

UAVBook Supplement

Feedforward Term for Orbit Following

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1 Feedforward with no wind

Chapter 10 in the book describes a vector field method for orbit following. The commanded course angle is given in Equation (10.13) as

$$\chi^c(t) = \varphi + \lambda \left[\frac{\pi}{2} + \tan^{-1} \left(k_{\text{orbit}} \left(\frac{d - \rho}{\rho} \right) \right) \right].$$

One of the disadvantages of this strategy is if the roll angle is used to command the course, and if the course angle is currently correct, then the roll angle will be zero. The controller can be improved by using a feedforward term on the roll angle.

If the UAV is on the orbit and there is no wind, then the desired heading rate is given by

$$\dot{\psi}^d = \frac{V_a}{R}.$$

Assuming a coordinated turn condition, the kinematics of the UAV are given by Equation (9.14) as

$$\dot{\psi} = \frac{g}{V_a} \tan \phi.$$

Setting these expressions equal to each other, and solving for the roll angle, gives the feedforward term

$$\phi_{ff} = \tan^{-1} \left(\frac{V_a^2}{gR} \right). \quad (1)$$

In other words, if the UAV is currently on the orbit and is banked at ϕ_{ff} , and assuming that airspeed and altitude are being maintained by the autopilot, then the UAV will continue to fly on the orbit.

2 Feedforward with wind

When wind is present, the situation becomes more complicated. We will assume in this section that the wind vector $\mathbf{w} = (w_n, w_e, w_d)^\top$ is known. In the wind case, a stationary orbit of radius R relative the ground is described by the expression

$$\dot{\chi}^d(t) = \frac{V_g(t)}{R},$$

where V_g is the time varying ground speed. From Equation (9.10) in the book, the coordinated turn condition in wind is given by

$$\dot{\chi} = \frac{g}{V_g} \tan \phi \cos(\chi - \psi).$$

Equating these expressions and solving for ϕ results in the feedforward term

$$\phi_{ff} = \tan^{-1} \left(\frac{V_g^2}{gR \cos(\chi - \psi)} \right).$$

From the wind triangle expression given in Equation (2.12) we have that

$$\sin(\chi - \psi) = \frac{1}{V_a \cos \gamma_a} (w_e \cos \chi - w_n \sin \chi).$$

Using a simple identity from trigonometry we get

$$\cos(\chi - \psi) = \sqrt{1 - \left(\frac{1}{V_a \cos \gamma_a} (w_e \cos \chi - w_n \sin \chi) \right)^2}.$$

In a constant altitude orbit, the flight path angle $\gamma = 0$, and therefore Equation (2.11), which comes from the wind triangle, becomes

$$\sin \gamma_a = \frac{w_d}{V_a},$$

from which we get that

$$\cos \gamma_a = \sqrt{1 - \left(\frac{w_d}{V_a}\right)^2},$$

which implies that

$$\cos(\chi - \psi) = \sqrt{1 - \frac{(w_e \cos \chi - w_n \sin \chi)^2}{V_a^2 - w_d^2}}.$$

In a constant altitude orbit, the flight path angle $\gamma = 0$, and therefore Equation (2.10), which comes from the wind triangle, becomes

$$V_g^2 - 2(w_n \cos \chi + w_e \sin \chi) V_g + (V_w^2 - V_a^2) = 0.$$

Taking the positive root for V_g gives

$$\begin{aligned} V_g &= (w_n \cos \chi + w_e \sin \chi) + \sqrt{(w_n \cos \chi + w_e \sin \chi)^2 - V_w^2 + V_a^2} \\ &= (w_n \cos \chi + w_e \sin \chi) + \sqrt{V_a^2 - (w_n \sin \chi - w_e \cos \chi)^2 - w_d^2}. \end{aligned}$$

The term under the square root will be positive when the airspeed is greater than the windspeed, ensuring a positive groundspeed.

Therefore, the feedforward roll angle is given by

$$\phi_{ff} = \tan^{-1} \left(\frac{\left((w_n \cos \chi + w_e \sin \chi) + \sqrt{V_a^2 - (w_n \sin \chi - w_e \cos \chi)^2 - w_d^2} \right)^2}{gR \sqrt{\frac{V_a^2 - (w_n \sin \chi - w_e \cos \chi)^2 - w_d^2}{V_a^2 - w_d^2}}} \right), \quad (2)$$

which depends only on the wind speed, the current course angle, and the airspeed. When the wind is zero, i.e., $w_n = w_e = w_d = 0$, then Equation (2) simplifies to Equation (1).