ICPC Template

墨染空

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1 Default

```
// Skyqwq
#include <bits/stdc++.h>

#define pb push_back
#define fi first
#define se second
#define mp make_pair

using namespace std;

typedef pair<int, int> PII;
```

```cpp
typedef long long LL;

template <typename T> bool chkMax(T &x, T y) { return (y > x) ? x = y, 1 : 0; }
template <typename T> bool chkMin(T &x, T y) { return (y < x) ? x = y, 1 : 0; }

template <typename T> void inline read(T &x) {
    int f = 1; x = 0; char s = getchar();
    while (s < '0' || s > '9') { if (s == '-') f = -1; s = getchar(); }
    while (s <= '9' && s >= '0') x = x * 10 + (s ^ 48), s = getchar();
}

int main() {

    return 0;
}
```

2 图论

2.1 Hall 定理

完美匹配：集合 $\leq$ 邻域的并

最大匹配：总数 - max(集合 -邻域并)

2.2 矩阵树定理

度数 - 边

2.3 欧拉回路计数

Best 定理

内向生成树个数 $\prod(out_i - 1)$

2.4 树上路径交

从 $lca(u,x), lca(u,y), lca(v,x), lca(v,y)$ 四个点找深度最大的两个点，记为 p_1, p_2。

* 若 $p_1 = p_2$ 且 $dep_{p1} < \max(dep_{lca(x,y)}, dep_{lca(u,v)})$ 那么无相交路径 * 否则相交路径就是 p_1 到 p_2

```
// 树上路径交

PII inline query(int u, int v, int x, int y) {
    int p[4] = { lca(u, x), lca(u, y), lca(v, x), lca(v, y)};
    int w = lca(u, v), z = lca(x, y);
    int p1 = 0, p2 = 0;
    for (int i = 0; i < 4; i++)
        if (dep[p[i]] > dep[p1]) p2 = p1, p1 = p[i];
        else if (dep[p[i]] > dep[p2]) p2 = p[i];
    if (p1 == p2 && (dep[p1] < dep[w] || dep[p1] < dep[z])) return mp(-1, -1);
    // p1 - p2 是子路径
    return mp(p1, p2);
}
```

2.5 圆方树

```
// 圆方树
int dfn[N], low[N], dfncnt, cnt;

int s[N], top;

void inline Add(int x, int y) {
    g[x].pb(y), g[y].pb(x);
}

void tarjan(int u, int fa) {
    dfn[u] = low[u] = ++dfncnt;
    s[++top] = u;
    for (int v: e[u]) {
        if (v == fa) continue;
        if (!dfn[v]) {
            tarjan(v, u);
            chkMin(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                int y; ++cnt;
                do {
                    y = s[top--], Add(y, cnt);
                } while (y != v);
                Add(cnt, u);
```

```
            }
        } else {
            chkMin(low[u], dfn[v]);
        }
    }
}
```

2.6 有向图 Tarjan

```


// 有向图 tarjan
void tarjan(int u) {
    dfn[u] = low[u] = ++dfncnt;
    s[++top] = u, ins[u] = true;
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (!dfn[v]) {
            tarjan(v), low[u] = min(low[u], low[v]);
        } else if (ins[v]) low[u] = min(low[u], dfn[v]);
    }
    if (low[u] == dfn[u]) {
        int v; ++cnt;
        do {
            v = s[top--], ins[v] = false, col[v] = cnt;
        } while (v != u);
    }
}
```

2.7 欧拉回路

```
// 欧拉回路
void dfs(int u) {
    for (int &i = head[u]; i; ) {
        int v = e[i].v;
        if(vis[i]) {
            i = e[i].next;
            continue;
```

```
        }

        vis[i] = true;
        if(t == 1) vis[i ^ 1] = true;

        i = e[i].next;
        dfs(v);
    }
}
```

2.8 最大流

```
// 最大流
namespace MF{
    int n, m, s, t, pre[N], cur[N], q[N];
    LL res, maxflow, d[N];
    int head[N], numE = 1;
    struct E{
        int next, v, w;
    } e[M << 1];

    void inline add(int u, int v, int w) {
        e[++numE] = (E) { head[u], v, w };
        head[u] = numE;
    }
    void inline init(int v, int a, int b) {
        for (int i = 1; i <= n; i++) head[i] = 0;
        numE = 1;
        n = v, s = a, t = b;
    }

    bool inline bfs() {
        int hh = 0, tt = -1;
        for (int i = 1; i <= n; i++) d[i] = 0;
        q[++tt] = s, d[s] = 1, cur[s] = head[s];
        while (hh <= tt) {
            int u = q[hh++];
            for (int i = head[u]; i; i = e[i].next) {
                int v = e[i].v;
```

```
            if (!d[v] && e[i].w) {
                cur[v] = head[v];
                q[++tt]= v, d[v] = d[u] + 1;
                if (v == t) return 1;
            }
        }
    }
    return 0;
}
LL dinic(int u, LL flow) {
    if (u == t) return flow;
    LL rest = flow;
    for (int i = cur[u]; i && rest; i = e[i].next) {
        cur[u] = i;
        int v = e[i].v;
        if (e[i].w&& d[v] == d[u] + 1) {
            int k = dinic(v, min((LL)e[i].w, rest));
            if (!k) d[v] = 0;
            rest -= k, e[i].w -= k,e[i ^ 1].w += k;
        }
    }
    return flow - rest;
}
void inline addE(int u, int v, int w) {
    add(u, v, w), add(v, u, 0);
}
LL inline work() {
    maxflow = 0;
    while (bfs())
        while (res = dinic(s, INF)) maxflow += res;
    return maxflow;
}
// Find min-cut
bool vis[N];

void dfs(int u) {
    //cerr << u << " dfs\n";
    vis[u] = 1;
    for (int i = head[u]; i; i = e[i].next) {
```

```
            int v = e[i].v;
            if (!vis[v] && e[i].w) dfs(v);
        }
    }

    void minCut() {
        for (int i = 1; i <= n; i++) vis[i] = 0;
        dfs(s);
    }
}
```

2.9 Prufer

```
void inline fToP() {
        for (int i = 1; i < n; i++) d[f[i]]++;
        for (int i = 1, j = 1; i <= n - 2; j++) {
                while (d[j]) j++;
                p[i++] = f[j];
                while (i <= n - 2 && --d[p[i - 1]] == 0 && p[i - 1] < j) p[i++] = f[p[i - 1]];
        }
}

void inline pToF() {
    for (int i = 1; i <= n - 2; i++) d[p[i]]++;
    p[n - 1] = n;
    for (int i = 1, j = 1; i < n; i++, j++) {
        while (d[j]) j++;
        f[j] = p[i];
        while (i < n - 1 && --d[p[i]] == 0 && p[i] < j) f[p[i]] = p[i + 1], ++i;
    }
}
```

2.10 长链剖分

```
int d[N], dep[N];
int g[N], son[N], fa[N][L], top[N];
LL res;
vector<int> U[N], D[N];
```

```
void dfs1(int u) {
    dep[u] = d[u] = d[fa[u][0]] + 1;
    for (int i = 1; fa[u][i - 1]; i++) fa[u][i] = fa[fa[u][i - 1]][i - 1];
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        dfs1(v);
        if (dep[v] > dep[u]) dep[u] = dep[v], son[u] = v;
    }
}

void dfs2(int u, int tp) {
    top[u] = tp;
    if (u == tp) {
        for (int x = u, i = 0; i <= dep[u] - d[u]; i++)
            U[u].push_back(x), x = fa[x][0];
        for (int x = u, i = 0; i <= dep[u] - d[u]; i++)
            D[u].push_back(x), x = son[x];
    }
    if (son[u]) dfs2(son[u], tp);
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (v != son[u]) dfs2(v, v);
    }
}

int inline query(int x, int k) {
    if (!k) return x;
    x = fa[x][g[k]], k -= (1 << g[k]) + d[x] - d[top[x]], x = top[x];
    return k < 0 ? D[x][-k] : U[x][k];
}
```

2.11 最小费用最大流

```
const int N = ?, M = ?;
const int INF = 0x3f3f3f3f;
int n, m, s, t, maxflow, cost, d[N], incf[N], pre[N];
int q[N];
int head[N], numE = 1;

```

```cpp
bool vis[N];

struct E{
    int next, v, w, c;
} e[M];

void inline add(int u, int v, int w, int c) {
    e[++numE] = (E) { head[u], v, w, c };
    head[u] = numE;
}

// Spfa ||
bool spfa() {
    memset(vis, false, sizeof vis);
    memset(d, 0x3f, sizeof d);
    int hh = 0, tt = 1;
    q[0] = s; d[s] = 0; incf[s] = 2e9;
    while (hh != tt) {
        int u = q[hh++]; vis[u] = false;
        if (hh == N) hh = 0;
        for (int i = head[u]; i; i = e[i].next) {
            int v = e[i].v;
            if (e[i].w && d[u] + e[i].c < d[v]) {
                d[v] = d[u] + e[i].c;
                pre[v] = i;
                incf[v] = min(incf[u], e[i].w);
                if (!vis[v]) {
                    q[tt++] = v;
                    vis[v] = true;
                    if (tt == N) tt = 0;
                }
            }
        }
    }
    return d[t] != INF;
}

void update() {
    int x = t;
```

```
    while (x != s) {
        int i = pre[x];
        e[i].w -= incf[t], e[i ^ 1].w += incf[t];
        x = e[i ^ 1].v;
    }
    maxflow += incf[t];
    cost += d[t] * incf[t];
}
```

2.12 KM

```
namespace KM{
    int n, va[N], vb[N], match[N], last[N];
    LL a[N], b[N], upd[N], w[N][N];
    bool dfs(int u, int fa) {
        va[u] = 1;
        for (int v = 1; v <= n; v++) {
            if (vb[v]) continue;
            if (a[u] + b[v] == w[u][v]) {
                vb[v] = 1, last[v] = fa;
                if (!match[v] || dfs(match[v], v)) {
                    match[v] = u; return true;
                }
            } else if (a[u] + b[v] - w[u][v] < upd[v])
                upd[v] = a[u] + b[v] - w[u][v], last[v] = fa;
        }
        return false;
    }
    void inline calc(int len, LL d[N][N]) {
        n = len;
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= n; j++) w[i][j] = d[i][j];
        for (int i = 1; i <= n; i++) {
            a[i] = -1e18, b[i] = 0;
            for (int j = 1; j <= n; j++)
                a[i] = max(a[i], w[i][j]);
        }
        for (int i = 1; i <= n; i++) {
            memset(va, 0, sizeof va);
```

```
            memset(vb, 0, sizeof vb);
            memset(upd, 0x3f, sizeof upd);
            int st = 0; match[0] = i;
            while (match[st]) {
                LL delta = 1e18;
                if (dfs(match[st], st)) break;
                for (int j = 1; j <= n; j++) {
                    if (!vb[j] && upd[j] < delta)
                        delta = upd[j], st = j;
                }
                for (int j = 1; j <= n; j++) {
                    if (va[j]) a[j] -= delta;
                    if (vb[j]) b[j] += delta;
                    else upd[j] -= delta;
                }
                vb[st] = true;
            }
            while (st) {
                match[st] = match[last[st]];
                st = last[st];
            }
        }
    }
}
```

2.13 有负圈/上下界费用流

```
// 有负圈 / 上下界
struct MCMF2{
    const int N = 205, M = 10005;
    const int INF = 0x3f3f3f3f;
    int n, m, s, t, maxflow, cost, d[N], incf[N], pre[N];
    int q[N], in, S, T;
    int head[N], a[N], numE = 1, a0, a1;
    bool vis[N];
    struct E{
        int next, v, w, c;
    } e[M << 2];
    void inline add(int u, int v, int w, int c) {
```

```
        e[++numE] = (E) { head[u], v, w, c };
        head[u] = numE;
    }
    void inline addE(int u, int v, int w, int c) {
        add(u, v, w, c), add(v, u, 0, -c);
    }
    bool spfa() {
        memset(vis, false, sizeof vis);
        memset(d, 0x3f, sizeof d);
        int hh = 0, tt = 1;
        q[0] = S; d[S] = 0; incf[S] = 2e9;
        while (hh != tt) {
            int u = q[hh++]; vis[u] = false;
            if (hh == N) hh = 0;
            for (int i = head[u]; i; i = e[i].next) {
                int v = e[i].v;
                if (e[i].w && d[u] + e[i].c < d[v]) {
                    d[v] = d[u] + e[i].c;
                    pre[v] = i;
                    incf[v] = min(incf[u], e[i].w);
                    if (!vis[v]) {
                        q[tt++] = v;
                        vis[v] = true;
                        if (tt == N) tt = 0;
                    }
                }
            }
        }
        return d[T] != INF;
    }
    void update() {
        int x = T;
        while (x != S) {
            int i = pre[x];
            e[i].w -= incf[T], e[i ^ 1].w += incf[T];
            x = e[i ^ 1].v;
        }
        maxflow += incf[T];
        cost += d[T] * incf[T];
```

```
    }

    void inline addEdge(int u, int v, int l, int d, int c) {
        a[v] += l, a[u] -= l;
        addE(u, v, d - l, c);
    }

    void inline work() {
        while (spfa()) update();
    }

    void inline ADD(int u, int v, int w, int c) {
        if (c >= 0) addEdge(u, v, 0, w, c);
        else a[v] += w, a[u] -= w, addEdge(v, u, 0, w, -c), a1 += c * w;
    }

    void inline solve() {
        for (int i = 1; i <= n; i++) {
            if (!a[i]) continue;
            if (a[i] > 0) addEdge(S, i, 0, a[i], 0);
            else addEdge(i, T, 0, -a[i], 0);
        }
        addEdge(T, S, 0, INF, 0);
        work();
        S = s, T = t;
        a1 += cost;
        maxflow = cost = 0;
        e[numE].w = e[numE - 1].w = 0;
        work();
        a0 += maxflow, a1 += cost;
    }
}
```

2.14 虚树

```
void insert(int x) {
    if (!top) { s[++top] = x; return; }
    int p = lca(x, s[top]);
    while (top > 1 && dep[s[top - 1]] >= dep[p]) e[s[top - 1]].pb(s[top]), top--;
```

```cpp
    if (s[top] != p) {
        e[p].pb(s[top]);
        s[top] = p;
    }
    s[++top] = x;
}


bool inline cmp(int x, int y) {
    return dfn[x] < dfn[y];
}
int inline build(vector<int> &A) {
    top = 0;
    sort(A.begin(), A.end(), cmp);
    for (int x: A) {
        insert(x);
    }
    for (int i = 1; i < top; i++)
        e[s[i]].pb(s[i + 1]);
    return s[1];
}
```

2.15 重链剖分 + LCA

```cpp
int sz[SZ], fa[SZ], dep[SZ], top[SZ], hson[SZ];

void dfs1(int u) {
    sz[u] = 1;
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (v == fa[u]) continue;
        dep[v] = dep[u] + 1, fa[v] = u;
        dfs1(v);
        sz[u] += sz[v];
        if (sz[v] > sz[hson[u]]) hson[u] = v;
    }
}

void dfs2(int u, int tp) {
```

```
    top[u] = tp;
    if (hson[u]) dfs2(hson[u], tp);
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (v == fa[u] || v == hson[u]) continue;
        dfs2(v, v);
    }
}

int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]]) swap(x, y);
        x = fa[top[x]];
    }
    if (dep[x] < dep[y]) swap(x, y);
    return y;
}
```

2.16 匈牙利

```
int match[N];
bool vis[N];

bool find(int u) {
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (vis[v]) continue;
        vis[v] = true;
        if (!match[v] || find(match[v])) {
            match[v] = u; return true;
        }
    }
    return false;
}
```

2.17 上下界网络流

```
// 上下界网络流
```

```
struct NF{
    int n, S, T, head[N], numE = 0, q[N], d[N], a[N], ans;

    struct E{
        int next, v, w;
    } e[M];

    void inline init(int len, int s, int t) {
        n = len, S = n + 1, T = n + 2, ans = 0;
        memset(head, 0, sizeof head);
        memset(a, 0, sizeof a);
        numE = 1;
    }

    void inline addEdge(int u, int v, int w) {
        e[++numE] = (E) { head[u], v, w };
        head[u] = numE;
    }

    void inline add(int u, int v, int c, int d) {
        a[v] += c, a[u] -= c;
        addEdge(u, v, d - c), addEdge(v, u, 0);
    }

    bool inline bfs() {
        memset(d, 0, sizeof d);
        int hh = 0, tt = 0; q[0] = S;
        d[S] = 1;
        while (hh <= tt) {
            int u = q[hh++];
            if (u == T) return true;
            for (int i = head[u]; i; i = e[i].next) {
                int v = e[i].v;
                if (e[i].w && !d[v]) {
                    d[v] = d[u] + 1;
                    q[++tt] = v;
                }
            }
```

```
    }
    return false;
}

int dinic(int u, int flow) {
    if (u == T) return flow;
    int rest = flow;
    for (int i = head[u]; i && rest; i = e[i].next) {
        int v = e[i].v;
        if (e[i].w && d[v] == d[u] + 1) {
            int k = dinic(v, min(rest, e[i].w));
            if (!k) d[v] = 0;
            e[i].w -= k, e[i ^ 1].w += k, rest -= k;
        }
    }
    return flow - rest;
}

void inline prework() {
    for (int i = 1; i <= n; i++)
        if (a[i] > 0) addEdge(S, i, a[i]), addEdge(i, S, 0), ans += a[i];
        else if (a[i] < 0) addEdge(i, T, -a[i]), addEdge(T, i, 0);
}

int inline run() {
    int res;
    addEdge(n, n - 1, INF);
    addEdge(n - 1, n, 0);
    while (bfs())
        while(res = dinic(S, INF)) ans -= res;
    if (ans) return -1;
    ans = e[numE].w;
    e[numE].w = e[numE - 1].w = 0;
    S = n - 1, T = n;
    while (bfs())
        while(res = dinic(S, INF)) ans += res;
    return ans;
}

```

```
} t;

int S = n + m + 1, T = n + m + 2;
t.init(T, S, T);
t.add(u, v, c, d);
t.prework();
output:: t.run()
```

2.18 $O(1)$ LCA

```
const int N = 5e5 + 5, L = 19;

int n, m, dfncnt, rt, st[L][N], Lg[N], dfn[N], d[N], fa[N];

vector<int> g[N];

void dfs0(int u) {
    st[0][dfncnt] = fa[u];
    dfn[u] = ++dfncnt;
    for (int v: g[u]) {
        if (v == fa[u]) continue;
        d[v] = d[u] + 1;
        fa[v] = u;
        dfs0(v);
    }
}

int inline cmp(int x, int y) {
    return d[x] < d[y] ? x : y;
}

void inline bd() {
    Lg[0] = -1;
    for (int i = 1; i <= n; i++)
        Lg[i] = Lg[i >> 1] + 1;
    for (int j = 1; j <= Lg[n]; j++)
        for (int i = 1; i + (1 << j) - 1 <= n; i++)
            st[j][i] = cmp(st[j - 1][i], st[j - 1][i + (1 << (j - 1))]);
}
```

```cpp

int inline lca(int x, int y) {
    if (x == y) return x;
    x = dfn[x], y = dfn[y];
    if (x > y) swap(x, y); --y;
    int k = Lg[y - x + 1];
    return cmp(st[k][x], st[k][y - (1 << k) + 1]);
}

void prework() {
    dfs0(rt);
    bd();
}

// Use lca(a, b)
```

2.19 点分治

```cpp
int val;

void findRoot(int u, int last, int &rt) {
    sz[u] = 1; int s = 0;
    for (int i = head[u]; i; i = e[i].next) {
        int v = e[i].v;
        if (st[v] || v == last) continue;
        findRoot(v, u, rt);
        sz[u] += sz[v], s = max(s, sz[v]);
    }
    s = max(s, S - sz[u]);
    if (s < val) val = s, rt = u;
}

void solve(int u) {
    if (st[u]) return;
    val = INF, findRoot(u, 0, u), st[u] = true;
    for (int i = head[u], j = 0; i; i = e[i].next) {
        int v = e[i].v;
        if (st[v]) continue;
        // Do sth
```

```
    }
    for (int i = head[u]; i; i = e[i].next) S = sz[e[i].v], solve(e[i].v);
}

S = n, solve(1);
```

3 Poly 多项式

3.1 1e18 多项式乘法

```
// 1e18 多项式乘法》。。。别用fft（mtt也不会写

#define I __int128_t
typedef vector<I> Poly;
const I P = 1945555039024054273ll, G = 5;
// p=1945555039024054273=27\times 2^{56}+1,g=5


I A[N], rev[N];
I lim = 1, len = 0;
LL W[19][N];

I inline power(I a, I b, I Mod = P) {
        I res = 1;
        while (b) {
                if (b & 1) res = res * a % Mod;
                a = a * a % Mod;
                b >>= 1;
        }
        return res;
}


void inline NTT(I c[], int lim, int o) {
        for (int i = 0; i < lim; i++)
                if (i < rev[i]) swap(c[i], c[rev[i]]);
        for (int k = 1, t = 0; k < lim; k <<= 1, t++) {
                for (int i = 0; i < lim; i += (k << 1)) {
```

```
                for (int j = 0; j < k; j++) {
                    I u = c[i + j], v = (I)c[i + k + j] * W[t][j] % P;
                    c[i + j] = u + v >= P ? u + v - P : u + v;
                    c[i + j + k] = u - v < 0 ? u - v + P : u - v;
                }
            }
        }
    if (o == -1) {
        reverse(c + 1, c + lim);
        I inv = power(lim, P - 2, P);
        for (int i = 0; i < lim; i++)
            c[i] = c[i] * inv % P;
    }
}

void inline setN(int n) {
    lim = 1, len = 0;
    while (lim < n) lim <<= 1, len++;
    for (int i = 0; i < lim; i++)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1));
}

Poly inline NTT(Poly a, int o) {
    int n = a.size();
    for (int i = 0; i < n; i++) A[i] = a[i];
    NTT(A, lim, o);
    a.clear();
    for (int i = 0; i < lim; i++) a.push_back(A[i]), A[i] = 0;
    return a;
}

Poly inline mul (Poly a, Poly b, int newn = -1) {
    if (newn == -1) newn = a.size() + b.size() - 1;
    setN(a.size() + b.size() - 1);
    Poly c = NTT(a, 1), d = NTT(b, 1);
    for (int i = 0; i < lim; i++) c[i] = (I)c[i] * d[i] % P;
    d = NTT(c, -1); d.resize(newn);
    return d;
}
```

```

// 用到的最大的 n
void inline init(int n) {
        setN(n);
        for (int k = 1, t = 0; k < lim; k <<= 1, t++) {
                I wn = power(G, (P - 1) / (k << 1));
                W[t][0] = 1;
                for (int j = 1; j < k; j++) W[t][j] = (I)W[t][j - 1] * wn % P;
        }
}

// --
```

3.2 正常多项式 + 线性递推

```
typedef vector<int> Poly;

#define pb push_back

const int N = 8e5 + 5, P = 998244353, G = 3;

int A[N], rev[N], mod, inv[N], fact[N], infact[N];
int lim = 1, len = 0, W[20][N];

int inline power(int a, int b, int Mod = P) {
        int res = 1;
        while (b) {
                if (b & 1) res = (LL)res * a % Mod;
                a = (LL)a * a % Mod;
                b >>= 1;
        }
        return res;
}

int Gi = power(G, P - 2, P), inv2 = power(2, P - 2, P);

void inline NTT(int c[], int lim, int o) {
        for (int i = 0; i < lim; i++)
                if (i < rev[i]) swap(c[i], c[rev[i]]);
```

```
	for (int k = 1, t = 0; k < lim; k <<= 1, t++) {
		for (int i = 0; i < lim; i += (k << 1)) {
			for (int j = 0; j < k; j++) {
				int u = c[i + j], v = (LL)c[i + k + j] * W[t][j] % P;
				c[i + j] = u + v >= P ? u + v - P : u + v;
				c[i + j + k] = u - v < 0 ? u - v + P : u - v;
			}
		}
	}
	if (o == -1) {
		reverse(c + 1, c + lim);
		int inv = power(lim, P - 2, P);
		for (int i = 0; i < lim; i++)
			c[i] = (LL)c[i] * inv % P;
	}
}

void inline setN(int n) {
	lim = 1, len = 0;
	while (lim < n) lim <<= 1, len++;
	for (int i = 0; i < lim; i++)
		rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1));
}

Poly inline NTT(Poly a, int o) {
	int n = a.size();
	for (int i = 0; i < n; i++) A[i] = a[i];
	NTT(A, lim, o);
	a.clear();
	for (int i = 0; i < lim; i++) a.push_back(A[i]), A[i] = 0;
	return a;
}

Poly inline mul (Poly a, Poly b, int newn = -1) {
	if (newn == -1) newn = a.size() + b.size() - 1;
	setN(a.size() + b.size() - 1);
	Poly c = NTT(a, 1), d = NTT(b, 1);
	for (int i = 0; i < lim; i++) c[i] = (LL)c[i] * d[i] % P;
	d = NTT(c, -1); d.resize(newn);
```

```
        return d;
}

// 用到的最大的 n
void inline init(int n) {
        setN(2 * n);
        for (int k = 1, t = 0; k < lim; k <<= 1, t++) {
                int wn = power(G, (P - 1) / (k << 1));
                W[t][0] = 1;
                for (int j = 1; j < k; j++) W[t][j] = (LL)W[t][j - 1] * wn % P;
        }
}

// f[0 ... n] 线性递推第 b 项
// g[1 ~ k] 为递推多项式

int inline LRS(int b, Poly f, Poly g) {
        int k = g.size() - 1;
        g[0] = 1;
        for (int i = 1; i <= k; i++) g[i] = (P - g[i]) % P;
        Poly h = mul(f, g, k);
        while (b) {
                Poly g2 = g;
                for (int i = 0; i < g2.size(); i += 2)
                        g2[i] = (P - g2[i]) % P;
                Poly t = mul(g2, g); g.clear();
                for (int i = 0; i < t.size(); i += 2)
                        g.pb(t[i]);
                t = mul(g2, h); h.clear();
                for (int i = (b & 1); i < t.size(); i += 2)
                        h.pb(t[i]);
                b >>= 1;
        }
        return (LL)h[0] * power(g[0], P - 2) % P;
}
```

4 字符串

4.1 AC 自动机

```
struct ACAutomation{
	int tr[SZ][26], nxt[SZ], idx, q[SZ];
	void inline insert(char s[]) {
		int p = 0;
		for (int j = 0; s[j]; j++) {
			int ch = s[j] - 'a';
			if(!tr[p][ch]) tr[p][ch] = ++idx;
			p = tr[p][ch];
		}
	}
	void build() {
		int hh = 0, tt = -1;
		for (int i = 0; i < 26; i++)
			if (tr[0][i]) q[++tt] = tr[0][i];
		while (hh <= tt) {
			int u = q[hh++];
			for (int i = 0; i < 26; i++) {
				int v = tr[u][i];
				if (!v) tr[u][i] = tr[nxt[u]][i];
				else nxt[v] = tr[nxt[u]][i], q[++tt] = v;
			}
		}
	}
}
```

4.2 KMP

```
struct KMP{
	int n, nxt[SZ];
	void inline build(char s[]) {
		n = strlen(s + 1);
		nxt[1] = 0;
		for (int i = 2, j = 0; i <= n; i++) {
			while (j && s[j + 1] != s[i]) j = nxt[j];
			if (s[j + 1] == s[i]) j++;
```

```
                nxt[i] = j;
            }
        }
        void inline match(char a[], int m) {
            for (int i = 1, j = 0; i <= m; i++) {
                while (j && s[j + 1] != a[i]) j = nxt[j];
                if (s[j + 1] == a[i]) j++;
                if (j == n) {
                    j = nxt[j];
                }
            }
        }
} kmp;
```

4.3 Manacher

```
// 中间添加 #
char s[N], g[N];

void change() {
        n = strlen(s + 1) * 2;
        g[0] = 0;
        for (int i = 1; i <= n; i++) {
                if (i % 2) g[i] = 1;
                else g[i] = s[i >> 1];
        }
        g[++n] = 1, g[n + 1] = 2;
        manacher();
}

void manacher() {
        int r = 0, mid = 0;
        for (int i = 1; i <= n; i++) {
                p[i] = i <= r ? min(r - i + 1, p[2 * mid - i]) : 1;
                while (g[i - p[i]] == g[i + p[i]]) ++p[i];
                if (i + p[i] - 1 > r) mid = i, r = i + p[i] - 1;
                ans = max(ans, p[i] - 1);
        }
}
```

4.4 SA

```
struct SA{
	int rk[SZ], sa[SZ], cnt[SZ], oldrk[SZ], id[SZ], n, m, p, height[SZ];
	bool inline cmp(int i, int j, int k) {
		return oldrk[i] == oldrk[j] && oldrk[i + k] == oldrk[j + k];
	}
	void inline build(char s[]) {
		n = strlen(s + 1), m = 221;
		for (int i = 1; i <= n; i++) cnt[rk[i] = s[i]]++;
		for (int i = 1; i <= m; i++) cnt[i] += cnt[i - 1];
		for (int i = n; i; i--) sa[cnt[rk[i]]--] = i;
		for (int w = 1; w < n; w <<= 1, m = p) {
			p = 0;
			for (int i = n; i > n - w; i--) id[++p] = i;
			for (int i = 1; i <= n; i++)
				if (sa[i] > w) id[++p] = sa[i] - w;
			for (int i = 1; i <= m; i++) cnt[i] = 0;
			for (int i = 1; i <= n; i++) cnt[rk[i]]++, oldrk[i] = rk[i];
			for (int i = 1; i <= m; i++) cnt[i] += cnt[i - 1];
			for (int i = n; i; i--) sa[cnt[rk[id[i]]]--] = id[i];
			p = 0;
			for (int i = 1; i <= n; i++) {
				rk[sa[i]] = cmp(sa[i], sa[i - 1], w) ? p : ++p;
			}
			if (p == n) break;
		}
		for (int i = 1; i <= n; i++) {
			int j = sa[rk[i] - 1], k = max(0, height[rk[i - 1]] - 1);
			while (s[i + k] == s[j + k]) k++;
			height[rk[i]] = k;
		}
	}
};
```

4.5 哈希 Hash

```
// 哈希

```

```
struct Hash{
        int b, P, p[N], h[N];
        int inline get(int l, int r){
            return (h[r] - (LL)h[l - 1] * p[r - l + 1] % P + P) % P;
        }
        void inline build(int n, int tb, int tp) {
                b = tb, P = tp;
                p[0] = 1;
            for(int i = 1; i <= n; i++){
                p[i] = (LL)p[i - 1] * b % P;
                h[i] = ((LL)h[i - 1] * b + s[i]) % P;
            }
        }
}
```

4.6 最小表示法

```
// 切记复制一倍到后面，最小表示法，返回开始下标
int inline minExp(int a[], int n) {
        int i = 1, j = 2;
        while (i <= n && j <= n) {
                int k;
                for (k = 0; k < n && a[i + k] == a[j + k]; k++);
                if (k == n) break;
                if (a[i + k] < a[j + k]) j += k + 1;
                else i += k + 1;
                if (i == j) i++;
        }
        return min(i, j);
}
```

4.7 Z 函数

```
// Z 函数
z[1] = n;
for (int i = 2, r = 0, j = 0; i <= n; i++) {
        if (i <= r) z[i] = min(r - i + 1, z[i - j + 1]);
        while (i + z[i] <= n && a[i + z[i]] == a[1 + z[i]]) z[i]++;
```

```
        if (i + z[i] - 1 > r) r = i + z[i] - 1, j = i;
}

for (int i = 1, r = 0, j = 0; i <= m; i++) {
        if (i <= r) p[i] = min(r - i + 1, z[i - j + 1]);
        while (i + p[i] <= m && b[i + p[i]] == a[1 + p[i]]) p[i]++;
        if (i + p[i] - 1 > r) r = i + p[i] - 1, j = i;
}
```

4.8 SAM

```
struct SAM{
        int idx, last;
        struct SAM_{
                int nxt[26], len, link;
        } t[N];
        void inline init() {
                last = idx = 1;
        }

        void inline extend(int c) {
                int x = ++idx, p = last; sz[x] = 1;
                t[x].len = t[last].len + 1;
                while (p && !t[p].nxt[c])
                        t[p].nxt[c] = x, p = t[p].link;
                if (!p) t[x].link = 1;
                else {
                        int q = t[p].nxt[c];
                        if (t[p].len + 1 == t[q].len) t[x].link = q;
                        else {
                                int y = ++idx;
                                t[y] = t[q], t[y].len = t[p].len + 1;
                                while (p && t[p].nxt[c] == q)
                                        t[p].nxt[c] = y, p = t[p].link;
                                t[q].link = t[x].link = y;
                        }
                }
                last = x;
        }
```

```
} t;
```

4.9 广义 SAM

```
struct GSAM{
	int idx, last;
	struct SAM{
		int ch[26], len, link;
	} t[N];
	void inline init() {
		last = idx = 1;
	}
	void inline insert(int c) {
		int p = last;
		if (t[p].ch[c]) {
			int q = t[p].ch[c];
			if (t[q].len == t[p].len + 1) last = q;
			else {
				int y = ++idx; t[y] = t[q];
				t[y].len = t[p].len + 1;
				while (p && t[p].ch[c] == q)
					t[p].ch[c] = y, p = t[p].link;
				t[q].link = y;
				last = y;
			}
			return;
		}
		int x = ++idx; t[x].len = t[p].len + 1;
		while (p && !t[p].ch[c]) t[p].ch[c] = x, p = t[p].link;
		int q, y;
		if (!p) t[x].link = 1;
		else {
			q = t[p].ch[c];
			if (t[q].len == t[p].len + 1) t[x].link = q;
			else {
				int y = ++idx; t[y] = t[q];
				t[y].len = t[p].len + 1;
				while (p && t[p].ch[c] == q)
					t[p].ch[c] = y, p = t[p].link;
```

```cpp
                        t[q].link = t[x].link = y;
                        last = y;
                    }
                }
                last = x;
            }
} t;
```

4.10 回文自动机

```cpp
// 回文自动机
struct PAM{
    int n, ch[N][26], fail[N], len[N], sz[N], idx = -1, last;
    char s[N];
    void clr() {
        n = 0;
        for (int i = 0; i <= idx; i++) {
            sz[i] = len[i] = fail[i] = 0;
            for (int j = 0; j < 26; j++)
                ch[i][j] = 0;
        }
        idx = -1;
        last = 0;
    }

    int newNode(int x) { len[++idx] = x; return idx; }
    int getFail(int x) {
        while (s[n - len[x] - 1] != s[n]) x = fail[x];
        return x;
    }
    int insert(char c) {
        int k = c - 'a';
        s[++n] = c;
        int p = getFail(last), x;
        if (!ch[p][k]) {
            x = newNode(len[p] + 2);
            fail[x] = ch[getFail(fail[p])][k];
            ch[p][k] = x, sz[x] = 1 + sz[fail[x]];
        } else x = ch[p][k];
```

```
        last = x;
        return x;
    }
    void bd() {
        // -1:idx jigen
        newNode(0), newNode(-1);
        s[0] = '$', fail[0] = 1, last = 0;
    }
} pam;
```

5 数学

5.1 单位根反演

$$[n|k] = \frac{1}{n}\sum_{i=1}^{n-1} w_n^{ik}$$

5.2 积分表

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{1}$$

$$\int \frac{1}{x} dx = \ln|x| \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \tag{15}$$

$$\begin{aligned} \int \frac{x}{ax^2+bx+c} dx = {} & \frac{1}{2a} \ln |ax^2+bx+c| \\ & - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \end{aligned} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a} dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{20}$$

$$\int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x\pm a}} dx = \frac{2}{3}(x \mp 2a)\sqrt{x\pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b} dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\begin{aligned}\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \Big[&(2ax+b)\sqrt{ax(ax+b)} \\ &-b^2 \ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\Big]\end{aligned} \tag{27}$$

$$\begin{aligned}\int \sqrt{x^3(ax+b)} dx = &\left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} \\ &+ \frac{b^3}{8a^{5/2}} \ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\end{aligned} \tag{28}$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2\pm a^2}dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right| \tag{36}$$

$$\begin{aligned}\int \sqrt{ax^2+bx+c}dx &= \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} \\ &+ \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right|\end{aligned} \tag{37}$$

$$\begin{aligned}\int x\sqrt{ax^2+bx+c} &= \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c}\right. \\ &\times\left(-3b^2+2abx+8a(c+ax^2)\right) \\ &\left.+3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right)\end{aligned} \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln \left| 2ax+b+2\sqrt{a(ax^2+bx+c)} \right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax dx = x \ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2} \left(\ln ax\right)^2 \tag{43}$$

$$\int \ln(ax+b)dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{44}$$

$$\int \ln(x^2+a^2) \text{ dx} = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2) \text{ dx} = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right) dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2-b^2x^2\right) dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2-\frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{49}$$

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \tag{50}$$

$$\int \sqrt{x} e^{ax} dx = \frac{1}{a}\sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}\left(i\sqrt{ax}\right), \quad \text{where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{51}$$

$$\int x e^{x} dx = (x-1) e^{x} \tag{52}$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax} \tag{53}$$

$$\int x^2 e^{x} dx = \left(x^2 - 2x + 2\right) e^{x} \tag{54}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax} \tag{55}$$

$$\int x^3 e^{x} dx = \left(x^3 - 3x^2 + 6x - 6\right) e^{x} \tag{56}$$

$$\int x^n e^{ax} \mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \mathrm{d}x \tag{57}$$

$$\int x^n e^{ax} \, \mathrm{d}x = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \quad \text{where } \Gamma(a,x) = \int_x^{\infty} t^{a-1} e^{-t} \, \mathrm{d}t \tag{58}$$

$$\int e^{ax^2} \, \mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2} \, \mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int x e^{-ax^2} \, \mathrm{dx} = -\frac{1}{2a} e^{-ax^2} \tag{61}$$

$$\int x^2 e^{-ax^2}\ \mathrm{dx} = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a}\cos ax \tag{63}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{64}$$

$$\begin{aligned} &\int \sin^n ax dx = \\ &-\frac{1}{a}\cos ax\ \ {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \end{aligned} \tag{65}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{66}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax \tag{67}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{68}$$

$$\begin{aligned} \int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times \\ {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \end{aligned} \tag{69}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{70}$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{71}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{72}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x \tag{73}$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{74}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a}\cos^3 ax \tag{75}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{76}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{77}$$

$$\int \tan ax dx = -\frac{1}{a}\ln\cos ax \tag{78}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax \tag{79}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{80}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{81}$$

$$\int \sec x dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{82}$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax \tag{83}$$

$$\int \sec^3 x \,\mathrm{dx} = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{84}$$

$$\int \sec x \tan x dx = \sec x \tag{85}$$

$$\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x \tag{86}$$

$$\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{87}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{88}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax \tag{89}$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{90}$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{91}$$

$$\int \sec x \csc x dx = \ln|\tan x| \tag{92}$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x \sin x \tag{93}$$

$$\int x \cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{94}$$

$$\int x^2 \cos x dx = 2x \cos x + \left(x^2 - 2\right) \sin x \tag{95}$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3} \sin ax \tag{96}$$

$$\begin{aligned}\int x^n cosxdx = -\frac{1}{2}(i)^{n+1} \left[\Gamma(n+1, -ix)\right.\\ \left.+(-1)^n \Gamma(n+1, ix)\right]\end{aligned} \tag{97}$$

$$\begin{aligned}\int x^n cosaxdx = \frac{1}{2}(ia)^{1-n} \left[(-1)^n \Gamma(n+1, -iax)\right.\\ \left.-\Gamma(n+1, ixa)\right]\end{aligned} \tag{98}$$

$$\int x \sin x dx = -x \cos x + \sin x \tag{99}$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \tag{100}$$

$$\int x^2 \sin x dx = \left(2 - x^2\right) \cos x + 2x \sin x \tag{101}$$

$$\int x^2 \sin ax dx = \frac{2 - a^2x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \tag{102}$$

$$\int x^n \sin x dx = -\frac{1}{2}(i)^n \left[\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)\right] \tag{103}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) \tag{104}$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \tag{105}$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) \tag{106}$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2+b^2} e^{bx} (a \sin ax + b \cos ax) \tag{107}$$

$$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \tag{108}$$

$$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \tag{109}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a} \sinh ax \tag{110}$$

$$\int e^{ax} \cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a \cosh bx - b \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{111}$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax \tag{112}$$

$$\int e^{ax} \sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b \cosh bx + a \sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{113}$$

$$\int e^{ax} \tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)} {}_2F_1\left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad - \dfrac{1}{a} e^{ax} {}_2F_1\left[\dfrac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{114}$$

$$\int \tanh ax\,dx = \frac{1}{a}\ln\cosh ax \tag{115}$$

$$\begin{aligned}\int \cos ax \cosh bx dx = \frac{1}{a^2+b^2}\left[a \sin ax \cosh bx\right.\\ \left.+b\cos ax \sinh bx\right]\end{aligned} \tag{116}$$

$$\begin{aligned}\int \cos ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b \cos ax \cosh bx+\right.\\ \left.a\sin ax \sinh bx\right]\end{aligned} \tag{117}$$

$$\begin{aligned}\int \sin ax \cosh bx dx = \frac{1}{a^2+b^2}\left[-a \cos ax \cosh bx+\right.\\ \left.b\sin ax \sinh bx\right]\end{aligned} \tag{118}$$

$$\begin{aligned}\int \sin ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b \cosh bx \sin ax-\right.\\ \left.a\cos ax \sinh bx\right]\end{aligned} \tag{119}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a}\left[-2ax+\sinh 2ax\right] \tag{120}$$

$$\begin{aligned}\int \sinh ax \cosh bx dx = \frac{1}{b^2-a^2}\left[b \cosh bx \sinh ax\right.\\ \left.-a\cosh ax \sinh bx\right]\end{aligned} \tag{121}$$

5.3 扩域

```
// 扩域
struct C{
    int x, y;
    // x + y * sqrt(o);
```

```
};

int o = 2;

// fn = Aa^n + Bb^n

int inline power(int a, int b) {
    int ret = 1;
    while (b) {
        if (b & 1) ret = 1ll * ret * a % P;
        a = 1ll * a * a % P;
        b >>= 1;
    }
    return ret;
}




int mod(int x) {
    return x >= P ? x - P : x;
}

C operator + (const C &a, const C &b) {
    return (C) { mod(a.x + b.x), mod(a.y + b.y) };
};

C operator * (const C &a, const C &b) {
    C c;
    c.x = (1ll * a.x * b.x + 1ll * a.y * b.y % P * o) % P;
    c.y = (1ll * a.x * b.y + 1ll * a.y * b.x) % P;
    return c;
};

C operator * (const C &a, const int &b) {
    C c;
    c.x = 1ll * a.x * b % P;
    c.y = 1ll * a.y * b % P;

```

```
    return c;
};


C inline power(C a, int b) {
    C ret = (C) { 1, 0 } ;
    while (b) {
        if (b & 1) ret = ret * a;
        a = a * a;
        b >>= 1;
    }
    return ret;
}

C operator / (const C &a, const C &b) {
    C c, d;
    c = a;
    d = b;
    d.y = mod(P - d.y);
    c = c * d;
    int I = (((LL)b.x * b.x - (LL)b.y * b.y * o) % P + P) % P;
    I = power(I, P - 2);
    c = c * I;
    return c;
};
```

5.4 原根

```
// 原根 / 封装不太好


int n, D, phi[N], primes[N], tot, d[N], len;
int ans[N], cnt;

bool st[N], pr[N];

void inline init() {
        phi[1] = 1, pr[2] = pr[4] = true;
        for (int i = 2; i < N; i++) {
```

```
            if (!st[i]) primes[tot++] = i, phi[i] = i - 1;
            for (int j = 0; i * primes[j] < N; j++) {
                st[i * primes[j]] = true;
                if (i % primes[j] == 0) {
                    phi[i * primes[j]] = phi[i] * primes[j];
                    break;
                }
                phi[i * primes[j]] = phi[i] * (primes[j] - 1);
            }
        }
        for (int i = 1; i < tot; i++) {
            for (LL j = primes[i]; j < N; j *= primes[i]) pr[j] = true;
            for (LL j = 2 * primes[i]; j < N; j *= primes[i]) pr[j] = true;
        }
}


void inline factor(int m) {
        len = 0;
        for (int i = 0; i < tot && primes[i] * primes[i] <= m; i++) {
            int j = primes[i];
            if (m % j == 0) {
                d[len++] = j;
                while (m % j == 0) m /= j;
            }
        }
        if (m > 1) d[len++] = m;
}

int inline power(int a, int b, int P) {
        int res = 1;
        while (b) {
            if (b & 1) res = (LL)res * a % P;
            a = (LL)a * a % P;
            b >>= 1;
        }
        return res;
}

```

```
bool inline check(int x, int P) {
        if (power(x, phi[P], P) != 1) return false;
        for (int i = 0; i < len; i++)
                if(power(x, phi[P] / d[i], P) == 1) return false;
        return true;
}

// 输入 P，返回最小原根

int inline get(int P) {
        for (int i = 1; i < P; i++)
                if (check(i, P)) return i;
        return 0;
}
//-
```

5.5 $O(n)$ 预处理逆元

```
void inline preInv(int n) {
    inv[1] = 1;
    for (int i = 2; i <= n; i++)
        inv[i] = ((LL)P - P / i) * inv[P % i] % P;
}
```

5.6 Exgcd 扩展欧里几得

```
LL inline exgcd(LL a, LL b, LL &x, LL &y) {
        if (b == 0) {
                x = 1, y = 0;
                return a;
        }
        LL d = exgcd(b, a % b, y, x);
        y -= a / b * x;
        return d;
}
```

5.7 扩展中国剩余定理 exCRT

```cpp
// 扩展中国剩余定理 exCRT
typedef pair<LL, LL> PLL;

LL gcd(LL a, LL b) {
	return b ? gcd(b, a % b) : a;
}

LL exgcd(LL a, LL b, LL &x, LL &y) {
	if (!b) {
		x = 1, y = 0;
		return a;
	}
	LL d = exgcd(b, a % b, y, x);
	y -= a / b * x;
	return d;
}

LL mul(LL x, LL y, LL P) {
	return (__int128)x * y % P;
//	return x * y % P;
}


// x mod m = a (m1, a1) (m2, a2) return x


PLL inline merge(PLL A, PLL B) {
	LL a1 = A.fi, b1 = A.se;
	LL a2 = B.fi, b2 = B.se;
	LL a = a1 / gcd(a1, a2) * a2;
	LL x, y;
	LL d = exgcd(a1, a2, x, y);
	assert((b2 - b1) % d == 0);
	x = mul(x, (b2 - b1) / d, a);
	if (x < 0) x += a;
	LL o = mul(x, a1, a) + b1;
	if (o >= a) o -= a;
	PLL c = mp(a, o);
	return c;
```

```
}
```

5.8 BSGS

```
// BSGS

unordered_map<int, int> mp;

int BSGS(int a, int b, int P) {
	int t = sqrt(P) + 1; mp.clear(); b %= P;
	for (int j = 0, s = b; j < t; j++)
		mp[s] = j, s = (LL)s * a % P;
	a = power(a, t, P);
	for (int i = 1, s = 1; i <= t; i++) {
		s = (LL)s * a % P;
		if (mp.count(s) && i * t - mp[s] >= 0)
			return i * t - mp[s];
	}
	return -1;
}

int exBSGS(int a, int b, int P) {
	int x, y, d, A = 1, k = 0;
	while ((d = gcd(a, P)) > 1) {
		if (b % d) return -1;
		b /= d, P /= d, k++, A = (LL)A * (a / d) % P;
		if (A == b) return k;
	}
	exgcd(A, P, x, y); x = (x % P + P) % P;
	int res = BSGS(a, (LL)b * x % P, P);
	return res == -1 ? -1 : res + k;
}
```

5.9 杜教筛

```
const int N = 5000005, S = 3000;
const LL INF = 9e18;

```

```cpp
LL p1[N], p2[S], m1[N], m2[S];

int n, primes[N], tot;

bool vis[N];

// 杜教筛 phi
LL s1(int x) {
	if (x < N) return p1[x];
	else if (p2[n / x] != INF) return p2[n / x];
	LL res = x * (x + 1ll) / 2;
	for (LL l = 2, r; l <= x; l = r + 1) {
		r = x / (x / l);
		res -= (r - l + 1) * s1(x / l);
	}
	return p2[n / x] = res;
}

// 杜教筛 mu

LL s2(int x) {
	if (x < N) return m1[x];
	else if (m2[n / x] != INF) return m2[n / x];
	LL res = 1;
	for (LL l = 2, r; l <= x; l = r + 1) {
		r = x / (x / l);
		res -= (r - l + 1) * s2(x / l);
	}
	return m2[n / x] = res;
}
```

5.10 Min25

```cpp
// Min25

int inv2 = power(2, P - 2), inv6 = power(6, P - 2);

// 求 g_k 函数: <= x 的和
int inline getS(LL x, int k) {
```

```
        if (k == 1) return (x % P * (x % P + 1ll) % P * inv2 + P - 1ll)  % P;
        if (k == 2) return (P - 1ll + x % P * (x % P + 1ll) % P * (2ll * x % P + 1) % P * inv6) %
             P;
}

int inline getV(LL x, int k) {
        if (k == 1) return x % P;
        if (k == 2) return (LL)x % P * x % P;
}

bool vis[M];

int primes[M], tot;

void inline linear(int n) {
        for (int i = 2; i <= n; i++) {
                if (!vis[i]) primes[++tot] = i;
                for (int j = 1; primes[j] <= n / i; j++) {
                        vis[i * primes[j]] = true;
                        if (i % primes[j] == 0) break;
                }
        }
}

// 预处理 g_k 处所有 n / i 形式的质数前缀和
struct MP1{
        int m, g[M], pos1[M], pos2[M], len, id;
        LL n, d[M];
        int inline getPos(LL x) {
                return x <= m ? pos1[x] : pos2[n / x];
        }
        void inline add(LL v) {
                d[++len] = v;
                g[len] = getS(v, id);
                if (v <= m) pos1[v] = len;
                else pos2[n / v] = len;
        }
        void build(LL sum, int t) {
                m = sqrt(n = sum); id = t;
```

```
		for (LL i = 1, j; i <= n; i = j + 1) {
			LL v = n / i; j = n / v;
			if (v <= m) break;
			add(v);
		}
		for (int i = m; i; i--) add(i);
		for (int i = 1; i <= tot && (LL)primes[i] * primes[i] <= n; i++) {
			LL pr = primes[i];
			for (int j = 1; j <= len && pr * pr <= d[j]; j++) {
				int k = getPos(d[j] / pr);
				g[j] = (g[j] - (LL)getV(pr, id) * (g[k] - g[getPos(primes[i - 1])] + P) % P + P) % P;
			}
		}
	}
	int inline s(LL x) { return g[getPos(x)]; }
} t1, t2;

int inline get(LL x) {
	return (t2.s(x) - t1.s(x) + P) % P;
}

int inline calc(LL x) {
	return x % P * (x % P - 1ll + P) % P;
}

void inline add(int &x, int y) {
	(x += y) %= P;
}

int inline s(LL n, int t) {
	if (primes[t] >= n) return 0;
	int ans = (get(n) - get(primes[t]) + P) % P;
	for (int i = t + 1; i <= tot && (LL)primes[i] * primes[i] <= n; i++) {
		int pr = primes[i];
		LL v = pr;
		for (int j = 1; v <= n; v = v * pr, j++) {
			add(ans, (LL)calc(v) * ((j != 1) + s(n / v, i)) % P);
		}
```

```
        }
        return ans;
}
```

5.11 FMT / FWT

```
// FMT / FWT

void inline OR(int n, int a[], int o) {
        for (int w = 1; w < n; w <<= 1)
                for (int i = 0; i < n; i += (w << 1))
                        for (int j = 0; j < w; j++)
                                add(a[i + j + w], o * a[i + j]);
}

void inline AND(int n, int a[], int o) {
        for (int w = 1; w < n; w <<= 1)
                for (int i = 0; i < n; i += (w << 1))
                        for (int j = 0; j < w; j++)
                                add(a[i + j], o * a[i + j + w]);
}


// 反向传 1/2
void inline XOR(int n, int a[], int o) {
        for (int w = 1; w < n; w <<= 1)
                for (int i = 0; i < n; i += (w << 1))
                        for (int j = 0; j < w; j++) {
                                int u = a[i + j], v = a[i + j + w];
                                a[i + j] = ((LL)u + v + P) * o % P;
                                a[i + j + w] = ((LL)u - v + P) * o % P;
                        }
}
```

5.12 子集卷积

```
// 子集卷积

```

```
void inline SubConv(int n, int a[], int b[], int c[]) {
	for (int i = 0; i < (1 << n); i++) {
		f[get(i)][i] = a[i];
		g[get(i)][i] = b[i];
	}
	for (int i = 0; i <= n; i++)
		OR(1 << n, f[i], 1), OR(1 << n, g[i], 1);
	for (int i = 0; i <= n; i++)
		for (int j = 0; j <= i; j++)
			for (int k = 0; k < (1 << n); k++)
				add(h[i][k], (LL)f[j][k] * g[i - j][k] % P);
	for (int i = 0; i <= n; i++) OR(1 << n, h[i], -1);
	for (int i = 0; i < (1 << n); i++) c[i] = h[get(i)][i];
}
```

6 数据结构

6.1 线段树合并与分裂

```

// 线段树合并与分裂

struct Seg{
	int idx;

	struct T{
		int l, r;
		LL v;
	} t[N * 22];

	void inline pushup(int p) {
		t[p].v = t[t[p].l].v + t[t[p].r].v;
	}

	void build(int &p, int l, int r) {
		if (!p) p = ++idx;
		if (l == r) {
			t[p].v = a[l];
```

```cpp
			return;
		}
		int mid = (l + r) >> 1;
		build(t[p].l, l, mid);
		build(t[p].r, mid + 1, r);
		pushup(p);
	}

	void change(int &p, int &q, int l, int r, int x, int y) {
		if (x <= l && r <= y) {
			q = p; p = 0;
			return;
		}
		if (!q) q = ++idx;
		int mid = (l + r) >> 1;
		if (x <= mid) change(t[p].l, t[q].l, l, mid, x, y);
		if (mid < y) change(t[p].r, t[q].r, mid + 1, r, x, y);
		pushup(p); pushup(q);
	}

	void merge(int &p, int &q, int l, int r) {
		if (!p) return;
		if (!q) { q = p; return; }
		if (l == r) { t[q].v += t[p].v; return; }
		int mid = (l + r) >> 1;
		merge(t[p].l, t[q].l, l, mid);
		merge(t[p].r, t[q].r, mid + 1, r);
		pushup(q);
	}

	void insert(int &p, int l, int r, int x, int k) {
		if (!p) p = ++idx;
		if (l == r) { t[p].v += k; return ; }
		int mid = (l + r) >> 1;
		if (x <= mid) insert(t[p].l, l, mid, x, k);
		else insert(t[p].r, mid + 1, r, x, k);
		pushup(p);
	}

```

```
	LL query(int p, int l, int r, int x, int y) {
		if (!p) return 0;
		if (x <= l && r <= y) return t[p].v;
		int mid = (l + r) >> 1; LL res = 0;
		if (x <= mid) res += query(t[p].l, l, mid, x, y);
		if (mid < y) res += query(t[p].r, mid + 1, r, x, y);
		return res;
	}

	int kth(int p, int l, int r, int k) {
		if (l == r) return l;
		int mid = (l + r) >> 1;
		if (k <= t[t[p].l].v) return kth(t[p].l, l, mid, k);
		else return kth(t[p].r, mid + 1, r, k - t[t[p].l].v);
	}
}
```

6.2 ST 表

```
// ST 表
struct ST{
	void inline STPrework(int n) {
		g[0] = -1;
		for (int i = 1; i <= n; i++)
			f[i][0] = a[i], g[i] = g[i >> 1] + 1;
		for (int j = 1; j <= g[n]; j++)
			for (int i = 1; i + (1 << j) - 1 <= n; i++)
				f[i][j] = max(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
	}

	int inline query(int l, int r) {
		int k = g[r - l + 1];
		return max(f[l][k], f[r - (1 << k) + 1][k]);
	}
}
```

6.3 Fhq Treap

```cpp
// 用来动态开点的池
struct T{
        int l, r, val, rnd, sz;
} t[SZ];
int idx;

struct Fhq{
        int rt;
        void pushup(int p) {

        }
        // value(A) < value(B)
        int merge(int A, int B) {
                if (!A || !B) return A + B;
                else if(t[A].rnd > t[B].rnd) {
                        t[A].r = merge(t[A].r, B);
                        pushup(A);
                        return A;
                } else {
                        t[B].l = merge(A, t[B].l);
                        pushup(B);
                        return B;
                }
        }

        // 按值分裂
        void split(int p, int k, int &x, int &y) {
                if (!p) x = y = 0;
                else {
                        if (t[p].val <= k)
                        x = p, split(t[p].r, k, t[p].r, y);
                        else y = p, split(t[p].l, k, x, t[p].l);
                        pushup(p);
                }
        }
        int getNode(int val) {
                t[++idx] = (T) { 0, 0, val, rand(), 1 };
                return idx;
        }
```

```cpp

	void insert(int val) {
		int x, y;
		split(rt, val, x, y);
		rt = merge(merge(x, getNode(val)), y);
	}

	int get(int l, int r) {
		int x, y, z;
		split(rt, l - 1, x, y);
		split(y, r, y, z);
		int res = t[y].N;
		rt = merge(x, merge(y, z));
		return res;
	}

	void del(int val) {
		int x, y, z;
		split(rt, val - 1, x, y);
		split(y, val, y, z);
		y = merge(t[y].l, t[y].r);
		rt = merge(x, merge(y, z));
	}
}
```

6.4 线段树

```cpp
// 普通线段树

struct Seg{
	#define ls (p << 1)
	#define rs (p << 1 | 1)
	void inline pu(int p) {

	}

	void inline pd(int p) {

	}
```

```cpp

        void bd(int p, int l, int r) {
                if(l == r) {
                        return;
                }
                int mid = (l + r) >> 1;
            bd(ls, l, mid);
            bd(rs, mid + 1, r);
            pu(p);
        }
        void chg(int p, int l, int r, int x, int y, int k, int c) {
            if(x <= l && r <= y) {
                return ;
            }
            int mid = (l + r) >> 1;
            pd(p);
            if(x <= mid) chg(ls, l, mid, x, y, k, c);
            if(mid + 1 <= y) chg(rs, mid + 1, r, x, y, k, c);
            pu(p);
        }

        int qry(int p, int l, int r, int x, int y) {
            if(x <= l && r <= y) return ?;
            int mid = (l + r) >> 1, s = 0;
            pd(p);
            if(x <= mid) s += qry(ls, l, mid, x, y);
            if(mid + 1 <= y) s += qry(rs, mid + 1, r, x, y);
            return s % P;
        }
}
```

6.5 主席树

```cpp
// 主席树
struct PersisSeg{
        struct T{
                int l, r;
                LL v;
        } t[SZ];
```

```
	int rt[SZ], idx;

	void inline update(int &p, int q, int l, int r, int x, int k) {
		t[p = ++idx] = t[q];
		t[p].v += k;
		if (l == r) return;
		int mid = (l + r) >> 1;
		if (x <= mid) update(t[p].l, t[q].l, l, mid, x, k);
		else update(t[p].r, t[q].r, mid + 1, r, x, k);
	}

	LL inline query(int p, int l, int r, int x, int y) {
		if (!p || x > y) return 0;
		if (x <= l && r <= y) return t[p].v;
		int mid = (l + r) >> 1; LL res = 0;
		if (x <= mid) res += query(t[p].l, l, mid, x, y);
		if (mid < y) res += query(t[p].r, mid + 1, r, x, y);
		return res;
	}
}
```

6.6 树状数组：区间加区间求和

```
// 区间加 区间查的树状数组
struct exBIT{
	BIT t1, t2;
	int n;
	void inline init(int len, int a[]) {
		n = len;
		for (int i = 1; i <= n; i++)
			b[i] = a[i] - a[i - 1];
		t1.init(n, b);
		for (int i = 1; i <= n; i++) b[i] *= i;
		t2.init(n, b);
	}
	void inline add(int l, int r, LL c) {
		t1.add(l, c), t1.add(r + 1, -c);
		t2.add(l, c * l), t2.add(r + 1, -c * (r + 1));
```

```
        }
        LL inline ask(int x) {
                return (x + 1) * t1.ask(x) - t2.ask(x);
        }
        LL inline ask(int x, int y) { return ask(y) - ask(x - 1); }
};
```

6.7 LCT

```
struct LCT{
        #define get(x) (ch[fa[x]][1] == x)
        #define isRoot(x) (ch[fa[x]][0] != x && ch[fa[x]][1] != x)
        #define ls ch[p][0]
        #define rs ch[p][1]

        int ch[N][2], fa[N], mx[N], w[N], rev[N];

        void inline pushup(int p) {

        }

        void inline pushdown(int p) {
                if (rev[p]) { swap(ls, rs), rev[ls] ^= 1, rev[rs] ^= 1, rev[p] = 0; }
        }

        void inline rotate(int x) {
                int y = fa[x], z = fa[y], k = get(x);
                if (!isRoot(y)) ch[z][get(y)] = x;
                ch[y][k] = ch[x][!k], fa[ch[y][k]] = y;
                ch[x][!k] = y, fa[y] = x, fa[x] = z;
                pushup(y); pushup(x);
        }

        void inline update(int p) {
                if (!isRoot(p)) update(fa[p]);
                pushdown(p);
        }

        void inline splay(int p) {
```

```
		update(p);
		for (int f = fa[p]; !isRoot(p); rotate(p), f = fa[p])
			if (!isRoot(f)) rotate(get(p) == get(f) ? f : p);
	}

	void inline access(int x) {
		for (int p = 0; x; p = x, x = fa[x]) {
			splay(x), ch[x][1] = p, pushup(x);
		}
	}

	int inline find(int p) {
		access(p), splay(p);
		while (ls) pushdown(p), p = ls;
		splay(p);
		return p;
	}

	void inline makeRoot(int x) {
		access(x), splay(x), rev[x] ^= 1;
	}

	void inline split(int x, int y) {
		makeRoot(x), access(y), splay(y);
	}

	void inline link(int x, int y) {
		makeRoot(x), fa[x] = y;
	}

	void inline cut(int x, int y) {
		split(x, y);
		ch[y][0] = 0, fa[x] = 0;
		pushup(y);
	}

}
```

6.8 左偏树

```
// 左偏树
struct LeftistTree{
	struct T{
	    int l, r, v, d, f;
	    // l, r 表示左右儿子, v 表示值
	    // d 表示从当前节点到最近叶子节点的距离, f 表示当前节点的父亲
	} t[SZ];

	int find(int x) {
	    return t[x].f == x ? x : t[x].f = find(t[x].f);
	}

	int merge(int x, int y) { // 递归合并函数
	    if (!x || !y) return x + y;
	    if (t[x].v > t[y].v || (t[x].v == t[y].v && x > y)) swap(x, y);
	    rs = merge(rs, y);
	    if (t[ls].d < t[rs].d) swap(ls, rs);
	    t[x].d = t[rs].d + 1;
	    return x;
	}

	int work(int x, int y) { // 合并 x, y 两个堆。
	    if (x == y) return 0;
	        if (!x || !y) return t[x + y].f = x + y;
	    if (t[x].v > t[y].v || (t[x].v == t[y].v && x > y)) swap(x, y);
	    t[x].f = t[y].f = x;
	    merge(x, y); return x;
	}

	void del(int x) {
	    t[x].f = work(ls, rs), t[x].v = -1;
	}
}
```

6.9 李超树

```
// 李超树
```

```

struct LC{
	struct Tree{
		int l, r;
		Line v;
	} t[N << 2];
	LL inline calc(Line e, LL x) {
		return e.k * x + e.b;
	}
	int idx, rt;
	void inline clr() {
		idx = 0; rt = 0;
	}
	// 这里写法非常简洁的原因是，让计算机人工帮你判断了单调 / 需要 upd 的位置，事实上只会走一边。
	void inline ins(int &p, int l, int r, Line e) {
		if (!p) {
			t[p = ++idx] = (Tree) { 0, 0, e };
			return;
		}
		int mid = (l + r) >> 1;
		if (calc(t[p].v, mid) > calc(e, mid)) swap(e, t[p].v);
		if (calc(e, l) < calc(t[p].v, l)) ins(t[p].l, l, mid, e);
		if (calc(e, r) < calc(t[p].v, r)) ins(t[p].r, mid + 1, r, e);
	}
	LL ask(int p, int l, int r, int x) {
		if (!p) return INF;
		if (l == r) return calc(t[p].v, x);
		int mid = (l + r) >> 1; LL ret = calc(t[p].v, x);
		if (x <= mid) chkMin(ret, ask(t[p].l, l, mid, x));
		else chkMin(ret, ask(t[p].r, mid + 1, r, x));
		return ret;
	}

} ;
```

6.10 回滚莫队

```
// 莫队
```

```
int pos[N], L[N], R[N], t;

struct Q {
	int l, r, id;
	bool operator < (const Q &b) const {
		if (pos[l] != pos[b.l]) return pos[l] < pos[b.l];
		return r < b.r;
	}
} q[N];

t = sqrt(n);
for (int i = 1; i <= n; i++) {
	pos[i] = (i - 1) / t + 1;
	if (!L[pos[i]]) L[pos[i]] = i;
	R[pos[i]] = i;
}

sort(q + 1, q + 1 + m);

// 回滚

int l = 1, r = 0, last = -1;
for (int i = 1; i <= m; i++) {
	if (pos[q[i].l] == pos[q[i].r]) {
		// 块内暴力
		continue;
	}
	if (pos[q[i].l] != last) {
		// 新的左块
		res = 0, top = 0, r = R[pos[q[i].l]], l = r + 1;
		last = pos[q[i].l];
	}
	while (r < q[i].r) {
		++r;
		// insert r
	}
	int bl = l, tp = res; // 记录
	while (l > q[i].l) {
```

```
                --l;
                // insert l
        }
        // 恢复
        ans[q[i].id] = res; res = tp;
}
```

6.11 动态凸包

```
typedef pair<LL, LL> PII;
typedef set<PII>::iterator SIT;
typedef set<PII> SI;

PII operator - (const PII &a, const PII &b) {
        return mp(a.x - b.x, a.y - b.y);
}

LL inline cross(PII a, PII b) {
        return a.x * b.y - a.y * b.x;
}

LL inline cross(PII a, PII b, PII c) {
        PII u = b - a, v = c - a;
        return cross(u, v);
}

// 动态凸包

struct Hull {
        SI su, sd;
        bool inline query(SI &s, PII u, int o) {
                SIT l = s.upper_bound(u), r = s.lower_bound(u);
                if (r == s.end() || l == s.begin()) return false;
                l--;
                return cross(*l, u, *r) * o <= 0;
        }
        void inline insert(SI &s, PII u, int o) {
                if (query(s, u, o)) return;
                SIT it = s.insert(u).first;
```

```
                while (1) {
                        SIT mid = it;
                        if (mid == s.begin()) break; --mid;
                        SIT l = mid;
                        if (l == s.begin()) break; --l;
                        if (cross(*l, *mid, u) * o >= 0) break;
                        s.erase(mid);
                }
                while (1) {
                        SIT mid = it; ++mid;
                        if (mid == s.end()) break;
                        SIT r = mid; ++r;
                        if (r == s.end()) break;
                        if (cross(u, *mid, *r) * o >= 0) break;
                        s.erase(mid);
                }
        }
        void inline ins(PII u) {
                insert(su, u, 1), insert(sd, u, -1);
        }
        int inline chk(PII u) {
                return query(su, u, 1) && query(sd, u, -1);
        }
} t;
```

6.12 珂朵莉树

```
// 珂朵莉树？？

struct E{
    int l, r, v;
    bool operator < (const E &b) const {
        return r < b.r;
    }
};

set<E> s;

typedef set<E>::iterator SIT;
```

```cpp

void split(int i) {
    SIT u = s.lower_bound((E){ 0, i + 1, 0 });
    if (u == s.end()) return;
    if (u -> r > i && u -> l <= i) {
        E t = *u;
        s.erase(u);
        s.insert((E){ t.l, i, t.v });
        s.insert((E){ i + 1, t.r, t.v });
    }
}

void inline ins(int l, int r, int v) {
        split(l - 1), split(r);
    while (1) {
        SIT u = s.lower_bound((E){ 0, l, 0, 0 });
        if (u == s.end()) break;
        if (u -> r > r) break;

        s.erase(u);
    }
    s.insert((E){ l, r, v });
}
```

6.13 HashMap

```cpp
// Hashmap


struct E{
        int next, v, w;
};

const int MOD = 999997;

struct Hash{
        E e[MOD];
        int numE, head[MOD];
        void inline clear() {
```

```cpp
        for (int i = 1; i <= numE; i++)
            head[e[i].v % MOD] = 0;
        numE = 0;
    }
    int &operator[] (int x) {
        int t = x % MOD;
        for (int i = head[t]; i; i = e[i].next) {
            if (e[i].v == x) {
                return e[i].w;
            }
        }
        e[++numE] = (E) { head[t], x, 0 };
        head[t] = numE;
        return e[numE].w;
    }
} t
```

6.14 全局平衡二叉树

```cpp
// 全局平衡二叉树


vector<int> g[N];

int lim[N];

bool vis[N];

int fa[N], sz[N], son[N], d[N];

struct Mat{
    // 定义矩阵的地方
    Mat operator * (const Mat &b) const {

    }
};

void dfs1(int u) {
    sz[u] = 1;
```

```
	for (int v: g[u]) {
		if (v == fa[u]) continue;
		fa[v] = u;
		d[v] = d[u] + 1;
		dfs1(v);
		sz[u] += sz[v];
		if (sz[v] > sz[son[u]]) son[u] = v;
	}
}

int len, b[N], val[N], rt[N], ps[N];

struct T{
	int l, r, f;
	Mat v, s;
} t[N];

int inline getM(int x, int y) {
	int mn = 2e9, p = -1;
	for (int i = x; i <= y; i++)
		if (chkMin(mn, max(val[i - 1] - val[x - 1], val[y] - val[i]))) p = i;
	return p;
}

#define ls t[p].l
#define rs t[p].r

void pu(int p) {
	if (ls && rs) t[p].s = t[rs].s * t[p].v * t[ls].s;
	else if (ls) t[p].s = t[p].v * t[ls].s;
	else if (rs) t[p].s = t[rs].s * t[p].v;
	else t[p].s = t[p].v;
}

void inline bd(int &p, int l, int r, int F) {
	if (l > r) return;
	int mid = getM(l, r);
	p = b[mid];
	t[p].f = F;
```

```cpp
        bd(ls, l, mid - 1, p), bd(rs, mid + 1, r, p);
        pu(p);
}

void inline remake(int u) {
        // 更新 u 的子树了，更新矩阵
}

void inline updF(int v) {
        // u 的轻儿子 v 变了，更新轻儿子对自己的影响
}

void inline bd(int tp) {
        int x = tp; vector<int> z;
        while (x) z.pb(x), x = son[x];
        for (int u: z) {
                for (int v: g[u])
                        if (v != fa[u] && v != son[u]) bd(v), updF(v);
                remake(u);
        }
        len = 0;
        for (int v: z) b[++len] = v, val[len] = sz[v] - sz[son[v]];
        for (int i = 1; i <= len; i++) val[i] += val[i - 1];
        bd(rt[tp], 1, len, 0);
        ps[rt[tp]] = tp;
}

void inline sop(int x) {
        while (x) {
                remake(x); int p = x, y = 0;
                while (p) y = ps[p], pu(p), p = t[p].f;
                if (!fa[y]) break;
                updF(y), x = fa[y];
        }
}
```

7 计算几何

7.1 Basic

```
const double eps = 1e-4;
typedef pair<double, double> PDD;
struct Line{
    PDD s, t;
};

int inline cmp(double x, double y) {
    if (fabs(x - y) < eps) return 0;
    return x < y ? -1 : 1;
}

double inline cross(PDD a, PDD b) { return a.fi * b.se - a.se * b.fi; }
PDD operator - (const PDD &a, const PDD &b) { return make_pair(a.fi - b.fi, a.se - b.se); }
PDD operator + (const PDD &a, const PDD &b) { return make_pair(a.fi+ b.fi, a.se+ b.se); }
PDD operator / (const PDD &a, double b) { return make_pair(a.fi / b, a.se / b); }
PDD operator * (const PDD &a, double b) { return make_pair(a.fi * b, a.se * b); }
double inline area(PDD a, PDD b, PDD c) { return cross(b - a, c - a); }
double inline dot(PDD a, PDD b) { return a.fi * b.fi + a.se * b.se; }
double inline len(PDD a) { return sqrt(dot(a, a)); }
double inline project(PDD a, PDD b, PDD c) { return dot(b - a, c - a) / len(b - a); }
double inline dist(PDD a, PDD b) { return sqrt((a.fi - b.fi) * (a.fi - b.fi) + (a.se - b.se) * (a
    .se - b.se)); }
// 顺时针转 x
PDD inline rotate(PDD a, double x) { return make_pair ( cos(x) * a.fi + sin(x) * a.se, -sin(x) *
    a.fi + cos(x) * a.se ); }
PDD inline norm(PDD a) { return a / len(a); }
double angle(PDD a, PDD b) {
    return acos(dot(a, b) / len(a) / len(b));
}
int sign(double fi) {
    if (fabs(fi) < eps) return 0;
    if (fi < 0) return -1;
    return 1;
}
```

7.2 点到线段距离

```
LD getD(PDD a, PDD u, PDD v) {
        LD w = min(dis(a, u), dis(a, v));
        LD c = dot(a - u, v - u);
        LD t = dis(u, v);
        c /= t;
        if (cmp(c, 0) >= 0 && cmp(c, t) <= 0) {
                LD z = norm(u - a);
                LD val = sqrt(z - c * c);
                w = val;
        }
        return w;
}
```

7.3 线段交

```
bool segInter(PDD a1, PDD a2, PDD b1, PDD b2) {
    double c1 = cross(a2 - a1, b1 - a1), c2 = cross(a2 - a1, b2 - a1);
    double c3 = cross(b2 - b1, a2 - b1), c4 = cross(b2 - b1, a1 - b1);
    return sign(c1) * sign(c2) <= 0 && sign(c3) * sign(c4) <= 0;
}

bool cmp2 (const Line &a, const Line &b) {
    double A = getAngle(a), B = getAngle(b);
    if (A != B) return A < B;
    else return area(a.s, a.t, b.t) < 0;
}

PDD getInter(PDD p, PDD v, PDD q, PDD w) {
    PDD u = p - q;
    double t = cross(w, u) / cross(v, w);
    return make_pair(p.fi + t * v.fi, p.se + t * v.se);
}

PDD getInter(Line a, Line b) { return getInter(a.s, a.t - a.s, b.s, b.t - b.s); }

bool inline Right(Line a, Line b, Line c) {
    PDD u = getInter(b, c);
```

```
    return area(a.s, a.t, u) <= 0;
}
```

7.4 凸包

```
void inline andrew() {
    sort(p + 1, p + 1 + n);
    for (int i = 1; i <= n; i++) {
        while (top > 1 && area(p[s[top - 1]], p[s[top]], p[i]) < 0) {
            if (area(p[s[top - 1]], p[s[top]], p[i]) <= 0) st[s[top--]] = false;
            else top--;
        }
        st[i] = true, s[++top] = i;
    }
    st[1] = false;
    for (int i = n; i; i--) {
        if (!st[i]) {
            while (top > 1 && area(p[s[top - 1]], p[s[top]], p[i]) <= 0)
                st[s[top--]] = false;
            st[i] = true, s[++top] = i;
        }
    }
    for (int i = 0; i < top; i++) s[i] = s[i + 1];
    top--;
}
```

7.5 半平面交

```
struct Line{
    PDD s, t;
    int id;
} e[N];

// 半平面交
double HPI() {
    sort(e + 1, e + 1 + n, cmp2);
    int hh = 0, tt = -1;
    for (int i = 1; i <= n; i++) {
```

```
        if (i && getAngle(e[i]) == getAngle(e[i - 1])) continue;
        while (hh < tt && Right(e[i], e[q[tt - 1]], e[q[tt]])) tt--;
        while (hh < tt && Right(e[i], e[q[hh]], e[q[hh + 1]])) hh++;
        q[++tt] = i;
    }
    while (hh < tt && Right(e[q[hh]], e[q[tt - 1]], e[q[tt]])) tt--;
    while (hh < tt && Right(e[q[tt]], e[q[hh]], e[q[hh + 1]])) hh++;
    q[++tt] = q[hh];
    tot = 0;
    for (int i = hh; i < tt; i++)
        p[++tot] = getInter(e[q[i]], e[q[i + 1]]);
    double res = 0;
    for (int i = 1; i < tot; i++)
        res += area(p[1], p[i], p[i + 1]);
    return res / 2;
}
```

7.6 最小圆覆盖

```
Point inline getCircle(Point a, Point b, Point c) {
    return Inter((a + b) / 2, rotate(b - a, PI / 2), (a + c) / 2, rotate(c - a, PI / 2));
}

// 最小圆覆盖

void inline minCircle(PDD a[]) {
    random_shuffle(a + 1, a + 1 + n);
    double r = 0; Point u = a[1];
    for (int i = 2; i <= n; i++) {
        if (cmp(r, len(u - a[i])) == -1) {
            r = 0, u = a[i];
            for (int j = 1; j < i; j++) {
                if (cmp(r, len(u - a[j])) == -1) {
                    r = len(a[i] - a[j]) / 2, u = (a[i] + a[j]) / 2;
                    for (int k = 1; k < j; k++) {
                        if (cmp(r, len(u - a[k])) == -1) {
                            u = getCircle(a[i], a[j], a[k]), r = len(a[i] - u);
                        }
                    }
```

```
                }
            }
        }
    }
}
```

7.7 自适应辛普森积分

```
// 自适应辛普森积分
double inline f(double fi) {
    return ?;
}
double inline s(double l, double r) {
    double mid = (l + r) / 2;
    return (r - l) * (f(l) + 4 * f(mid) + f(r)) / 6;
}

double inline asr(double l, double r) {
    double mid = (l + r) / 2, v = s(l, r);
    double a = s(l, mid), b = s(mid, r);
    if (fabs(a + b - v) < eps) return v;
    else return asr(l, mid) + asr(mid, r);
}
```

7.8 极角排序

```
// https://codeforces.com/contest/1284/problem/E 的怨念 不丢精度的极角排序

LL inline cross(PII x, PII y) {
    return 1ll * x.fi * y.se - 1ll * x.se * y.fi;
}

int inline quad(PII x) {
    if (x.fi >= 0 && x.se >= 0) return 1;
    if (x.fi <= 0 && x.se >= 0) return 2;
    if (x.fi <= 0 && x.se <= 0) return 3;
    if (x.fi >= 0 && x.se <= 0) return 4;
    return 0;
```

```
}
```

7.9 Int 下凸包 + 闵可夫斯基和

```
// PII andrew + mincowf


LL operator * (PII a, PII b) {
    return (LL)a.fi * b.se - (LL)a.se * b.fi;
}

PII operator + (PII a, PII b) {
    return mp(a.fi + b.fi, a.se + b.se);
}

PII operator - (PII a, PII b) {
    return mp(a.fi - b.fi, a.se - b.se);
}

LL dot (PII a, PII b) {
    return (LL)a.fi * a.se + (LL)b.fi * b.se;
}

vector<PII> inline andrew(vector<PII> a) {
    int n = a.size();
    top = 0;
    sort(a.begin(), a.end());
    for (int i = 0; i < n; i++) {
        while (top > 1 && (a[i] - a[s[top - 1]]) * (a[s[top]] - a[s[top - 1]]) > 0) {
            vis[s[top--]] = 0;
        }
        vis[i] = 1, s[++top] = i;
    }
    vis[0] = 0;
    for (int i = n - 1; i >= 0; i--) {
        if (!vis[i]) {
            while (top > 1 && (a[i] - a[s[top - 1]]) * (a[s[top]] - a[s[top - 1]]) > 0)
                vis[s[top--]] = 0;
            vis[i] = 1, s[++top] = i;
```

```cpp
        }
    }
    --top;
    vector<PII> ret;
    for (int i = 1; i <= top; i++) ret.pb(a[s[i]]);
    for (int i = 0; i < n; i++) vis[i] = 0;
    return ret;
}

// 有

vector<PII> calc(vector<PII> a, vector<PII> b) {
    vector<PII> c;
    c.pb(a[0] + b[0]);
    vector<PII> dx, dy;
    for (int i = 1; i < a.size(); i++) dx.pb(a[i] - a[i - 1]);
    dx.pb(a[0] - a.back());
    for (int i = 1; i < b.size(); i++) dy.pb(b[i] - b[i - 1]);
    dy.pb(b[0] - b.back());
    int i = 0, j = 0;
    while (i < dx.size() && j < dy.size()) {
        if (dx[i] * dy[j] > 0)
            c.pb(c.back() + dx[i++]);
        else if (dx[i] * dy[j] == 0 && c.size() > 1) {
            // 共线放一起不然是错的！！！！
            if (dot(c.back() - c[c.size() - 2], dx[i]) > 0)
                c.pb(c.back() + dx[i++]);
            else c.pb(c.back() + dy[j++]);
        } else {
            c.pb(c.back() + dy[j++]);
        }
    }
    while (i < dx.size()) c.pb(c.back() + dx[i++]);
    while (j < dy.size()) c.pb(c.back() + dy[j++]);
    assert(c.back() == c[0]);
    c.pop_back();
    return c;
}
```