Ms. No.: JMVA-14-443R2
Title: Scale and Curvature Effects in Principal Geodesic Analysis
Corresponding Author: Mr. Drew Lazar
Authors:

Dear Mr. Lazar,

Thank you for submitting your manuscript to the Journal of Multivariate Analysis. The reviewers have made suggestions which the Editor feels would improve your manuscript. The Editor encourages you to consider these comments and make an appropriate revision of your manuscript. The reviewers' comments are below.

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Thank you, and we look forward to receiving your revised manuscript.

With kind regards,

Kavitha Balu
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Reviewers' comments:

FROM THE EDITOR'S DESK:

Dear Dr. Lazar,

The second revision of your paper has been reviewed by the same Associate Editor (AE) and referees as the previous versions. Meanwhile, however, I have taken over the editorship of the journal, which results in a change of style in the handling of submissions. Unfortunately, I must report that Reviewer #2 is still unhappy with the paper. He/she writes as follows:

"The author has changed the manuscript to incorporate some, but not all, of the
points of my second review. In particular, for point (6) (numbering in response), I wrote:

"Curvature" is a word in the title and a major focus of the paper. On the other hand, one has to read a substantial part of the paper before curvature actually appears (page 13). In the conclusion, the author writes "Note that as in (33) in section 4.3 all expansions in this paper can be expressed and interpreted strictly in geometric terms of angles, sectional curvatures and Riemannian curvature tensors". This way of expressing the expansions is however not used in the paper...

I believe these points limit the reach of the paper for two reasons: ...

* it is not clear before the end of the paper where curvature appears in the expansions and it thus hard to see how the non-linearity of the data spaces effects PGA.

The author has met this point to a very little degree. This choice is explained in the response:

"As I state in the conclusion, "generally sectional curvatures of larger magnitudes give larger higher-order terms" but I felt further interpretation of the expressions of all the expansions in this form would make the paper quite a bit longer and cluttered. Such interpretation can certainly be the subject of further inquiry with applications to be developed in subsequent papers."

Unfortunately, I still believe this is a weakness of the paper. The examples show specific forms of the relation between scale and curvature but it is hard to see how scale and curvature relate in the proposition (in Section 2) that is a main part of the paper. As an example, the word 'scale' is mentioned in the introduction (page 2) and the next occurrence is much later after the proposition (page 8). Curvature appears on page 4 and then again on page 14. I understand that thoroughly describing the relation between scale, curvature, and the expansions is hard but reading the statement of the

proposition in the current form, it is also hard to interpret what it is actually stating: (the expansions at first sight seems to be just Taylor expansions, and the concrete expressions are still only present in the proof despite my critique in the previous review. It is not described in the text that \epsilon is a scaling parameter (as far as I could find, 'scaling parameter' is
mentioned on page 2
and the next occurrence is on page 23)).

In his/her recommendation, the AE wrote: "I concur with the reviewer. We both believe that the author should make the presentation of the expansions in Section 2 clearer and include further descriptions of the overall approach with \epsilon a scaling parameter, write the statement of the proposition in a way so that the result is easy to interpret, and give more explanation as to how the result relates scale and curvature."

With these changes implemented, the AE would recommend your paper for publication. I am pleased to invite such a revision, which I regard as major, with the proviso that it should not lengthen the manuscript. Let me also chip in with minor comments:

a) Definition 3 (Intrinsic Mean): there should be a reference to papers by Fr\'echet and/or Karcher on the Fr\'echet/Karcher mean. (and Fr\'echet is typeset Fr\'echet in LaTeX)

b) Page 2, "In Section 3 we review the geometry of the space of positive definite matrices": Section 3 should be Section 4 as far as I can see.

Finally, in anticipation of acceptance, let me list a new set of typographical rules and conventions that should be adhered to in the third revision (it could be that some of them do not apply to your paper or that they have already been implemented).

1) Enumerations should list the first and last element only, i.e., write 1, \ldots, n (NOT 1, 2, \ldots , n). Also do note use \cdots in enumerations.

2) Respect the following priority rules for fences [{()}] unless the fences have a special meaning, i.e., write E{X(t)} but if you refer to the set {1, ..., n}, don't write (1, ..., n), because in the latter two contexts, {} and () have conventional meanings.

3) Use \^top for transposition (NOT ', ^T or ^\tau) and \ln for log, unless you mean log_10.

4) Use \mathcal{N} for the Normal distribution, and similar symbols for the Poisson, beta, etc., e.g., \mathcal{N}(0,1) or \mathcal{P}(\lambda).

5) Use \Pr for probability, so that it prints "Pr" in Roman characters.

6) Symbols like E, var, cov, corr, diag, trace, etc. should be in Roman characters, i.e., {\rm E}, etc. (NOT in

italics).

7) "cf." means "compare to", NOT "refer to". If you mean "refer to", write "see".

8) Avoid in-line fractions; if you need one, write them in the form a/b rather than \frac{a}{b}.

9) Last but not least, papers should be referred to by AUTHOR (YEAR) or AUTHOR & AUTHOR (YEAR), or AUTHOR et al. (YEAR), and the bibliography should not be numbered.

Good luck with the revision. I look forward to receiving it early in the new year.

Best regards,

Christian Genest, PhD, FRSC
Editor-in-Chief
Journal of Multivariate Analysis