Cosmology notes - Reheating

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INTRODUCTION

Nowadays, the most competitive cosmological model is Λ CDM model with a single field inflation. Reheating is a proposed process at the end of the period of accelerated expansion which is an important part of inflationary cosmology. It occurs through coupling the inflation scalar field ϕ, and inflaton particles may decay into Standard Model matter particles. In other words, the initial energy transfer from the inflation field to matter particles. We will discuss this process based on quantum mechanical production of matter particles with a classical background inflaton field.

INFLATION MODELS

In the context of general relativity as the theory describing space and time, inflation requires scalar field matter. Namely, the energy momentum tensor must be dominated by the potential energy density of ϕ. We have some scenarios with different kinds of inflation field, but it could not be serve as SM Higgs field. A standard example for this is

$$V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2 \tag{1}$$

where η is vacuum expectation value of ϕ. In this simple case, the SM Higgs must have a coupling constant λ which cannot sufficiently small to generate enough slow-roll time for the inflation.

Therefore, we should consider something beyond Standard Model. We consider some of them here.

- "New" inflation. In this case, it is regarded that inflation will take place when ϕ is undergoing the symmetry-breaking phase transition and slowly rolling to $\phi = \pm\eta$. However, it suffers from an initial condition problem.

- "Chaotic" (or "large-field") inflation. In this case, inflation is triggered by a slow rolling ϕ unrelated to a symmetry-breaking phase transition. A simple model for this is

  $$V(\phi) = \frac{1}{2}m^2\phi^2 \tag{2}$$

  It is assumed that ϕ starts out at large field values and slowly roos towards its vacuum state $\phi = 0$. Two condition must be satisfied if the inflation would be successful.

  - The energy density must be dominated by the potential energy term.
  - The acceleration term in $\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$ must be negligible.

  However, "Chaotic" inflation will meet a problem because it is required from the second condition that $|\phi| > m_{pl}$. Meanwhile, gravitational effects often steepen the potential for $|\phi|$ beyond m_{pl} and thus prevent slow-roll.

- "Hybrid" inflation. This is proposed to avoid the problem of "Chaotic" inflation. In this case, we add a second scalar field ψ and have

  $$V(\phi,\psi) = \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\psi^2\phi^2 + \frac{1}{4}\lambda\left(\psi^2 - v^2\right)^2 \tag{3}$$

  For large values of $|\phi|$, the potential in ψ has a minimum at $\psi = 0$, whereas for small values of $|\phi|$, $\psi = 0$ becomes an unstable point.

INFLATION DECAY

Reheating describe the production of SM matter at the end of the period of accelerated expansion. The energy density, at this time, is stored overwhelmingly in the oscillations of ϕ. To study the quantum mechanical production of matter particles with a classical background inflaton field, we should first consider ϕ in chaotic model we discussed above with

$$\phi(t) \to \frac{m_{pl}}{\sqrt{3\pi}mt}\sin(mt) \tag{4}$$

when $mt \gg 1$.

We assume that the inflaton ϕ is coupled to another scalar field χ. We will start from a classical mechanism of particle production, and modify it to a quantum one. We will assume a static background and move to an expanding one later. We will also focus on the number of χ particle.

The classical production

In this case, we consider the energy loss of the inflaton due to the production of χ particles. This process is taken into account by adding a damping term to the "chaotic" inflation (as a classical background) equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} = -V'(\phi) \tag{5}$$

where Γ is decay rate with the interaction Lagrangian as $\mathcal{L}_{\text{int}} = -g\sigma\phi\chi^2$. For small g, Γ is smaller than the Hubble parameter at the end of the inflation. Thus, χ particle production become effective only when H is comparable to Γ. We use the reheating temperature

$$T_R \sim (\Gamma m_{pl})^{1/2} \tag{6}$$

as temperature of SM fields after energy transfer to describe how much energy end up in χ particles. However, this perturbative reheating is slow and produces a very lower reheating temperature compared to the energy scale at which energy takes place. Besides, the violation of fluctuation theorem while using Equation 5 and ignorance of coherent nature of inflation field motivate a matter field treated quantum mechanically.

Preheating

In this subsection we will consider a quantum production of χ particles in classical ϕ background. Here we will use a new Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}g^2\chi^2\phi^2 \tag{7}$$

and neglect the space expansion.

We could expand $\hat{\chi}$ into different modes and obtain the equation of motion as

$$\chi_k'' + (A_k - 2q\cos 2z)\chi_k = 0 \tag{8}$$

where we introduce dimensionless time variable $z = mt$ and where

$$A_k = \frac{k^2 + m_\chi^2}{m^2} + 2q \qquad q = \frac{g^2\Phi^2}{4m^2}$$

For simplicity, we claim that exponential growth of mode functions will lead to exponential growth of the number of χ particle. From Equation 8 we have a instabilities and leads to exponential growth

$$\chi_k \propto \exp(\mu_k z) \tag{9}$$

For small q we have "narrow resonance" while for $q/gg1$ we have broad resonance. However, for broad resonance, if the k mode satisfy

$$k^2 \le \frac{2}{3\sqrt{3}}gm\Phi - m_\chi^2 \tag{10}$$

the so-called adiabaticity condition

$$\frac{\mathrm{d}\omega_k^2}{\mathrm{d}t} \le 2\omega_k^3 \tag{11}$$

breaks up and there will be particle production here. When $q \gg 1$, the resonance is much more efficient and it is called broad resonance process. We will focus on broad resonance process below.

Preheating in Expanding Background

Here we should consider an expanding background by taking the cosmological factor $a(t)$ into account. The equation of motion then become

$$\ddot{\chi} + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + g^2\Phi(t)^2\sin^2(mt)\right)\chi_k \tag{12}$$

The adiabaticity condition is violated when

$$\frac{k^2}{a^2} \le \frac{2}{3\sqrt{3}}gm\Phi(t) - m_\chi^2 \tag{13}$$

and the broad parametric resonance ends when $q \le 1/4$

For a more quantitative analysis, we consider a rescaling $X_k(t) = a^{3/2}(t)X_k(t)$ to eliminate the Hubble friction term. Thus, Equation 12 becomes

$$\ddot{X}_k + \omega_k^2 X_k = 0 \tag{14}$$

The adiabaticity condition is violated during short time interval around $t = t_j$ when $\phi = 0$.

In $[t_{j-1}, t_j]$, we have adiabatic evolution of X_k as

$$X_k^j(t) = \frac{\alpha_k^j}{\sqrt{2\omega_k}}e^{i\int\omega_k \mathrm{d}t} + \frac{\beta_k^j}{\sqrt{2\omega_k}}e^{-i\int\omega_k \mathrm{d}t} \tag{15}$$

where α_k^j and β_k^j are constant Bogoliubov coefficients with $|\alpha_k^j|^2 - |\beta_k^j|^2 = 1$.

For non-adabatic evolution, we use the approximation $\phi^2(t) \simeq \Phi^2 m^2 (t-t_j)^2$ and new variable

$$\tau = g\Phi m(t - t_j) \qquad \kappa^2 = \frac{(k^2/a^2) + m_\chi^2}{g\phi m}$$

and we have the equation of motion

$$\frac{\mathrm{d}^2 X_k}{\mathrm{d}\tau^2} + \left(\kappa^2 + \tau^2\right)X_k = 0 \tag{16}$$

With Bogoliubov transformation in non-abadatic case, we have the occupation number of k mode as

$$\begin{aligned} n_k^{j+1} &= \left(1 + 2e^{-\pi\kappa^2} + 2e^{-\frac{\pi}{2}\kappa}\sqrt{1+e^{-\pi k^2}}\sin\theta_{\text{tot}}^j\right)n_k^j \\ &= e^{2\pi\mu_k^j}n_k^j \end{aligned} \tag{17}$$

We introduce the Floquet exponent μ_k^j for further calculation. For a "average" Floquet exp onent, we have

$$\begin{aligned} \mu_k &= \frac{\pi}{m\Delta t}\sum_j \mu_k^j \\ &\approx \frac{1}{2\pi}\ln 3 - \mathcal{O}(\kappa^2) \end{aligned} \tag{18}$$

Thus we could derive the number of new particle χ

$$n_\chi(t) \approx \frac{(gm\phi_0)^{3/2}|\beta_m^0|^2 e^{2m\mu t}}{16\pi^3 a^3\sqrt{\frac{m\mu t}{2\pi} + 1}} \tag{19}$$

Termination of Preheating

In previous analysis, We have neglected the back-reaction of χ particles. Only under two condition back-reaction is negligible: (1) change Δm_ϕ^2 in the square mass of inflaton is smaller than m^2; (2)the energy in the χ particles is sub-dominant.

For the first condition, the presence of χ particles changes the effective mass of inflaton oscillations. In the Hartree approximation, the change in the inflaton mass is given by

$$\Delta m_\phi^2 = g^2 \langle \chi^2 \rangle \tag{20}$$

From Equation 15 and 20 we have

$$n_\chi(t) \leq \frac{m^2 \Phi(t)}{g} \tag{21}$$

This is the condition under which back-reaction is negligible.

For the second condition, ρ_χ is smaller than the potential energy of the inflaton field at the time t_1 as long as the value $q(t_1) > 1$. (q is from Equation 8).

Beyond Chaotic Inflation

Here we give a brief discussion of some variance of preheating beyond simple chaotic model. These new model have a efficient inflaton decay than simple chaotic inflation.

- Negative frequency of χ. In this case, we would get a an exponential instability. We could change the sign in Equation 7 to do so, or we could have a symmetry breaking potential for a certain time interval. In the latter case a tachyonic preheating occurs.
- Tachyonic preheating. In addition to the symmetry breaking potential case, we also have one for hybrid inflation. The fluctuations of ψ would have a tachyonic form in this case.
- A coupling χ. If χ particles are coupled linearly to fermions in $\mathcal{L}_{\text{int}} = -g\sigma\phi\chi^2$, the χ particles created when $\phi \sim 0$ decay after half a ϕ oscillation. This prevent χ to from slowing the decay of ϕ.