Lecture 7

Beam Search and Genetic Algorithms I

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Review

- **Last Class**
  - **Local Search**
    - **Hill-Climbing**
    - **Simulated Annealing**

- **This Class**
  - **Beam Search**
  - **Introduction to Genetic Algorithm**

- **Next Class**
  - **More on Genetic Algorithm**

Review
Local Search Algorithms

- **Local Search Problem**
  - no frontier set, no backtracking, no predefined goal states
  - each state is a potential solution
  - only current state, neighboring states, and objective function matter
  - Goal: maximize or minimize an objective function
- **General Procedure**
  - Keep only a single state in memory
  - Generate only the neighbours of that state
  - Keep one of the neighbors and discard others
- **Two strategies for choosing the state to visit next**
  - Hill climbing: continually move uphill
  - Simulated annealing: escape local optima by accepting worse moves within a probability
- **Then, an extension to multiple current states**
  - Beam search
  - Genetic algorithms

Beam Search

- Beam search
  - remembers a population of k states, where k > 1
  - each state is a potential solution
  - k current states, all neighboring states of these k states, and objective function matter
  - Goal: maximize or minimize an objective function
- General Procedure
  - Keep k states in memory
  - Generate all the neighbours of k states
  - Choose the best k states among all the neighbors to be the population and discard others
- The value of k
  - Affect the amount of memory
  - Control the level of parallelism

Beam Search

- **Q 1: What does Beam Search look like when k = 1?**

- **Q 2: How is Beam Search different from running Hill-Climbing with k random restarts in parallel?**

Beam Search

- Q 1: What does Beam Search look like when k = 1?
  - Hill-Climbing

- Q 2: How is Beam Search different from running Hill-Climbing with k random restarts in parallel?
  - If we run k random restarts, these random restarts are independent from each other, and each search updates its state independent of the other states.
  - With beam search, we are choosing the best k states among all the neighbors of the k current states, and the k states are not operating independently

- However, it is still greedy. Whenever we find a good region in the search space, we tend to cluster there and ignore other parts.

Stochastic Beam Search

- **Stochastic Beam search**
  - remembers a population of k states, where k > 1
  - The main difference is how it updates the k states

- **General Procedure**
  - Keep k states in memory
  - Generate all the neighbours of k states
  - Choose the next k states **probabilistically** among all the neighbors to be the population and discard others

- **The probability of choosing each state**
  - proportional to the fitness value of the state (maximization problem)
  - or inversely proportional to the cost of the state (minimization problem)

Stochastic Beam Search

- **Mimic the process of natural selection**
  - **Imagine that each state is an organism and the state’s neighbors are potential offspring**
  - **The survival of the potential offsprings depends on their fitness**
  - **Some will survive, and others will not**
  - **The offspring that survive will make up the next population**

Biological Background – Natural Selection

- The origin of species: “Preservation of favourable variations and rejection of unfavourable variations.” – Charles Darwin

- There are more individuals born than can survive, so there is a continuous struggle for life.

- Individuals with an advantage have a greater chance for survive: survival of the fittest.

Evolutionary Computation

- Natural Evolution
  - Generating a population of individuals with increasing fitness
  - Increasing ability to survive and reproduce in a specific environment

- Evolutionary Computation
  - Simulate the natural evolution on a computer
  - Generate a set of solutions to a problem of increasing quality

Genetic Algorithms

- **Genetic Algorithms**
  - A class of search or optimization algorithms
  - Inspired by the biological evolution process
  - As early as 1962, John Holland's work on adaptive systems laid the foundation for later developments.
  - By 1975, the publication of the book Adaptation in Natural and Artificial Systems, by Holland and his students and colleagues.

- **Widely-used today in business, scientific and engineering circles**

What is GA

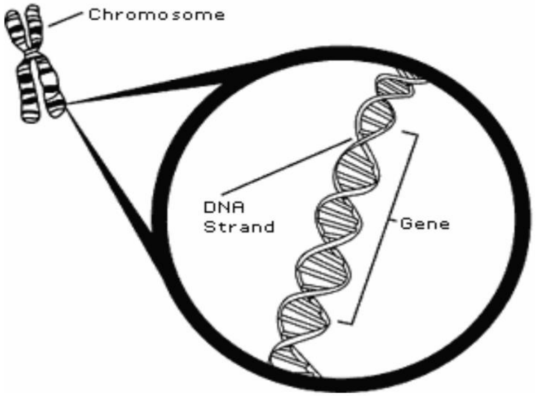
- **A genetic algorithm (or GA): a search technique used in computing to find true or approximate solutions to search or optimization problems.**

- **(GA)s are categorized as global search heuristics.**

- **(GA)s: a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, selection, crossover (also called recombination), and mutation.**

Terminology

- **Individual**
  - Any possible solution (or state) to a problem
- **Chromosome**
  - Representation of a solution
- **Gene**
  - Constituent entity of the chromosome
- **Population**
  - Set of individuals
- **Fitness Function**
  - Representation of how good a candidate solution is
- **Genetic operators**
  - Operators applied on chromosomes in order to create genetic variation (other chromosomes)



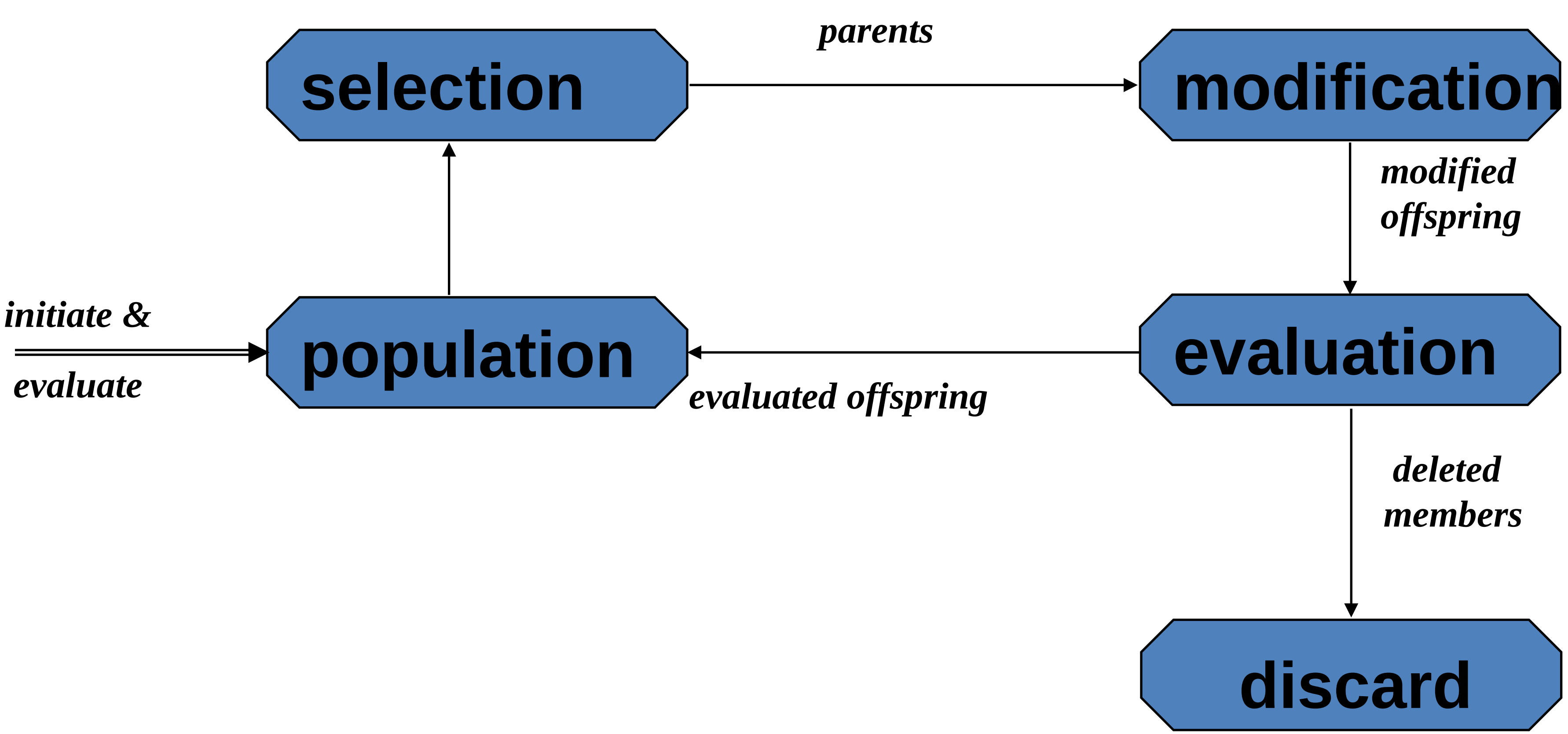
Basic Genetic Algorithms

- Start with n random solutions (the initial population)
- Repeatedly do the following:
  - Evaluate each of the solutions by the fitness function
  - Select a subset of the best solutions probabilistically
  - Use these solutions to generate a new population by
    - “crossover” with a rate
    - “mutation” with a rate
  - The new population is used in the next iteration
- Quit when you have a satisfactory solution (or you run out of time)

Terminology

- ***Selection***
  - replicates the most successful solutions found in a population at a rate proportional to their relative quality
- ***Crossover (Recombination)***
  - decomposes two distinct solutions (parents) and then randomly mixes their parts to form novel solutions (offspring)
- ***Mutation***
  - randomly perturbs a candidate solution

The Evolutionary Cycle



Example: the MAXONE problem

- Suppose we want to maximize the number of ones in a string of m binary digits

Is it a trivial problem?

- It may seem so because we know the answer in advance
- However, we can think of it as maximizing the number of correct answers, each encoded by 1, to m yes/no difficult questions

Example (cont.)

- **An individual is encoded (naturally) as a string of *m* binary digits**

- **The fitness *f* of a candidate solution to the MAXONE problem is the number of ones in its genetic code**

- **We start with a population of *n* random strings.**

  Suppose that *m* = 10 and *n* = 6

Example (Initialization)

- We toss a fair coin 60 times and get the following initial population:
- n = 6 individuals
  - each is encoded as a string of m = 10 binary digits

$$s_1 = 1111010101 \qquad f(s_1) = 7$$

$$s_2 = 0111000101 \qquad f(s_2) = 5$$

$$s_3 = 1110110101 \qquad f(s_3) = 7$$

$$s_4 = 0100010011 \qquad f(s_4) = 4$$

$$s_5 = 1110111101 \qquad f(s_5) = 8$$

$$s_6 = 0100110000 \qquad f(s_6) = 3$$

Example (Step1: Selection)

- Next, randomly (using a biased coin) select a subset of the individuals based on their fitness with the roulette wheel method:

$f(s_1) = 7 \quad p(s_1) = 7/34$

$f(s_2) = 5 \quad p(s_2) = 5/34$

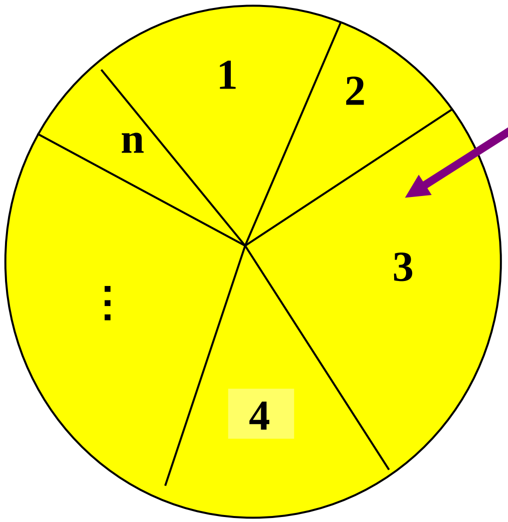
$f(s_3) = 7 \quad p(s_3) = 7/34$

$f(s_4) = 4 \quad p(s_4) = 4/34$

$f(s_5) = 8 \quad p(s_5) = 8/34$

$f(s_6) = 3 \quad p(s_6) = 3/34$

Individual *i* will have a probability to be chosen $\frac{f(i)}{\sum_i f(i)}$



Area is proportional to fitness value

We repeat the extraction as many times as the number of individuals we need to have the same parent population size (6 in our case).

Example (Selection Set)

Suppose that, after performing selection, we get the following population:

$$s_1` = 1111010101 \qquad (s_1)$$

$$s_2` = 1110110101 \qquad (s_3)$$

$$s_3` = 1110111101 \qquad (s_5)$$

$$s_4` = 0111000101 \qquad (s_2)$$

$$s_5` = 0100010011 \qquad (s_4)$$

$$s_6` = 1110111101 \qquad (s_5)$$

Example (Step2: Crossover)

- Next we mate strings for crossover.
  - For each couple we decide according to crossover probability (for instance 0.6) whether to actually perform crossover or not
  - Suppose that we decide to actually perform crossover only for couples (s1`, s2`) and (s5`, s6`). For each couple, we randomly extract a crossover point, for instance 2 for the first and 5 for the second

Example (Crossover result)

Before crossover:

s_1` = 1111010101 $s_2$` = 1110110101 s_5` = 0100010011 $s_6$` = 1110111101

Cross point = 2 Cross point = 5

After crossover:

s_1`` = 1110110101 $s_2$`` = 1111010101 s_5`` = 0100011101 $s_6$`` = 1110110011

Example (Step3: Mutations)

The final step is to apply random mutation: for each bit that we are to copy to the new population we allow a small probability of error (for instance 0.1)

Before applying mutation:

$s_1'' = 1110110101$

$s_2'' = 1111010101$

$s_3'' = 1110111101$

$s_4'' = 0111000101$

$s_5'' = 0100011101$

$s_6'' = 1110110011$

After applying mutation:

$s_1''' = 1110100101$	$f(s_1''') = 6$
$s_2''' = 1111110100$	$f(s_2''') = 7$
$s_3''' = 1110101111$	$f(s_3''') = 8$
$s_4''' = 0111000101$	$f(s_4''') = 5$
$s_5''' = 0100011101$	$f(s_5''') = 5$
$s_6''' = 1110110001$	$f(s_6''') = 6$

The new population is formed and will be used in the next iteration.

Example (and now, iterate)

In one generation, the total population fitness changed from 34 to 37, thus improved by ~9%

At this point, we go through the same process all over again, until a stopping criterion is met

Components of a GA

A problem definition as input, and

- Encoding principles (gene, chromosome)
- Initialization procedure (creation)
- Selection of parents (reproduction)
- Genetic operators (crossover, mutation)
- Evaluation function (fitness to environment)
- Termination condition

Summary

- **Beam Search**
- **Genetic Algorithm**
  - **Crossover**
  - **Mutation**
  - **Application in MAXONE Problem**

What I want you to do

- **Review Course Slides**