Technical and Structural Change

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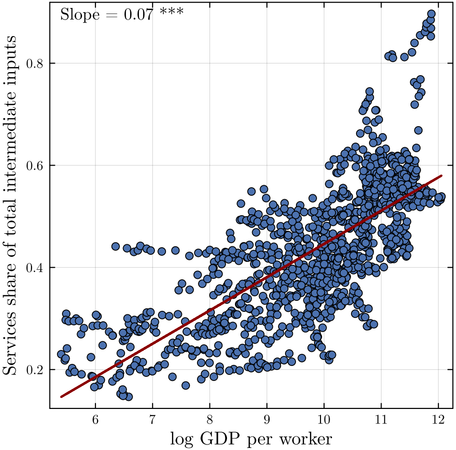
Motivation: Structural Change in Intermediate Inputs



- As countries develop, the services share of total intermediates rises.

U.S. data

Motivation: Structural Change in Intermediate Inputs

- What drives the rise of service intermediates?
- Standard explanation from the literature: **substitution** effect via relative prices. (e.g., Valentinyi, 2021 and Gaggl et al., 2023)
- In this work:
  - Substitution does not fully account for the rise of services intermediates.
  - New mechanism: **biased technical change**.

Motivation: Bias in Technical Change

- Technical progress affects
  1. Production efficiency
  2. **Input composition**
- Technology is often incorporated into intermediates (e.g., Acemoglu and Azar, 2020)
- Example - the production function of an economist in
  - 1965: research $= f(\text{paper, chalk, physical journals, ...})$
  - 2025: research $= f(\text{MATLAB license, ChatGPT, ...})$
- Has technical change driven intermediate demand toward services?
  ($\Leftrightarrow$ is technical change **service-biased**?)

Research Questions

- Research questions:
  1. How much of the rising share of service intermediates is driven by biased technical change?
     Preview: $\sim$ 40%, U.S. 1965-2014
  2. Does it affect other measures of structural change?
     Preview: $\sim$ 50% of employment, $\sim$ 20% of rise in final expenditures service share
  3. What is its contribution to aggregate GDP growth?
     Preview: $\sim$ 25% *decline* in real GDP, U.S. 1965-2014
- Methodology:
  1. Cross-country evidence (WIOD Input-Output data, 40 countries over 1965-2014)
  2. Quantitative model (calibrated using U.S. data)

Contribution to the Literature

1. Sectoral composition of input-output linkages
   Berlingieri (2013); Sposi (2019); Valentinyi (2021); Gaggl et al. (2023)
   - Rise of service intermediates is driven by within sub-sector changes.
2. Mechanisms of structural transformation
   Kongsamut et al. (2001); Ngai and Pissarides (2007); Herrendorf et al. (2014)
   - New mechanism: **biased technical change**.
3. Biased technical change
   Acemoglu (2002); León-Ledesma et al. (2010);
   - **Intermediate-biased** technical change.
4. Productivity propagation via input-output linkages
   Acemoglu et al. (2012); Baqaee and Farhi (2019); Baqaee and Rubbo (2023)
   - Propagation of **intermediate-specific** productivities.

Empirical Evidence - Shift-Share Decomposition

- Data source: World Input-Output Database (WIOD) details
  - 10 sectors, 40 countries, 1965-2014
- Structural change in intermediates might be driven by
  1. Sectors becoming more service intensive (**within**).
  2. Reallocation of output toward service-intensive sectors (**between**).
- The services share of aggregate intermediates (S_{ct}) is

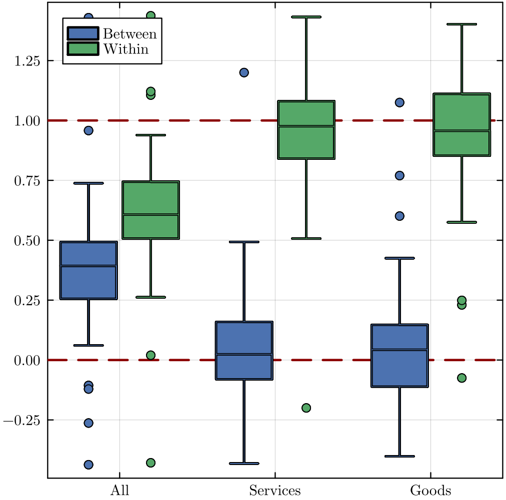
$$\Delta \ln S_{ct} \approx \underbrace{\sum_j \bar{\omega}_{jct} \Delta \chi_{jct}}_{\text{"Within"}} + \underbrace{\sum_j \bar{\chi}_{jct} \Delta \omega_{jct}}_{\text{"Between"}}$$

where

- ω_{jct}: industry's share of total sectoral intermediate inputs
- χ_{jct}: service share of industry's intermediates

Empirical Evidence - Shift-Share Decomposition (Cross-Country Mean)

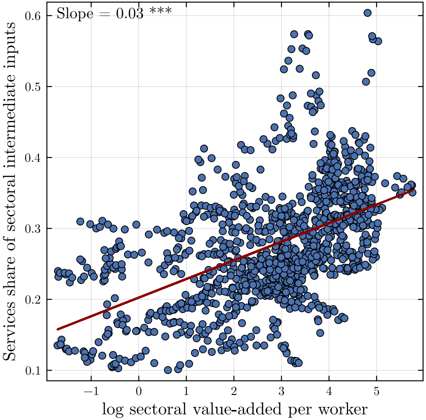
- Across all industries: between $\sim$ within.
- Grouping by sector: within $\gg$ between
- $\Rightarrow$ All industries become more service intensive.
- $\Rightarrow$ Reallocation mostly across broad-sector



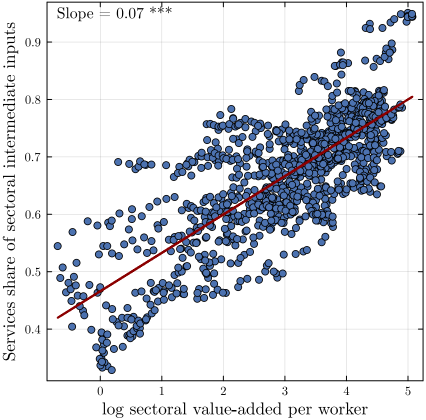
U.S. data

Empirical Evidence - Services Shares of Intermediates, by Broad Sector

(a) Goods



(b) Services



- The services share of total sectoral intermediates rises more in the service-producing sector.

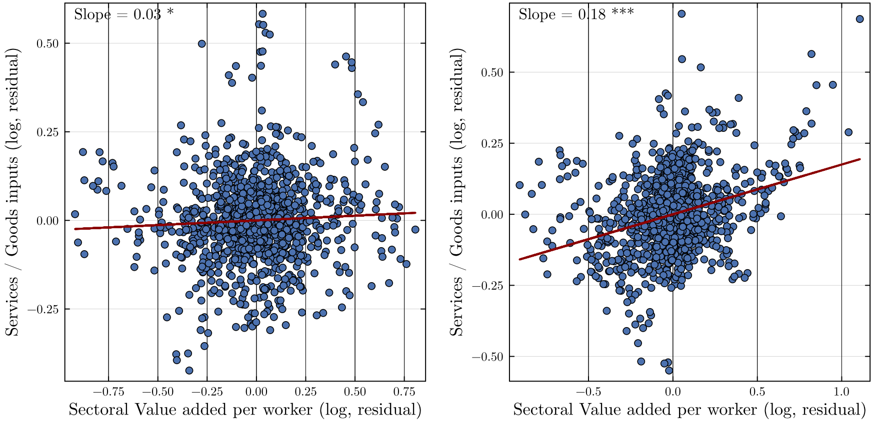
Empirical Evidence - Do Prices Drive Structural Change in Intermediates?

Sector:	Goods			Services		
	Services share of sectoral inputs (log)					
	(1)	(2)	(3)	(4)	(5)	(6)
Relative Prices (log, Services/ Goods)	0.213***	0.216***	0.191***	0.138***	0.112***	0.100***
	(0.020)	(0.022)	(0.023)	(0.012)	(0.013)	(0.012)
GDP per worker (log)		-0.004			0.022***	
		(0.008)			(0.005)	
Sect. value added per worker (log)			0.025*			0.072***
			(0.011)			(0.006)
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
N	1,181	1,181	1,181	1,181	1,181	1,181
R^2	0.924	0.924	0.924	0.925	0.926	0.933
Within-R^2	0.091	0.091	0.095	0.112	0.127	0.215

- After controlling for prices, **correlation with sectoral value added remains significant**.
- **Controlling for sectoral value added improves fit** (within R^2).

Empirical Evidence: Residual Correlation

methodology U.S. data



- After partialling-out prices, **residual correlation remains significant**.
- Correlation is stronger in the service-producing sector.

Empirical Evidence – Summary of Results

- Structural change in intermediates is **not reallocation-driven**
  - Broad-based increase in service intensity across industries.
  - Reallocation of aggregate service intermediate use is at the sector level.
  - $\Rightarrow$ **Two-sector framework (Goods vs. Services)** captures the main stylized facts.
- Controlling for prices shows **residual correlation** between services use and value added:
  - $\Rightarrow$ **Relative prices do not fully explain structural change in intermediates**.
- Heterogeneity in the size of residual correlation between sectors.

Model Outline

- Dynamic 2-sector ("g", "s") model
- **Input-output network**: sectoral output can be used for
  1. Final consumption (household)
  2. Intermediate for any sector
- **Biased technical change**: heterogeneous intermediate-specific productivities.
- Productivities are exogenous.
- Standard representative household ("c") (homothetic CES preferences)

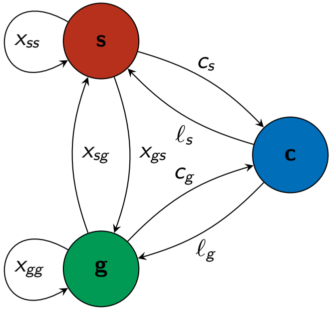


Figure: Graph of the input-output structure

Model: Technology

- **Sectoral production function**:

$$y_{i,t} = A^{go}_{i,t} \cdot M_{i,t} \left(\mathbf{a}_{i,t} \circ \mathbf{x}_{i,t}\right)^{\alpha} \left(\ell_{i,t}\right)^{1-\alpha} \qquad \forall i \in [g, s].$$

  where:

  - $A^{go}_{i,t}$ is (gross-output) *sectoral TFP*
  - $\mathbf{a}_{i,t} = [a_{ig,t}, a_{is,t}]$ is a vector of *input-specific productivities*.
  - $\mathbf{x}_{i,t} = [x_{ig,t}, x_{is,t}]$ is a vector of *intermediate inputs*.

- **Intermediate aggregator**:

$$M\left(\mathbf{a}_{i,t} \circ \mathbf{x}_{i,t}\right) = C_{i,t}\left[\gamma_i (a_{ig,t} x_{ig,t})^{\frac{\sigma_i - 1}{\sigma_{i,t}}} + (1-\gamma_i)(a_{is,t} x_{is,t})^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}} \quad \forall i \in [g, s].$$

- Intermediates are **gross-complements**: $\sigma_i < 1$.

Model: Structural Change in Intermediates

- The (log) sectoral relative share of services intermediates:

$$\ln \frac{p_{is,t} x_{is,t}}{p_{ig,t} x_{ig,t}} = \sigma_i \ln\left(\frac{1-\gamma_i}{\gamma_i}\right) + \underbrace{(1-\sigma_i)(\ln a_{ig,t} - \ln a_{is,t})}_{\text{bias in technical change} = \phi_{i,t}} + \underbrace{(1-\sigma_i)(\ln p_{is,t} - \ln p_{ig,t})}_{\text{substitution effect}}, \quad \forall i \in [g, s].$$

- **Takeaway**: due to gross-complementarity ($\sigma_i < 1$) intermediate shares move toward
    1. the **more expensive** intermediate.
    2. the **least productive** intermediate.

Model: Value Added

- Sectoral value added (VA):

$$Y_{i,t} = \underbrace{\left[A^{go}_{i,t} \cdot \theta^{\alpha}_{i,t} \cdot \left(\phi^{\chi_{is,t}}_{i,t}\right)^{-\alpha}\right]^{\frac{1}{1-\alpha}}}_{A^{va}_{i,t}} \cdot \ell_{i,t}$$

- Aggregate value added (GDP) in terms of the numeraire - consumer price index:

$$Y_t = \underbrace{\mathcal{C}^Y \left[\sum_{j\in[g,s]} \eta_j^{\sigma_c} \left(\frac{(\mathcal{P}^M_{j,t}(\mathbf{A},\phi))^{\alpha}}{A^{go}_{j,t}}\right)^{1-\sigma_c}\right]^{\frac{1}{(1-\sigma_c)(1-\alpha)}}}_{\mathcal{A}_t} \cdot L_t$$

- **Takeaway**: biased technical change ($\phi_{i,t} \neq 1$) affects
  1. Sectoral VA via services' intermediate shares.
  2. Aggregate VA via intermediate price indexes.

Model: Role of I-O Structure and Bias upon TFP

1. **Input-output network**: if $\alpha = 0$ (no intermediates) then

$$\mathcal{A}_t = \left[\sum_{j \in g,s} \eta_j^{\sigma_c} \left(A_{j,t}\right)^{1-\sigma_c} \right]^{\frac{1}{\sigma_c - 1}} \implies \text{Aggr. TFP} = \text{CES mean of sectoral TFP}$$

2. **Bias in technical change**, if $\phi_s = \phi_g \; \forall i \in [g, s]$

$$\mathcal{A}_t = \left[\sum_{j \in [g,s]} \eta_j \left(\frac{A_{j,t}}{\bar{A}_t^{\frac{\alpha}{1-\alpha}}} \right)^{\sigma_c - 1} \right]^{\frac{1}{\sigma_c - 1}}$$

where $\bar{A}_t$ is a (biased) CES of sectoral productivities:

$$\bar{A}_t \equiv \frac{1}{\theta} \left[\gamma^{\sigma} \left(A_{g,t}\right)^{\sigma-1} + \phi(1-\gamma)^{\sigma} \left(A_{s,t}\right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.$$

Model: (First Order) Aggregate Dynamics

- Growth rate of aggregate TFP:

$$\Delta \ln \mathcal{A}_t \approx \underbrace{\sum_{i\in[g,s]} \lambda_{i,t} \left(\Delta \ln A^{go}_{i,t} + \alpha \Delta \ln \theta_{i,t}\right)}_{\text{TFP component}} - \underbrace{\sum_{i\in[g,s]} s_{i,t} \Delta \ln \phi_{i,t}}_{\text{Bias component}} \qquad \lambda_{i,t}, s_{i,t} > 0 \qquad \forall i, t$$

$\Rightarrow$ **Aggregate TFP is declining in the bias term.**

- Evolution of relative prices:

$$\Delta \ln p_{i,t} = \overbrace{\sum_{j\in[g,s]} \hat{\Omega}_{ij,t} \Omega_{is,t} \Delta \ln \phi_{j,t}}^{\text{Bias component}} - \overbrace{\sum_{j\in[g,s]} \hat{\Omega}_{ij,t} (\Delta \ln A^{go}_{j,t} + \alpha \Delta \ln \theta_{i,t})}^{\text{TFP component}} - \overbrace{\Delta \ln Y_t}^{\text{Numeraire adjustment}}$$

$\Rightarrow$ **Sectoral prices are increasing in the bias term.**

Model: Structural Transformation

- Sectoral intermediate:

$$\Delta \ln \frac{\text{Share}_{i,s}}{\text{Share}_{i,g}} = \phi_i + (1 - \sigma_i)\Delta \ln \hat{P}(\phi)$$

$\Rightarrow$ Service share of intermediate depends on ϕ both directly and via relative prices ($\hat{P}$).

- Final demand:

$$\Delta \ln \frac{\text{Share}_{C,s}}{\text{Share}_{C,g}} = (1 - \sigma_C)\Delta \ln \hat{P}(\phi)$$

$\Rightarrow$ Service share of final expenditure depends on ϕ via relative prices ($\hat{P}$).

Model: Asymptotic Aggregate Balanced Growth Path

- Assumptions:
  1. Sectoral productivities grow at constant rates -
  $$\Delta \ln \tilde{A}^{go}_{i,t} = g(\tilde{A}^{go}_i) \quad \text{and} \quad \Delta \ln \phi_{i,t} = g(\phi_i) \qquad \forall i \in [g,s] \text{ and } \forall t.$$
  2. Technical change is such that -
  $$g(\tilde{A}^{go}_g) > g(\tilde{A}^{go}_s) \geq 0 \qquad \text{and} \qquad g(\phi_s) > g(\phi_g) \geq 0.$$
- Asymptotic Aggregate Balanced Growth Path (AABGP):

$$\begin{aligned}\lim_{t\to\infty} \Delta \ln \mathcal{A}_t &= \lim_{t\to\infty} \Delta \ln A^{va}_{s,t} \qquad (\Rightarrow \textbf{Service sector takes over the economy}) \\ &= \frac{1}{1-\alpha} \cdot \left(\Delta \ln A^{go}_{s,t} + \alpha \Delta \ln \theta_{s,t}\right) - \underbrace{\frac{\alpha}{(1-\sigma_s)(1-\alpha)}}_{>1} \cdot \Delta \ln \phi_{s,t},\end{aligned}$$

$\Rightarrow$ **Bias in technical change slows down asymptotic growth**

Calibration: Strategy

- **Data:**
  - U.S. data (1965–2014)
  - Sources: *BEA's NIPA* (employment, aggregate values) and *WIOD* (I-O network, prices).
- **Methodology:**
  - Sectoral ES (σ_i) and biased technical change ($\phi_{i,t}$)
    - Jointly via GMM, using lagged inputs as instruments (Wooldridge, 2009, León-Ledesma et al., 2010, Lashkari et al., 2024).
    - Productivities modeled as random walks with drift.
    - Monte Carlo simulations validate estimator's finite-sample performance details & results.
  - Sectoral total factor productivity: residual from growth accounting (given ϕ and θ)
  - Relative final demand elasticity (σ_c) estimated via OLS from demand function.

Calibration: Measuring Bias in Technical Change methodology

	σ	$\Delta \ln \phi$
Goods	0.001 (0.008)	0.001 (0.005)
Services	0.002 (0.002)	0.012** (0.004)

- Intermediates are perfect complements ($\Leftrightarrow$ intermediates aggregator is Leontief)
- Technical change is
  - **neutral** in the goods sector.
  - **services-biased** in the services sector.

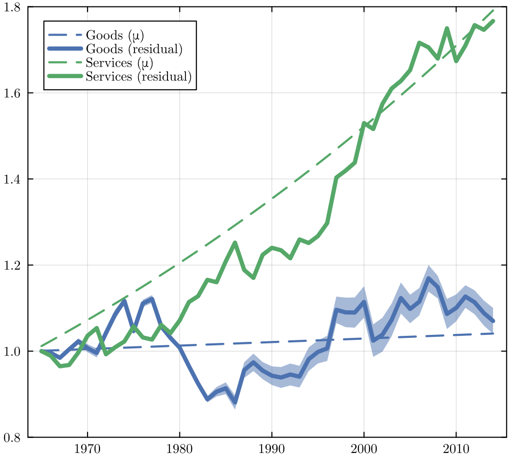
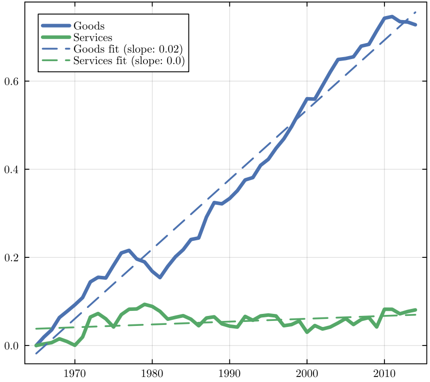


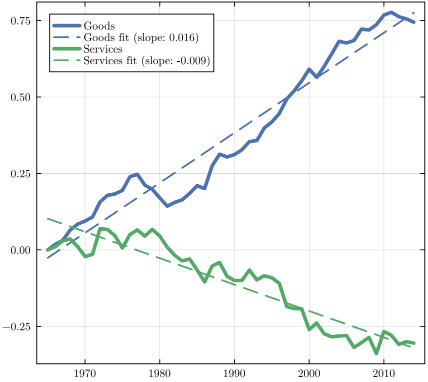
Figure: Evolution of residual ϕ by sector

Calibration: Growth Accounting

(a) (Gross-output) productivity ($\ln A^{go}$)



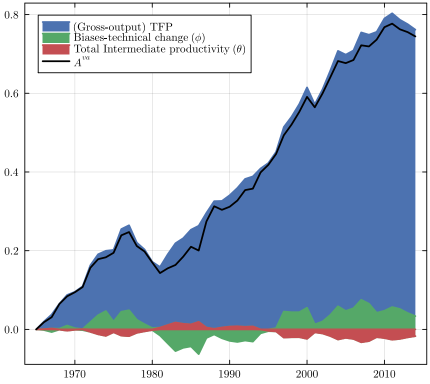
(b) (Value-added) labor productivity ($\ln A^{va}$)



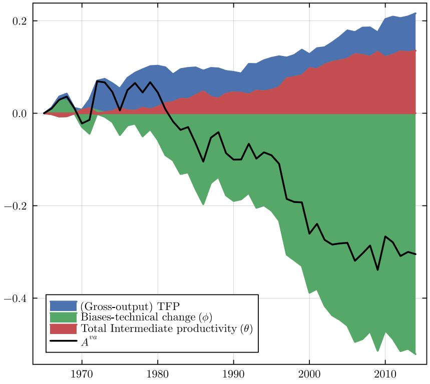
Calibration: Decomposition of Sectoral Value-Added Labor Productivity

- Why is the value-added labor prod. declining in the services-producing sector?

$$\Delta \ln A^{VA} = \underbrace{\frac{\Delta \ln A^{go}_{i,t}}{1-\alpha_i}}_{\text{Gross-output TFP}} + \underbrace{\frac{\alpha_i \Delta \ln \theta_{i,t}}{1-\alpha_i}}_{\text{Total intermediate}} - \underbrace{\frac{\alpha_i \chi_{is,t} \Delta \ln \phi_{i,t}}{1-\alpha_i}}_{\text{Bias}}$$



(a) Goods



(b) Services

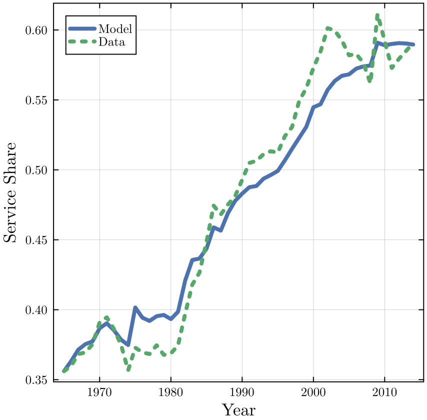
Quantitative Exercise - Parameter Table

Parameter	Value	Source
α_G	0.52	Average intermediate share of nominal output, (1965-2014, WIOD)
α_S	0.27	Average intermediate share of nominal output, (1965-2014, WIOD)
Γ_G	0.77	Goods share of nominal output, (1965, WIOD)
Γ_S	0.41	Goods share of nominal output, (1965, WIOD)
Γ_C	0.79	Goods share of nominal final household expenditure (1965, WIOD)
σ_G	0.00	Estimated via GMM
σ_S	0.00	Estimated via GMM
σ_C	0.17	Estimated via OLS

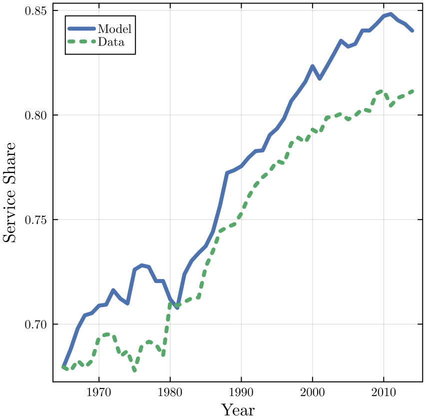
Table: Calibrated parameters

Quantitative Exercise - Model Fit

(a) Services share of total intermediates (nominal)



(b) Services share of final expenditures (nominal)



- Leontief $M \Rightarrow$ intermediates quantity $\propto \phi$: model captures relative prices well!
- Missing mechanism in final expenditure (non-homothetic preferences)

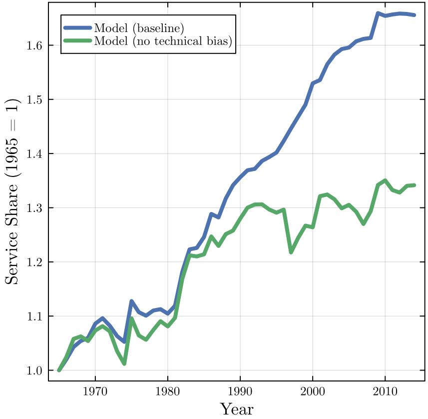
Quantitative Exercise - Counterfactual Exercise 1

- Research questions:
  1. How much of the rising share of service intermediates is driven by biased technical change?
  2. Does it affect other measures of structural change?
  3. What is its contribution to aggregate GDP growth?
- Strategy: solve the model two times:
  1. With estimated ϕs ("baseline")
  2. With $\phi = 1$ for all sectors ("counterfactual")

     $\Rightarrow$ **Unbiased technical change**

Counterfactual Exercise - Structural Change in Intermediates

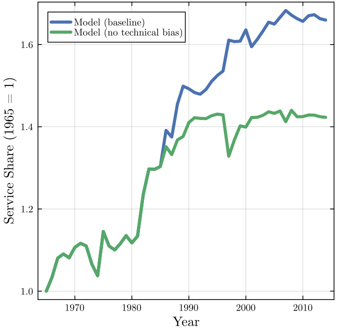
(a) Services' share of total intermediates



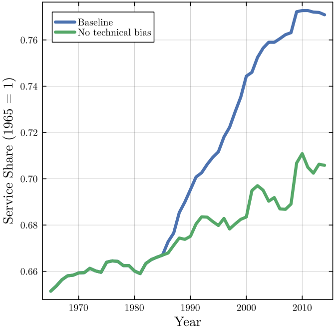
- Biased technical change accounts for $\sim 50\%$ of the rise in the **services' share of total intermediates** in the U.S. between 1965-2014.

Counterfactual Exercise - Structural Change

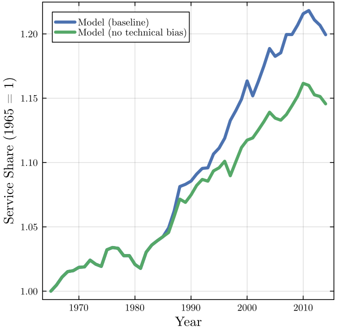
(a) Value added (services' share)



(b) Employment (services' share)



(c) Final exp. (services' share)

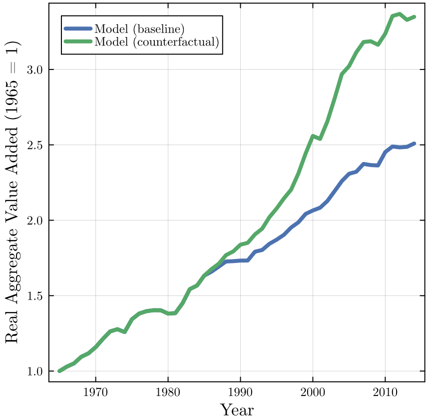


- Biased technical change accounts for:
  1. $\sim$ 30% of the change in the service share of **aggregate value added**.
  2. $\sim$ 50% of the change in the service share of **aggregate employment**.
  3. $\sim$ 20% of the change in the service share of **final expenditures**.

  in the U.S. between 1965 and 2014

Counterfactual Exercise (GDP per Capita)

(a) Aggregate real value added



- Biased technical change accounts for a $\sim 30\%$ *decline* in **aggregate value added**

Daron Acemoglu. Directed technical change. *The review of economic studies*, 69(4):781–809, 2002.

Daron Acemoglu and Pablo D Azar. Endogenous production networks. *Econometrica*, 88(1): 33–82, 2020.

Daron Acemoglu, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012.

David Baqaee and Elisa Rubbo. Micro propagation and macro aggregation. *Annual Review of Economics*, 15(1):91–123, 2023.

David Rezza Baqaee and Emmanuel Farhi. The macroeconomic impact of microeconomic shocks: Beyond hulten's theorem. *Econometrica*, 87(4):1155–1203, 2019.

Giuseppe Berlingieri. Outsourcing and the rise in services. 2013.

Paul Gaggl, Aspen Gorry, and Christian Vom Lehn. Structural change in production networks and economic growth. 2023.

Berthold Herrendorf, Richard Rogerson, and Akos Valentinyi. Growth and structural transformation. *Handbook of economic growth*, 2:855–941, 2014.

Rainer Klump, Peter McAdam, and Alpo Willman. The normalized ces production function: theory and empirics. *Journal of Economic surveys*, 26(5):769–799, 2012.

Piyabha Kongsamut, Sergio Rebelo, and Danyang Xie. Beyond balanced growth. *The Review of Economic Studies*, 68(4):869–882, 2001.

Danial Lashkari, Arthur Bauer, and Jocelyn Boussard. Information technology and returns to scale. *American Economic Review*, 114(6):1769–1815, 2024.

Miguel A León-Ledesma, Peter McAdam, and Alpo Willman. Identifying the elasticity of substitution with biased technical change. *American Economic Review*, 100(4):1330–1357, 2010.

L Rachel Ngai and Christopher A Pissarides. Structural change in a multisector model of growth. *American economic review*, 97(1):429–443, 2007.

Michael Sposi. Evolving comparative advantage, sectoral linkages, and structural change. *Journal of Monetary Economics*, 103:75–87, 2019.

Akos Valentinyi. Structural transformation, input-output networks, and productivity growth. *Structural Transformation and Economic Growth (STEG) Pathfinding paper*, 2021.

Jeffrey M Wooldridge. On estimating firm-level production functions using proxy variables to control for unobservables. *Economics letters*, 104(3):112–114, 2009.

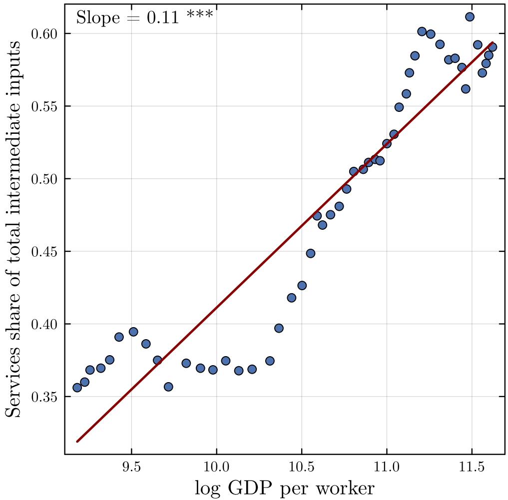
Conclusion

- This work:

1. Documents rise in the service share of total intermediates.
2. Argues that mechanism from literature cannot fully account for it.
3. Introduces a novel mechanism: biased technical change.
4. Shows how sectoral biased technical change affects aggregate performance
5. Shows that biased technical change is a key mechanism of structural change across all common dimensions.

Appendix

Appendix: Structural Transformation in the U.S.



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Appendix: Data 1

- Source: Harmonized WIOD (40 countries, 12 industries)
- Aggregated into two sectors: **goods** vs **services**
- Abstract from trade: use total inputs by country–sector
- Consistent growth rates via chain-linking across WIOD vintages
- Deflation with sectoral price indices (normalized to 1965)
- Final series: nominal & real inputs/outputs + deflators

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Appendix: Data 2

ISIC3 code	Broad sector	Description
AtB	Goods	Agriculture, hunting, forestry, and fishing
C	Goods	Mining and quarrying
D	Goods	Total manufacturing
E	Goods	Electricity, gas and water supply
F	Goods	Construction
G	Services	Wholesale and retail trade
H	Services	Hotels and restaurants
I	Services	Transport, storage, post and telecommunications
J	Services	Financial intermediation
K	Services	Real estate, renting and business activities
LtQ	Services	Community social and personal services

Table: WIOD sector classification

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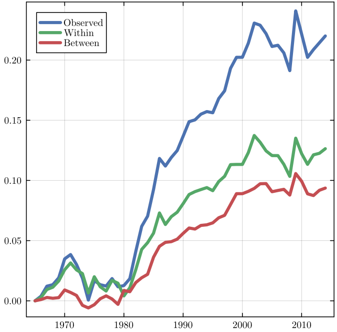
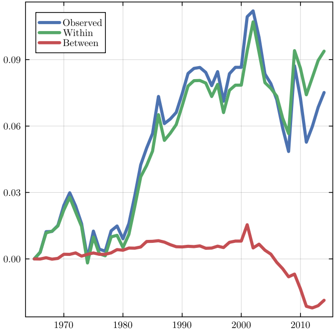
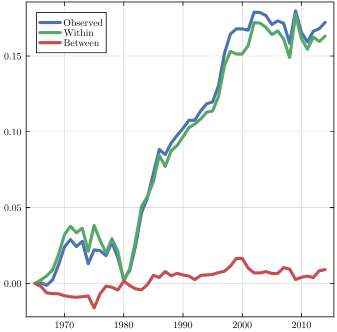
Appendix: Shift-Share Decomposition (USA)



(a) All sub-sectors



(b) Goods



(c) Services

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Appendix: Residual Correlation (Methodology)

- For each sector $s \in \{\text{Goods}, \text{Services}\}$ and country c, retrieve residuals $\hat{\varepsilon}_{cst}$ from:

$$\log \frac{\text{input}_{cst}^{serv.}}{\text{input}_{cst}^{goods}} = \beta_s \, \text{RelPrice}_{cst} + \gamma_{cs} + \varepsilon_{cst}, \qquad \forall s \in [goods, services].$$

- Similarly I obtain fitted residuals $\hat{\varepsilon}_{cst}$ from

$$\log(\text{Value added})_{cst} = \beta_s^Y \, \text{RelPrice}_{cst} + \gamma_{cs}^Y + \varepsilon_{cst}^Y, \qquad \forall s \in [goods, services].$$

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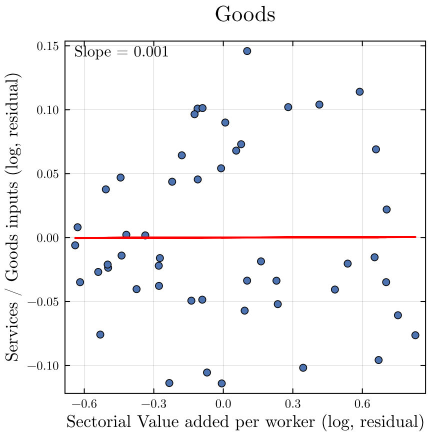
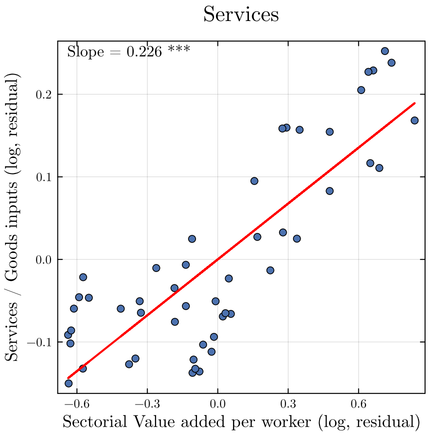
Appendix: Residual Correlation (USA)





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Appendix: Calibration Methodology - 1

- Due to the role of complementarity, bias and σ must be jointly estimated.
- **Re-parametrization**:

$$\exp\phi_{i,t} = \frac{a_{is,t}}{a_{ig,t}} \qquad \text{and} \qquad \exp\theta_{i,t} = a_{ig,t}\cdot C_{i,t}, \qquad \forall i \in [g,s].$$

- **Normalized intermediates aggregator** (Klump et al., 2012):

$$\bar{M}_{i,t} = \theta_{i,t}\cdot\left[\Gamma_{i,t}\cdot\bar{x}_{ig,t}^{\frac{\sigma_i-1}{\sigma_i}} + (1-\Gamma_{i,t})(\exp(\bar{\phi}_{i,t})\bar{x}_{is,t})^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}},$$

where $\bar{x}_{i,t} \equiv x_{i,t}/x_{i0}$ and

$$\Gamma_{i,t} \equiv \frac{p_{g,0}x_{ig,0}}{p_{g,0}x_{ig,0}+p_{s,0}x_{is,0}}$$

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Appendix: Calibration Methodology - 2

- **Bias terms solve:**

$$\begin{cases} \phi_{i,t} = (\ln \bar{p}_{is,t} - \ln \bar{p}_{ig,t}) - \frac{1}{1-\sigma_i} \cdot \ln \frac{\bar{p}_{s,t}\bar{x}_{is,t}}{\bar{p}_{g,t}\bar{x}_{ig,t}} \\ \theta_{i,t} = \ln \bar{M}_{i,t} - \ln \left\{ \left[\Gamma_{i,t} \bar{x}_{ig,t}^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \Gamma_{i,t})(\exp(\phi_{i,t}(\bar{\mathbf{p}}_{i,t}, \bar{\mathbf{x}}_{i,t})) \cdot \bar{x}_{is,t})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}} \right\} \end{cases}$$

- **Assumption**:

$$\phi_{ij,t} = \phi_{ij,t-1} + \mu_{ij}^{\phi} + \epsilon_{ij,t}^{\phi} \qquad j \in [g, s]$$
$$\theta_{ij,t} = \theta_{ij,t-1} + \mu_{ij}^{\theta} + \epsilon_{ij,t}^{\theta} \qquad j \in [g, s]$$

- This system can be solved using GMM (Wooldridge, 2009, Lashkari et al., 2024)

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Appendix: Monte Carlo 1

- **Data generation** (León-Ledesma et al., 2010):
  1. Technology processes:
  $$\ln \tilde{\psi}_t = \ln \tilde{\psi}_{t-1} + \mu^{\psi} + \epsilon_{\psi,t}, \quad \ln \theta_t = \ln \theta_{t-1} + \mu^{\theta} + \epsilon_{\theta,t}$$
  with shocks $\epsilon \sim \mathcal{N}(0, \sigma)$.
  2. Input series:
  $$z_{i,t} = z_{i,t-1} \exp(\mu^{z} + \epsilon_{z,t}), \quad z \in \{x_g, x_s\}.$$
  3. Equilibrium output & prices: solved from CES production function and FOCs.

- **Estimation:**
  - Parameters $(\sigma, \mu^{\phi}, \mu^{\theta})$ estimated over **5,000 replications**, each with 50 observations.
  - Shocks: $\{\epsilon_{\phi,t}, \epsilon_{\theta,t}, \epsilon_{x_g,t}, \epsilon_{x_s,t}\}$.
  - Variances: $[\sigma_{\phi}, \sigma_g, \sigma_{x_g}, \sigma_{x_s}] = [0.025, 0.015, 0.1, 0.2]$.
  - Performance assessed via bias, RMSE, and convergence frequencies.

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Appendix: Monte Carlo 2

Parameter	True	Mean	Std	Bias	RMSE	Conv Rate
σ	0.600	0.597	0.038	-0.003	0.038	1.00
μ_ϕ	0.020	0.020	0.008	-0.000	0.008	1.00
μ_θ	0.010	0.010	0.003	0.000	0.003	1.00

Table: Monte Carlo Results (N=50, Sims=5000)

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