta} \equiv B_t \qquad \forall i \in [g,s].$$

Therefore,

$$Y_t = \left(\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}}\right) B_t^{\frac{1}{1-\alpha}} K_t^{(1-\beta)} \underbrace{\sum_{i\in[g,s]} \ell_{i,t}}_{=1}.$$

From our choice of numeraire, we have that.

$$\mathcal{P}^C_{it} = \left[\eta^{\sigma_c} \cdot \left(p_{g,t}\right)^{1-\sigma_c} + (1-\eta)^{\sigma_c} \cdot (p_{s,t})^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}} = 1.$$

Rearranging:

$$p_{i,t} = \left[\sum_{j\in[g,s]} \eta_j^{\sigma_c} \left(\frac{p_{j,t}}{p_{i,t}}\right)^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}}.$$

Therefore

$$\frac{p_{j,t}}{p_{i,t}} = \frac{A^{go}_{i,t}(\mathcal{P}^M_{i,t})^{\alpha}}{A^{go}_{j,t}/(\mathcal{P}^M_{j,t})^{\alpha}},$$

and thus

$$B_t = \frac{A_{i,t}^{go} p_{i,t}}{(\mathcal{P}_{i,t}^M)^\alpha} = p_{i,t} = \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}_{j,t}^M}{A_{j,t}^{go}} \right)^{\sigma_c - 1} \right]^{\frac{1}{1-\sigma_c}}.$$

It follows that

$$Y_t = \underbrace{\mathcal{C}^Y \left[\sum_{j \in [g,s]} \eta_j^{\sigma_c} \left(\frac{\mathcal{P}_{j,t}^M}{A_{j,t}^{go}} \right)^{\sigma_c - 1} \right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}}_{\mathcal{A}_t} \cdot K_t^{1-\beta}.$$

From the sectoral FOCs, we have the following information.

$$r_t = (1-\beta)(1-\alpha) \frac{p_{i,t} y_{i,t}}{\kappa_{i,t}} = (1-\beta) \frac{\mathcal{P}_{it}^Y Y_{i,t}}{\kappa_{i,t}}.$$

Replacing equation 35, we have

$$r_t = (1-\beta)(1-\alpha) \frac{p_{i,t} y_{i,t}}{\kappa_{i,t}} = (1-\beta) \left(\alpha^{\frac{\alpha}{1-\alpha}} + \alpha^{\frac{1}{1-\alpha}} \right) \cdot \left(\frac{A_{i,t}^{go} p_{i,t}}{(P_{i,t}^M)^\alpha} \right)^{\frac{1}{1-\alpha}} \left(\frac{\kappa_{i,t}}{\ell_{i,t}} \right)^{-\beta},$$

and thus

$$r_t = (1-\beta) \mathcal{A}_t K_t^{-\beta} \qquad \text{and} \qquad w_t = \beta \mathcal{A}_t K_t^{\beta},$$

where the latter follows from the same algebraic steps as the labor's FOC. ∎

Proposition 11. *The growth rate of aggregate value-added reads:*

$$\Delta \ln Y_t = \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A_{i,t}^{va} + \Delta \ln K_t.$$

Keeping prices constant, the evolution of real GDP reads

$$\Delta \ln Y_t = \sum_{i \in [g,s]} \frac{\mathcal{P}_{i,t}^Y Y_{i,t}}{Y_t} \Delta \ln Y_{i,t} = \sum_{i \in [g,s]} \frac{(1-\alpha) p_{i,t} y_{i,t}}{Y_t} \Delta \ln Y_{i,t},$$

where we exploited that $\mathcal{P}_{i,t}^Y Y_{i,t} = (1-\alpha) p_{i,t} y_{i,t}$ as in the sectoral FOCs. Define the domar weights $\lambda_{i,t} \equiv p_{i,t} y_{i,t} / Y_t$, thus

$$\begin{aligned} \Delta \ln Y_t &= (1-\alpha) \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln Y_{i,t} \\ &= \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A_{i,t}^{va} + (1-\beta)(1-\alpha) \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln \kappa_{i,t} + \beta(1-\alpha) \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln \ell_{i,t}. \end{aligned}$$

Since $(1-\beta)(1-\alpha)y_{i,t}p_{i,t} = r_t\kappa_{i,t}$ and $\beta(1-\alpha)y_{i,t}p_{i,t} = w_t\ell_{i,t}$ from the FOCs, we have that

$$\Delta \ln Y_t = \sum_{i\in[g,s]} \lambda_{i,t}\Delta \ln A^{va}_{i,t} + \frac{r_t}{Y_t}\underbrace{\sum_{i\in[g,s]} \Delta\kappa_{i,t}}_{=\Delta K_t} + \frac{w_t}{Y_t}\underbrace{\sum_{i\in[g,s]} \Delta\ell_{i,t}}_{=0}.$$

Finally notice that $\frac{r_t}{Y_t} = (1-\beta)K_t$ and thus:

$$\Delta \ln Y_t = \sum_{i\in[g,s]} \lambda_{i,t}\Delta \ln A^{va}_{i,t} + \Delta \ln K_t.$$

Proposition 12. *An aggregate balanced growth path (ABGP) exists where all aggregate variables grow at a constant and symmetric rate:*

$$\Delta \ln K = \Delta \ln W = \Delta \ln Y = \Delta \ln C = \Delta \ln K = \frac{\Delta \ln \mathcal{A}}{\beta} \equiv \frac{\sum_{i\in[g,s]} \lambda_{i,t}\Delta \ln A^{va}_{i,t}}{\beta}.$$

Proof. Assume r to be constant, as per the Kaldor Facts. From the first order condition, we know that $\kappa_{i,t}/\ell_{i,t}$ equate across sectors. Therefore, $K_t/L_t = r/w_t$ due to free mobility of factors. Via normalization, we have that $L_t = 1$, $\forall t$. Therefore,

$$g(K) = g(W).$$

Since $r_t = (1-\beta)\mathcal{A}_t K_t^{-\beta}$ we have that

$$g(K) = \frac{g(\mathcal{A})}{\beta}.$$

Since $Y_t = \mathcal{A}K_t^{1-\beta}$,

$$g(Y) = g(\mathcal{A}) + (1-\beta)g(K) = g(\mathcal{A}) + \left(\frac{1}{\beta} - 1\right)g(\mathcal{A}) = \frac{1}{\beta}g(\mathcal{A}) = g(K).$$

Therefore, we have that $g(\mathcal{A})$ must be constant. From the investment equation:

$$\frac{K_{t+1}}{K_t} = (1+\delta) - \frac{\mathcal{I}_t}{K_t}.$$

Rearranging, we have that

$$\frac{\mathcal{I}_t}{K_t} = \text{Constant.} \qquad \Rightarrow \qquad g(\mathcal{I}) = g(K).$$

From the Euler equation, we know that, for constant r, $g(C)$ is also constant. In terms of the numeraire, we have that $Y = C + \mathcal{I}$ and thus

$$g(Y) = g(C) + g(\mathcal{I}).$$

It follows that

$$g(K) = g(W) = g(Y) = g(C) = g(\mathcal{I}) = \frac{1}{\beta} g(\mathcal{A}).$$

Finally, notice that since

$$\Delta \ln Y_t = \frac{\sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A_{i,t}^{va}}{\beta} + (1 - \beta) \Delta \ln K_t.$$

Then, it must be that

$$\mathcal{A}_t = \sum_{i \in [g,s]} \lambda_{i,t} \Delta \ln A_{i,t}^{va}.$$

■