UAVBOOOK Supplement.
Additional thoughts on propeller thrust model

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Figure 1: Typical propeller used on a small UAS.

Source material for the following discussion on thrust produced by the propeller is adapted from [1].

A simple schematic of a propeller is shown in Figure 2. The free stream velocity of the air entering the propeller is given by V_a and the discharge velocity is given by V_d. The thrust produced by the propeller is given by

$$T_{\text{outlet}} = \rho V_d Q_d,$$

where ρ is the density of air, and Q_d is the quantity of air at the discharge of the propeller given in units of volume per unit of time. The quantity of

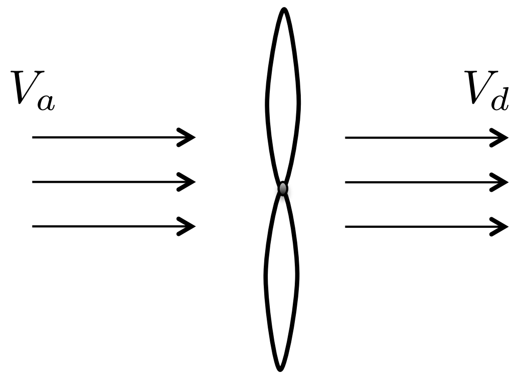

Figure 2: Propeller schematic.

air at the discharge can be given by

$$Q_d = C_{\text{prop}} A_{\text{prop}} V_d,$$

where C_{rotor} is a unitless coefficient that quantifies the efficiency of the propeller, and A_{prop} is the area swept out by the propeller given by $A_{\text{prop}} = \pi R_{\text{prop}}^2$, where R_{prop} is the radius of the propeller. The thrust at the outlet of the propeller is therefore

$$T_{\text{outlet}} = \rho \pi R^2 C_{\text{prop}} V_d^2.$$

The propeller increases the pressure and velocity of the air at its inlet. If the velocity of the air at the inlet is nonzero, or in other words if the free stream velocity $V_a \neq 0$, then the propeller is unable to increase the velocity of the air by that amount. The resulting drag force, which is called the momentum drag is given by

$$T_{\text{drag}} = \rho V_a Q_d = \rho \pi R_{\text{prop}}^2 C_{\text{prop}} V_a V_d.$$

Therefore, the thrust produced by the propeller is

$$T_{\text{prop}} = T_{\text{outlet}} - T_{\text{drag}} = \rho \pi R_{\text{prop}}^2 C_{\text{prop}} V_d (V_d - V_a).$$

To obtain an expression that relates the throttle command δ_t to the thrust T_{prop}, we need a model that expresses the discharge velocity V_d as a function of δ_t.

When the airspeed is zero, the discharge velocity is given by

$$V_d = k_m \delta_t,$$

where k_m is the motor constant measured in units of m/s. The throttle δ_t is assumed to take values between zero and one, where $\delta_t = 1$ is full throttle, and $\delta_t = 0$ is no throttle. Therefore, a simple method to measure k_m is to place a pitot tube attached to an autopilot in the outlet stream of the propeller, and to command full throttle. The gain k_m is the resulting airspeed measured by the pitot tube.

At full throttle, the velocity of the discharge will be k_m, roughly independent of the ambient airspeed V_a. However, when the throttle is zero, the propeller stops spinning (we are assuming an electric motor) and the discharge velocity will be the ambient airspeed V_a. Using a linear interpolation model, we get that the discharge velocity is given by

$$V_d = k_m \delta_t + (1 - \delta_t) V_a = V_a + \delta_t (k_m - V_a).$$

Therefore, the thrust model for the prop as a function of the throttle input is given by

$$T_{\text{prop}} = \rho \pi R_{\text{prop}}^2 C_{\text{prop}} (V_a + \delta_t (k_m - V_a))(\delta_t (k_m - V_a)).$$

References

[1] P. FitzPatrick, "Calculation of thrust in a ducted fan assembly for hovercraft," tech. rep., Hovercraft Club of Great Britain, 2003.