Lab 4: Bonus Questions

- There are two sets of bonus questions for this lab, each worth 3 points, i.e. you can earn up to 6 points from the bonus questions on this lab. For all labs, you can gain up to 18 points. Obtaining 12 out of the 18 points will improve your grade by one if you fulfill all requirements to pass the course.
- You have to work on the questions on your own and have to hand in your solutions individually. Hence, **no collaboration** is allowed and you are only allowed to ask for help from the teaching assistants or the lecturer. If you use non-living sources, like the internet or books, you must add a reference. It is not allowed to copy solutions, not even partially, from other sources (your fellow students, internet, Large Language Models or whatever).
- Your answers will be graded on a continuous scale, i.e., we try to reward partially correct answers.
- Please hand in your solutions as a PDF through Canvas.

1 Camera Geometry, 3 points

(a) Consider the following situation: You have a set of 2D-3D correspondences between features in an image and 3D points on a plane for a camera with an unknown focal length f. The plane is parallel to the X-Y plane of the local coordinate system of the camera and the Z-axis of the local camera coordinate system intersects the plane at the point $(0, 0, z_{\text{plane}})^T$, i.e., the camera has a fronto-parallel view of the plane. However, the distance z_{plane} of the plane to the origin of the local camera coordinate system is unknown.
Derive an algorithm that computes both the unknown focal length f and the distance z_{plane} from a given set of 2D-3D correspondences or prove that no such algorithm exists. You can assume that all matches are correct.

(b) An explicit representation of a line in 3D is given by

$$\mathbf{x}(\lambda) = \mathbf{o} + \lambda \mathbf{d} \ ,$$

where $\mathbf{o} \in \mathbb{R}^3$ is some point on the line, $\mathbf{d} \in \mathbb{R}^3$ is the direction of the line, and $\lambda \in \mathbb{R}$ is a scaling factor.
Consider the set of all parallel lines in a given direction $\mathbf{d}$ in the local coordinate system of the camera. Assume that the direction $\mathbf{d}$ is not lying in the X-Y plane of the local coordinate system of the camera. The *vanishing point* corresponding to this set of lines is the point in the image in which the projections of all these lines intersect (i.e., the point in which all lines intersect after they have been projected into the image). A classical example for a vanishing point is a camera looking down a railroad track, leading to a point in the image where the two lines defining the track intersect.
Show that the vanishing point corresponding to parallel lines in direction $\mathbf{d}$ (in the local camera coordinate system) corresponds to the intersection of the line $(0, 0, 0)^T + \lambda \mathbf{d}$ with the image plane for an arbitrary focal length, denoted f.
Hint: Consider the projection of points that are infinitively far away along the lines.

(c) Assume that you know the *absolute* poses $[\mathtt{R}_1|\mathbf{t}_1]$ and $[\mathtt{R}_2|\mathbf{t}_2]$ of two images, i.e., $[\mathtt{R}_i|\mathbf{t}_i]$ transforms a point from a global coordinate system into the local camera coordinate system of the i-th image. For a third image, you only know the *relative* poses $[\mathtt{R}_{31}|\mathbf{t}_{31}]$ and $[\mathtt{R}_{32}|\mathbf{t}_{32}]$, where $[\mathtt{R}_{3i}|\mathbf{t}_{3i}]$ transforms a point from the local coordinate system of image i to the local coordinate system of the third image. Since you obtained the relative poses by decomposing essential matrices, $\mathbf{t}_{31}$ and $\mathbf{t}_{32}$ are only known up to a scaling factor, i.e., assume that $||\mathbf{t}_{31}|| = ||\mathbf{t}_{32}|| = 1$. Describe how you can compute the absolute pose $[\mathtt{R}_3|\mathbf{t}_3]$ of the third image from this given information.

2 Epipolar geometry, 3 points

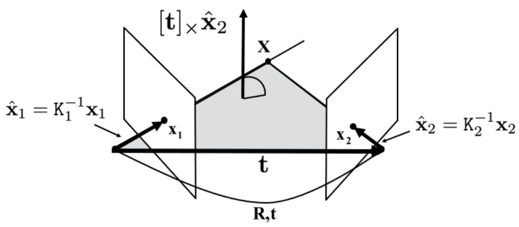

Figure 1: Setup in task 2a-b.

(a) Study the epipolar geometry in Figure 1. Explain what the figure illustrates, i.e. what the schematics and the variables represent. Use the figure to interpret/explain the following equation:

$$\lambda_2 \hat{\mathbf{x}}_2 = R\lambda_1 \hat{\mathbf{x}}_1 + \mathbf{t}$$

(b) Derive the epipolar constraints from the equation in (a). Make sure you motivate all steps in your derivation with means of the figure, equations and/or explanations.

(c) Derive an expression for the two epipoles using only the variables in Figure 1. Also provide an interpretation of what the epipoles represent.