ORIE 5730 - OPTIMISATION MODELLING IN FINANCE

Multi-style rotation strategies using Augmented Black Litterman

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Assigned by:

Prof. James Renegar

Submitted by:

Yeon Soo Jenny Jee (yj275)

Niharika Dalsania (nd322)

Sukriti Kumar (sk3335)

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Abstract

This study uses the S&P500 constituents to make an active portfolio using a multi style rotation strategy. The strategies that have been considered are market, size, value and momentum investing. Augmented Black Litterman model has been used to incorporate our views on the expected returns on these portfolios.

We have used the market premium for market factor, market capitalisation for size factor, PE ratio for value factor and for momentum factor the past 12 months (barring the last month's return) have been used. Weighing each of these factor using Markowitz, we obtain a portfolio and then compare its returns with the benchmark ETFs for each of the strategy used. The cumulative returns show the superior performance of our portfolio compared to each individual factor portfolio as well as each of the benchmarks over the test time period.

1 Introduction

Over the years, the aim of active portfolio management has been to consistently beat market and other benchmarks. It has been observed that the value, size and momentum strategies obtain superior results. However, the timing of superior returns of each of these strategies is different and hence rotating between these strategies would theoretically be able to fetch better returns. This is also because these strategies keep moving in and out of favor because of various economic and financial risk factors. The market also will not allow a strategy to perform consistently well since its alpha will deteriorate because of the market forces. It thus makes sense to have a multi-style rotating strategy. The main aim of our study is to formulate active equity strategies based on market, size, value and momentum factor.

2 Background

The traditional Markowitz portfolio optimisation (Harry Markowitz, 1952) involves taking a group of assets to form a portfolio and constructing vector of expected returns as well as its corresponding covariance matrix. This then fed into conventional mean-variance optimiser to get the optimal weights of assets. Although this sounds perfect in theory, studies have shown that portfolios built on this basis generally fetch very poor returns. A major reason for this is the poor estimate of future expected returns. The Black-Litterman model (Black, F., Litterman, R., 1992) tries to overcome this limitation by allowing to smartly construct the vector for expected returns using prior knowledge. It allows investors to incorporate their personal views on expected returns, to build augmented returns, and resolve the issue of input-sensitivity and estimation error maximization, hence resulting in intuitive and diversified portfolios as compared to traditional mean variance optimisation.

Black Litterman model is based on the idea of a *view* - an adjusted forecast of future returns obtained after including personal insights. Although typically used to optimise multi-asset strategies, Black Litterman has alternative useful applications in algorithmic trading by optimally assigning weights to signals (trigger for some action in financial markets) rather than assets. In this project, we treat the signals or the different factor strategies as assets. Multiple factors are found to be predictive of future price movements and it is possible that we have multiple signals on the same asset. In this situation, we construct a portfolios of signals from each factor, where each signal pertains to a small basket of stocks and try to assign optimal weights to these signals, while assigning equal weights for the stocks in their respective baskets.

The following subsections looks at the factors considered in this study and then explains the mathematical details of Markowitz Model and Black Litterman Model.

2.1 Factor variables

The 4 factors considered for this study are market, size, value and momentum.

For market factor investing, we use the excess market premium over risk free rate. This is from the Capital Asset Pricing Model (Sharpe, 1964).

For size investing, Market capitalisation has been used. This has been included because of the historical tendency of the stocks of firms with smaller market capitalisation to outperform the stocks of the firms with larger market capitalisation.

For value investing, Price to earnings (PE) ratio has been used. This measure helps in determining the market value of a stock compared to its earnings, to distinguish between value stocks and growth stocks.

For momentum factor investing, monthly price momentum has been used. Price momentum is calculated as a price return over a period (Leivo and Pätäri, 2011). For our model, we have considered the monthly momentum where the lags vary from 2 months to 12 months (Jegadeesh and Titman, 1993). This is because the latest returns tend to show reversal or contrarian effect. So to avoid it, the first month's lagged returns have been removed. Momentum factor is used because it is a measure that accounts for persistence of the stock returns. This also signifies a weak form of Efficient Market Hypothesis.

All these factors are from the three factor Fama French model.

2.2 Markowitz Model

This is the traditional mean variance portfolio optimiser. The model is expressed as an optimization problem with the following objective and constraint:

$$\min\ w^T \Sigma w$$
$$\text{s.t. } e^T w = 1$$

Here, w is weight vector containing weights assigned to each factor signals, and

$x^T \Sigma x$ is the variance that we try to minimize.

Note that Σ is the covariance matrix of the factor portfolio returns.

2.3 Black Litterman Model

Using this model, we incorporate our views to build augmented returns and then we update factor portfolio returns before weighing factor signals by Markowitz as discussed above. The estimated returns and covariance matrix updated by Black Litterman are given by the following:

$$E[R] = (\tau\Sigma^{-1} + P^T\Omega^{-1}P)^{-1}(\tau\Sigma^{-1}\pi + P^T\Omega^{-1}Q)$$
$$\Sigma' = (\tau\Sigma^{-1} + P^T\Omega^{-1}P)^{-1}$$

Here,

$E[R]$ is the updated estimated returns;

τ is a scalar representing uncertainty ratio of our views;

Σ is the covariance matrix of augmented returns;

P is a mapping matrix to identify the stocks involved in a building a specific factor view;

Ω is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view;

π is the implied equilibrium return vector which is given by $\pi = \lambda \Sigma w_{mkt}$ where λ is the risk aversion coefficient and w_{mkt} is the market capitalization weight of the assets;

Q is the view vector;

Σ' is the updated covariance matrix;

This updated covariance is then passed into Markowitz model to get optimal weights for each factor signal.

3 Dataset

We have used all equities which are constituents of S&P 500 index. For this analysis, monthly data from 2000 to 2020 has been used. We have let the universe of stocks grow and not restricted it to only the ones which are present from the start of 2000.

3.1 Train-Validation-Test Data Split

We then divided the dataset into 3 splits namely, train, validation and test.

1. The train time frame is used to find loadings for each factor strategy using Linear Regression.

2. The validation time frame is used to find the misalignment in actual returns in that time frame and the forecasted returns obtained in that time frame using the factor loadings obtained above. These misalignments are used to learn a pattern using polynomial/exponential regression of how these misalignments might look like in future.

3. The test time frame is divided into two subparts: validation-test time frame and actual-test time frame. The validation-test time frame is used to forecast misalignments as well as forecast returns and form our views that is to be passed into Black Litterman, followed by Markowitz. We look at the portfolio return in this validation-test time to validate/adjust our trading strategy accordingly. The actual-test time frame is now finally used to test the adjusted strategy against the benchmarks.

First, we keep all data from January 2018 to December 2019 untouched as the test data. During this time frame, January 2018 to December 2018 becomes the validation-test data while January 2019 to December 2019 becomes the actual-test data. Next, of the remaining data prior to January 2018, we consider 80% of the available history of each company as the train data for that company and the remaining 20% of the companies' history forms the part of validation set. Now this means that the validation time frame for each company may start at a different time instance. So, for each factor, to have a *uniform validation time frame* for building views, we use the time period from earliest time the validation time period started for one company to 2018-01 as the validation time frame.

3.2 Data Sources

The company specific data of market cap for size, PE ratio for value has been sourced from Bloomberg. Market factor has been sourced from Fama French's website and the index levels of benchmarks have been sourced from yahoo finance.

4 Data Preprocessing

We consider using data of monthly frequency keeping in mind that re-balancing a portfolio more frequently than on monthly basis might be unrealistic. The market factor obtained from Fama French website already has monthly data. For value, size and momentum factor variables, we have take a mean of daily levels in a month. In case of missing information, the monthly levels have then been front filled.

For the dependent variable, the monthly returns of each company is similarly computed by taking mean of the daily prices of the companies in a month and computing the returns using the formula:

$r_t = (p_t - p_{t-1})/p_{t-1}$

where r_t is the monthly return

p_t is the monthly price at instance t

p_t is the monthly price at instance t-1

For momentum factor, the lagged returns of past 2 to 12 months have been used.

5 Methodology

In this section, we look at the steps involved in initiating the active portfolio construction process.

5.1 Constructing factor Mimicking Portfolio

As the first step of our portfolio construction process, we generate the factor returns of each of the individual factors.

5.1.1 Linear Regression for factor loadings

This is done by running a linear regression model on the training time frame first to get the factor loadings (β) for each of the factors for each of the companies. Linear regression to find factor loadings in run for a subset of companies only - the ones which have at least 45 months of data for returns.

$E(R_i - R_f) = R_f + \beta(factor)$

where R_i is the return of the company i, and

R_f is the risk free rate of return

We then use these factor loadings to forecast the returns for the validation time frame. This gives us the forecasted factor returns for each of the four factors for each of the companies.

5.1.2 Long-Short Portfolio

For creating the factor mimicking portfolios, for each factor, we construct long short portfolios using the forecasted returns for each of the companies. For each factor, at each time instance, for creation of a long-short portfolio, we consider the top 10% stocks and the bottom 10% stocks (on the basis of forecasted returns) as the stocks which will be longed and shorted in that factor portfolio. Note that, the stocks are equally weighted in these portfolios.

Figure 1: Misalignments in Factors

5.2 Misalignment in actual and forecasted returns

To build our *views* on each factor, we compute the misalignment between the actual portfolio returns and the forecasted portfolio returns during the validation time frame. This misalignment is simply given as follows:

$$Misalignment = actual\ returns_p - forecasted\ returns_p$$

where $actual\ returns_p$ is the actual returns of the portfolio p,
$forecasted\ returns_p$ is the forecasted returns of the portfolio p chosen by a particular factor.

This misalignment between the actual and the forecasted returns are then used to forecast misalignment during validation-test time frame.

5.2.1 Polynomial/ Exponential Regression

To forecast misalignments for the validation-test time frame, we first try to fit polynomial/exponential/sinusoidal models on the misalignments in the validation time frame.

As the independent variables, we have used the past six lags of the misalignments. We split the validation time frame into 80% and 20%. The first 80% of the validation time frame is used to fit the models. We use polynomial regression upto degree d to fit misalignments from each of the factors. We also fit exponential and sinusoidal model on these misalignments. The best model (i.e. the one with lowest MSE on the remaining 20% of the data in the validation time frame) out of these models is chosen for forecasting misalignments independently from each of the factors.

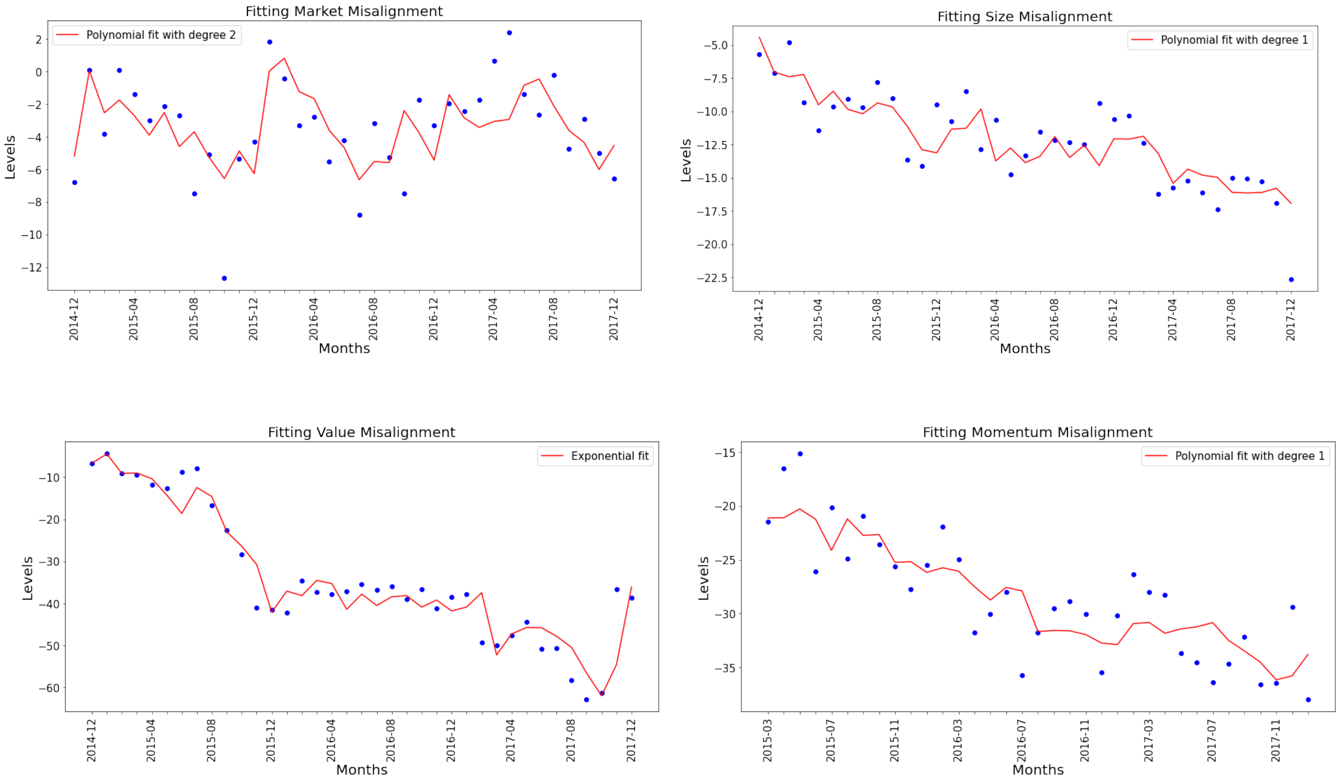

Figure 2: Polynomial and Exponential fit on the validation time frame

6 Validation Results

We now step by step look at how our portfolio is constructed, weighted and evaluated. First we forecast returns followed by forecasting misalignments. These forecasted misalignments are used to get the augmented returns which is then used for updating covariance matrix using Black Litterman. The updated covariance matrix now is used for weighing of trading signals. We will now delve deeper into these steps.

6.1 Forecasting Returns

For each factor, using the factor loadings from Linear Regressions, we forecast returns for each factor, for each company in the validation-test time frame.

6.2 Forecasting misalignments and augmented returns

For forecasting the misalignments on the validation-test time frame, we use the model with the best MSE. In our case, for the market factor, this model is a polynomial model with degree 2. For the size factor, the best model turns out to be the exponential model. For value factor and momentum factor, a linear model is chosen.

Once, we have the forecasted misalignments we add them to the forecasted returns of the portfolios for the validation-test time frame to get the augmented returns (Note, factor mimicking portfolios are constructed at each time instance in validation-test time frame again). This is because we initially defined misalignment as the difference of forecasted portfolio return from actual portfolio return. So adding this misalignment to the forecasted portfolio return gives the augmented returns, which should give a close proxy to how actual portfolio return should look like. These augmented returns for each factor is what we call our *views* for that factor.

6.3 Updating covariance matrix using Black Litterman

Having, all the forecasts available, we revisit the updated covariance matrix using Black Litterman. Note that from here on, N = n+f, where f is the number of factor so f = 4 in our case. Also, n = n1+n2+n3+n4 where n1 is the number of stocks chosen by the portfolio formed by market factor, n2 is the number of stocks chosen by the portfolio formed by size factor, n3 is the number of stocks chosen by the portfolio of formed by value factor and n4 is the number of stocks chosen by the portfolio formed by momentum factor. Note that the same stocks might be chosen by different factors, i.e. that same stocks might be counted in n1, n2, n3 and n4, but we treat them as different stocks and simply consider it more than once in the total n stocks if needed. This is because of the assumption that no factors has any correlation with the other ones using the assumption of linear regression that no multi-collinearity exists in the Fama French three factor model. Now, looking at the updated covariance matrix at each time instance:

$$\Sigma'_{[N\times N]} = (\tau\Sigma^{-1} + P^T\Omega^{-1}P)^{-1}$$

Taking a close look at each component used here, including its dimensions:

- τ is a scalar representing uncertainty ratio of our views. This is ideally obtained from expert judgement. It is constant for all factors, all time instances throughout. We can take an average of the uncertainty we have for each of the factors. Suppose, we are 95% certain about our market views, 90% about our value views, 90% about size views and 95% about momentum views, then τ will (100 - 92.5)/100 = 0.075 in this case.

- $\Sigma_{[N\times N]}$ is the covariance matrix of augmented returns which is given by:

$$\Sigma_{[N\times N]} = \begin{bmatrix} \Sigma_{[n\times n]} & B_{[n\times f]}\Sigma_{[f\times f]} \\ (B_{[n\times f]}\Sigma_{[f\times f]})^T & \Sigma_{[f\times f]} \end{bmatrix}$$

  Looking at each subcomponent of this matrix:

  - $\Sigma_{[n\times n]}$ is the covariance matrix of returns of n stocks until this time instance (use actual returns until test time frame, then use the forecasted returns).

  - $B_{[n\times f]}$ is the matrix of factor loadings with factor loadings of stocks in market portfolio in first n1 rows in column 1, factor loadings of stocks in size portfolio in next n2 rows in column 2, factor loadings of stocks in value portfolio in next n3 rows in column 3 and factor loadings of stocks in momentum portfolio in next n4 rows in column 4. Other entries in matrix remains zero. Note that there is a single factor loading for each stock using market, value and size factor, but for momentum factor, there are 11 factor loadings from the lagged terms and there exponential weighted average is chosen to get a single number representation of the factor loading with the rationale being that most recent factor is given the most weight/importance. All other entries in matrix are zero.

  - $\Sigma_{[f \times f]}$ is the covariance matrix of our views until this time instance (forecast returns for time instances before test time frame, use these forecasted returns until test time frame, then use the augmented returns as the views).

- $P_{[f \times N]}$ is a mapping matrix of 0s and 1s depicting what stocks corresponding to each factor are chosen in the factor mimicking portfolios. The first row will have 1s in first n1 columns, the second row 1s in next n2 columns, the third row will have 1s in next n3 columns and the fourth row will have 1s in next n4 columns. In order to map which stocks belong to which factor portfolio, there is 1 in row 1 column n+1, 1 in row 2 column n+2, 1 in row 3 column n+3 and 1 in row 4 column n+4. All other entries in matrix are zero.

- $\Omega_{[f \times f]}$ is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view. Suppose as we initially considered, we are 95% certain about our market views, 90% about our size views, 90% about value views and 95% about momentum views. Then the view uncertainty for market factor is 0.05*(market view). Similarly 0.1*(size view), 0.1*(value view) and 0.05*(momentum view). Then find a covariance matrix from these views and set non-diagonal values as zero because we assume each factor view to be independent, so the view estimation errors in factors are independent too.

Having all these components, we now get the updated covariance matrix from Black Litterman at each time instance that we can pass through Markowitz. As we saw above, it has a dimension $[N \times N]$, where N = n+f, we can potentially weigh each factor portfolio as well as the n stocks i.e. n1, n2, n3, n4 stocks within each factor portfolio respectively. But since we are considering equal weighted stocks here, we move ahead with only weighing the factor signals, which was our initial goal. So the covariance matrix to be passed in Markowitz is $\Sigma_{[f \times f]}$.

6.4 Weighing signals using Markowitz

We pass the above discussed $\Sigma_{[f \times f]}$ through Markowitz mean variance portfolio using cvxpy. The weights thus obtained are the weights of the portfolios built using the four factors.

6.5 Portfolio Returns of Validation

The weights of each factor portfolio obtained above are applied to the returns of factor portfolios built at each time instance in validation-test time frame. On the validation-test time frame, the cumulative expected portfolio return from January 2018 till December 2018 turn out to be -100.52%. We use this validation-test time frame to effectively fine-tune the hyperparameters of our model/strategy so as to improve its expected performance on out of sample actual-test time frame. Seeing the results, we decide to reverse the positions of the strategies; taking short position if we were taking long position initially and vice versa. This can be done by just reversing the weights obtained from the Markowitz model.

7 Conclusion

Finally, we test the results of our model on the actual-test time frame (i.e. from January 2019 to December 2019). The cumulative returns of our portfolio during this time frame is 62.71% i.e. if $1 was invested in our portfolio at the start of January 2019, the value of the investment at the end of December 2019 would have been $1.63.

7.1 Comparison to Factor Benchmarks

These results were them compared with factor benchmarks to see if our strategy results in superior performance. Next for benchmarks, we have factor based ETFs, the cumulative returns for which have been given in the table below:

Name of Portfolios	Cumulative Returns
S&P 500 core ETF Market Factor	23.99%
iShares MSCI USA Size Factor ETF	20.37%
iShares MSCI USA Value Factor ETF	17.70%
SPDR S&P 1500 Momentum Tilt ETF	22.66%
Our portfolio	62.71%

Table 1: Benchmark ETFs

Figure 3 shows the evolution of the cumulative returns of the portfolio constructed vs the benchmark ETFs that we chose. From the figure, it is clearly visible that our portfolio performs quite well at the start of the year 2019 but the cumulative returns slow done later in the year. Very high volatility is observed through the figure.

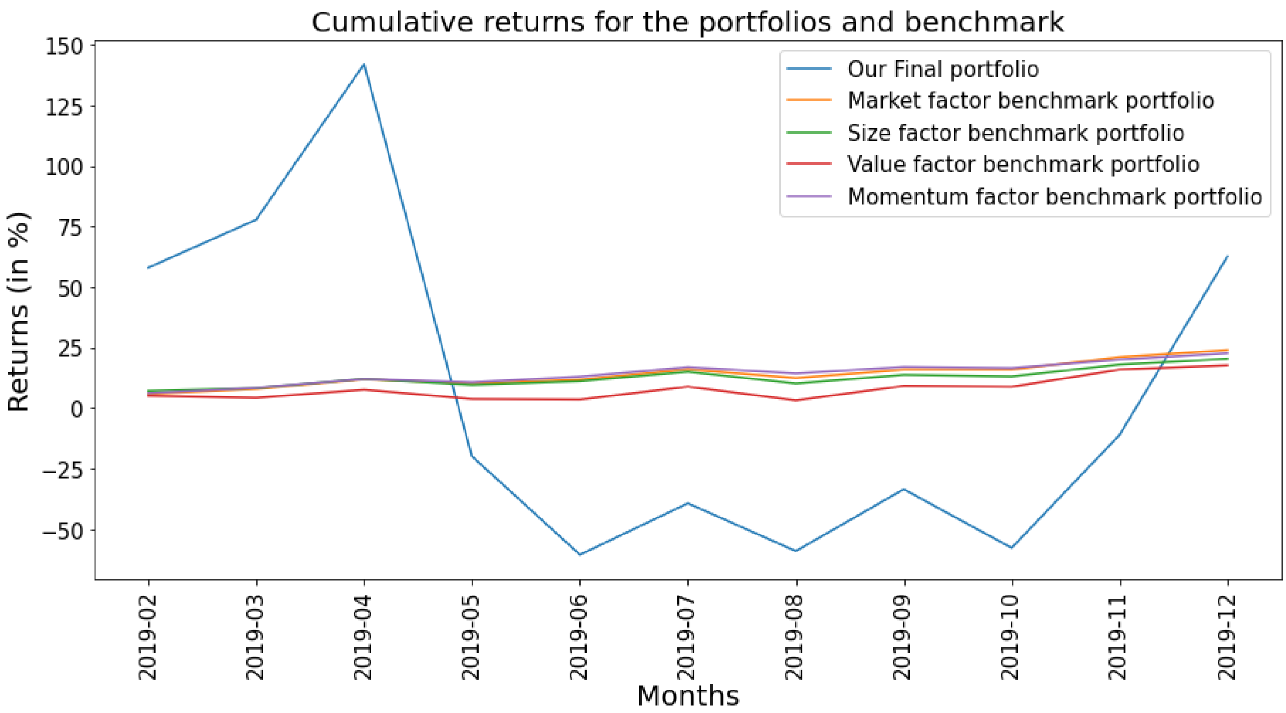

Figure 3: Performance of our portfolio compared to benchmarks

7.2 Test Result Analysis

The graph above shows that the returns on our portfolio are very volatile. We hereby try to explore the possible reason for the same by looking at what sectors dominate the factor portfolios and how weights allocation to these factor portfolios change over time.

7.2.1 Sector Based Analysis

We also performed a sector based analysis to see which sectors outperform in our portfolio of long and short equities.

For the market factor, information technology and consumer discretionary performed the best for the long portfolio, whereas consumer staples and information technology performed the best for the short portfolio. Utilities sector was under-performing in the long portfolio but over-performs in the short portfolio.

For the size factor, information technology and health care performed the best for the long portfolio, whereas financial sector and industrial sector performed the best for the short portfolio. Financial sector was under-performing in the long portfolio but over-performs in the short portfolio.

For the value factor like the size factor, information technology and health care performed the best for the long portfolio, whereas financial sector and industrial sector performed the best for the short portfolio. Financial sector was under-performing in the long portfolio but over-performs in the short portfolio.

For the momentum factor, information technology and consumer discretionary performed the best for both the long portfolio as well as the short portfolio.

It was observed from the above graphs that information technology sector companies account for a significant part of both the long and the short portfolio for all factors. So our final portfolio would also have a significant amount of IT sector companies. In general, the information sector companies are known to be volatile so if the portfolio selects majority of the information technology companies the volatility of the portfolio is expected to increase.

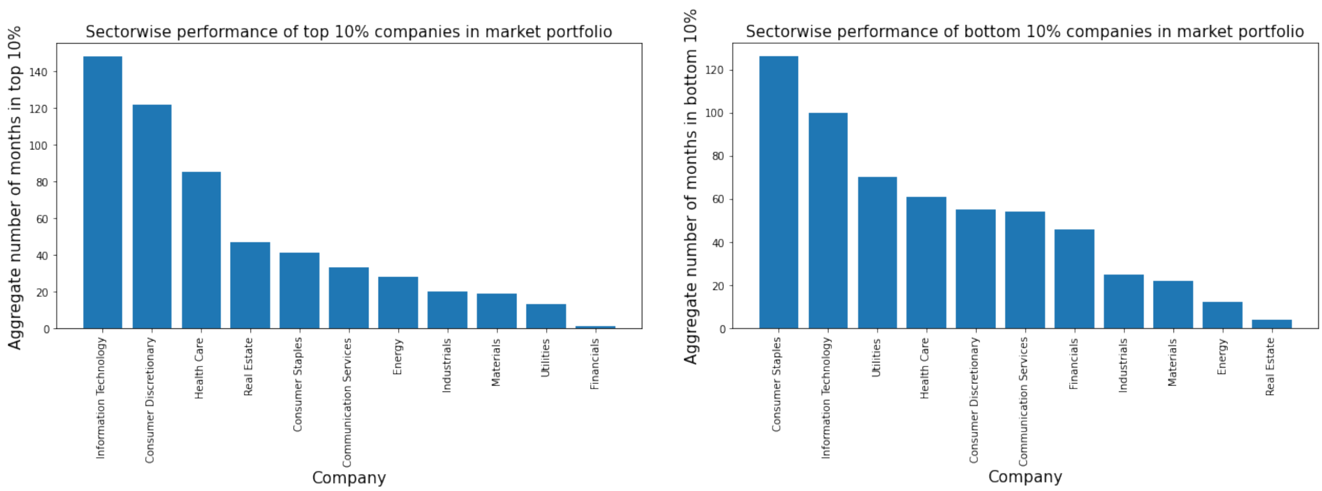

Figure 4: Distribution of sectors of companies for market factor

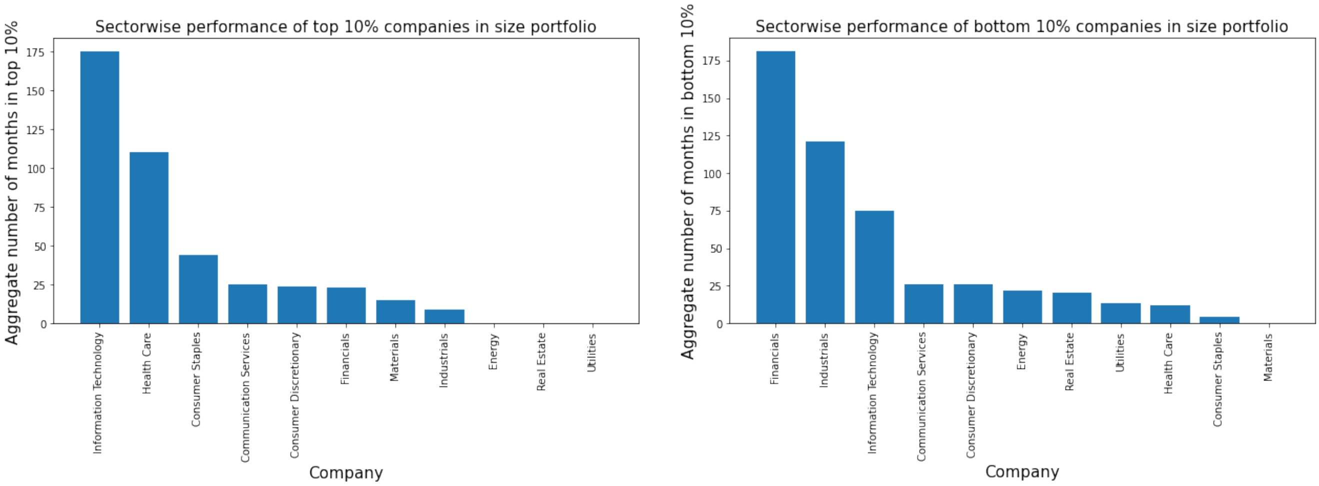

Figure 5: Distribution of sectors of companies for size factor

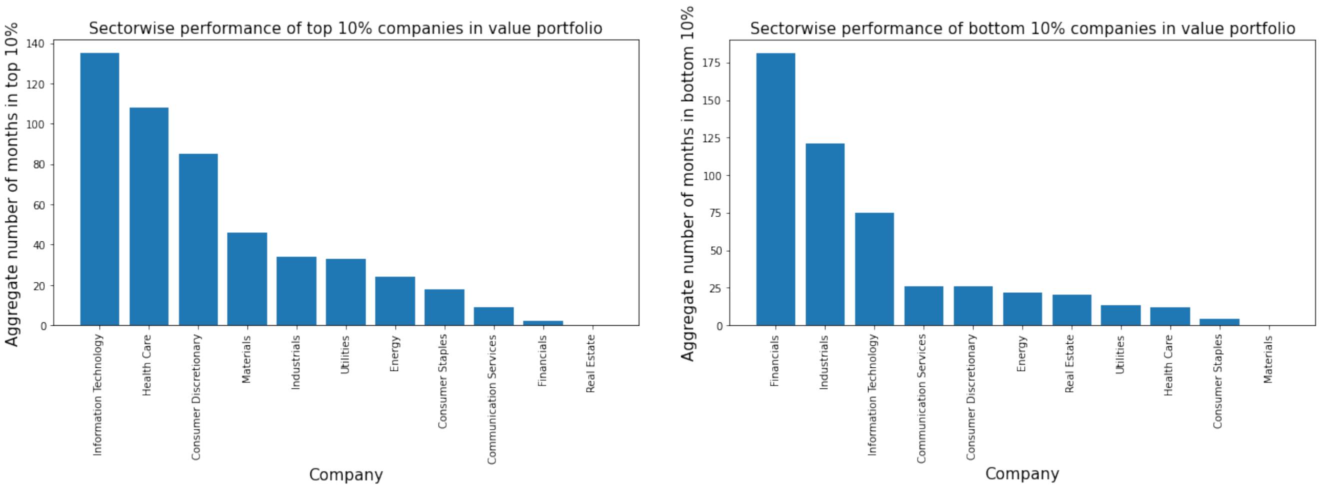

Figure 6: Distribution of sectors of companies for value factor

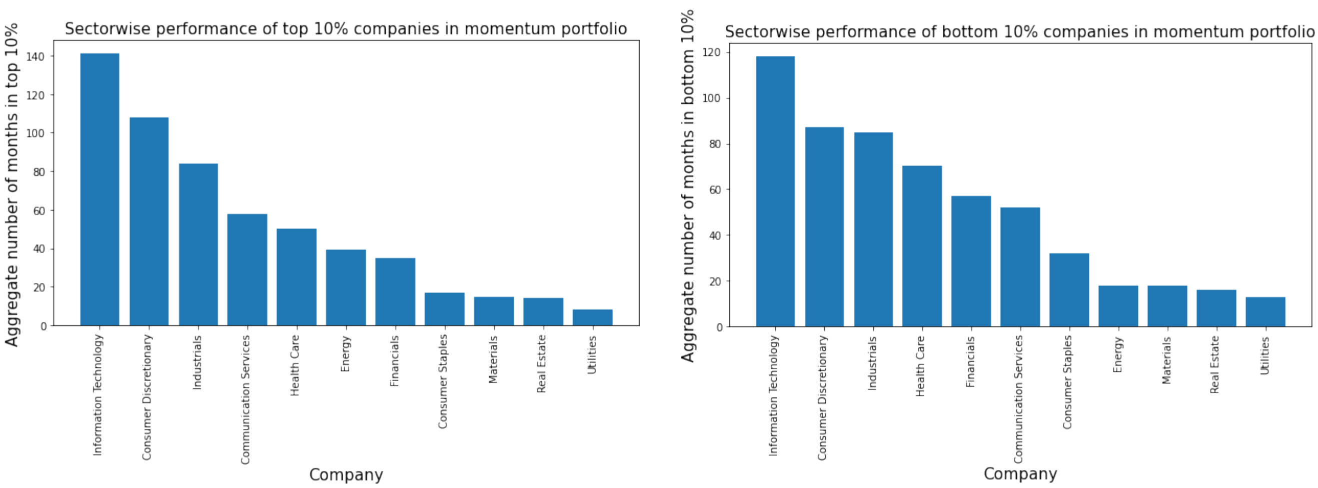

Figure 7: Distribution of sectors of companies for momentum factor

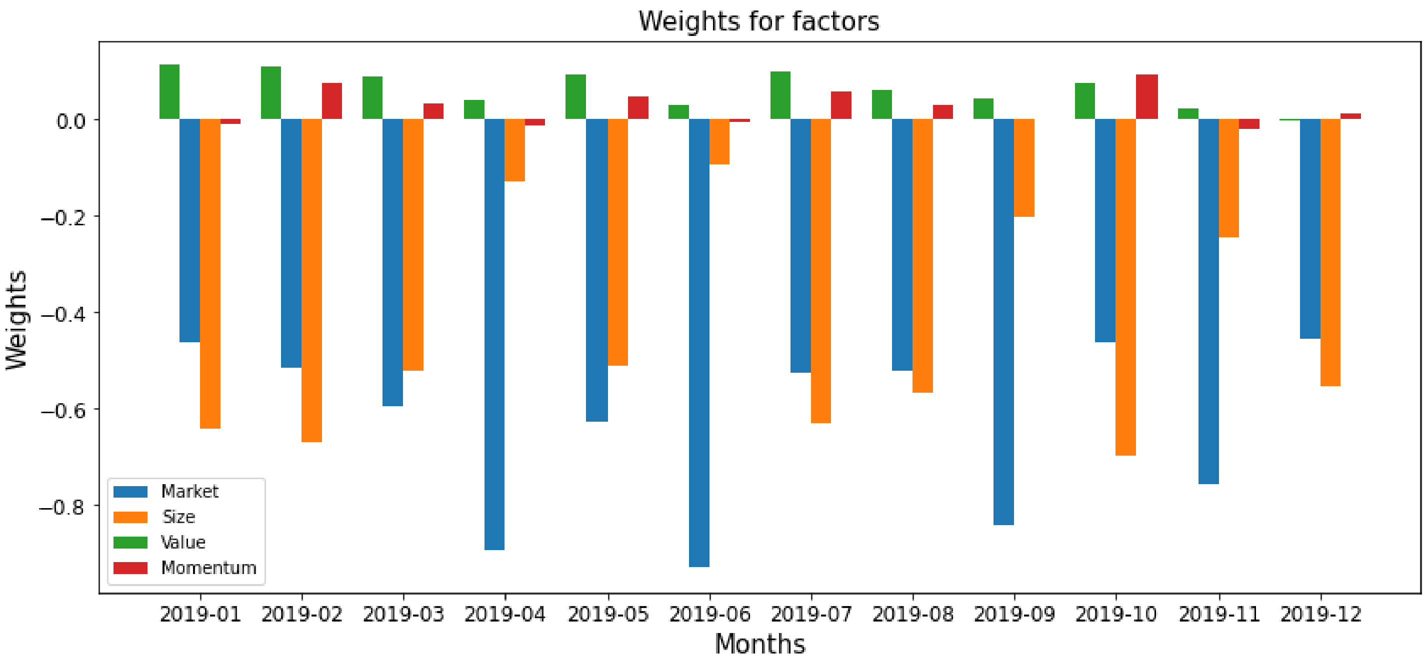

Figure 8: Weights assigned to each factor signal in test time frame

7.2.2 Weight Based Analysis

Knowing the dominant sectors for each factor portfolio, we now look at how the weight allocation to each signal changes over time.

From Figure 8, we see that market and size factor almost alternatively in terms of weight. So our strategy continuously switches between market and size signal, i.e. it allocates higher weights to market factor trading strategy in one month followed by allocating higher weights to size factor trading strategy in next month. Now we have seen above the the sectors dominating market factor were IT and Consumer Staples while sectors dominating size factor were IT and Financials. So as the weights of the factor signals changes, the composition of the final portfolio also changes over time. This could be another possible explanation of the high volatility in the returns of our portfolio.

In the real world, the transaction cost might erode the profit that is being made by this strategy and also stabilise the volatility that is seen by this portfolio.

The results from this study depict that multi style rotation strategies combined with personal insights for active investors outperform the market as well as the individual factor based portfolios (benchmark factor ETFs).

8 Future Works

In this section we explore the initial broader scope that was considered for the project and then look at other future areas of developments.

Initial Thoughts

In the early stages of the project, we considered including macroeconomic mimicking portfolios by using additional factors such as industrial production growth and inflation innovation and incorporating shifts in economic regime too.

The current study only focuses on equity and its factor like market, size, value and momentum. However, we also planned to include other asset classes like fixed income with factors such as quality, value, momentum, volatility etc and commodities with factors such as carry, value, momentum, defensive etc.

We also narrowed down our project from having stock-specific as well as factor-specific views initially to now including only factor-specific views. The initial ideas of including more factors or combining two or more factors for superior performance are also yet to be explored.

Further Extensions

An important thing to note is that Markowitz optimisation starts with random weights so results are dependent on the initial seed values. To neutralize this effect, we can run multiple Monte Carlo simulations of generating Weights from Markowitz and hence the returns, and then average those out. This might also help reduce the high volatility in returns. This approach though requires significant computational power which we lacked.

A generalisation of the current study could be allowing unequal weights on stocks, and choosing them via Markowitz mean variance optimisation too for each factor portfolio to further improve performance.

This study ignores transaction costs involved while monthly re-balancing of the portfolio. However, given the highly volatile nature of returns, the transaction costs might not be insignificant. So for more realistic estimates of returns, we can include transaction costs in the calculations.

A loophole in this study, as in most of the mean-variance optimisation problems is that this gives us a single-period optimiser and has no potential to automatically adjust to new information. One needs to decide a time frame over which views should be formed and return vectors should be estimated (in our case this is monthly), and then use Black Litterman and Markowitz for optimal weight allocation. When this time period ends, one has to repeat the process, now taking the new available data as well, and re-balance positions accordingly. This adds additional degree of freedom in algorithmic trading which is not preferable and results in high volatility. So as a potential future improvement, one can transition from single-period optimiser to multi-period optimiser using complex theoretical base.

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