AMA1131 Calculus for Engineers Assignment 2. (Pa1) XIANG Yufan (Frank) 23096511D.

1.

$\lim_{x\to0^+} f(x) = e^{-\frac{1}{0^+}} = 0.$

$\lim_{x\to0^-} f(x) = a\cdot0^- + b = b$

$\therefore b=0.$

$f'(0^-) = \lim_{h\to0^-}\frac{f(h)-f(0)}{h} = \lim_{h\to0^-}\frac{ah}{h} = a.$

$f'(0^+) = \lim_{h\to0^+}\frac{f(h)-f(0)}{h} = \lim_{h\to0^+}\frac{e^{-\frac{1}{h}}}{h} \to 0$

$= e^{\lim_{h\to0^+} -\frac{1}{h} - \ln h}.$

$t = \lim_{h\to0^+} -\frac{1}{h} - \ln h.$

$= -\lim_{h\to0^+}\frac{1+\ln h\cdot h}{h} = -\infty$

$\lim_{h\to0^+}\frac{\ln h}{\frac{1}{h}} \overset{L'H}{=} \lim_{h\to0^+}\frac{\frac{1}{h}}{-\frac{1}{h^2}} = -h = 0$

$= e^t = 0$

$\therefore a=0,\ b=0$

2.

$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} x^2\sin(\frac{1}{x^2}) + \lim_{x\to0^-} b\cos x$

$= 0 + b = b$

$\because -1 \le \sin(\frac{1}{x^2}) \le 1$

$\therefore 0 = -\lim_{x\to0^-}x^2 \le \lim_{x\to0^-} x^2\sin(\frac{1}{x^2}) \le \lim_{x\to0^-}x^2 = 0$

By sandwich theorem $\lim_{x\to0^-} x^2\sin(\frac{1}{x^2}) = 0$

$\lim_{x\to0^+} f(x) = (0^++1)^{0^+} + a(0^++3)$

$= 1+3a.$

$1+3a=b.$

$f'(0^+) = \lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}$

$= \lim_{x\to0^+}\frac{(x+1)^x + ax}{x}$

$= a + \lim_{x\to0^+}\frac{e^{x\ln(x+1)}-1}{x}$ $(\frac{0}{0})$

$\overset{L'H}{=} a + \lim_{x\to0^+}\frac{e^{x\ln(x+1)}\cdot(\ln(x+1)+\frac{x}{x+1})}{1}$

$= a.$

$f'(0^-) = \lim_{x\to0^-}\frac{f(x)-f(0)}{x-0}$

$= \lim_{x\to0^-}\frac{x^2\sin(\frac{1}{x^2})+b\cos(x)-b}{x}$

$= \lim_{x\to0^-} x\sin(\frac{1}{x^2}) + b\lim_{x\to0^-}\frac{\cos(x)-1}{x}$ $(\frac{0}{0})$

$\overset{L'H}{=} 0 + b\lim_{x\to0^-}\frac{-\sin(x)}{1}$

$= 0$

$\therefore a=0,\ b=1.$

3.

(a). $f'(x) = e^x(x^2\sin x + 2x\cdot\sin x + x^2\cos x)$

$= xe^x(x\sin x + 2\sin x + x\cos x).$

(b). $f'(x) = e^x\ln x + \frac{e^x}{x} + \cos x$

(c). $f'(x) = \frac{1}{\tan^{-1}(e^{x^2}-1)}\cdot\frac{1}{1+(e^{x^2}-1)^2}\cdot e^{x^2}\cdot 2x$

(d). $f(x) = [\ln(1+x^2)]^e$

$f'(x) = e[\ln(1+x^2)]^{e-1}\cdot\frac{1}{1+x^2}\cdot 2x$

$f'(v) = e[\ln(1+v^2)]^{e-}\cdot\frac{1}{1+v^2}\cdot 2v$

(e). $f(x) = e^{\tan(x)\ln[\cos(x)+3]}.$

$f'(x) = e^{\tan(x)\ln[\cos(x)+3]}$

$\cdot\left[\frac{1}{\cos^2 x}\ln[\cos(x)+3] + \frac{\tan(x)\cdot[-\sin(x)]}{\cos(x)+3}\right]$

4.

(a) $= \lim_{x\to 1} \frac{e^{1-x}}{x^2+x} \cdot \lim_{x\to 1} \frac{(\sqrt{2x-x^4} - \sqrt[3]{x}) \to 0}{1-\sqrt[4]{x^3} \to 0}$

$(2x-x^4)^{\frac{1}{2}} - x^{\frac{1}{3}}$; $1 - x^{\frac{3}{4}}$

$\because \frac{0}{0}$

$\overset{L'H'}{=} \frac{1}{2} \cdot \lim_{x\to 1} \frac{\frac{1}{2}(2x-x^4)^{-\frac{1}{2}} \cdot (2-4x^3) - \frac{1}{3}x^{-\frac{2}{3}}}{-\frac{3}{4}x^{-\frac{1}{4}}}$

$= \frac{1}{2} \cdot \frac{\frac{1}{2}\cdot(-2) - \frac{1}{3}}{-\frac{3}{4}}$

$= \frac{1}{2} \times (-\frac{4}{3}) \cdot (-\frac{4}{3}) = \frac{8}{9}$

(b) $= \lim_{x\to\infty} \frac{\ln x \to \infty}{\sqrt{2x+\ln x} + \sqrt{2x} \to \infty}$

$(\sqrt{2x+\ln x} + \sqrt{2x})$

$\overset{L'H'}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2+\frac{1}{x}}{\sqrt{2x+\ln x}} + \frac{2}{\sqrt{2x}}} = \lim_{x\to\infty} \frac{1}{\frac{2x+1}{\sqrt{2x+\ln x}} + \sqrt{2x}}$

$= 0$

(c) $= e^{\lim_{x\to 0^+} \frac{1}{x^2}\ln(1+x) - \frac{1}{x}} = e^{-\frac{1}{2}}$.

$\lim_{x\to 0^+} \frac{\ln(1+x) - x}{x^2} \to \frac{0}{0}$

$\overset{L'H'}{=} \frac{\frac{1}{1+x} - 1}{2x} \to \frac{0}{0}$ $(1+x)^{-1}$

$\overset{L'H'}{=} \frac{-1 \cdot \frac{1}{(1+x)^2}}{2} = -\frac{1}{2}$.

(d) $= e^{\lim_{x\to\infty} \frac{\ln(\frac{\pi}{2} - \arctan x)}{\ln x}}$ $\to -\infty$, $\to +\infty$

$\overset{L'H'}{=} e^{\lim_{x\to\infty} \frac{-\frac{1}{(\frac{\pi}{2}-\arctan x)(1+x^2)}}{\frac{1}{x}}}$

$\overset{L'H'}{=} e^{\lim_{x\to\infty} \frac{-x \cdot \frac{1}{(1+x^2)}}{(\frac{\pi}{2} - \arctan x)}}$ $\to 0$, $\to 0$

$\frac{0}{0}$ $\overset{L'H'}{=} e^{\lim_{x\to\infty} \frac{\frac{(1+x^2) - x\cdot 2x}{-(1+x^2)^2}}{-\frac{1}{1+x^2}}}$

$= e^{\lim_{x\to\infty} \frac{1-x^2}{(1+x^2)}} = e^{-1} = \frac{1}{e}$

(e) $= e^{\lim_{x\to 0^+} \frac{10}{x}\ln(1+10x)} = e^A = e^{100}$

let $A = \lim_{x\to 0^+} \frac{10}{x}\ln(1+10x)$

$\overset{L'H'}{\underset{(\frac{0}{0})}{=}} \lim_{x\to 0^+} \frac{10 \cdot \frac{1}{1+10x} \cdot 10}{1}$

$= 100$

(f) $\because$ let $A = \lim_{x\to 0} x \arctan(\frac{1}{x})$

$\because -\frac{\pi}{2} \leq \arctan(x) \leq \frac{\pi}{2}$

$\therefore$ $0 = -\frac{\pi}{2}\lim_{x\to 0} x \leq A \leq \frac{\pi}{2}\lim_{x\to 0} x = 0$

$\therefore$ By sandwich theorem.

$A = 0$.

(g). let $u = 2x$

$= \lim_{u\to\infty} \frac{(1+\frac{1}{u})^u}{e^u} = \lim_{u\to\infty} \frac{e}{e^u}$

By definition $= 0$

(h) $= \lim_{x\to(\frac{\pi}{2})^-} \frac{1-\sin x}{\cos x}$

$\overset{L'H'}{\underset{(\frac{0}{0})}{=}} \lim_{x\to(\frac{\pi}{2})^-} \frac{-\cos x}{-\sin x} = \frac{1}{\tan x} = 0$.

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5. let. $f(x) = e^x - \frac{1}{2} - \cos(2x) + 2\sin x$.

$f(0) = 1 - \frac{1}{2} - 1 + 0 = -\frac{1}{2} < 0$.

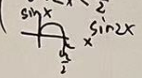
$f(\frac{\pi}{4}) = e^{\frac{\pi}{4}} - \frac{1}{2} - 0 + \sqrt{2} > 0$.

$\because (\sqrt{2} > \frac{1}{2})$

$\therefore f'(x) = e^x + 2\sin 2x + 2\cos x$.

$\forall 0 < x < \frac{\pi}{4}$, $\sin 2x > 0$; $e^x > 0$, $\cos x > 0)$

$(0 < 2x < \frac{\pi}{2}$



$\therefore \forall 0 < x < \frac{\pi}{4}$, $f'(x) > 0$

$\therefore f(x)$ is increasing at $(0, \frac{\pi}{4})$.

$\therefore \exists$ unique x, $f(x) = 0$.

6. Differenciate on both side:

$-\sin(x^2+2y) \cdot (2x+2y') + 5e^y + 5x \cdot e^y \cdot y'$

$= \frac{1}{1+y^2} \cdot y' + 6y'$

$x=0, y=0$: $5 = 7y'$

$y' = \frac{5}{7}$

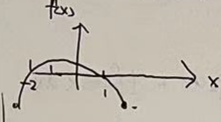
0.

7.

$f'(x) = -6x^2 - 6x + 12$.

$= -6(x^2+x-2)$

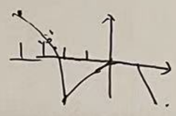
$= -6(x-1)(x+2)$.



$\because [-4, -2]$, $f'(x) < 0$, decreasing.

$[-2, 1]$, $f'(x) > 0$, increasing

$[1, 2]$, $f'(x) < 0$, decreasing



$f(-4) = +128 - 48 - 48 - 7 = 25$

$f(-2) = 16 - 12 - 24 - 7 = -27$

$f(1) = 0$.

$f(2) = -11$.

$\therefore$ global max: $f(-4) = 25$

global min. $f(-2) = -27$

local max: $f(1) = 0$

local min.: $f(-2) = -27$

(a)

$= 2\int x\,dx - 4\int x\,dx + 7\int x^{-2}dx + 3\int \sin x\,dx$

$= 2\cdot\frac{x^2}{2} - 4\cdot\ln|x| + 7\cdot\frac{x^{-1}}{-1} + 3(-\cos x)$

$= x^2 - 4\ln|x| - \frac{7}{x} - 3\cos x + C$.

(b)

$= 5\int e^x dx - 8\int\frac{1}{\sqrt{x^2+1^2}}dx + 9\int\cos x\,dx$

$= 5e^x - 8\ln|x+\sqrt{x^2+1}| + 9\sin x + C$.

(c) let $u = x^2+3$

$\frac{du}{dx} = 2x \Leftrightarrow dx\cdot x = \frac{du}{2}$.

$= \frac{4}{2}\int u^5 du$

$= 2\cdot\frac{u^6}{6}$

$= \frac{(x^2+3)^6}{3} + C$

(d) let $u = 5x^2$.

$\frac{du}{dx} = 10x \Leftrightarrow x\cdot dx = \frac{du}{10}$.

$= \frac{1}{10}\int e^u du$

$= \frac{e^u}{10}$

$= \frac{e^{5x^2}}{10} + C$

(e). let $u = x+8$

$du = dx$, $x = u-8$.

$= \int\frac{u-8}{u^{\frac{1}{3}}}du$.

$= \int u^{\frac{2}{3}}du - 8\int u^{-\frac{1}{3}}du$.

$= \frac{u^{\frac{5}{3}}}{\frac{2}{3}} - \frac{8u^{\frac{2}{3}}}{\frac{2}{3}}$

$= \frac{3}{5}(x+8)^{\frac{5}{3}} - 12.(x+8)^{\frac{2}{3}} + C$

(f) let $u = e^{2x}$

$\frac{du}{dx} = 2e^{2x} \Leftrightarrow dx\cdot e^{2x} = \frac{du}{2}$

$= \frac{1}{2}\int\frac{1}{\sqrt{1^2-u^2}}du$

$= \frac{1}{2}\sin^{-1}u = \frac{1}{2}\sin^{-1}e^{2x} + C$.

(g)

$= \frac{1}{3}\int x^{-2}dx + \int\frac{1-\cos 12x}{2}dx$.

let $u = 12x$, $\frac{du}{dx} = 12$, $dx = \frac{du}{12}$

$= \frac{1}{3}\cdot\frac{x^{-1}}{-1} + \frac{x}{2} - \frac{1}{2}\cdot\frac{1}{12}\int\cos u\,du$

$= -\frac{1}{3x} + \frac{x}{2} - \frac{\sin u}{24}$

$= -\frac{1}{3x} + \frac{x}{2} - \frac{\sin 12x}{24} + C$

(h) let $u = 3x$

$dx = \frac{du}{3}$

$= \frac{1}{3}\int\frac{1}{2^2+u^2}du$

$= \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{u}{2}$

$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + C$.

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(i). let $u = 4x$.

$dx = \frac{du}{4}$.

$= \frac{1}{4}\int \sin 2u \cdot \sin u \, du$.

$= \frac{1}{2}\int \sin^2 u \cos u \, du$.

let $t = \sin^2 u = \sin 4x$

$\frac{dt}{du} = \cos u \Leftrightarrow \cos u \, du = dt$

$= \frac{1}{2}\int t^2 dt$.

$= \frac{1}{2} \frac{t^3}{3}$

$= \frac{(\sin 4x)^3}{6} + C$.

(j) let $u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Leftrightarrow \frac{1}{\sqrt{x}} \cdot dx = 2du$

$= 2\int \frac{1}{4-u^2} du$

$= -2\int \frac{1}{u^2 - 2^2} du$

$= -2 \cdot \frac{1}{4}\ln\left|\frac{u-2}{u+2}\right| + C = -\frac{1}{2}\ln\left|\frac{\sqrt{x}-2}{\sqrt{x}+2}\right| + C$.

(k). $= \int e^{\frac{1-\cos 2x}{2}} \cdot \sin(2x) \, dx$

let $u = \frac{1-\cos 2x}{2}$

$\frac{du}{dx} = -\frac{1}{2}\cdot(-\sin 2x)\cdot 2 \Leftrightarrow du = dx \cdot \sin 2x$

$= \sin 2x$

$= \int e^u \cdot du$

$= e^u = e^{\frac{1-\cos 2x}{2}} + C$.

(l). let $u = \sin x$ let $A = \int \cos^5 x \, dx = \int (1-u^2)^2 \cdot du$.

$\frac{du}{dx} = \cos x$.

$= \int u^4 du - 2\int u^2 du + \int 1 du$

$= \frac{u^5}{5} - \frac{2u^3}{3} + u$

$= \frac{\sin^5 x}{5} - \frac{2\sin^3 x}{3} + \sin x$

$\frac{1}{5}$ $-\frac{2}{15} + \frac{2}{9} = \frac{6+10}{45}$ $\frac{16}{45}$ —

$\frac{1}{5} + \frac{2}{3} = \frac{3+10}{15}$

$-\frac{1}{5} + \frac{2}{3}$

$\int x \cos^5 x \, dx = \int x \, dA$

$u = x$ $v = A$.

$du = dx$ $dv = dA$

$= x \cdot A - \int A \, dx = \frac{x\sin^5 x}{5} - \frac{2x\sin^3 x}{3} + x\sin x$

$+ \left(\frac{1}{5}\int \sin^5 x \, dx - \frac{2}{3}\int \sin^3 x dx + \int \sin x dx\right)$.

$= \frac{x\sin^5 x}{5} - \frac{2x\sin^3 x}{3} + x\sin x$

$+ \frac{\cos^5 x}{25} + \frac{4\cos^3 x}{45} + \frac{8}{15}\cos x$

$\int \sin^3 x \, dx$

$= \int -(1-u^2) du$

$= +\frac{u^3}{3} - u$.

$= +\frac{\sin^3 x}{3} - \sin x$

$\int \sin^4 x \cdot \sin x \, dx$

$u = \cos x$ $du = dx \cdot (-\sin x)$

$= \int -(1-u^2)^2 du$

$= -\int u^4 du + 2\int u^2 du - \int 1 du$

$= -\frac{u^5}{5} + \frac{2u^3}{3} - u$

$= -\frac{\sin^5 x}{5} + \frac{2\sin^3 x}{3} - \sin x$

(m) $= 8\int \sin^2 2x \, dx$

$= 4\int 1 - \cos 4x \, dx$

let $u = 4x$ $dx = \frac{du}{4}$

$= 4x - \int \cos u \, du$.

$= 4x - \sin u$.

$= 4x - \sin 4x + C$

(o) let $A = \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} = \frac{B}{x+3} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}$

$= \frac{B(x-1)^2 + (x+3)(x-1)C + D(x+3)}{(x+3)(x-1)^2}$.

$3x^2 - 8x + 13 = x^2(B+C) + x(-2B + 2C + D) + B - 3C + 3D$.

$\begin{cases} B + C = 3 \\ 2C - 2B + D = -8 \\ B - 3C + 3D = 13 \end{cases}$ $\begin{cases} B = 4 \\ C = -1 \\ D = 2 \end{cases}$

$\int A = \int \frac{4}{x+3} d(x+3) + \int \frac{-1}{x-1} d(x-1) + \int \frac{2}{(x-1)^2} d(x-1)$

$= 4\ln(x+3) - \ln(x-1) - \frac{2}{(x-1)} + C$

(p) $\int \frac{2}{x(x^2+1)} dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+1} dx = \int \frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)} dx$

$x^2(A+B) + Cx + A = 2$.

$\begin{cases} A + B = 0 \\ C = 0 \\ A = 2 \end{cases}$ $\begin{cases} A = 2 \\ B = -2 \\ C = 0 \end{cases}$

$= \int \frac{2}{x} dx - \int \frac{2x}{x^2+1} dx$

let $u = x^2+1$ $\frac{du}{dx} = 2x$

$= 2\ln|x| - \int \frac{1}{u} du$

$= 2\ln|x| - \ln|x^2+1| + C$