Week 4 Homework

```
library('GGally')
library('tidyverse')
library('randomForest')
library('rpart')


#Crime data file
crimeData <- read.table("http://www.statsci.org/data/general/uscrime.txt", header = TRUE)

#Set seed for reproducibility
set.seed(156)
```

Question 1

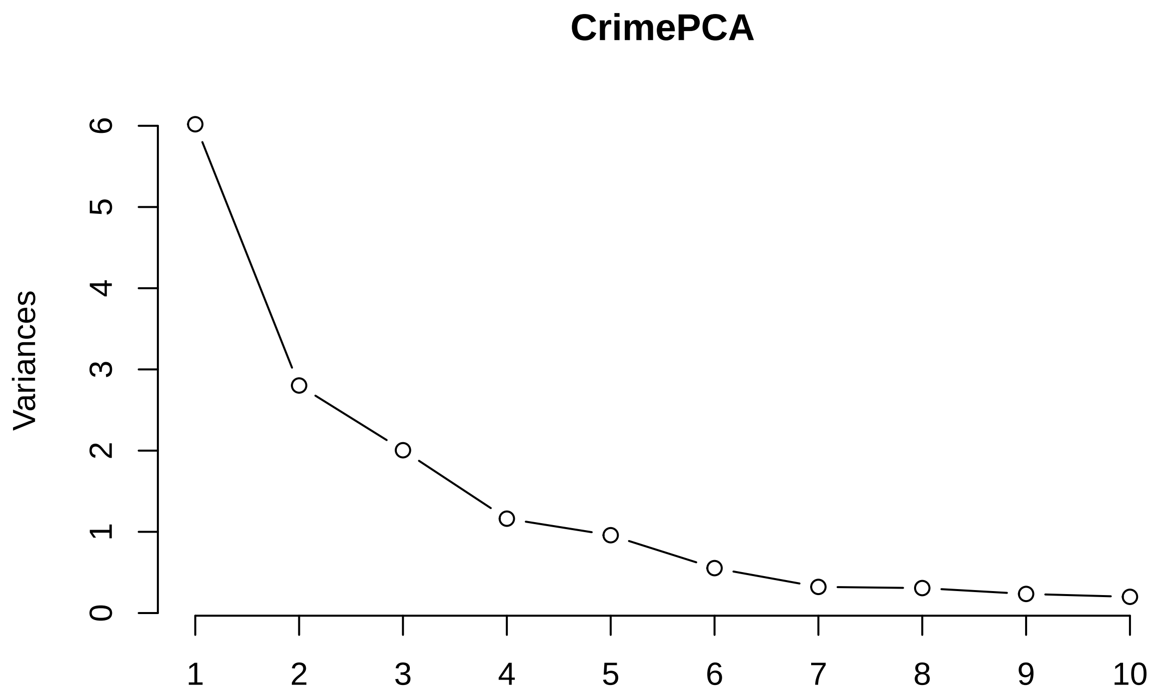
apply Principal Component Analysis and then create a regression model using the first 4 principal components. Specify your new model in terms of the original variables (not the principal components), and compare its quality to that of your solution to Homework 3 Question 4.

The results I obtained indicate that my original model from Week 3 was a more accurate model.

```
#Performing PCA on original crime dataset, removing response variable.
CrimePCA <- prcomp(crimeData[,-16], center = TRUE, scale. = TRUE)

#The first four principal components account for 79% of the variance in the data. This can be seen with
summary(CrimePCA)
```

```
## Importance of components:
##                           PC1    PC2    PC3     PC4     PC5     PC6
## Standard deviation     2.4534 1.6739 1.4160 1.07806 0.97893 0.74377
## Proportion of Variance 0.4013 0.1868 0.1337 0.07748 0.06389 0.03688
## Cumulative Proportion  0.4013 0.5880 0.7217 0.79920 0.86308 0.89996
##                            PC7     PC8     PC9    PC10    PC11    PC12
## Standard deviation     0.56729 0.55444 0.48493 0.44708 0.41915 0.35804
## Proportion of Variance 0.02145 0.02049 0.01568 0.01333 0.01171 0.00855
## Cumulative Proportion  0.92142 0.94191 0.95759 0.97091 0.98263 0.99117
##                           PC13   PC14    PC15
## Standard deviation     0.26333 0.2418 0.06793
## Proportion of Variance 0.00462 0.0039 0.00031
## Cumulative Proportion  0.99579 0.9997 1.00000
```

```
screeplot(CrimePCA, type = "lines")
```




```r
#rotation matrix of first 4 Principal Components
CrimePCA$rotation[,1:4]
```

```
##                PC1         PC2           PC3         PC4
## M      -0.30371194  0.06280357  0.1724199946 -0.02035537
## So     -0.33088129 -0.15837219  0.0155433104  0.29247181
## Ed      0.33962148  0.21461152  0.0677396249  0.07974375
## Po1     0.30863412 -0.26981761  0.0506458161  0.33325059
## Po2     0.31099285 -0.26396300  0.0530651173  0.35192809
## LF      0.17617757  0.31943042  0.2715301768 -0.14326529
## M.F     0.11638221  0.39434428 -0.2031621598  0.01048029
## Pop     0.11307836 -0.46723456  0.0770210971 -0.03210513
## NW     -0.29358647 -0.22801119  0.0788156621  0.23925971
## U1      0.04050137  0.00807439 -0.6590290980 -0.18279096
## U2      0.01812228 -0.27971336 -0.5785006293 -0.06889312
## Wealth  0.37970331 -0.07718862  0.0100647664  0.11781752
## Ineq   -0.36579778 -0.02752240 -0.0002944563 -0.08066612
## Prob   -0.25888661  0.15831708 -0.1176726436  0.49303389
## Time   -0.02062867 -0.38014836  0.2235664632 -0.54059002
```

```r
#Checking to ensure that principal components are orthogonal - perfect correlation across the diagonal
cor(CrimePCA$x[,1:4])
```

```
##               PC1           PC2           PC3          PC4
## PC1  1.000000e+00 -1.273307e-16 -1.825724e-16 2.298165e-16
## PC2 -1.273307e-16  1.000000e+00 -5.694249e-16 3.269637e-16
## PC3 -1.825724e-16 -5.694249e-16  1.000000e+00 1.177395e-16
```

```
## PC4  2.298165e-16  3.269637e-16  1.177395e-16 1.000000e+00
```

```r
# creating dataframe with principal components and response variable
CrimePCAData <- cbind(crimeData[,16],data.frame(CrimePCA$x[,1:4]))
colnames(CrimePCAData)[1] <- 'Crime'

#creating linear model using 1st 4 principal components - R squared value is only 0.3. PC3 and PC4 seem
CrimePCA.lm <- lm(Crime ~., data = CrimePCAData)
summary(CrimePCA.lm)
```

```
##
## Call:
## lm(formula = Crime ~ ., data = CrimePCAData)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -557.76 -210.91  -29.08  197.26  810.35
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   905.09      49.07  18.443  < 2e-16 ***
## PC1            65.22      20.22   3.225  0.00244 **
## PC2           -70.08      29.63  -2.365  0.02273 *
## PC3            25.19      35.03   0.719  0.47602
## PC4            69.45      46.01   1.509  0.13872
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 336.4 on 42 degrees of freedom
## Multiple R-squared:  0.3091, Adjusted R-squared:  0.2433
## F-statistic: 4.698 on 4 and 42 DF,  p-value: 0.003178
```

```r
#converting rotation matrix to model coefficients
betas <- CrimePCA$rotation[,1:4] %*% CrimePCA.lm$coefficients[-1]
colnames(betas)[1] <- 'coefficients'
betas
```

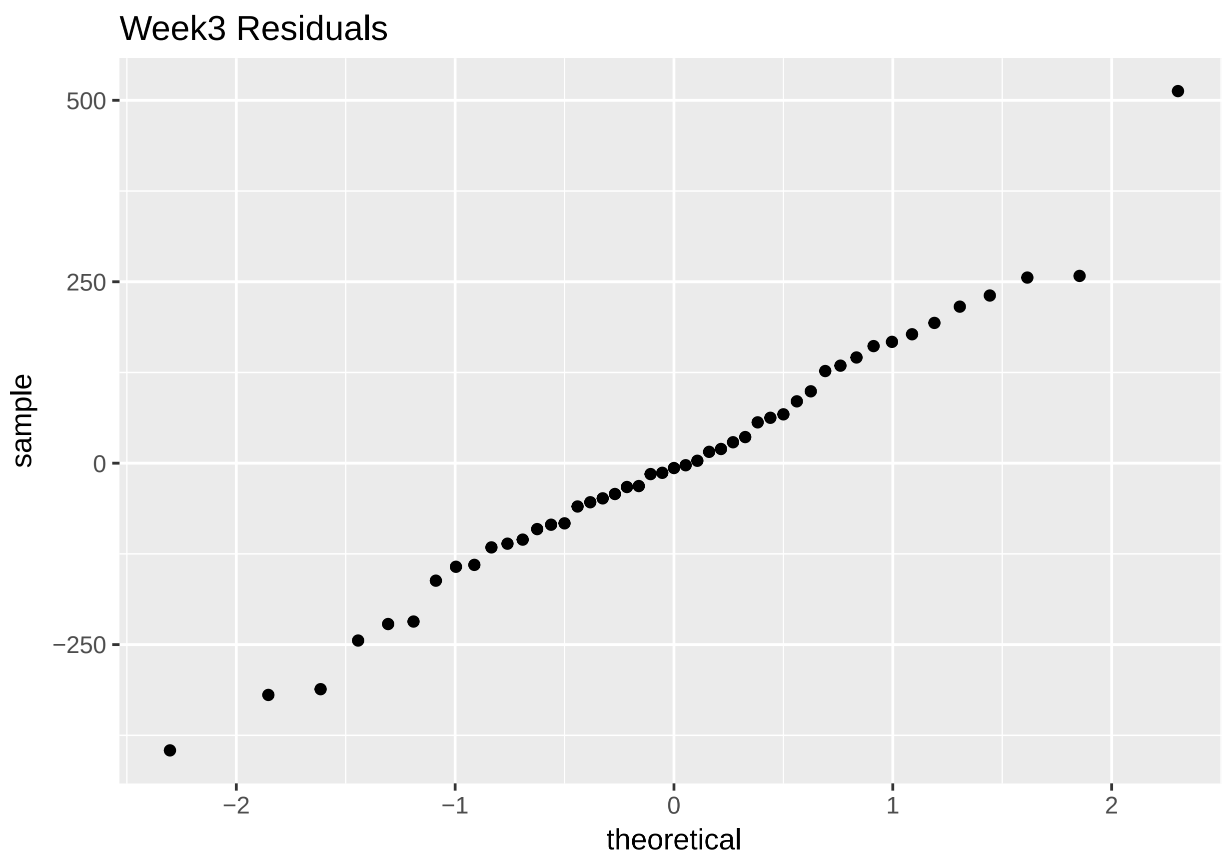
```
##        coefficients
## M        -21.277963
## So        10.223091
## Ed        14.352610
## Po1       63.456426
## Po2       64.557974
## LF       -14.005349
## M.F      -24.437572
## Pop       39.830667
## NW        15.434545
## U1       -27.222281
## U2         1.425902
## Wealth    38.607855
## Ineq     -27.536348
## Prob       3.295707
## Time      -6.612616
```

```r
#original linear model from Week 3
crimeModel <- lm(Crime ~ ., data = crimeData)
```

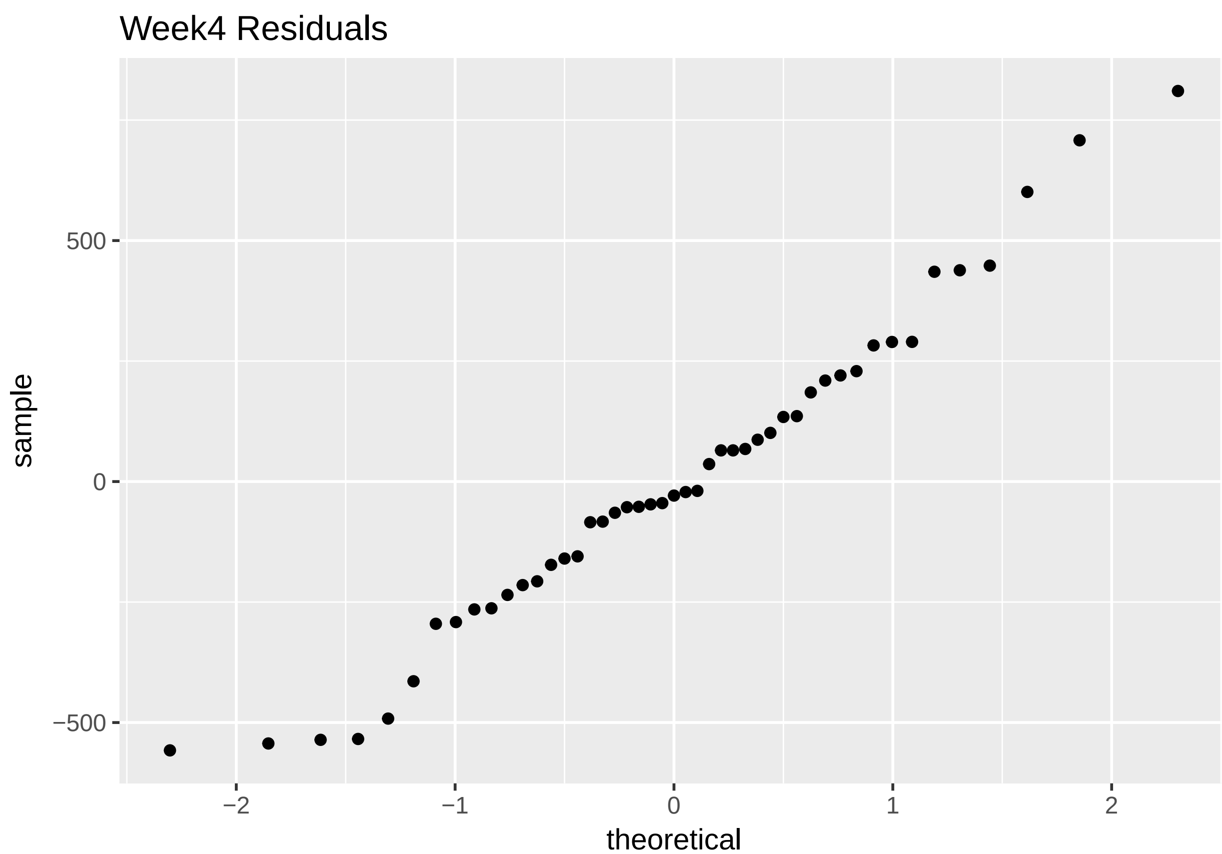
```r
#using ANOVA to compare the PCA model vs the original model. This shows the P value being 7.857e-06 whi
anova(CrimePCA.lm,crimeModel)
```

```
## Analysis of Variance Table
##
## Model 1: Crime ~ PC1 + PC2 + PC3 + PC4
## Model 2: Crime ~ M + So + Ed + Po1 + Po2 + LF + M.F + Pop + NW + U1 +
##     U2 + Wealth + Ineq + Prob + Time
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1     42 4753950
## 2     31 1354946 11   3399004 7.0697 7.857e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
crimeData$predictedWeek3 <-  predict(crimeModel)
crimeData$predictedWeek4 <- predict(CrimePCA.lm)

# Obtain  residual values
crimeData$residualsWeek3 <- residuals(crimeModel)
crimeData$residualsWeek4 <- residuals(CrimePCA.lm)

#Creating residual df for plotitng
modelResiduals <- data.frame(data=(cbind(residuals(crimeModel),residuals(CrimePCA.lm))))
colnames(modelResiduals) <- c('Week3', 'Week4')

#qqplots of the residuals of both models show they are fairly normally distributed.
modelResiduals %>%
  ggplot(aes(sample=modelResiduals$Week3)) +
  stat_qq() +
  labs(title = "Week3 Residuals")
```




```
modelResiduals %>%
  ggplot(aes(sample=modelResiduals$Week4)) +
  stat_qq() +
  labs(title = "Week4 Residuals")
```



Question 2

Using the same crime data set as in Homework 3 Question 4, find the best model you can using (a) a regression tree model, and (b) a random forest model. For each model, describe one or two qualitative takeaways you get from analyzing the results

Rpart Insights: The variable importance from the model is telling me that Po1 and Po2 are both very important, but very similar. Meaning only one is actually needed. Wealth and Ineq are also very similar but importance. Prob and M are the next two most importance variables. RPart chose very similar variables as most important compared to randomForest.

Random Forest Insights: The plot showing MSE tells me that Po1 and Po2 are very similar in importance so only one is really needed in the model. NW is the next most important variable followed by Prob, then Wealth. Node purity also tells me that Po1 and Po2 are very similar in importance, followed by Prob, and Wealth and NW are very similar, followed by Pop. Both of these show that most of the variance can be explained with Po1, Prob, Wealth, and NW. It also shows that there are many variables that are similar to each other in importance and we could throw some out if need be while still maintaining the same level of accuracy.

My random forest model is able to predict on average 73% of the actual crime values.

Another insight that I take from these models is that they show collinearity of predictors. This is a very useful tool which can help reduce dimensionality in models. The collinearity that is being shown in these models is also what PCA and a VIF test are showing as well. This is a universal takeaway from the tree family models that can be applied to modeling the same dataset with other methods.

```r
CrimeData.2 <- read.table("http://www.statsci.org/data/general/uscrime.txt", header = TRUE)
crimeDataRF <- CrimeData.2
crimeDataRpart <- CrimeData.2

#(b) regression tree model
Crime.Rpart <- rpart(Crime ~ ., data = crimeDataRpart, method = "anova")
summary(Crime.Rpart)
```

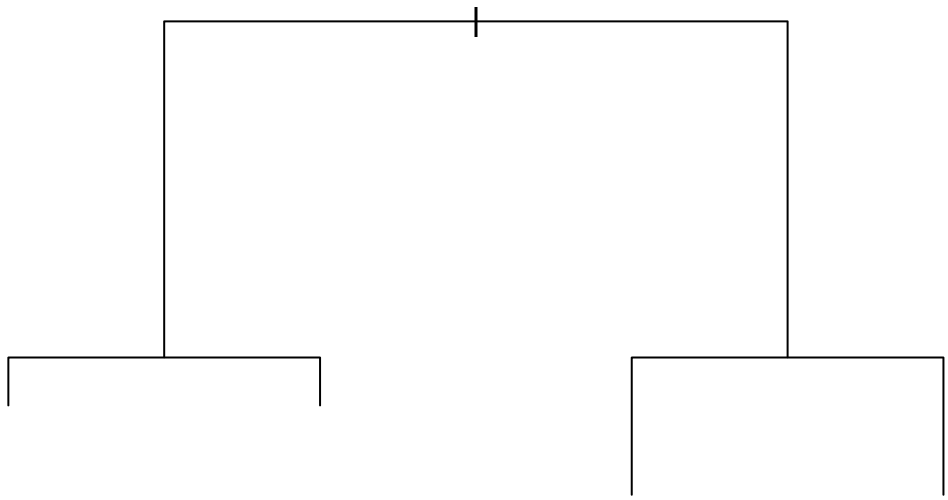
```
## Call:
## rpart(formula = Crime ~ ., data = crimeDataRpart, method = "anova")
##   n= 47
##
##           CP nsplit rel error    xerror      xstd
## 1 0.36296293      0 1.0000000 1.0085866 0.2534839
## 2 0.14814320      1 0.6370371 0.8648446 0.1874900
## 3 0.05173165      2 0.4888939 1.0148268 0.2337991
## 4 0.01000000      3 0.4371622 0.9076756 0.2252342
##
## Variable importance
##    Po1    Po2 Wealth   Ineq   Prob      M     NW    Pop   Time     Ed
##     17     17     11     11     10     10      9      5      4      4
##     LF     So
##      1      1
##
## Node number 1: 47 observations,    complexity param=0.3629629
##   mean=905.0851, MSE=146402.7
##   left son=2 (23 obs) right son=3 (24 obs)
##   Primary splits:
##       Po1    < 7.65      to the left,  improve=0.3629629, (0 missing)
##       Po2    < 7.2       to the left,  improve=0.3629629, (0 missing)
##       Prob   < 0.0418485 to the right, improve=0.3217700, (0 missing)
##       NW     < 7.65      to the left,  improve=0.2356621, (0 missing)
##       Wealth < 6240      to the left,  improve=0.2002403, (0 missing)
##   Surrogate splits:
##       Po2    < 7.2       to the left,  agree=1.000, adj=1.000, (0 split)
##       Wealth < 5330      to the left,  agree=0.830, adj=0.652, (0 split)
##       Prob   < 0.043598  to the right, agree=0.809, adj=0.609, (0 split)
##       M      < 13.25     to the right, agree=0.745, adj=0.478, (0 split)
##       Ineq   < 17.15     to the right, agree=0.745, adj=0.478, (0 split)
##
## Node number 2: 23 observations,    complexity param=0.05173165
##   mean=669.6087, MSE=33880.15
##   left son=4 (12 obs) right son=5 (11 obs)
##   Primary splits:
##       Pop < 22.5      to the left,  improve=0.4568043, (0 missing)
##       M   < 14.5      to the left,  improve=0.3931567, (0 missing)
##       NW  < 5.4       to the left,  improve=0.3184074, (0 missing)
##       Po1 < 5.75      to the left,  improve=0.2310098, (0 missing)
##       U1  < 0.093     to the right, improve=0.2119062, (0 missing)
##   Surrogate splits:
##       NW   < 5.4       to the left,  agree=0.826, adj=0.636, (0 split)
##       M    < 14.5      to the left,  agree=0.783, adj=0.545, (0 split)
##       Time < 22.30055  to the left,  agree=0.783, adj=0.545, (0 split)
##       So   < 0.5       to the left,  agree=0.739, adj=0.455, (0 split)
```

```
##       Ed   < 10.85     to the right, agree=0.739, adj=0.455, (0 split)
##
## Node number 3: 24 observations,    complexity param=0.1481432
##   mean=1130.75, MSE=150173.4
##   left son=6 (10 obs) right son=7 (14 obs)
##   Primary splits:
##       NW   < 7.65      to the left,  improve=0.2828293, (0 missing)
##       M    < 13.05     to the left,  improve=0.2714159, (0 missing)
##       Time < 21.9001   to the left,  improve=0.2060170, (0 missing)
##       M.F  < 99.2      to the left,  improve=0.1703438, (0 missing)
##       Po1  < 10.75     to the left,  improve=0.1659433, (0 missing)
##   Surrogate splits:
##       Ed   < 11.45     to the right, agree=0.750, adj=0.4, (0 split)
##       Ineq < 16.25     to the left,  agree=0.750, adj=0.4, (0 split)
##       Time < 21.9001   to the left,  agree=0.750, adj=0.4, (0 split)
##       Pop  < 30        to the left,  agree=0.708, adj=0.3, (0 split)
##       LF   < 0.5885    to the right, agree=0.667, adj=0.2, (0 split)
##
## Node number 4: 12 observations
##   mean=550.5, MSE=20317.58
##
## Node number 5: 11 observations
##   mean=799.5455, MSE=16315.52
##
## Node number 6: 10 observations
##   mean=886.9, MSE=55757.49
##
## Node number 7: 14 observations
##   mean=1304.929, MSE=144801.8
```

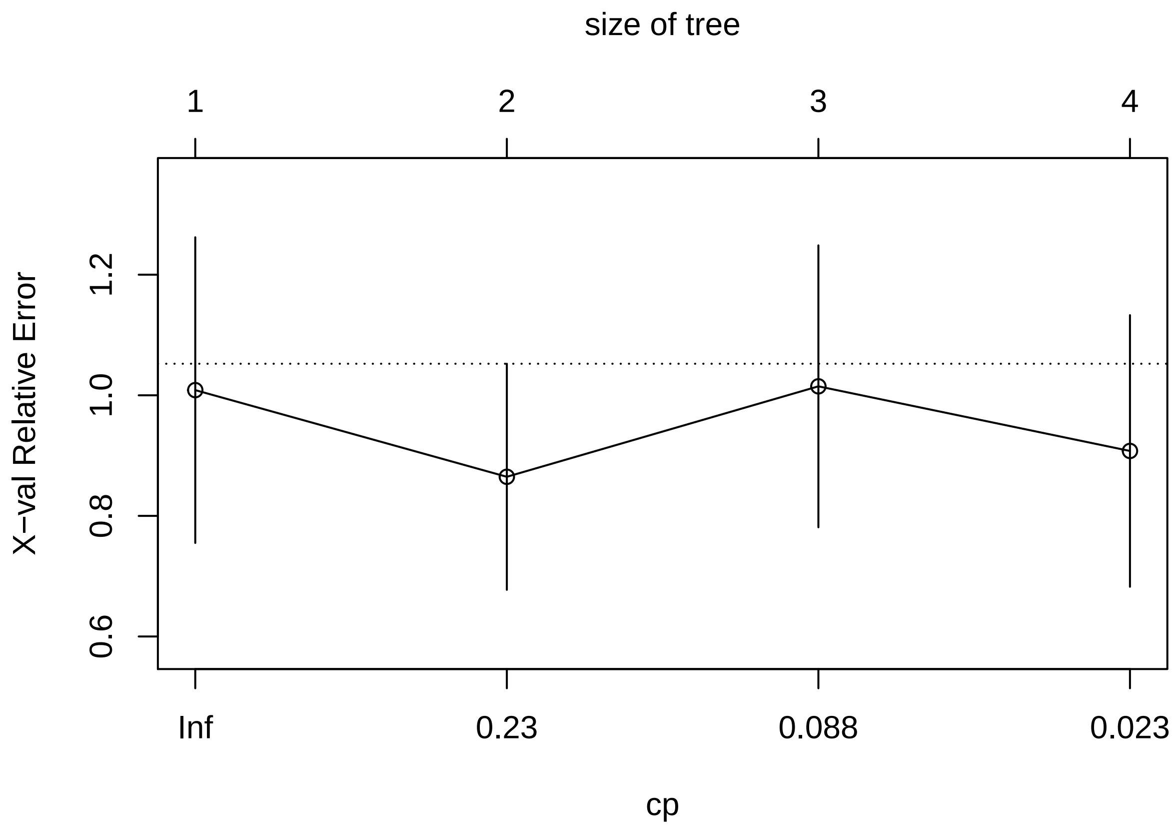
```
Crime.Rpart$variable.importance
```

```
##       Po1       Po2    Wealth      Ineq      Prob         M        NW
## 2497521.7 2497521.7 1628818.5 1602212.0 1520230.6 1388627.8 1245883.8
##       Pop      Time        Ed        LF        So
##  661770.6  601906.0  569545.9  203872.5  161800.8
```

```
plot(Crime.Rpart)
```

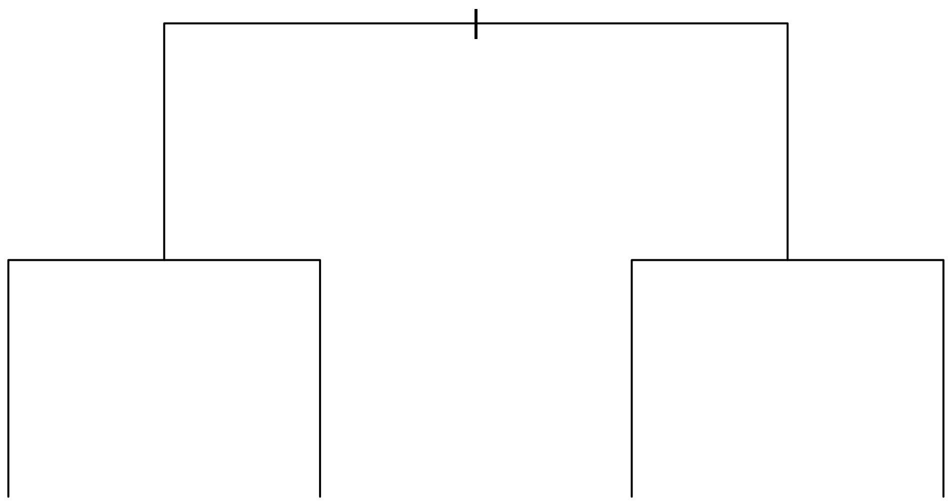


```
plotcp(Crime.Rpart)
```




```
plot(Crime.Rpart, uniform=TRUE,
    main="Regression Tree for Crime ")
```

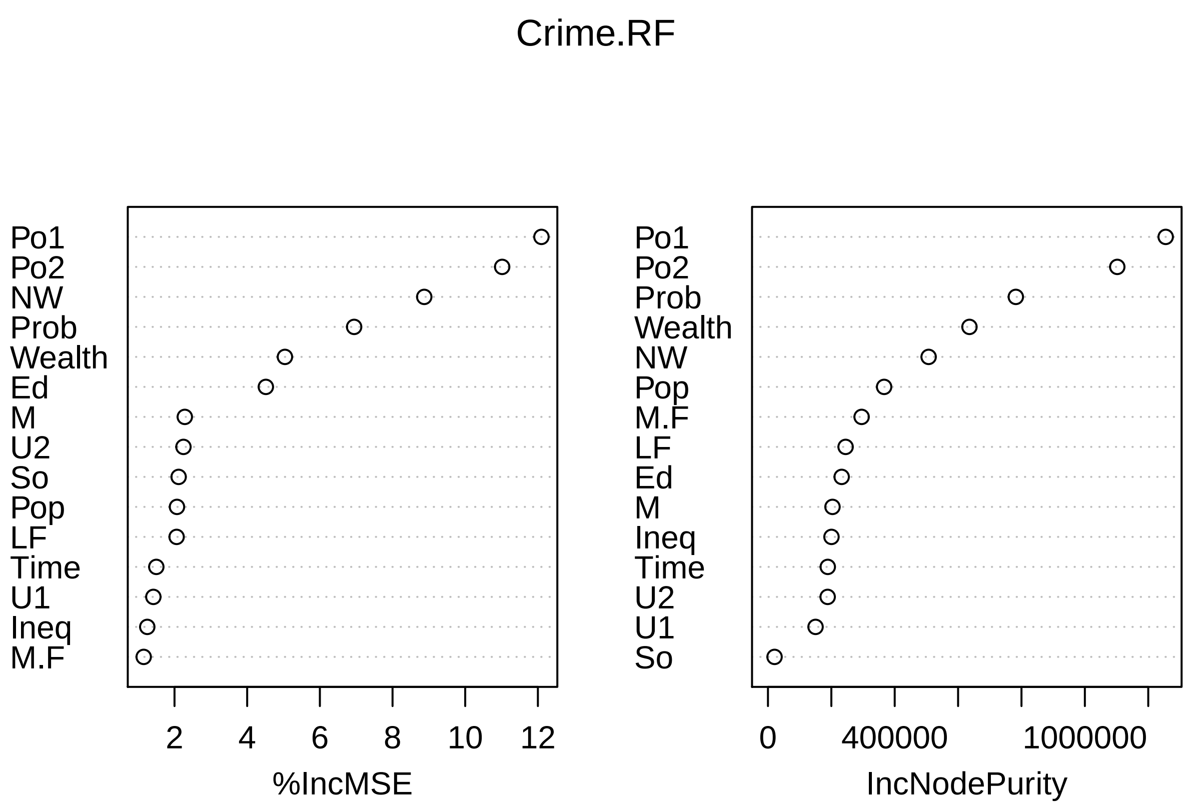
Regression Tree for Crime



```r
#------------------------------------------------------------------------------------
#(a) RandomForest Model
Crime.RF <- randomForest(Crime ~ ., data = crimeDataRpart, importance = TRUE)
Crime.RF
```

```
##
## Call:
##  randomForest(formula = Crime ~ ., data = crimeDataRpart, importance = TRUE)
##                Type of random forest: regression
##                      Number of trees: 500
## No. of variables tried at each split: 5
##
##           Mean of squared residuals: 84928.76
##                     % Var explained: 41.99
```

```r
#plotting the importance of variables in the Randomforest model.
varImpPlot(Crime.RF)
```




```
RFpred <- predict(Crime.RF)

crimeDataRF[,17] <- predict(Crime.RF)
colnames(crimeDataRF)[17] <- 'RFprediction'

#showing predictions and percent correct to prediction
crimeDataRF$predictionVariance <- abs(crimeDataRF$RFprediction - crimeDataRF$Crime)
crimeDataRF$predictionPrcntCorrect <- 1 - (round(crimeDataRF$predictionVariance / crimeDataRF$Crime,2))

#This tells me that my random Forest is on average 73% correct in predicting crime.
mean(crimeDataRF$predictionPrcntCorrect)
```

```
## [1] 0.7374468
```