# Nonlinear Total Energy Control for the Longitudinal Dynamics of an Aircraft

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Abstract—Many common altitude and airspeed control schemes for an aircraft assume that the altitude and airspeed dynamics are decoupled which leads to errors. The Total Energy Control System (TECS) is an approach that controls the altitude and airspeed by manipulating the total energy rate and energy distribution rate, of the aircraft, in a manner which accounts for the dynamic coupling. Typically TECS control schemes are PI controllers. In this paper, a nonlinear controller based on the TECS principles are derived. Simulation results show that the nonlinear controller has better performance than the PI TECS control schemes.

#### I. Introduction

A common approach to control the airspeed and altitude for a fixed wing aircraft is to assume that the are airspeed and altitude dynamics are decoupled. Using this assumption, the altitude is controlled by elevator while the thrust controls the airspeed. An example of this type of controller is in [1]. While this type of controller has been used successfully for decades, the underlying assumption that the airspeed and altitude dynamics are decoupled is inherently false. For example, consider the simple case when an aircraft pitches up while the thrust does not change. If the dynamics were truly decoupled, the aircraft's altitude would increase while the airspeed would remain unchanged. However, as should be obvious, the airspeed will decrease while the altitude increases. In other words, some of the aircraft's kinetic energy is converted to potential energy.

In the early 1980's Lambregts et al. realized that the airspeed and altitude of an aircraft could be controlled by manipulating the kinetic and potential energy of the system [2], [3], [4]. Approaching the problem this way allowed the coupling between the altitude and airspeed to be taken into account. Controllers based on this idea, called the Total Energy Control System (TECS), have been been successfully used and tested on a variety of airframes [5], [6]. While most of these controllers are

of PI type, other variants have been developed such as adding pitch damping [7]. These ideas have also been explored to see if cockpit displays could be developed to present pilots this information [8]. TECS concepts have also been applied to the lateral control of an aircraft [9], [10] and for the longitudinal control of a helicopter [11].

In this paper, we derive a nonlinear altitude and airspeed controller based on the TECS concept. Two variants of this controller are compared to the original TECS controller, a variant of the TECS controller, and a standard decoupled controller through several simulations. These simulations show that the nonlinear TECS controllers account for the coupling between the airspeed and altitude dynamics while having smaller oscillations in the response compared to the original TECS control scheme.

This paper is organized in the following manner. Section II defines the various energy values and their derivatives. The original TECS control scheme is described in Section III. Section IV contains the motivation and derivation for our nonlinear TECS controller. Two other controllers, for comparison, are briefly described in Section V. The simplified three degree of freedom dynamics model is described in Section VI and the simulation results are shown in Section VII. Finally, conclusions are presented in Section VIII.

## II. ENERGY DEFINITIONS

If the airplane is modeled as a point mass then the total energy is the kinetic energy plus the potential energy or

$$E_T \triangleq \mathsf{m}gh + \frac{1}{2}\mathsf{m}v^2. \tag{1}$$

The total energy rate is

$$\dot{E}_T = mg\dot{h} + mv\dot{v}. \tag{2}$$

The energy difference is defined to be the potential energy minus the kinetic energy or

$$E_D \triangleq \mathbf{m}gh - \frac{1}{2}\mathbf{m}v^2 \tag{3}$$

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and the energy distribution rate, or the energy difference rate, is

$$\dot{E}_D = \mathbf{m}g\dot{h} - \frac{1}{2}\mathbf{m}v\dot{v}.\tag{4}$$

## III. ORIGINAL TECS

Every TECS altitude and airspeed controller shares the same basic principles. First, energy is conserved. Kinetic energy, or the airplane's velocity, can be converted into potential energy, or altitude, and vice versa. Second, assuming a point mass model of the aircraft, the thrust generated by the propulsion system is the only way to add energy to the system and drag is the only way that energy is removed. TECS controllers also assume that the angle of attack,  $\alpha$  is low and the flight path angle,  $\gamma$ , does not influence the total drag. Using these basic principles and these assumptions the basic TECS longitudinal controller can be developed.

Typically TECS controllers use a scaled version of the total energy rate and energy distribution [12]. Rearranging (2) provides

$$\dot{E}_1 = \frac{\dot{E}_T}{\text{m}qv} = \frac{\dot{v}}{q} + \frac{\dot{h}}{v}.$$
 (5)

If we assume that  $\gamma$  is small then (5) becomes

$$\dot{E}_1 = \frac{\dot{v}}{q} + \gamma$$

which is the standard total energy rate value used in TECS control schemes. Likewise, the energy distribution rate, (4), can be rearranged to

$$\dot{E}_2 = \frac{\dot{E}_D}{\text{m}gv} = -\frac{\dot{v}}{g} + \gamma.$$

As mentioned earlier, the total energy rate is controlled by the thrust which implies that the commanded thrust should be proportional to  $\dot{E}_1$ . Likewise, the elevator control is approximately energy conservative and allows kinetic energy and potential energy to be converted to each other. This means that the elevator should be used to control the energy distribution rate.

The thrust command is

$$T^c = T_D + \Delta T$$

where  $T_D$  is the trim thrust needed to counteract drag and

$$\Delta T = k_{1,i} \int_{t_0}^t (\dot{E}_1^c - \dot{E}_1) \delta t + k_{1,p} \dot{E}_1^c,$$
 (6)

where  $k_{1,i}$  and  $k_{1,p}$  are the integral and proportional gains respectively and  $\dot{E}_1^c$  is the commanded total energy rate computed from the commanded acceleration and

flight path angle. Likewise, the pitch command is

$$\theta^c = k_{2,i} \int_{t_0}^t (\dot{E}_2^c - \dot{E}_2) \delta t + k_{2,p} \dot{E}_2^c,$$

where  $k_{2,i}$  and  $k_{2,p}$  are the integral and proportional gains respectively and  $\dot{E}_2^c$  is the commanded energy distribution rate.

This control scheme assumes that there are underlying fast low level control loops that control T and  $\theta$ . In addition, the thrust and pitch dynamics must have a similar bandwidth and be as close as possible.

The commanded acceleration and flight path angle are created by a simple proportional term [3]

$$\dot{h}^c = k_h(h^c - h), \tag{7}$$

$$\gamma^c = \frac{\dot{h}^c}{v},$$

$$\dot{v}^c = k_v(v^c - v). \tag{8}$$

## IV. NONLINEAR TECS

The TECS concept can also be used to derive an adaptive nonlinear control scheme where the controller attempts to control  $E_T$  and  $E_D$  directly, The total energy, of the system, can only change due to the thrust or drag on the aircraft otherwise the energy is just transitioning between kinetic and potential. This means that the total energy rate can be derived by looking at the case where the velocity changes and the altitude is constant. Assuming  $\dot{h}=0$ , the total energy rate is

$$\dot{E}_T = \mathbf{m}v\dot{v} = vF,$$

where F is the net non-conservative force on the aircraft. Assuming that the angle of attack is low and that the thrust and drag are aligned, then

$$\dot{E}_T = v(T - D). \tag{9}$$

In general, when  $\dot{h} \neq 0$ ,

$$mg\dot{h} + mv\dot{v} = \dot{E}_T = v(T - D)$$

which means

$$\dot{E}_D = 2\mathsf{m}gh - v(T - D). \tag{10}$$

Because the drag is unknown, we model it as

$$D = \hat{D} + \boldsymbol{\phi}^T(\boldsymbol{x})\boldsymbol{\Psi}$$

where  $\hat{D}$  is the aerodynamic model's estimate of drag,  $\phi(x)$  is a vector of known bounded basis functions, and  $\Psi$  is a vector of unknown parameters.

Let the Lyapunov function be

$$V = \frac{1}{2}\Gamma_T \tilde{E}_T^2 + \frac{1}{2}\Gamma_D \tilde{E}_D^2 + \tilde{\boldsymbol{\Psi}}^T \tilde{\boldsymbol{\Psi}}, \tag{11}$$

where  $\tilde{E} \triangleq E^d - E$  is the error,  $(\cdot)^d$  denotes the desired value,  $\Gamma_T > 0$ , and  $\Gamma_D > 0$ . Taking the derivative of (11) and using  $\dot{h} = v \sin(\gamma)$ , (9), and (10) gives

$$\dot{V} = \Gamma_T \tilde{E}_T \left( \dot{E}_T^d - \left( T - \hat{D} - \boldsymbol{\phi}^T \boldsymbol{\Psi} \right) v \right) 
+ \Gamma_D \tilde{E}_D \left( \dot{E}_D^d - 2 mgv \sin \gamma + \left( T - \hat{D} - \boldsymbol{\phi}^T \boldsymbol{\Psi} \right) v \right) 
+ \tilde{\boldsymbol{\Psi}}^T \dot{\hat{\boldsymbol{\Psi}}},$$
(12)

where we assume that the unknown parameters are slowly varying. If the thrust is chosen to be

$$T^c = \hat{D} + \boldsymbol{\phi}^T \hat{\boldsymbol{\Psi}} + \frac{\dot{E}_T^d}{v} + k_T \frac{\tilde{E}_T}{v}, \tag{13}$$

where  $k_T > 0$ , then (12) becomes

$$\dot{V} = -k_T \Gamma_T \tilde{E}_T^2 
+ \Gamma_D \tilde{E}_D \Big( \dot{E}_D^d - 2 \mathsf{m} g v \sin \gamma + \dot{E}_T^d + k_T \tilde{E}_T \Big) 
+ \tilde{\Psi}^T \Big( \dot{\hat{\Psi}} + \Big( -\Gamma_T \tilde{E}_T + \Gamma_D \tilde{E}_D \Big) \phi v \Big).$$
(14)

If the flight path angle is chosen to be

$$\gamma^c = \sin^{-1}\left(\frac{\dot{E}_D^d + \dot{E}_T^d + k_T\tilde{E}_T + k_D\tilde{E}_D}{2mgv}\right), \quad (15)$$

where  $k_D > 0$ , then (14) becomes

$$\dot{V} = -k_T \Gamma_T \tilde{E}_T^2 - k_D \Gamma_D \tilde{E}_D^2 
+ \tilde{\Psi}^T \left( \dot{\tilde{\Psi}} + \left( -\Gamma_T \tilde{E}_T + \Gamma_D \tilde{E}_D \right) \phi v \right).$$
(16)

Note that (15) can be simplified, by using  $\dot{h} = \frac{\dot{E}_D + \dot{E}_T}{2 \text{mg}}$ , to

$$\gamma^c = \sin^{-1}\left(\frac{\dot{h}^d}{v} + \frac{1}{2mgv}\left(k_T\tilde{E}_T + k_D\tilde{E}_D\right)\right). \tag{17}$$

and that the commanded flight path angle can be converted to a pitch command by  $\theta^c = \gamma^c + \alpha$ . The parameter adaption rate is chosen to be

$$\dot{\hat{\Psi}} = \left(\Gamma_T \tilde{E}_T - \Gamma_D \tilde{E}_D\right) \phi v \tag{18}$$

so (16) becomes

$$\dot{V} = -k_T \Gamma_T \tilde{E}_T^2 - k_D \Gamma_D \tilde{E}_D^2 \tag{19}$$

which is negative semi-definite. By realizing that V is lower bounded and assuming that the desired altitude and airspeed are bounded we know that  $V(t) \leq V(t_0) \ \forall \ t \geq t_0$ . This implies that  $\tilde{E}_T$ ,  $\tilde{E}_D$ , and  $\tilde{\Psi}$  are bounded. The derivative of (19) is

$$\ddot{V} = -2k_T \Gamma_T \tilde{E}_T \dot{\tilde{E}}_T - 2k_D \Gamma_D \tilde{E}_D \dot{\tilde{E}}_D.$$
 (20)

Inserting the error rates and the commanded thrust and

flight path angle into (20) provides

$$\ddot{V} = -2k_T^2 \Gamma_T \tilde{E}_T^2 - 2k_D^2 \Gamma_D \tilde{E}_D^2 + 2\tilde{\Psi}^T \left( k_T \Gamma_T \tilde{E}_T - k_D \Gamma_D \tilde{E}_D \right) \phi v$$

which is bounded and finite. From this we can conclude, using Barbalat's Lemma, that  $\dot{V} \to 0$  which implies  $\tilde{E}_T \to 0$  and  $\tilde{E}_D \to 0$ .

There are several interesting things about the non-linear TECS controller given by (13) and (17). If we assume that the drag is known and the thrust has been trimmed then the commanded change in thrust is

$$\Delta T = \frac{\dot{E}_T^d}{v} + k_T \frac{\tilde{E}_T}{v}$$

which can be thought of as a PI controller where the proportional gain is 1 and the integral gain is  $k_T$ . Other than the fixed proportional gain, this is exactly the original TECS thrust control scheme (6).

Using the energy definitions (1) and (3) (17) becomes

$$\sin(\gamma^c) = \frac{\dot{h^d}}{v} + \frac{1}{2mqv} \Big( (k_T - k_D) \tilde{E}_K + (k_T + k_D) \tilde{E}_P \Big),$$

where

$$\tilde{E}_K \triangleq \frac{1}{2} \mathbf{m} \left( \left( v^d \right)^2 - v^2 \right),$$

$$\tilde{E}_P \triangleq \mathbf{m} g \left( h^d - h \right).$$

This controller has three distinct behaviors depending on the relative size of  $k_T$  and  $k_D$ . If  $0 > k_D > k_T$  then

$$\sin(\gamma^c) = \frac{\dot{h}^d}{v} + \frac{1}{2mgv} \left( k_1 \tilde{E}_K + k_2 \tilde{E}_P \right),$$

where  $k_1 = |k_T - k_D|$  and  $k_2 = k_T + k_D$ . If  $\dot{v}^d > 0$  then  $\gamma^c$  will increase which is the exact opposite of the desired behavior. If  $k_D = k_T$  then

$$\sin(\gamma^c) = \frac{\dot{h^d}}{v} + \frac{k_2 \tilde{h}}{v}.$$

In this case,  $\gamma^c$  only depends on the desired climb rate and the altitude error and cannot take into account the coupling between the altitude and airspeed. Finally, if  $0 > k_T > k_D$  then

$$\sin(\gamma^c) = \frac{h^d}{v} + \frac{1}{2mgv} \left( -k_1 \tilde{E}_K + k_2 \tilde{E}_P \right).$$

This case gives us the desired behavior in that  $\gamma^c$  will increase if  $\dot{v}^d < 0$  or  $\dot{h}^c > 0$ .

It is interesting to note that while the nonlinear thrust controller matches the original thrust controller the flight path angle controllers do not match. The nonlinear controller uses the  $\dot{h}^d$  instead of  $\dot{E}_D^d$  and

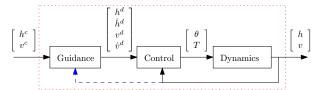


Fig. 1: Overall structure for the nonlinear non-adaptive TECS controller. The guidance block, which generates the desired altitude, climb rate, velocity, and acceleration, can have feedback represented by the dashed blue line. The entire system, enclosed by the dotted red line, is linearized.

it does not use the true energy difference. Instead, it uses a weighted energy difference where the kinetic and potential energy differences are weighted differently. It is impossible to force this controller to use the true energy difference because it would require  $k_D=0$ . In addition, the potential energy error,  $k_2\in(k_T,2k_T)$ , is always weighted heavier than the kinetic energy error,  $k_1\in(0,k_T)$ .

Unlike the linear TECS controller, the nonlinear controller requires a desired airspeed, acceleration, altitude and climb rate. There are two different ways that the commands can be generated. The first is by using a reference model for the thrust and altitude where the desired climb rate and acceleration are generated by

$$\dot{h}^d = k_h (h^c - h^d), \tag{21}$$

$$\dot{v}^d = k_v (v^c - v^d). \tag{22}$$

The desired altitude and airspeed are computed by

$$h^{d} = \int_{t_0}^{t} \dot{h}^{d}(t)dt + h^{d}(t_0),$$
  
$$v^{d} = \int_{t_0}^{t} \dot{v}^{d}(t)dt + v^{d}(t_0),$$

where the desired values are initialized as  $h^d(t_0) = h(t_0)$  and  $v^d(t_0) = v(t_0)$  .

The second approach uses the current state to generate the desired rates. The desired climb rate and acceleration are generated by (7) and (8) respectively.

One interesting feature of the second approach is that  $h^c$  does not need to equal  $h^d$  in steady state. The dynamic response of the desired values can cause the nonlinear adaptive TECS control algorithm to go unstable, however, it removes the steady state error for a step input when the adaptive component is not used. We will show this for a linearized version of the system.

Consider the system shown in Figure 1 where the

dynamics are [1]

$$\dot{h} = v \sin(\gamma),$$
  

$$\dot{v} = -g \sin(\gamma) - D + T,$$

the controller is (13) with  $\phi=0$  and (17), and the guidance block can have feedback, given by (7) and (8), or no feedback, given by (21) and (22). Note that this ignores the pitch and thrust dynamics. These systems, with and without feedback, were linearized about the operating point h=0 and v=15m/s with  $k_T=0.2$ ,  $k_D=0.25$ ,  $k_h=0.2$ ,  $k_v=0.2$ , and  $\hat{D}=0$ . Both systems are stable and the eigenvalues of the state space matrix A are

$$eig(A_{noFeedback}) = [-0.22, -0.12, -0.2, -0.2],$$
  
 $eig(A_{feedback}) = [-0.4, -0.23, -0.2, -0.02].$ 

The final value theorem states that

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s).$$

If we define

$$E(s) \triangleq R(s) - Y(s) = R(s) - C(s\mathbf{I}_4 - A)^{-1}BR(s),$$

where R(s)=1/s for a step input, then H(S)=sE(s) for these systems are

$$H_{noFeedback}(s) = \begin{bmatrix} \frac{s(s+0.34s+.0.025)}{s^3+.54s^2+.094s+.0052} & \frac{-0.0043s+.0061}{s^3+0.5383s^2+0.0933s+0.0052} \\ \frac{-s(0.024s+0.0054)}{s^3+.54s^2+.094s+.0052} & \frac{s^3+0.5383s^2+0.017s+0.0036)}{s^3+0.5383s^2+0.0933s+0.0052} \end{bmatrix}$$

$$H_{feedback}(s) =$$

$$\begin{bmatrix} \frac{s(s^3 + 0.65s^2 + 0.1s + .0016)}{d(s)} & \frac{s(-0.0043s + .0006)}{d(s)} \\ \frac{2.88s^2(-0.024s - 0.0054}{d(s)} & \frac{s(s^3 + 0.54*s^2 + 0.095s + 0.0052)}{d(s)} \end{bmatrix},$$

where

$$d(s) = s^4 + 0.85s^3 + 0.23s^2 + 0.022s + .0003.$$

Note that the rows correspond to the outputs h and v and the columns correspond to the inputs  $h^c$  and  $v^c$ . Taking the limit as  $s \to 0$  shows that the no feedback case has a steady state error in both altitude and airspeed due to the airspeed command and that adding the feedback removes the steady state error completely.

# V. OTHER CONTROLLERS

Two other controllers were implemented in order to provide a better comparison. The first controller uses two PID loops on the total energy and the energy difference instead of the total energy rate and energy distribution rate that the original TECS controller uses. These control loops are given by

$$T^{c} = k_{p,t}\tilde{E}_{T} - k_{d,t}\dot{\tilde{E}}_{T} + k_{i,t}\int_{t_{0}}^{t}\tilde{E}_{T},$$
  
$$\theta^{c} = k_{p,\theta}\tilde{E}_{D} - k_{d,\theta}\dot{\tilde{E}}_{D} + k_{i,\theta}\int_{t_{0}}^{t}\tilde{E}_{D}.$$

The second controller is a standard decoupled successive loop closure control scheme where the thrust is controlled by a PI loop based on the airspeed error and the pitch is controlled by a PI loop based on the altitude error [1]. The commanded thrust and pitch are

$$T^{c} = k_{p,t}\tilde{v} + k_{i,t} \int_{t_{0}}^{t} \tilde{v},$$
  
$$\theta^{c} = k_{p,\theta}\tilde{h} + k_{i,\theta} \int_{t_{0}}^{t} \tilde{h}.$$

#### VI. DYNAMICS

These control algorithms were tested using a simple two dimension flight simulator for a small Zagi airplane. The dynamic equations are

$$\begin{split} \dot{h} &= \sin(\theta) u - \cos(\theta) w, \\ \dot{u} &= -q w + \frac{F_x}{\mathsf{m}}, \\ \dot{w} &= q u + \frac{F_z}{\mathsf{m}}, \\ \dot{\theta} &= q, \end{split}$$

where u and w are the forward and vertical velocities in the body frame and q is the pitch rate. The horizontal and vertical forces are

$$F_x = -mg\sin(\theta) - \cos(\alpha)F_D + \sin(\alpha)F_L + T,$$
  

$$F_z = mg\sin(\theta) - \sin(\alpha)F_D - \cos(\alpha)F_L,$$

where the lift and drag forces are given by

$$F_D = \frac{1}{2}\rho Sv^2 \left( C_{D0} + C_{D\alpha}\alpha + \frac{c}{2v} C_{Dq} \right),$$
  
$$F_L = \frac{1}{2}\rho Sv^2 \left( C_{L0} + C_{L\alpha}\alpha + \frac{c}{2v} C_{Lq} \right).$$

The angle of attack and airspeed are

$$\alpha = \tan^{-1}\left(\frac{w}{u}\right) \text{ and } v = \sqrt{u^2 + w^2}$$
 (23)

respectively. The aerodynamic coefficients and other parameters are shown in Table I. Instead of adding lowlevel pitch and thrust control loops, the pitch and thrust responses are modeled as second order systems by

$$\dot{q} = -2\zeta_{\theta}\omega_{n\theta}q + \omega_{n\theta}(\theta^c - \theta)$$
$$\ddot{T} = -2\zeta_{T}\omega_{nT}\dot{T} + \omega_{nT}(T^c - T).$$

TABLE I: The Zagi airframe parameters from [1]

Parameter	Value	Parameter	Value
$C_{D0}$	0.01631	$C_{L0}$	0.09167
$C_{D\alpha}$	0.2108	$C_{L\alpha}$	3.5016
$C_{Dq}$	0	$C_{Lq}$	2.8932
m	1.56 kg	S	$0.2589 \ m^2$
c	0.3302 m		

where  $\zeta_{n\theta} = \zeta_{nT} = 0.707$  and  $\omega_{n\theta} = \omega_{nT} = 5$ .

# VII. SIMULATION RESULTS

Each control algorithm, original TECS, nonlinear without adaptive but with guidance feedback, nonlinear adaptive, decoupled, and TECS PID, was run for a variety of scenarios some of which are shown in Figure 2. Each controller was tuned such that they had comparable rise times. Note that the vehicle's initial states were slightly out of trim which explains the initial drop in the altitude and airspeed. The estimated drag was  $\hat{D}=0.8D$  for the nonlinear and adaptive TECS controllers.

Figure 2a shows the response for a positive altitude step command while the desired velocity is constant and demonstrates the main weakness of the traditional decoupled control approach. Notice that when the desired altitude increases, the airspeed for the decoupled controller immediately starts to decrease. This issue is also shown in Figures 2b and 2d.

The original TECS controller does not have this problem because it accounts for the coupling in the dynamics. However, it tends to have the largest overshoot and, as shown in Figures 2a and 2c, the state that is commanded to be held constant oscillates while the other variable undergoes the step command. The oscillation and overshoot can be reduced by using the TECS PID based on the energy instead of the PI controllers based on the energy rate.

The adaptive TECS controller was able to correct for its incorrect drag model but it has a minor oscillation in its airspeed response. However, this oscillation is significantly less than the original TECS controller's oscillation. This oscillation can be removed completely by using the nonlinear controller without the adaptive element but with the guidance feedback.

#### VIII. CONCLUSIONS

In this paper we derived a nonlinear altitude and airspeed controller for a fixed wing aircraft based on TECS principles. The adaptive variant of this controller was shown to be able to track a desired airspeed and altitude using Lyapunov stability arguments and the non-adaptive variant was shown to have zero steady state

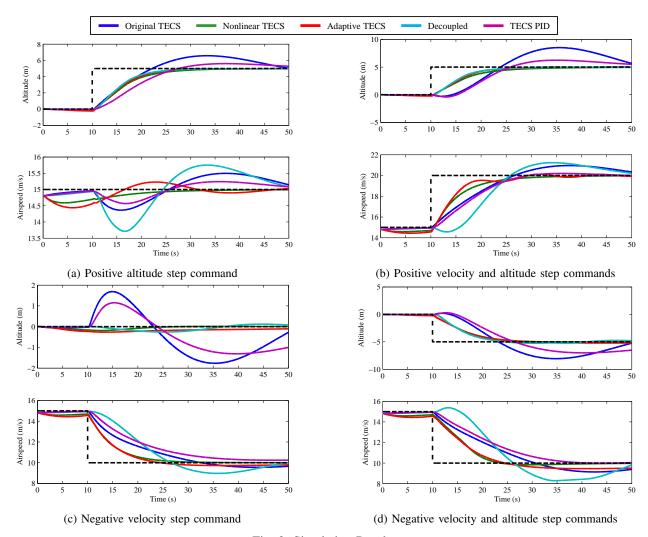


Fig. 2: Simulation Results

error for a step input based on the final value theorem. Simulation results have shown that both variants handle the coupling between the altitude and airspeed dynamics and have a better response than the original TECS control scheme.

There are several aspects of this research that can be continued. First, it would be useful to include the pitch and thrust control loops instead of a model of the responses. Second, the nonlinear controllers have only been tested in simulation for a small aircraft. More simulation tests should be performed on a variety of airframes and extensive flight tests should be conducted.

# IX. ACKNOWLEDGMENTS

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