Question 1

1.(a).

fast food 3   other. 6.   toy 2.

$_3P_3 \cdot {_6P_6} \cdot {_2P_2} = 3! \cdot 6! \cdot 2!$

$= 6 \times 720 \cdot 2 = 8640$

4

(b).

fast   toy.

pack them together.

So there are $11 \underbrace{-3+1}_{\text{fast.}} \underbrace{-2+1}_{\text{toy}} = 8$.

$_8P_8 \cdot {_3P_3} \cdot {_2P_2} = 8! \cdot 3! \cdot 2!$

$= 4.8384 \cdot 10^5$ (483840)

5

(c). ← 6 other

7 space to insert. 5 (2+3).

$_6P_6 \times {_7P_5} = 6! (7 \times 6 \times 5 \times 4 \times 3)$

$= 720 \cdot 2520 = 1814400$.

5 are not shown one after the other.

5

2.(a). HARMONIZE

9 letters.

$\binom{9}{5} = \frac{9!}{5!4!} = 126.$ 3

(b). HARMONIZE

vowel ✓ ✓ ✓ ✓ ✓

4 vowel, 5 not vowel.

$\binom{4}{1}\binom{5}{4} + \binom{4}{0}\binom{5}{5} = 4 \times 5 + 1 = 21.$ 3

0 vowel 1 vowel

c. (i). $\binom{4}{2}\binom{5}{3} \cdot {}_5P_5 = 6 \cdot 10 \cdot 5!$ 4

$= 7200$

2 vowels

(ii) $\binom{4}{2}\binom{5}{3} \cdot {}_3P_3 \cdot {}_4P_2 = 6 \times 10 \cdot 6 \cdot 12$ 6

$= 4320$

arrange 3.

insert 2 vowels

3 not vowels

4 space

12·60

(iii). $\binom{4}{2}\binom{5}{3} \cdot {}_4P_4 \cdot {}_2P_2 = 6 \times 10 \times 24 \cdot 2$ 2

$= 2880$

package 2 together

(now $5 - 2 + 1 = 4$ items) you can also see this (4320+2880=7200) (ii+iii=i).

Question 1

3.(a). let X = number of duck.

$X \sim H(18, 8, 5)$.

$P(X \geq 4) = P(X=4) + P(X=5)$

$$= \frac{\binom{8}{4}\binom{10}{1} + \binom{8}{5}\binom{10}{0}}{\binom{18}{5}} = \frac{700+56}{8568} = 0.0823.$$

4

(ii) $5 - X > X \Leftrightarrow 2X < 5 \Leftrightarrow X \leq 2.$

$\uparrow$ pig

$P = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$$= \frac{\binom{8}{0}\binom{10}{5} + \binom{8}{1}\binom{10}{4} + \binom{8}{2}\binom{10}{3}}{\binom{18}{5}}$$

$$= \frac{\cancel{2520+}\,252 + 1680 + 3360}{8568.} = \frac{5292}{8568} \approx 0.617.$$

You can also use

$P(X \leq 2) = 1 - P(X \geq 4) - P(X=3)$

$= 1 - 0.0823 - \frac{\binom{8}{3}\binom{10}{2}}{\binom{18}{5}}$

$\frac{2520}{8568} = 0.294$

$\approx 0.62.$

4

(b). X duck in group I.

$X \sim H(18, 8, 9)$.

(i). $P(X=4) = \frac{\binom{8}{4}\binom{10}{5}}{\binom{18}{9}} = \frac{70 \cdot 252}{48620} = 0.36$

4

(ii) $9 - X > X \Rightarrow 2X < 9 \Rightarrow X \leq 4.$

7

$$P(X \leq 4) = \frac{\binom{8}{0}\binom{10}{9} + \binom{8}{1}\binom{10}{8} + \binom{8}{2}\binom{10}{7} + \binom{8}{3}\binom{10}{6} + \binom{8}{4}\binom{10}{5}}{\binom{18}{9}} = \frac{10+360+3360+11760+176}{48620} = \frac{33130}{48620} = 0.68$$

I Question II III

2 white 3 black 4 w 1 b 3 w 4 b

4.

let. A is: Urn I was selected

B is ball is white.

$$P(B) = \frac{1}{3}\left(\frac{2}{2+3} + \frac{4}{4+1} + \frac{3}{3+4}\right)$$

rar I II III

$$= \frac{1}{3}\cdot\left(\frac{2}{5}+\frac{4}{5}+\frac{3}{7}\right) = 0.542.$$

$$" \frac{14+28+15}{35} = \frac{57}{35} = \frac{19}{35}.$$

$$P(AB) = \frac{1}{3}\cdot\frac{2}{2+3} = 0.133. = \frac{2}{15}$$

$$P(A|B) = \frac{P(AB)}{P(B)} \approx \frac{0.133}{0.542} = 0.246.$$

$$= \frac{2}{15}\cdot\frac{35}{19} = \frac{14}{57}$$

5(a). X : number of items are defectives

(i) $X \sim B(3, \frac{1}{3})$.

(i). $P(X=1) = \binom{3}{1} \cdot \frac{1}{3} \cdot (1-\frac{1}{3})^2 = 3 \cdot \frac{1}{3} \cdot (\frac{2}{3})^2 = \frac{4}{9}$

(ii). $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{3}{0}(1-\frac{1}{3})^3 = 1 - \frac{8}{27}$

$= \frac{19}{27} = 0.7037$

(b). X : number of malfunction.

$X \sim H(6, 2, 2)$.

$P(X=2) = \frac{\binom{2}{2}\binom{4}{0}}{\binom{6}{2}} = \frac{1}{15} = 0.067$.

3

4

4

8

,6 (1+6) (2+5) (3+4) (4+3) (5+2) (6+1)

6.

(a). $P = \frac{6}{6\times 6} = \frac{1}{6}$

total of 7 occurs in 1 tim

$X \sim B(n, \frac{1}{6})$.

3

(b). $X \sim B(24, \frac{1}{6})$.

(i) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \binom{24}{0}(1-\frac{1}{6})^{24} - \binom{24}{1}\frac{1}{6}\cdot(1-\frac{1}{6})^{23}$

4

$= 0.937$. 0.9270 O.K.

(ii) $P(15 \leq X \leq 20) = \binom{24}{15}(\frac{1}{6})^{15}(\frac{5}{6})^{9} + \binom{24}{16}(\frac{1}{6})^{16}(\frac{5}{6})^{8} + \binom{24}{17}(\frac{1}{6})^{17}(\frac{5}{6})^{7}$
$+ \binom{24}{18}(\frac{1}{6})^{18}(\frac{5}{6})^{6} + \binom{24}{19}(\frac{1}{6})^{19}(\frac{5}{6})^{5}$
$+ \binom{24}{20}(\frac{1}{6})^{20}(\frac{5}{6})^{4} = \frac{\sum_{i=15}^{20}\binom{24}{i}5^{24-i}}{6^{24}} = \frac{2.8703\cdot 10^{12}}{6^{24}} = 6.05\cdot 10^{-7}$

8

also since $np = 4 < 5$ we can use possion distribution

$X \sim P(4)$. $P(15 \leq X \leq 20) = \frac{1}{e^4}\left(\frac{4^{15}}{15!} + \frac{4^{16}}{16!} + \frac{4^{17}}{17!} + \frac{4^{18}}{18!} + \frac{4^{19}}{19!} + \frac{4^{20}}{20!}\right)$
$= \frac{1}{5.4576}(1.088\cdot 10^{-3})$
$\approx 1.99\cdot 10^{-4}$

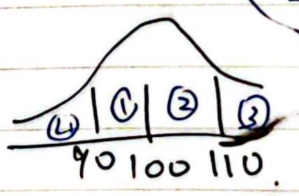
(c). $np = 100 > 5$, use normal distribution to approximate.

$6^2 = (100\cdot\frac{5}{6})$

$X \sim N(100, \frac{500}{6})$.

2

$6 \approx 9.129$



89.5 109.5

① = ② ①+② = 1 − 2·③
③ = ④

$P(90 \leq X < 110) = P(X \geq 90) - P(X \geq 110)$

$= 1 - 2P(X \geq 110)$ $\frac{1.015}{}$

$= 1 - 2P(Z \geq \frac{110-100}{9.129}) = 1 - 2\cdot 0.1379 = 0.724$

3

Bonus.:

(a).

$$c\sum_{x=1}^{\infty}\left(\frac{2}{3}\right)^x$$

$$= c\cdot\frac{2}{3}\cdot\sum_{x=0}^{\infty}\left(\frac{2}{3}\right)^x$$

$$= \frac{2}{3}c\cdot\frac{1-\left(\frac{2}{3}\right)^{\infty}}{1-\frac{2}{3}} = \frac{2}{3}c\cdot\frac{1}{\frac{1}{3}} = 2c = 1$$

$$\therefore c = \frac{1}{2}.$$

2

(b) ① $E(x) = \sum_{x=1}^{\infty}\frac{1}{2}\cdot x\cdot\left(\frac{2}{3}\right)^x = \frac{1}{2}\left(1\cdot\left(\frac{2}{3}\right) + 2\cdot\left(\frac{2}{3}\right)^2 + \cdots\right).$

② $\frac{2}{3}E(x) = \frac{1}{2}\sum_{x=1}^{\infty} x\cdot\left(\frac{2}{3}\right)^{x+1} = \frac{1}{2}\left(1\cdot\left(\frac{2}{3}\right)^2 + 2\cdot\left(\frac{2}{3}\right)^3\right).$

① − ② $= \frac{1}{3}E(x) = \frac{1}{2}\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right).$

$$= \frac{1}{2}\cdot\frac{2}{3}\cdot\underbrace{\frac{1-\left(\frac{2}{3}\right)^{\infty}}{1-\frac{2}{3}}}_{3} = 1.$$

5

$\therefore E(x) = 3.$

3

(c). $P(X\le n) = \sum_{x=1}^{n}\frac{1}{2}\cdot\left(\frac{2}{3}\right)^x = \frac{1}{3}\cdot\frac{1-\left(\frac{2}{3}\right)^n}{1-\frac{2}{3}} = 1-\left(\frac{2}{3}\right)^n$, $n \in N^+$

n is positive integer (N^+).

let $F(x)$ be the cdf of X.

$$F(x) = \begin{cases} 1-\left(\frac{2}{3}\right)^n & \text{if } n\le x<n+1 \\ 0 & \text{else when} \end{cases}$$