#ACORN

7. **Grading Scheme and Schedule**:

Rubrics: >= 40 for D/D+; >= 55 for C-/C/C+; >= 70 for B-/B/B+; >= 85 for A-/A/A+

Continuous Assessment (CA): 40% = 2 Assignments (10% +10%) + Midterm (20%)

Final Exam: 60%

Tentatively, we will use the following schedule:

Midterm (20%): 15:30--17:00, HKT, Oct 21, Monday, ~~N001~~ Details to be announced.

Assignment 1 (10%) due on Friday, Oct 25, 17:00;

Assignment 2 (10%) due on Friday, Nov 22, 17:00.

Assignments should be submitted online via Blackboard. Late submissions will NOT be marked.

The final exam will be organized according to University/Department instructions when they are available.

1. 最短路和最小生成树

Dijkstra $d, b(\text{从哪个转移}), V(\text{已完毕}), U(\text{没访问})$
Prim's Algorithm: $V(\text{在的集合}), F(\text{生成树}),$

Dij

Init:
V, d
Iteration： c, V, U, upd, d, b

Prim

V, F
每次加一个最个最小的

2. 最大流

x 是 flow， $|x|$ 是最大流的值
增广路 augmenting path. P
$R(P, x)$ 增广路边权的 min
Ford-Fulkerson algorithm
$O(|E| \cdot \hat{x})$
最大流=最小割 他也不证
最小鸽：V：包含 s 的那个集合

Augument: Residual .

3. Project Network (CPM and PERT)

CPM: Critical Path Method (确定)
PERT: Program Evaluation and Review Technique (概率)
project: 消耗时间和资源
Project Network:

- 边：任务 / 假（确保顺序
- 点：标记 全完成了...
  不能重边，一个活动最多一条边
  拓扑排a序

CPM

拓扑完递推 (dp)
Latest Time：倒着推一遍
每个点二元组 (st, ed)
slack: $ed - st$
critical path: slack = 0 (ed = st

event: node 事件
activity: $(a, b) \to l_b - e_a - t$ 把点对写出来
act = 0

4. 线性规划 (LP)

Def

- n 个变量：$x_{1\sim n}$
- m 个不等式（每行一个限制，系数矩阵 [technological coefficients] $a_{m\times n}$：矩阵 A）
- 最小要求 $b_{1\sim m}$ （向量 b）
  默认大于等于，非负整数（non-negativity constraints）
  $c_{1\sim n}$ 最优化的系数（向量 c）：

$$\min c^T x$$
$$s.t.\ Ax \geq b$$
$$x \geq 0$$

  用矩阵，不用扣分。
  合法解。feasible region 是 polytope 多面体
  bounded 有界 / unbounded 无界
  最短路 网络流都可以写成线性规划

2D：画个图就行
corner/vertex: 观察
facet：$(n-1)$ 维度。
vertex：face
Canonical form 规范形式：$\min, \geq$ 或 $\max, \leq$
标准形式：等于并且 $b \geq 0$。

典型形式转成标准形式：加松弛向量 slack variable
Free variables：大减小也能转化成无线值域
3 Graphical solutions for 2D LP

Lecture 5 线性规划对偶

shadow prices 经济
primal problem 原始问题
dual problem 对偶问题
Canonical Form
注意变成标准形式，b也要全正，要反转 ☆

$\min c^T x$
$Ax \geq b$
$x \geq 0$

对偶
$\max b^T y$
$A^T y \leq c$
$y \geq 0$

注意 swap(b, c) 然后转置换一下，大于号小于号换一下
标准形式对偶：

1. $\min, \geq$ 变成 $\leq$ 去掉限制 y

dual pair
等式 - 变量
尝试对偶：

1. 把每个等式给个变量；等式变成没限制，不等式变成 ≥ 0
2. 把每个变量凑个等式：≥ 0 变成不等式（取决于原来 min 那边就小于等于），无限制就变成等式

一个视角

对于任何 y, x
$b^Ty \le (Ax)^Ty = x^T(A^Ty) \le c^Tx$
有感觉取等最优。注意必须都是 ≥ 0！！！

记住：min 那边 c^Tx 上界，max 那边 b^Ty 手搓出恒等式就好了。两部放用到的分别是 $Ax \ge b$ 和 $A^Ty \le c$

弱对偶性 Weak Duality

即便可以无界什么的，永远是打的不一定相等

再来一遍
$\min c^Tx, Ax \ge b$ 和 $\max b^Ty, A^Ty \le c$
$c^T\overline{x} \ge (A^T\overline{y})^T\overline{x} = \overline{y}^T(A\overline{x}) \ge \overline{y}^Tb = b^T\overline{y}$
就代入两步，神奇。和（上面玩去哪一样的，注意加上划线，任意一组解
能找到相等的那就最优解（显然

最优解无限(无界) - 没解了
但没解不能说另一个无界。。

$Ax \ge b, A^Ty \le c$

per kilo 就有限制

polytope 不封闭也是对的

Fundamental Theorem of Duality

- 要么两个都是有有能求出来最优解
- 一个无界 - 一个不可行
- 都不可行

强对偶性

一个问题有有限的最优解或者俩都可以可行的，就是强对偶性，可以取等

Strong Duality

		Dual		
		Unbounded Obj.	Bounded Opt.	Infeasible
Primal	Unbounded Obj.	×	×	✓
	Bounded Opt.	×	✓	×
	Infeasible	✓	×	✓

- Legend:
  - Unbounded Obj.: The objective value can go unbounded.
  - Bounded Opt.: The optimal value is attained by a (finite) optimal solution.
  - Infeasible: The feasible region is empty.

Kuhn-Tucker (optimality) conditions

都可行切相等 充分必要（也叫 Karush-Kuhn-Tucker (KKT) conditions.）

互补松弛（③值一样）

Another Formulation of The Complementary Slackness

Let $$v^* = c - A^T y^*$$

Then the complementary slackness condition tells us that

$$(y^*)^T b = b^T y^* = c^T x^* = (A^T y^* + v^*)^T x^* = (y^*)^T A x^* + (v^*)^T x^*,$$

which is equivalent to

$$(y^*)^T (b - Ax^*) = (v^*)^T x^* = (c - A^T y^*)^T x^*.$$

In addition, from the first two conditions, we have

- $y^* \geq 0$ and $b - Ax^* \leq 0 \quad \Rightarrow \quad (y^*)^T (b - Ax^*) \leq 0,$
- $(c - A^T y^*)^T \geq 0$ and $x^* \geq 0 \quad \Rightarrow \quad (c - A^T y^*)^T x^* \geq 0.$

Hence we obtain

$$(y^*)^T (b - Ax^*) = 0, \quad (c - A^T y^*)^T x^* = 0,$$

which is an alternative formulation for the Complementary Slackness condition

这个黄金代还有点搞不明白，有点巧妙 $v = c - A^T y*$
最后获得的结果就是 y 和 左边不等式点积和 x 和 c - A 点积都是 0，这是充要条件吗。这个组合式就等价于那个恒等式，不过感觉完全没必要aaa，都是对于主条件把左边移动过去。

Complementary Slackness

The first complementary slackness condition is

$$(y^*)^T(b - Ax^*) = 0.$$

Let $A_{i\bullet}$ be the ith row of A. Then

$$\underbrace{y_1^*(b_1 - A_{1\bullet}x^*)}_{\leq 0} + \cdots + \underbrace{y_i^*(b_i - A_{i\bullet}x^*)}_{\leq 0} + \cdots + \underbrace{y_m^*(b_m - A_{m\bullet}x^*)}_{\leq 0} = 0$$

$$\Downarrow$$

$$\underbrace{y_1^*(b_1 - A_{1\bullet}x^*)}_{=0} + \cdots + \underbrace{y_i^*(b_i - A_{i\bullet}x^*)}_{=0} + \cdots + \underbrace{y_m^*(b_m - A_{m\bullet}x^*)}_{=0} = 0$$

For every $i = 1, \cdots, m$, we have either

$$y_i^* = 0 \quad \text{or} \quad \underbrace{A_{i\bullet}x^* = b_i}_{\text{constraints equality}}.$$

这就是把他拆开，说明了就是相当于要么那一维直接对上要么 y*就得是0，二选一

Complementary Slackness

In conclusion, we have the following.

1. For every $i = 1, \cdots, m$,
   - if $y_i^* > 0$, then $A_{i\bullet}x^* = b_i$;
   - if $A_{i\bullet}x^* > b_i$, then $y_i^* = 0$.
2. For every $j = 1, \cdots, n$,
   - if $x_j^* > 0$, then $A_{\bullet j}^T y^* = c_j$;
   - if $A_{\bullet j}^T y^* < c_j$, then $x_j^* = 0$.

最后这个结论就是说你这里等式成立，那边变量自由了，还是那个劲，两个吧 bound 住一个就行！

注意到如果等式一两个然后变量多，那不如对偶，变量少永远是好的，然后你再回来。

所以松弛互补为啥怎么反过来把最优解方案搞出来来着

没明白，就是能 plug 的东西只有 y>0 的等式取等和那个 n个等式没去等的这边变量等=0

The Optimality Conditions for Inequality Constraints

The Kuhn-Tucker optimality conditions for inequality constraints can now be written as:

1. **Primal Feasibility** $Ax^* \geq b, x^* \geq 0;$
2. **Dual Feasibility** $A^T y^* \leq c, y^* \geq 0;$
3. **Complementary Slackness** $(y^*)^T(b - Ax^*) = 0, \quad (c - A^T y^*)^T x^* = 0.$

- y^* and v^* $(= c - A^T y^*)$ are called the Lagrangian Multipliers corresponding to the (primal) inequality constraints and the (primal) non-negativity constraints, respectively.

The Optimality Conditions for Equality Constraints

Consider the standard form of the linear program:

$$\min \ c^T x;$$
$$s.t. \ Ax = b,$$
$$x \geq 0.$$

The Kuhn-Tucker optimality conditions for equality constraints:

1. **Primal Feasibility** $\quad Ax^* = b, x^* \geq 0;$
2. **Dual Feasibility** $\quad A^T y^* \leq c;$
3. **Complementary Slackness** $\quad (c - A^T y^*)^T x^* = 0.$

就对面的等式和自己的变量还活着，不明白

Lecture 6 单纯形法

$\min, =$ 注意 $b \geq 0$ 否则调整一下 。
$m < n$ 如果多直接解完了。（高斯消元？？。
$rank(A) = m$ 相同的消掉了。
full rank: $rank(A) = min(n, m)$
所以有 m 个独立的列，重排一下让前 m 个列独立。
$A = (B, N)$，B 为 $m \times m$ 的非奇异矩阵，他的每列是独立的

Basic Feasible Solutions

- Consider the system $Ax = b$ and $x \geq 0$.
- Since $rank(A) = m$, there are m linearly independent columns in A.
- Rearranging the columns of A (and rearranging the order of the variables), so that the first m columns in A are linearly independent.

$$A = \overbrace{\left(\begin{array}{c|c} \underbrace{\quad B \quad}_{m} & \underbrace{\ N\ }_{n-m} \end{array} \right)}^{n} \Bigg\} m$$

- Write $A = (B, N)$, where B is an $m \times m$ matrix.
- All columns of B are linearly independent.
- B is non-singular.

x 也可以切成两块，前一半块是 m，后一半是 $n - m$

Basic Feasible Solutions

$$\text{Let } x = \underbrace{\begin{pmatrix} x_B \\ x_N \end{pmatrix}}_{1} \begin{array}{l} \Big\} m \\ \Big\} n-m \end{array} \qquad \text{where } x_B = B^{-1}b \text{ and } x_N = 0.$$

Then $Ax = (B, N)\begin{pmatrix} x_B \\ x_N \end{pmatrix} = Bx_B + Nx_N = B(B^{-1}b) + N \cdot 0 = b.$

We find a vector $x = \begin{pmatrix} x_B \\ x_N \end{pmatrix} = \begin{pmatrix} B^{-1}b \\ 0 \end{pmatrix}$ satisfies $Ax = b$.

$x_B = B^{-1}b, x_N = 0$ 考虑这个解好像是合法的。

所以上面这个就叫基础解 basic solution
basic feasible solution (BFS)：如果 $x_B \geq 0$（要满足这个
B 叫基础矩阵（basic matrix / basis）： N：non basic matrix
basic variables,(x_B 基础量)，nonbasic variables (x_n 全是 0)
sets of basic and nonbasic variables：B, N
非退化基本可行解：nondegenerate basic feasible solution（全 > 0，反过来如果有一个是 0 就是degenerate basic feasible solution退化可行基解

感觉主要就是选对 B 确定是消不掉然后分开然后反过来乘以下。

也可以选若干的列不用重排，然后对应位置给岸上 x 就行。

For a linear program, BFS ⇔ vertices of its feasible polytope region. 就你那些合法解就覆盖polytope的所有端点，不用管那些 N 了。。（?
如果是有限解，那一定在端点上 （vertex，所以一定有最优的 BFS

其他版本：

- 从这个 BFS 到另外一个（Pivoting!!
- 有无界解
- 我们找到了最优解
  Pivoting 怎么找相邻的？消掉一个元？？

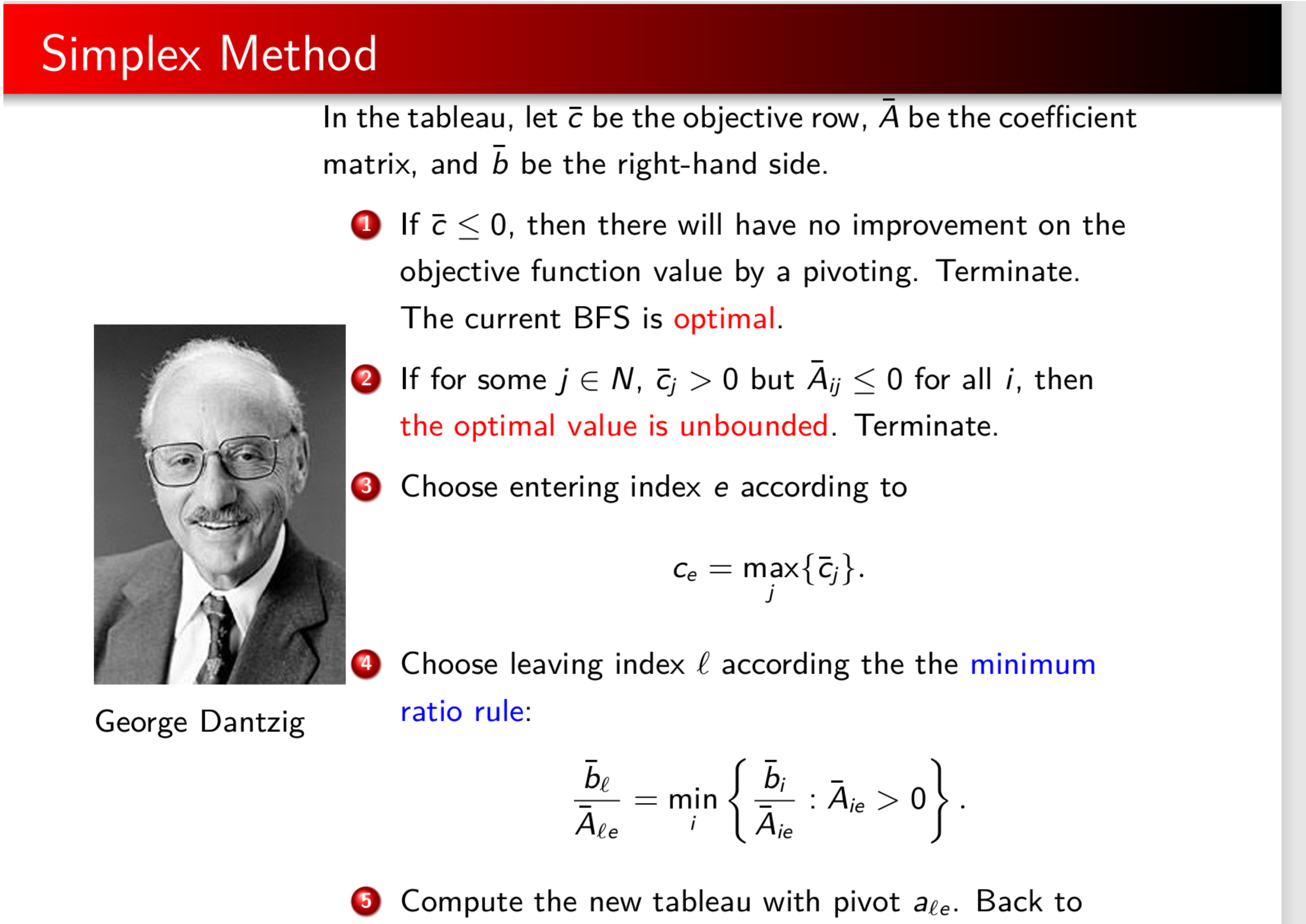
tableau ：就等于直接消元就行，把那列变成 1 然后其他消空，你做完这样又是一个单位矩阵的，有可以逆了，所以很方便就在这上面做就行了。（解就是他对应的那几个变量对应赋值成右边的 b）
A common pivoting rule: 每次选正的最大的（这样最大，消掉比较对。选大的不一定对，但看上去比较贪心对？？
这相当于每次选若干列是单位矩阵，就 tableau 这么消，挺舒服的。
为什么要把上面的系数也消掉：这样直接看常数项就行了，很方便（（qwq

每次选的行我们应该选单位值最小的，不然右边就变成负的了。（就不会提供 BFS 了。

So:

1. 每次选系数最大的
2. 选单位对应 RHS 最小的

当所有负数都变成非负的。（没贡献了?



系数矩阵的逆，如果全负就没有贡献就最优解。否则如果有系数整的但是矩阵那列都是 < 0 那就不能要了。不然选最大系数的那个，然后选正的里面所有单位最小（系数除以自己）的那个消除一下

Two-Phase：
先让能找到单位矩阵（添加冗余变量，就是每行得找到一个列使得他是 1 其他都是 0，然后 min 怼上去），然后 min sum 冗余变量，如果找到 0 解那就有最初解了，如果没有就无解。找到最优解了以后把 > 0 的几行消掉，就得到了单位矩阵（注意最上面也要把那些代还完）（然后把哪些冗余变量位删掉就可以了哈哈

每次一开始保证是 BFS 就保证那个表达式不能出现哪些 1（替换掉）（

第九周的这时候开始 23min 他说 10 分会考啥?

Lecture 7 整数规划 Integer Programming

线性规划取值必须是整数。（MIP Mixed 如果只有一部分限制是整数）

Binary variables / BIP (Binary Integer Programming)

取值只有 $0/1$

OR 条件

对于两个 ≤ 0 条件，考虑加入一个巨大的 M 和 01 变量 y 分别是 $yM, (1-y)M$。这样就是让一个满足就可以了！（或者理解为在打的那边加个 0 or INF 松弛)

RHS Choice

建若干 binary 加起来是 1 相当于限制了最多选一个，然后 offer choice 就好了

二进制表示：找到每个变量的值域然后换成二进制的 01 这样就可以变成 binary

LP Relaxation LP 放松

Total Unimodularity

矩阵 unimodular （幺模：1 ; -1)
totally unimodular (全幺模（删若干行列）：所有子矩阵行列式都是 -1, 0, 1)
定义 polytope $Ax = b$，如果 b 是整数并且 A 是全幺模，那么 x 都是整数向量

全幺模：

- 值域就是 $-1, 0, 1$
- 每列最多两个非 0
- 俩行有相同符号列，必须不同，两个在同列的元素不同，他们对应的行必须相同（（

Lecture 8 Inventory Theory

Ordering cost (or setup cost): 有就加常量 K
Purchasing cost：原材料 传输
Holding cost：包括保险
Shortage cost:

1. backlogged demand (backlogging)
2. a lost sale (no backlogging).
   Salvage value 残值
   discount rate

- continuous review， 刚到预期一下就会马上order
- discrete 隔几天执行

Economic Order Quantity

Deterministic Continuous-review Modelsdemand 每次删 a 哥（需求，删的，每单位时间-这么多，没了再补充）
Q 哥同时到
order cost K
continuous
No shortage
hold cost h
买一个 c
及时补充 最小化花费

Basic EOQ Model

Therefore, the total cost per cycle is

$$K + cQ + \frac{hQ^2}{2a}.$$

The total cost per unit time (running cost) is

$$T(Q) = \frac{K + cQ + hQ^2/(2a)}{Q/a} = \frac{aK}{Q} + ac + \frac{hQ}{2}.$$

We note that

$$T'(Q) = \frac{-aK}{Q^2} + \frac{h}{2}.$$

The only critical point is obtained by solving $T'(Q) = 0$ and we have

$$Q^* = \sqrt{\frac{2aK}{h}}.$$

这样平均每分钟的花销就是 $(K + cQ + h \times \frac{Q}{2} \times \frac{Q}{a})/(Q/a)$

$$\frac{Ka}{Q} + ac + \frac{hQ}{2}$$

对钩函数。。笨死了

Basic EOQ Model

- The EOQ does NOT depend on the unit purchasing price c, because no matter what is the size of each order, one has to purchase a units per unit time.
- From the formula we note that EOQ increases when the demand rate a or the ordering cost K increases.

  If the unit holding cost increases, then the EOQ decreases and the system holds less inventory.
- Note that when EOQ is applied, the average holding cost per unit time is

$$\frac{hQ^*}{2} = \frac{h}{2}\sqrt{\frac{2aK}{h}} = \sqrt{\frac{haK}{2}}$$

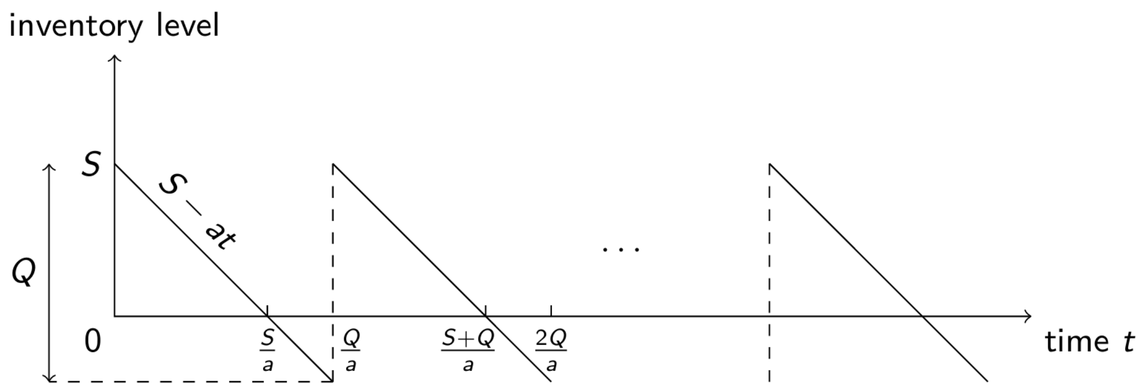
  and the ordering cost per unit time is

$$\frac{K}{Q^*/a} = Ka\sqrt{\frac{h}{2aK}} = \sqrt{\frac{haK}{2}}.$$

  Therefore the holding cost per unit time is equal to the ordering cost per unit time.

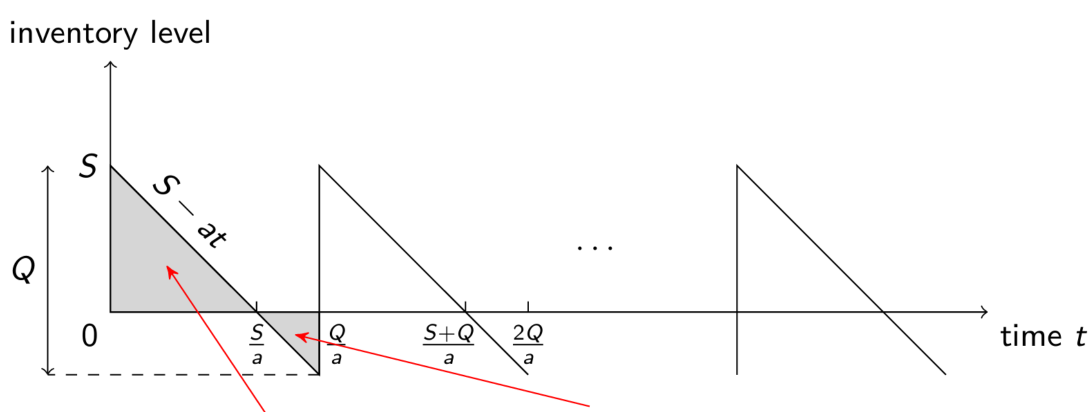
EOQ Model with Planned Shortages

backorder



$$
\begin{aligned}
p &= \text{shortage cost per unit short per unit of time short,} \\
S &= \text{inventory level just after a batch of } Q \text{ units is added to inventory,} \\
Q - S &= \text{shortage in inventory just before a batch of } Q \text{ units is added.}
\end{aligned}
$$

EOQ Model with Planned Shortages



Holding cost per cycle $= \dfrac{hS}{2}\dfrac{S}{a} = \dfrac{hS^2}{2a}$

Shortage cost per cycle

$$= \frac{p(Q-S)}{2}\frac{Q-S}{a} = \frac{p(Q-S)^2}{2a}$$

$$\text{Total cost per cycle} = K + cQ + \frac{hS^2}{2a} + \frac{p(Q-S)^2}{2a}$$

Therefore the total cost per unit time is minimized when

$$Q^* = \sqrt{\frac{2aK(p+h)}{ph}} = \sqrt{\frac{2aK}{h}}\sqrt{\frac{p+h}{p}} = \text{EOQ}\sqrt{\frac{p+h}{p}}$$

and

$$S^* = S^*_{Q^*} = \left(\frac{p}{p+h}\right)Q^* = \sqrt{\frac{2aKp}{h(p+h)}} = \text{EOQ}\sqrt{\frac{p}{p+h}}.$$

EOQ Model with Planned Shortages

Remarks:

- The optimal cycle length t^* is given by

$$t^* = \frac{Q^*}{a} = \sqrt{\frac{2K}{ah}}\sqrt{\frac{p+h}{p}}.$$

- The maximum allowable shortage is

$$Q^* - S^* = \sqrt{\frac{2aK}{p}}\sqrt{\frac{h}{p+h}}.$$

- The fraction of time for no shortage is

$$\frac{S^*/a}{Q^*/a} = \frac{p}{p+h}.$$

- As $p \to \infty$, both Q^* and S^* tends to the EOQ. In this case the maximum allowable shortage $Q^* - S^*$ tend to zero.

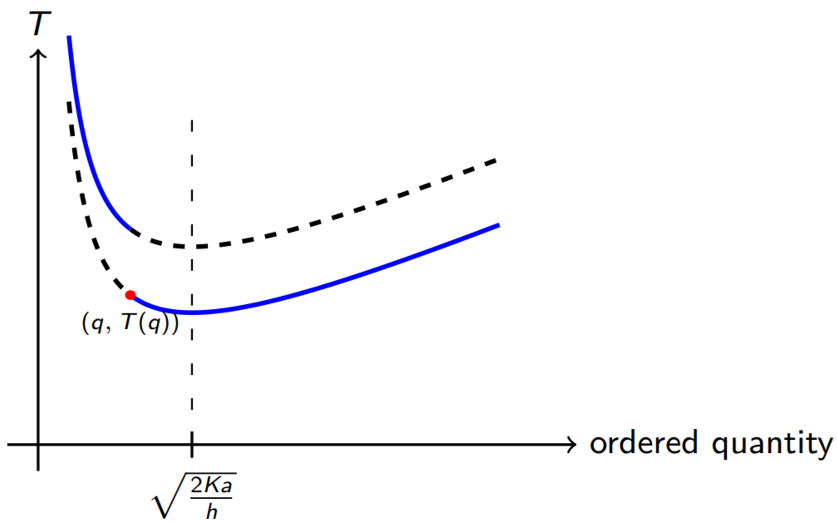
$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Lecture 9. Inventory Theory II

EOQ Model with Quantity Discounts

There are price breaks.

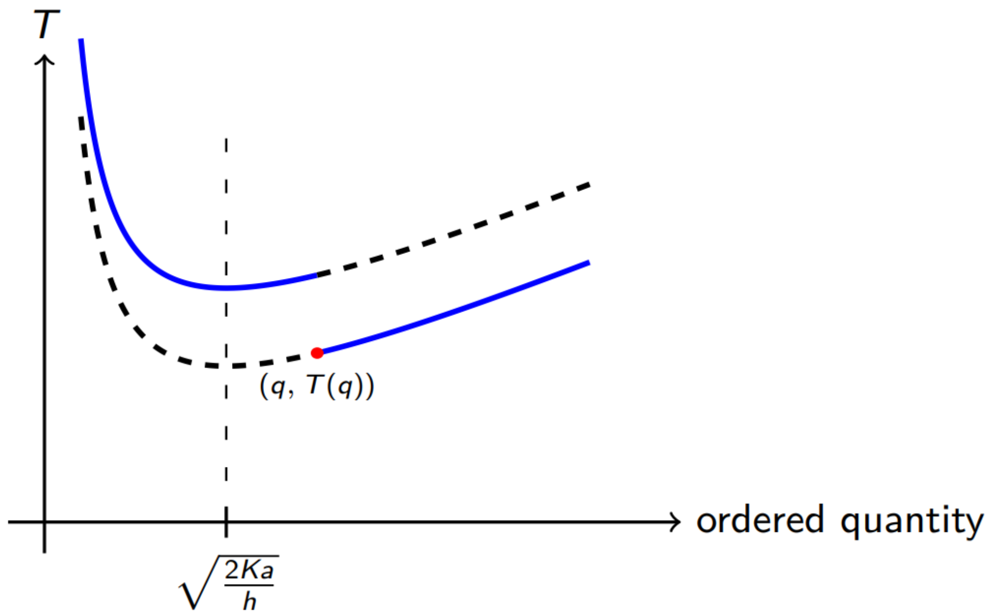
$Q < q, c_1\ else\ c_2$ 一般 c_2 便宜



If $\sqrt{\frac{2Ka}{h}} \geq q$, the minimum value is $T(\sqrt{\frac{2Ka}{h}})$.

注意 c 不影响这个决策点

Minimization of the Total Cost per Unit Time



If $\sqrt{\frac{2Ka}{h}} < q$, compare the values of $T(\sqrt{\frac{2Ka}{h}})$ and $T(q)$.

比较 $\sqrt{\frac{2Ka}{h}}$ 和 q，大的那个是最小值（分讨

Stochastic Model

$K = 0$ 残值
y (需要决定的 order 数量)
D （随机变量需求
c, h(拿剩下的)$, p$

$cy + p\max(0, D-y) + h\max(0, y-D)$
PDF: φD 算期望：直接积分就行

Single-period Model without Setup Cost

In other words we must determine the value y^* which satisfies

$$\int_0^{y^*} \varphi_D(\xi)\, d\xi = \frac{p-c}{p+h}. \tag{2}$$

- $p - c$ = unit cost of underordering
  = decrease in profit that results from failing to order a unit that could have been sold during the period.
- $c + h$ = unit cost of overordering
  = decrease in profit that results from ordering a unit that could not be sold during the period.

N.B.: If D is a discrete random varaible that takes only integer values, the optimal order value y^* is given by the smallest integer that satisfies

$$\sum_{d=0}^{y^*} P_D(d) \geq \frac{p-c}{p+h}.$$

要大于等于。

Lecture 10 - Queueing Theory I

Kendall's Notation

It is convenient to use a shorthand notation (introduced by D.G.Kendall) of the form $a/b/c/d$ to describe queueing models, where a specifies the inter-arrival time, b specifies the service time, c is the number of servers and d is the number of waiting space. For example,

1. G/M/s/n : General (independent) input, exponential (Markov) service time, s servers, n waiting space;
2. M/G/s/n : exponential (Markov) inter-arrival time, General (independent) service time, s servers, n waiting space;
3. M/D/s/n : exponential (Markov) inter-arrival time, constant (Deterministic) service time, s servers, n waiting space;
4. E_k/M/s/n: k-phase Erlangian inter-arrival time, exponential (Markov) service time, s servers, n waiting space;
5. M/M/s/n: exponential inter-arrival time, exponential service time, s servers, n waiting space.

到达时间 / 服务时间 / servers 数量 / 等待空间

前两个可以填的东西：G (普通 / 独立) / M （Markov / 指数） / D(确定) / E_k (k-phase Erlangian)

Revision: The Exponential Distribution

A continuous r.v. T is an exponential r.v. with parameter $\alpha > 0$ if its probability density function (pdf) is defined by

$$\boxed{f(t) = \begin{cases} \alpha e^{-\alpha t} & t \geq 0 \\ 0 & t < 0. \end{cases}}$$

The distribution function is given by

$$\boxed{F(t) = Prob(T \leq t) = \begin{cases} 1 - e^{-\alpha t} & t \geq 0 \\ 0 & t < 0. \end{cases}}$$

We note that

$$\boxed{E[T] = \alpha^{-1} \quad \text{and} \quad \mathrm{Var}(T) = \alpha^{-2}.}$$

The exponential distribution plays an important in modeling the inter-arrival and service time in a Markovian queueing system.

No-Memory Property

No-Memory Property

The previous example demonstrates one of the important features of the exponential distribution. This is called the no-memory property and is described precisely in the following proposition. It says that the time until the next arrival does not depend on when the last arrival occurred.

Proposition 10.2 (No-Memory Property)

If T has an exponential distribution, then for all nonnegative values of t and δt,

$$Prob(T > t + \delta t | T > t) = Prob(T > \delta t),$$

and

$$Prob(T \leq t + \delta t | T > t) = Prob(T \leq \delta t).$$

It can be shown that no other distribution satisfies this condition.

No-Memory Property

Proof.

$$\begin{aligned} Prob(T > t + \delta t | T > t) &= \frac{Prob(T > t + \delta t \ \& \ T > t)}{Prob(T > t)} \\ &= \frac{Prob(T > t + \delta t)}{Prob(T > t)} \\ &= \frac{e^{-\alpha(t+\delta t)}}{e^{-\alpha(t)}} \\ &= e^{-\alpha \delta t} \\ &= Prob(T > \delta t) \end{aligned}$$

The other equality can be obtained directly from the above one.

Properties of Exponential Distribution

Proposition 10.3

The minimum of several independent exponential random variables has an exponential distribution.

Proof. Suppose that $U = \min\{T_1, T_2, \ldots, T_n\}$, where each T_i is an exponential random variable with parameter α_i. Then

$$\begin{aligned}
Prob(U > t) &= Prob(T_1 > t, T_2 > t, \ldots, T_n > t) \\
&= Prob(T_1 > t)Prob(T_2 > t) \ldots Prob(T_n > t) \\
&= e^{-\alpha_1 t} e^{-\alpha_2 t} \ldots e^{-\alpha_n t} \\
&= \exp\left(-\sum_{i=1}^{n} \alpha_i t\right)
\end{aligned}$$