Ms. No.: JMVA-14-443R1
Title: Scale and Curvature Effects in Principal Geodesic Analysis
Corresponding Author: Mr. Drew Lazar
Authors:

Dear Mr. Lazar,

Thank you for submitting your manuscript to the Journal of Multivariate Analysis. The reviewers have made minor suggestions and comments which the Editor believe would improve your manuscript. The Editors have recommended acceptance pending an appropriate revision. The reviewers' comments are below.

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With kind regards,

Kavitha Balu
Journal Manager
Journal of Multivariate Analysis
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Reviewers' comments:

Reviewer #2: In the revised version of the paper, the recursive principal geodesic analysis (RPGA) procedure is left out and the focus is entirely on the expansions in the scale parameter of the PGA objective function and the principal directions. These expansions are novel and gives insight into the differences between the Euclidean and the non-linear situation.

To the extend I can evaluate, the paper is technically correct.

"Curvature" is a word in the title and a major focus of the paper. On the other hand, one has to read a substantial part of the paper before curvature actually appears (page 13). In the conclusion, the author writes "Note that as in (33) in section 4.3 all expansions in this paper can be expressed and interpreted strictly in geometric terms of angles, sectional curvatures and Riemannian curvature tensors". This way of expressing the expansions is however not used in the paper.

The introduction of the paper has little explanation of the problem that is the focus of the paper and the reason why relating scale and curvature is an interesting problem. Figures explaining the setup and the expansions are sparse and far into the paper (page 15).

I believe these points limit the reach of the paper for two reasons: * the message that scale of data is important when evaluating curvature effects is more or less lost because the very technical proposition is stated without good motivation; * it is not clear before the end of the paper where curvature appears in the expansions and it thus hard to see how the non-linearity of the data spaces effects PGA.

The experiment in table 1 (page 10) is carried out with data sampled from a Von-Mises Fisher distribution on the sphere. Does it make sense to compute principal directions from such a distribution? Isn't it isotropic (identity*scalar covariance matrix) around \mu and the principal directions thus not defined? (implying that the directions resulting from the PGA analysis are random)

In conclusion: the experiment on page 10 should be redone with anisotropic data, and I strongly recommend that the role of curvature is emphasized and that the introduction is extended/rewritten to better illustrate the point of the paper (including illustrative figures).

Minor comments:
* the u_1,\ldots,u_{j-1} in the proposition at page 4 are defined before the proposition which makes it confusion the read the proposition on its own.

* still in the proposition, g_{j,4} and again e.g. f_{k,4} in eq (24) are only defined in the proof of the proposition.

* page 3, "We will assume throughout ...". Doesn't the existence and uniqueness almost everywhere fail on e.g. the torus where almost every geodesic geodesic is dense an existence thus not defined?

* page 5: "As given in ...". Is immersed submanifold needed here? Doesn't the convergence happen on any Riemannian manifold?