Path-finding

Daniel Nogueira

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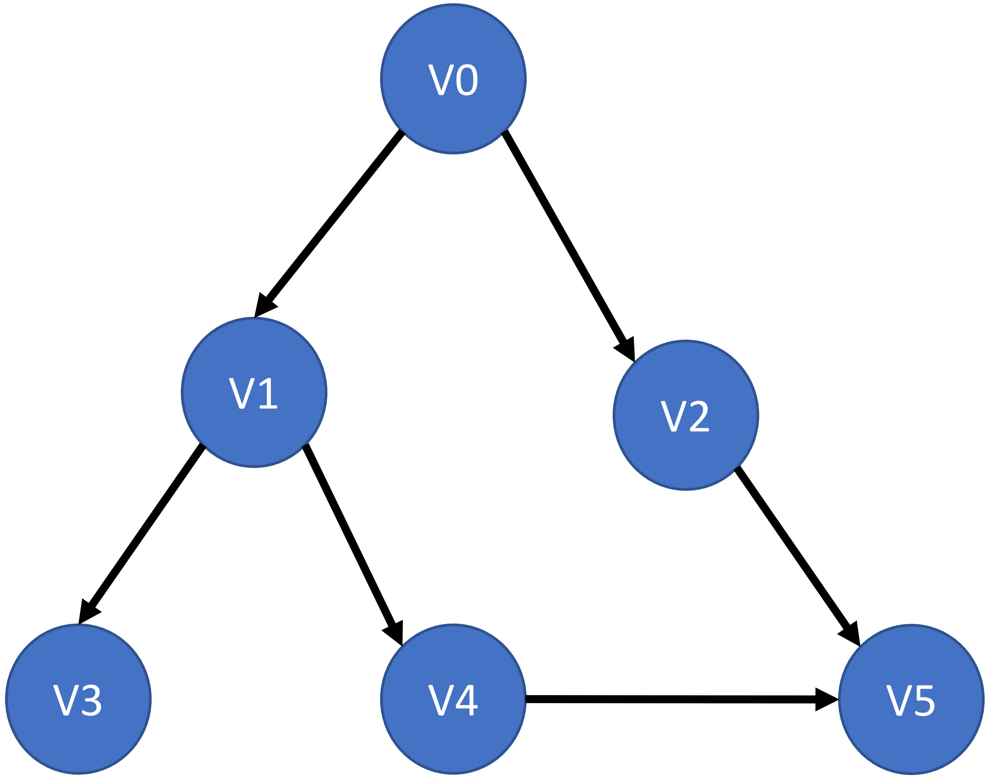
Algorithms

Depth-First Search (DFS)

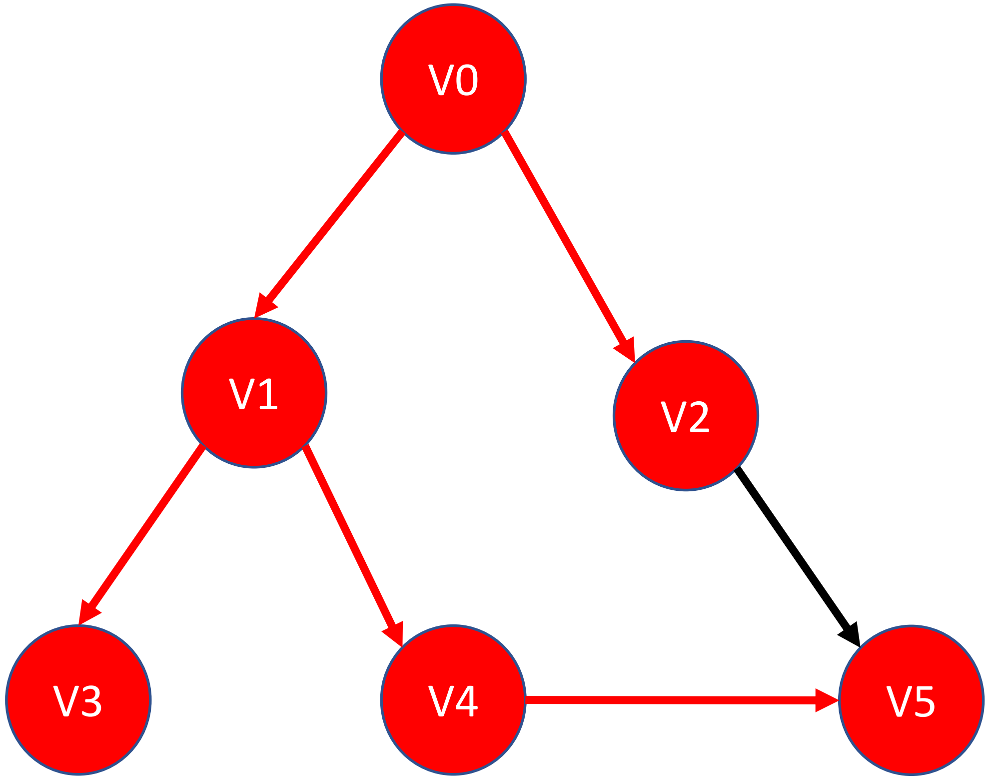


The Algorithm

1.Set a start node
2. While this is not an objective or final node (node whose adjacency has already been visited):
 - Choose an adjacent node not yet visited
 - Visit it
3. If it is a non-objective end node:
 - Return to this father
 - If there is a father, repeat. If there is no parent, choose another start node

Algorithms

Depth-First Search (DFS)



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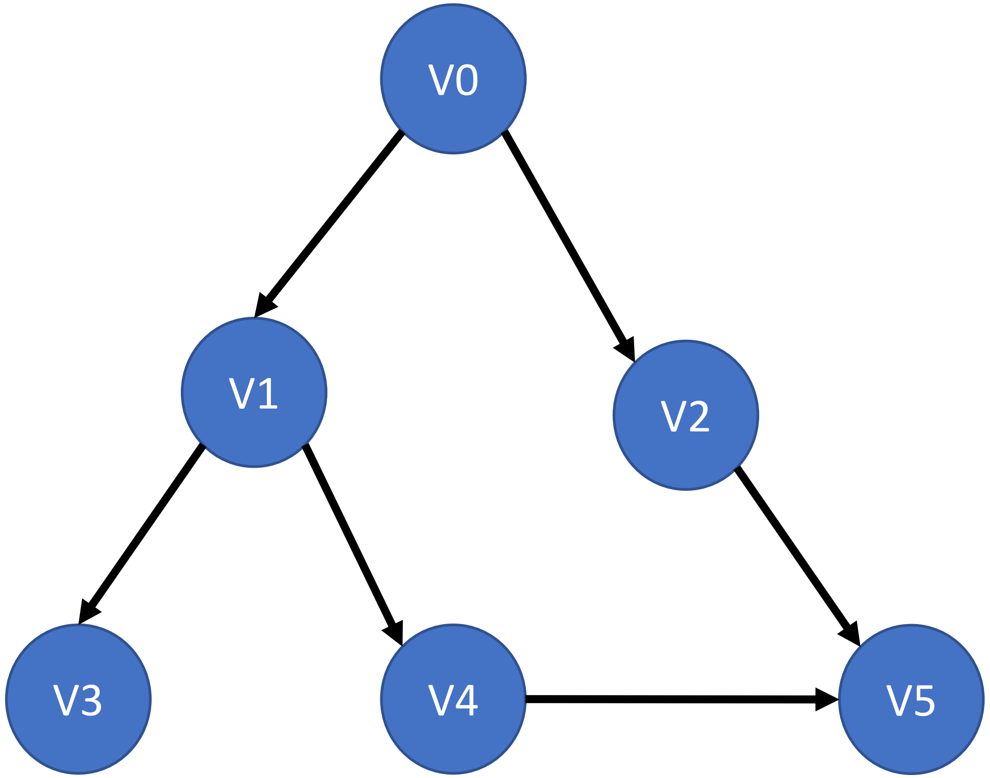
Breadth-First Search (BFS)

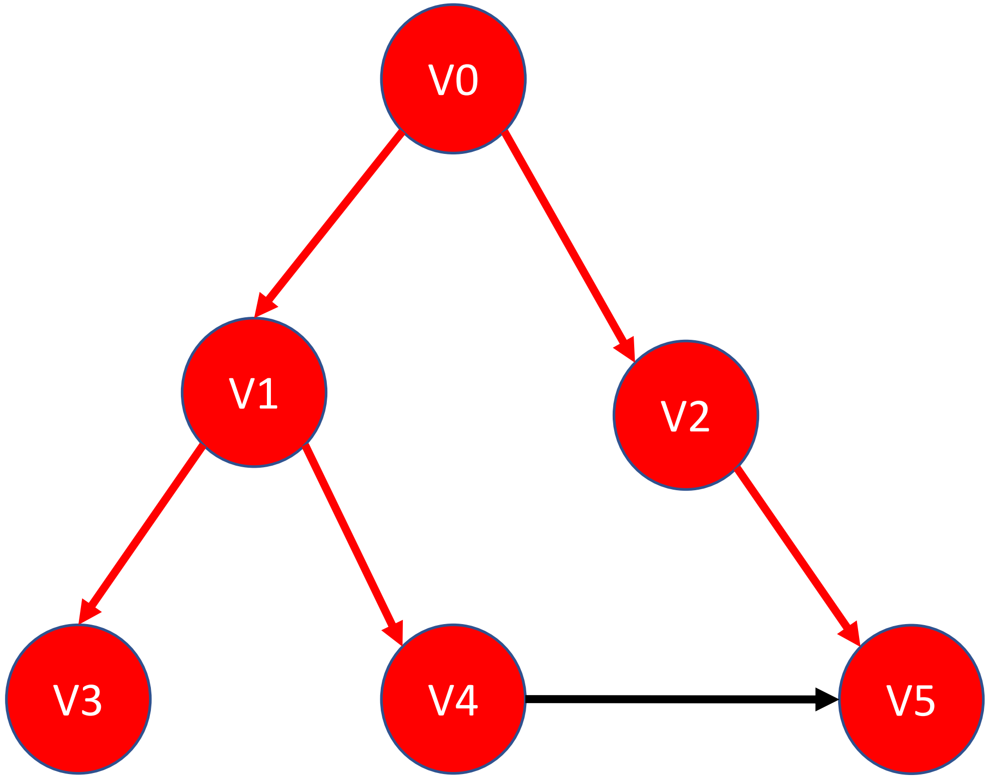


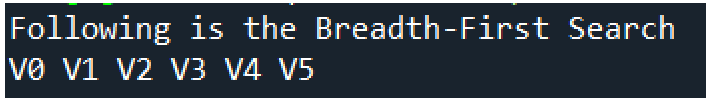
The Algorithm

1.Define an initial node, marking it as explored
2. Put it on the list
3. As long as the queue is not empty:
  - Remove the 1st node from the queue, u
  - For each neighbour v of u:
    * If v is not explored:
      ** Mark v as explored
      ** Put v at the end of the queue
4. Repeat from another starting node, if there is one

Algorithms

Breadth-First Search (BFS)





DFS V0 V1 V3 V4 V5 V2

BFS V0 V1 V2 V3 V4 V5

Algorithms

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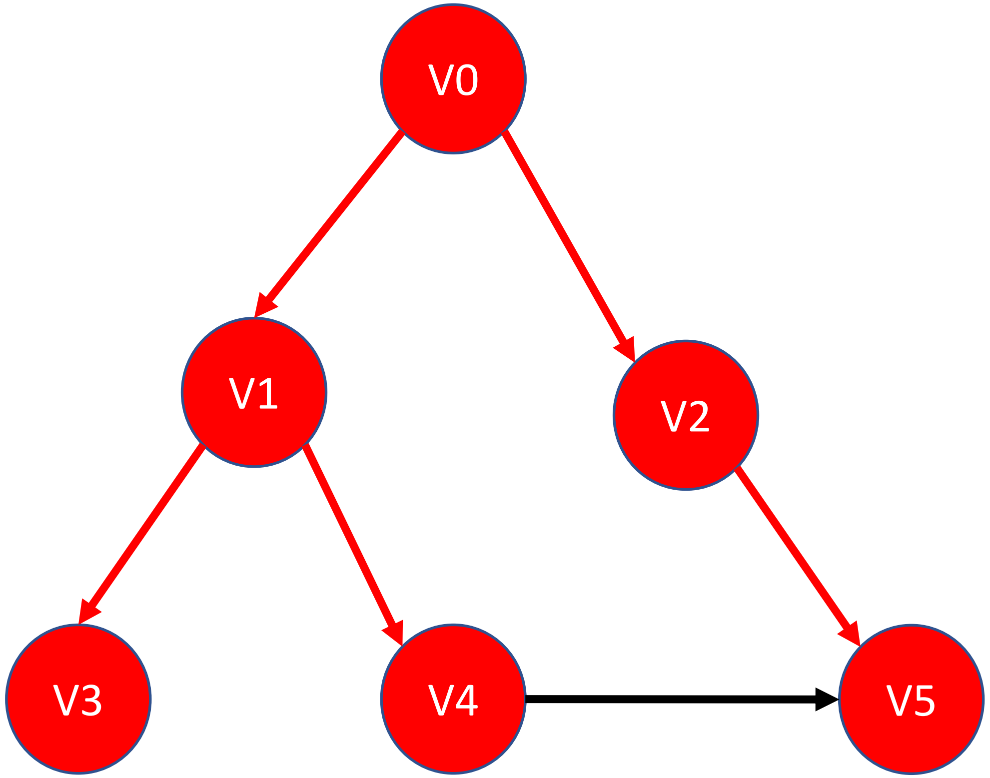

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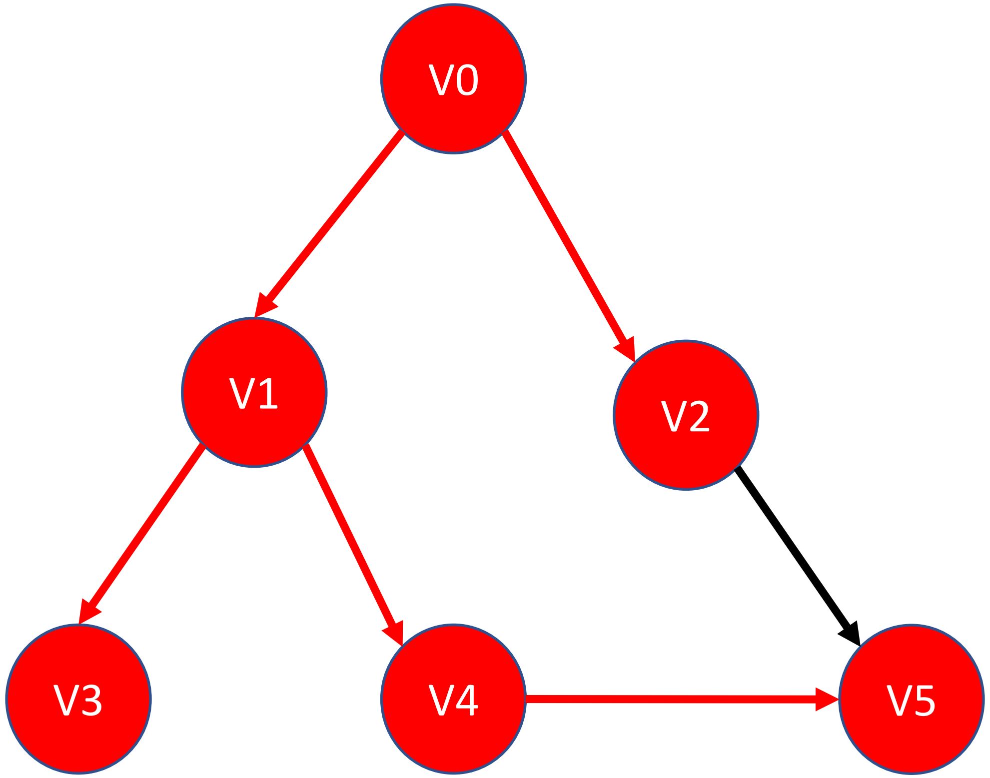

Algorithms

Dijkstra → Dijkstra's algorithm can be used to find the **shortest distance** from the source vertex to all other vertices in a weighted graph. For each vertex *v* of the graph, we maintain an attribute ***d(v)*** which is an upper bound on the weight of the shortest path from the initial **node *s* to *v*.**

The Algorithm

1. Initialize the graph with d(s) = 0, d(v) = INF, for all v ≠ s, and p(v) = -1 for all *v*
2. Make open(v) = True for every v in the graph
3. As long as there is an open vertex:
   * Choose *u* whose estimate is the smallest among the open
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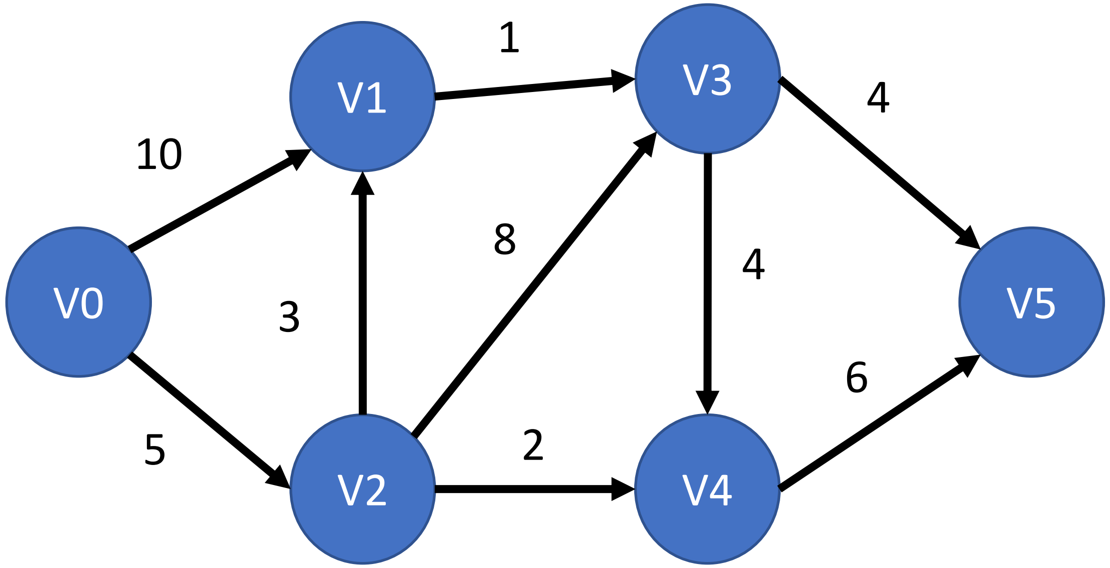
Algorithms

Dijkstra



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Algorithms

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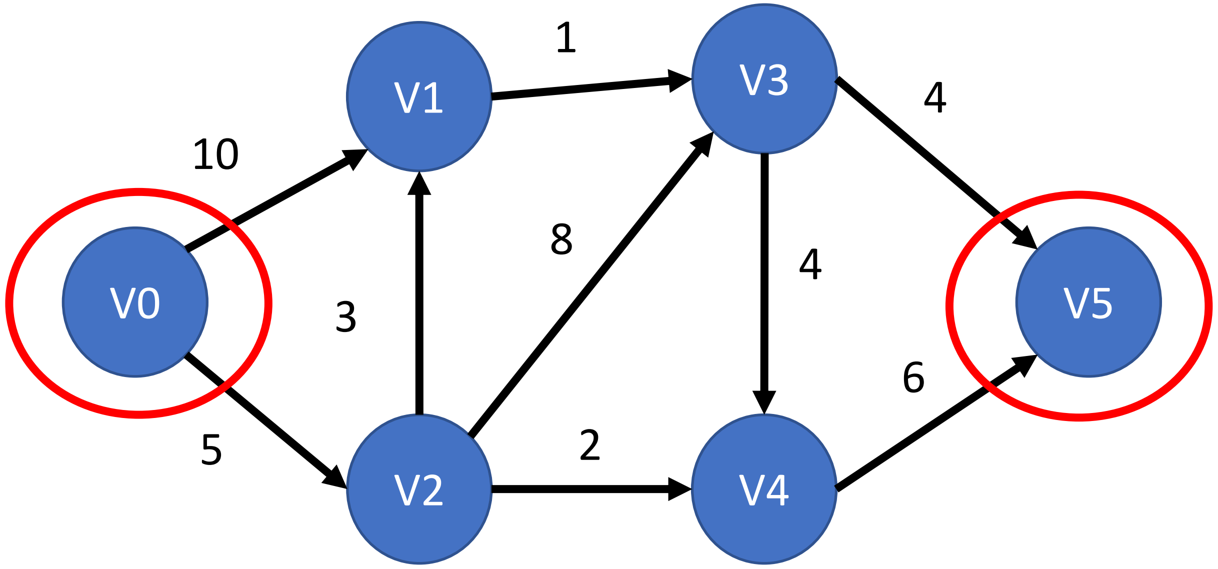

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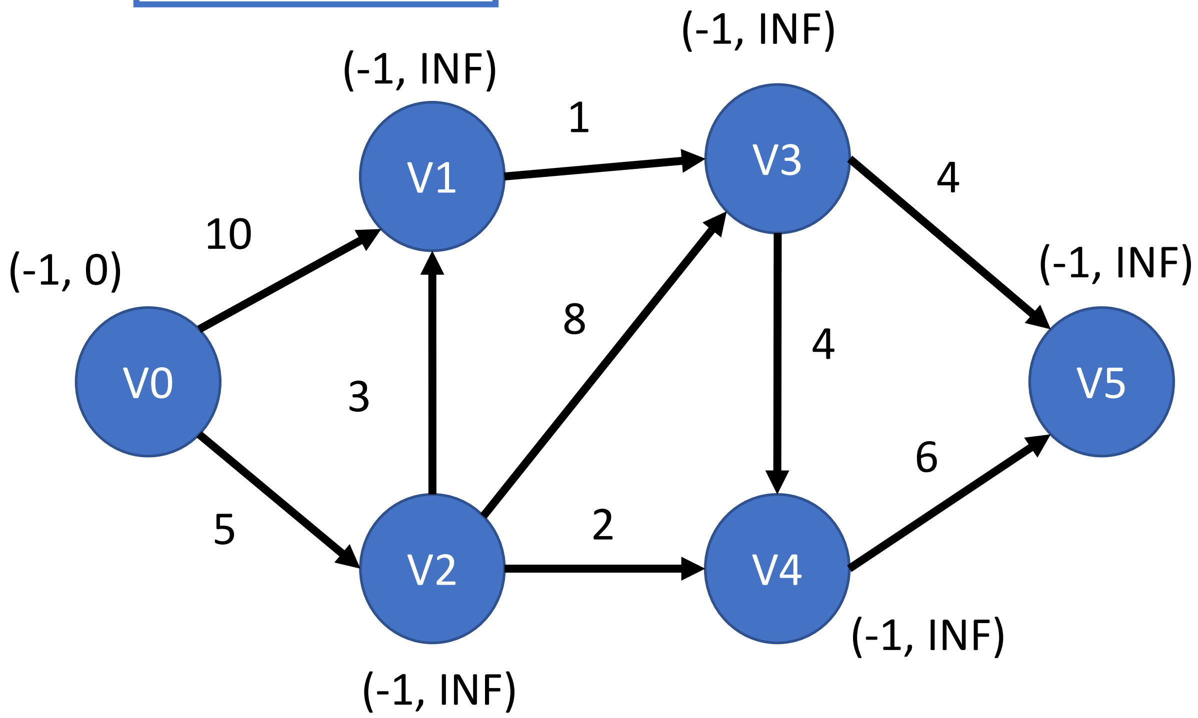

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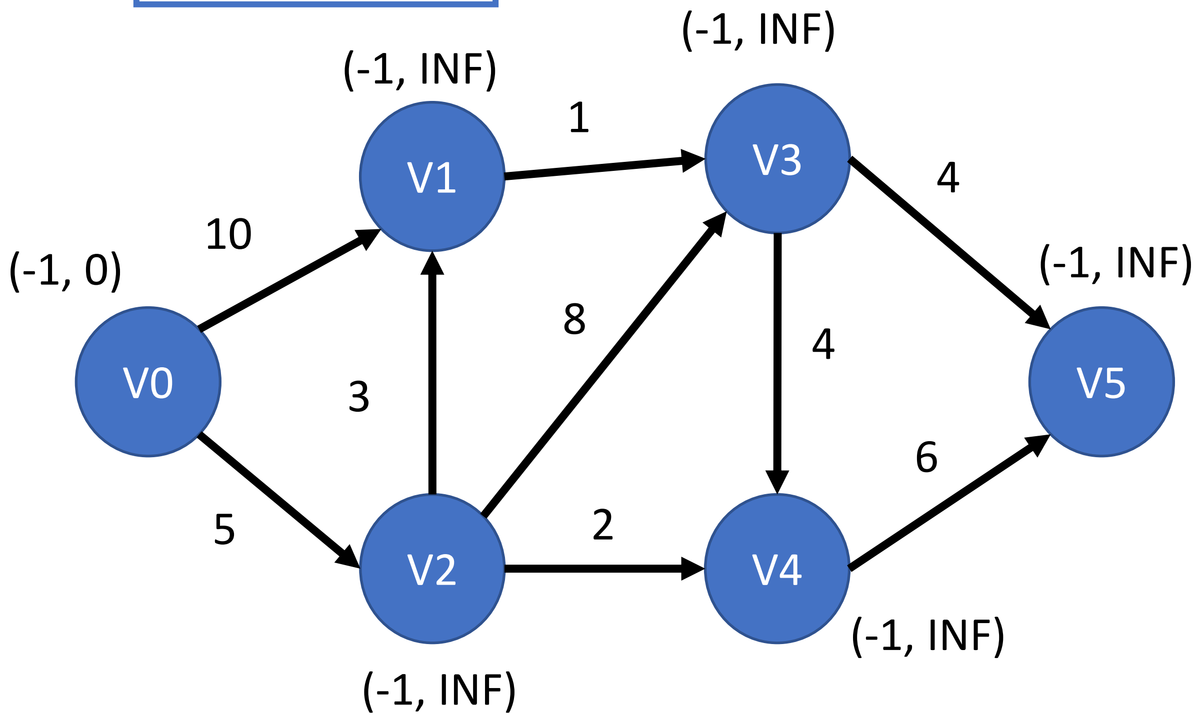

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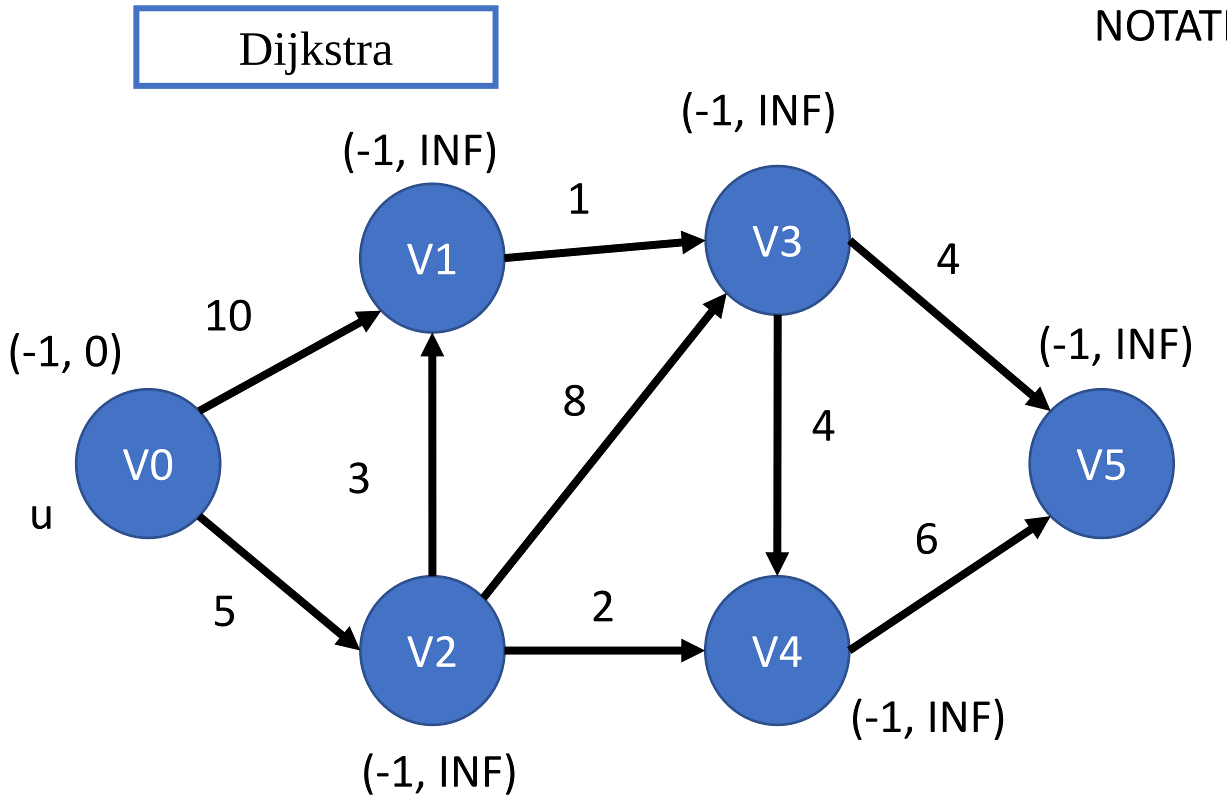

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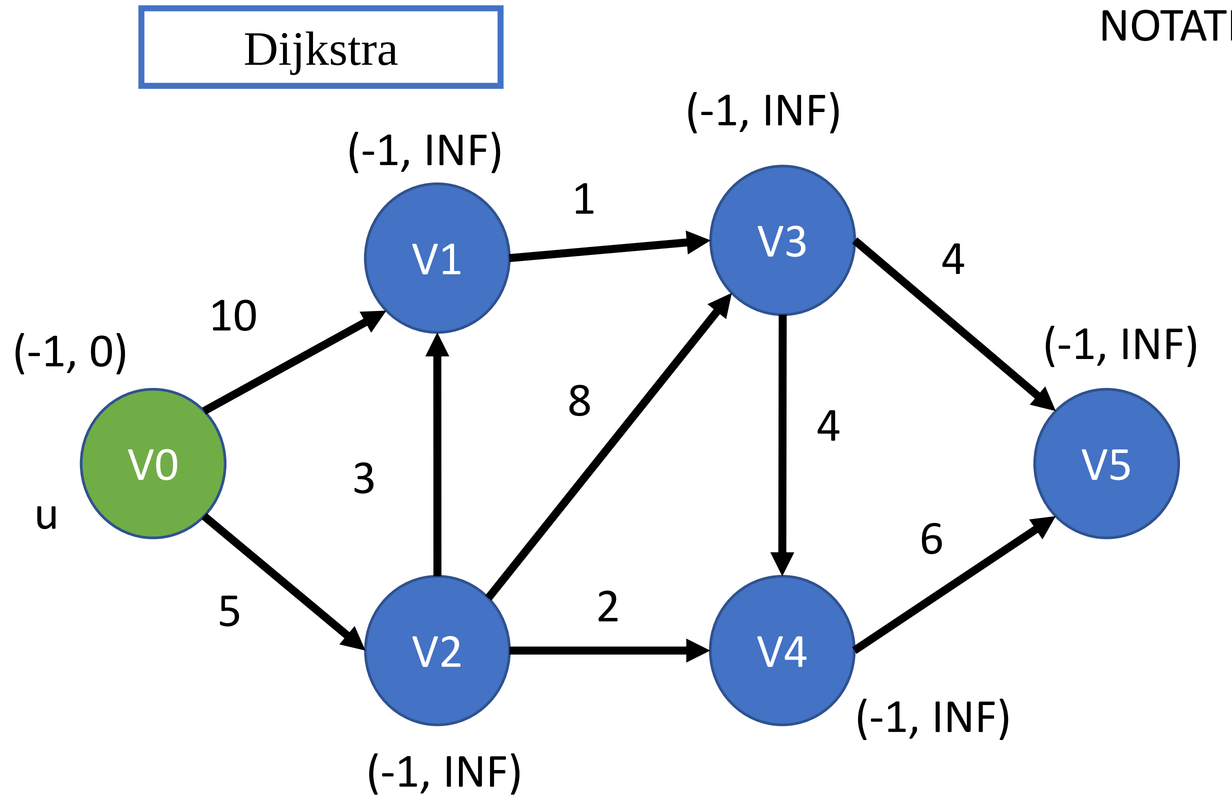

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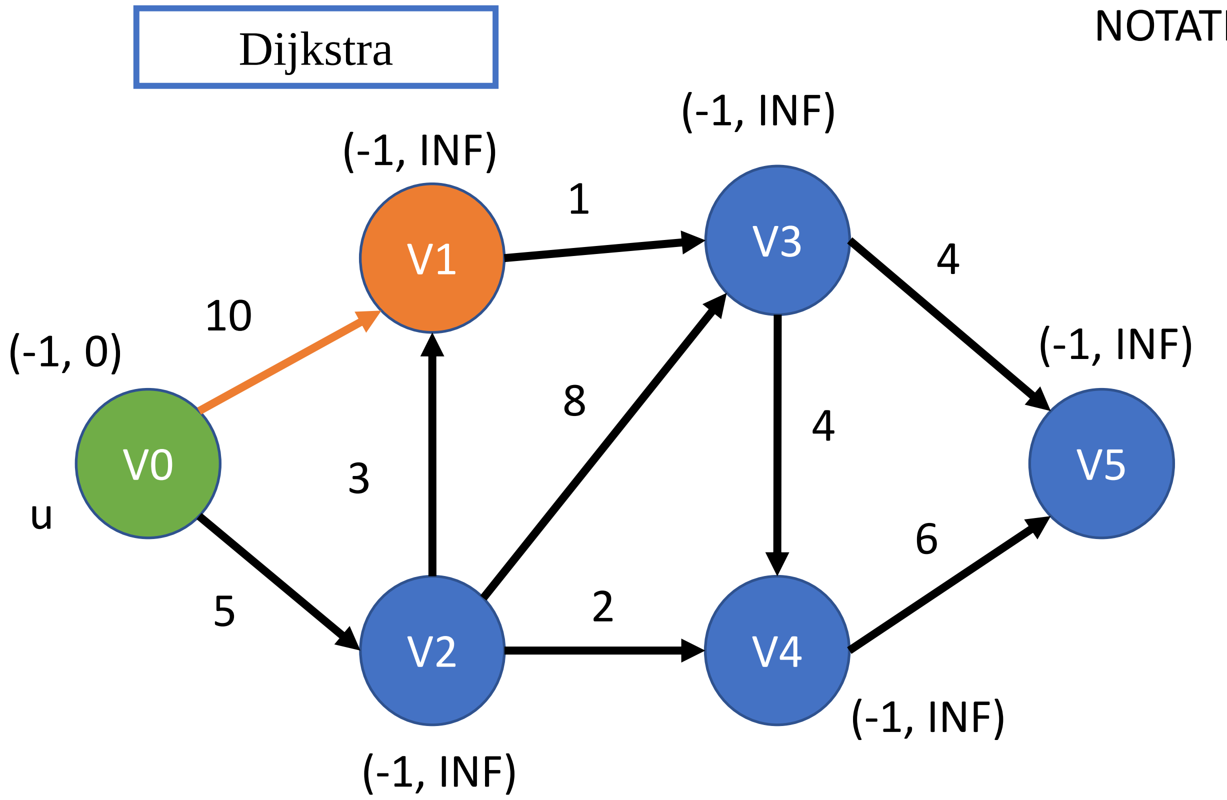

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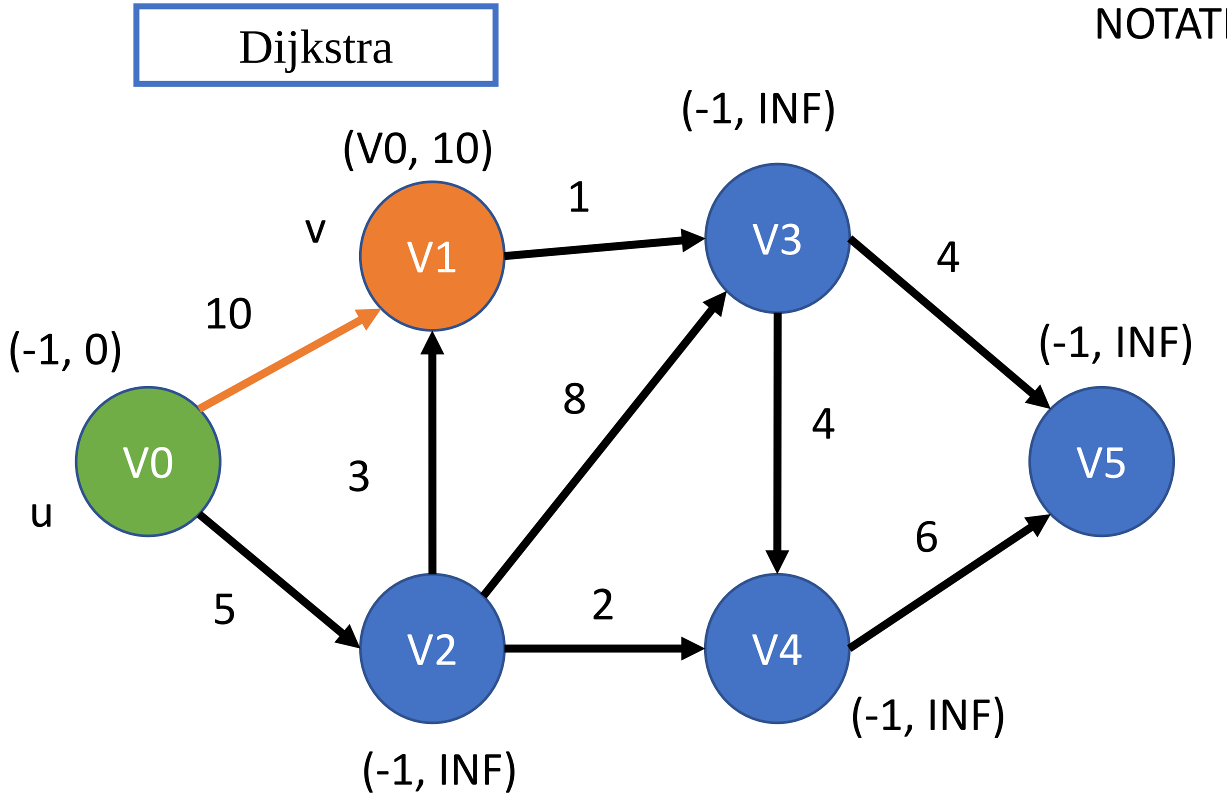

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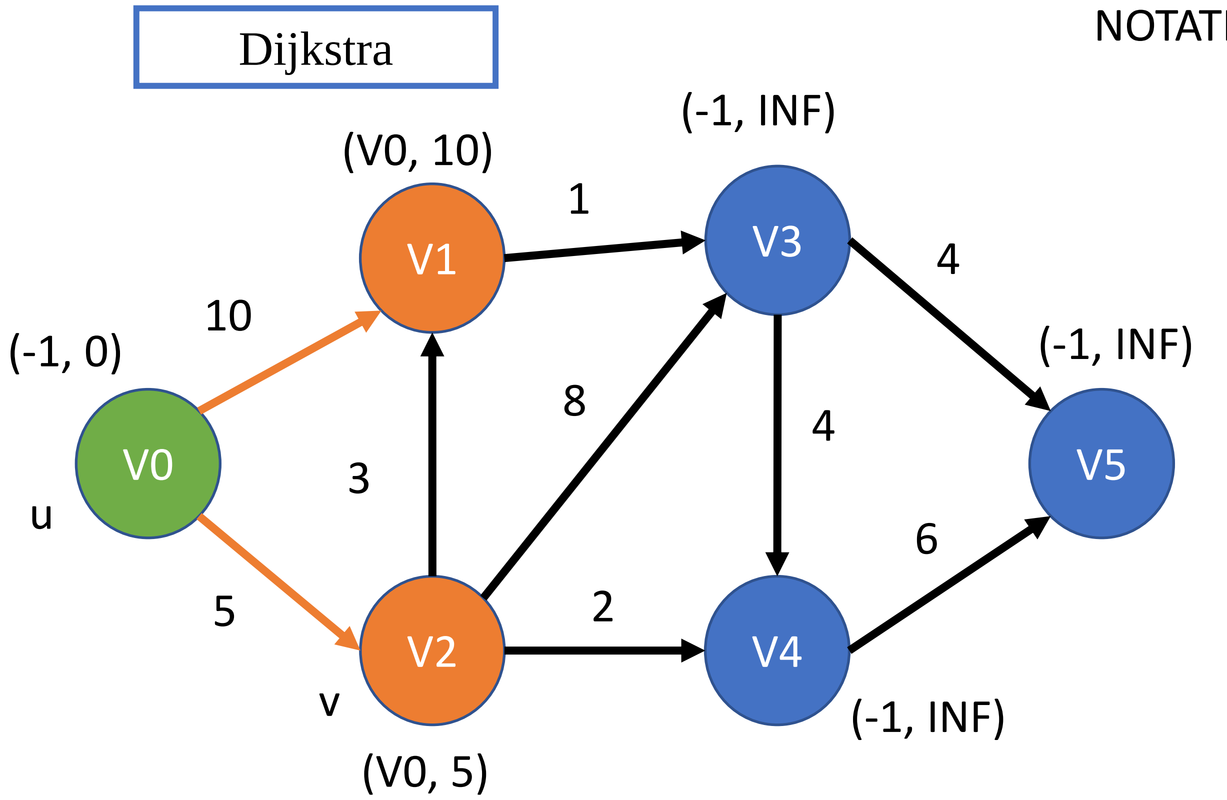

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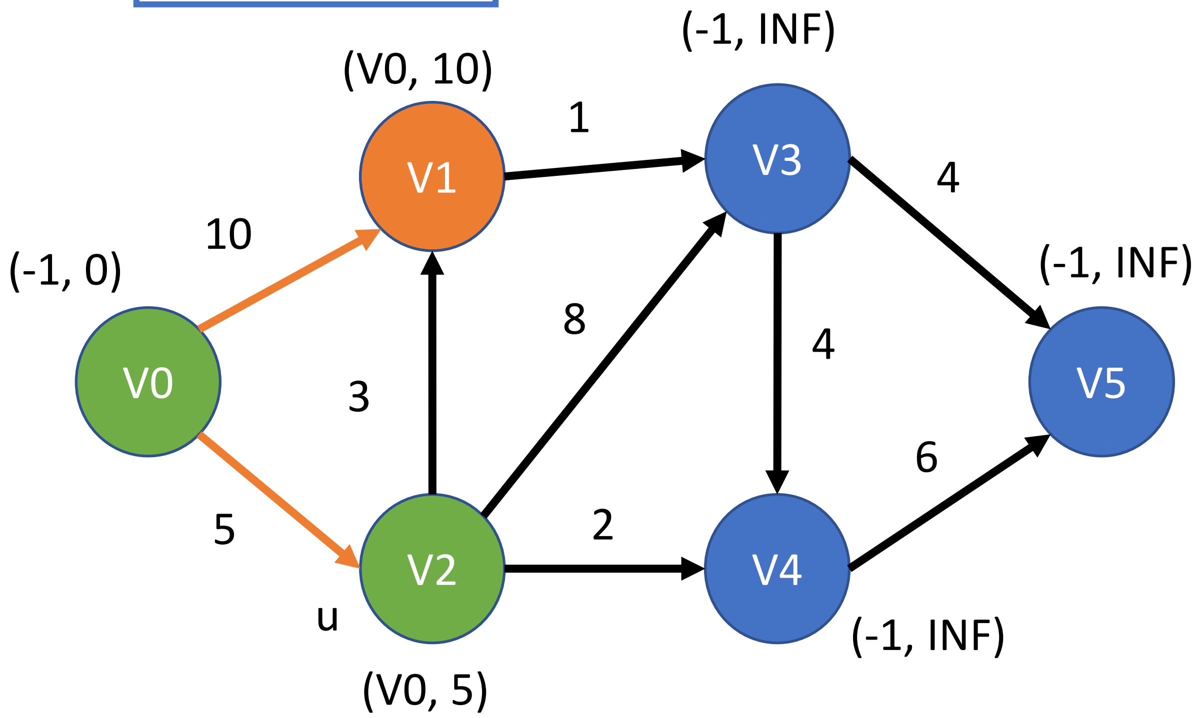

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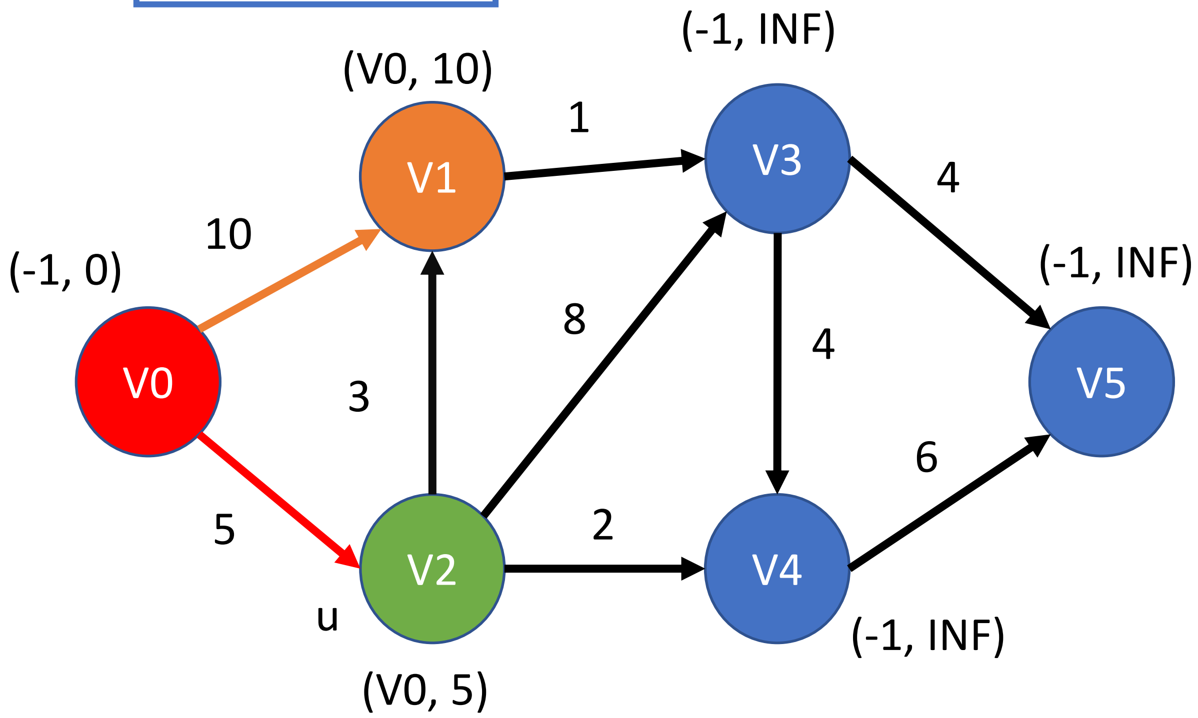

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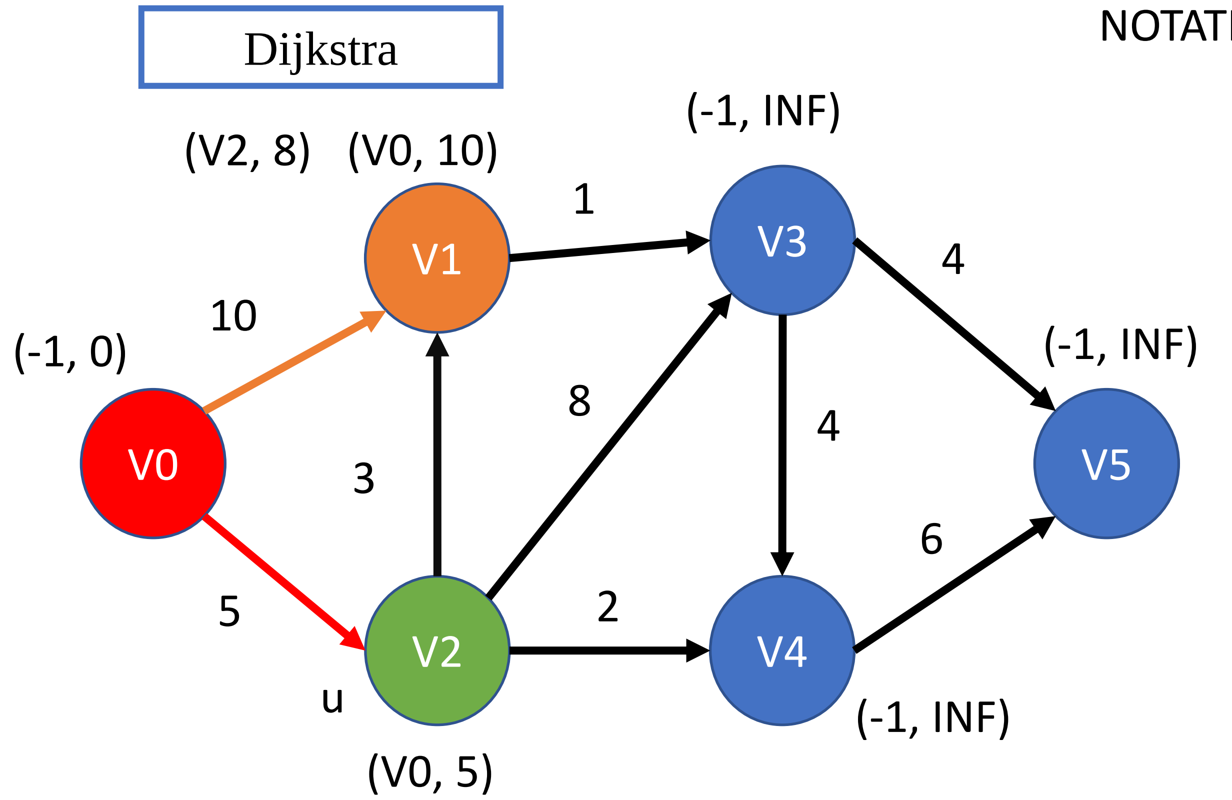

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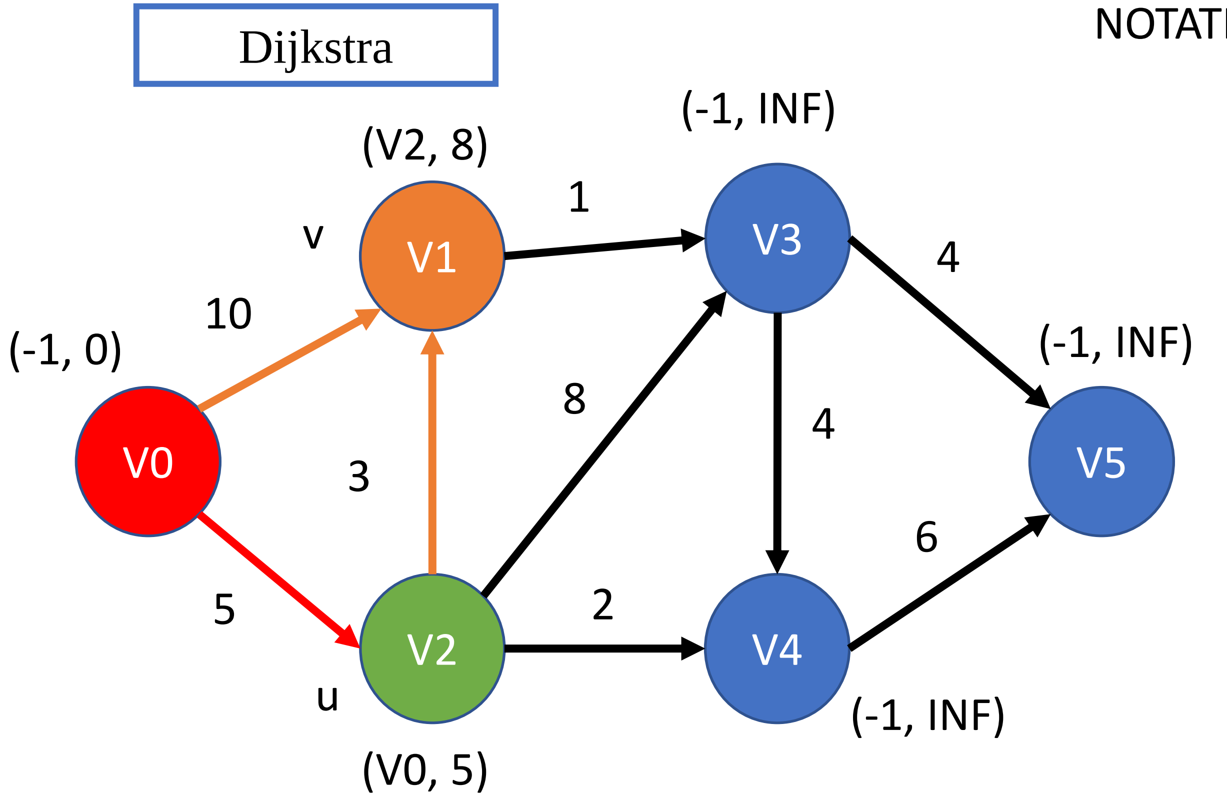

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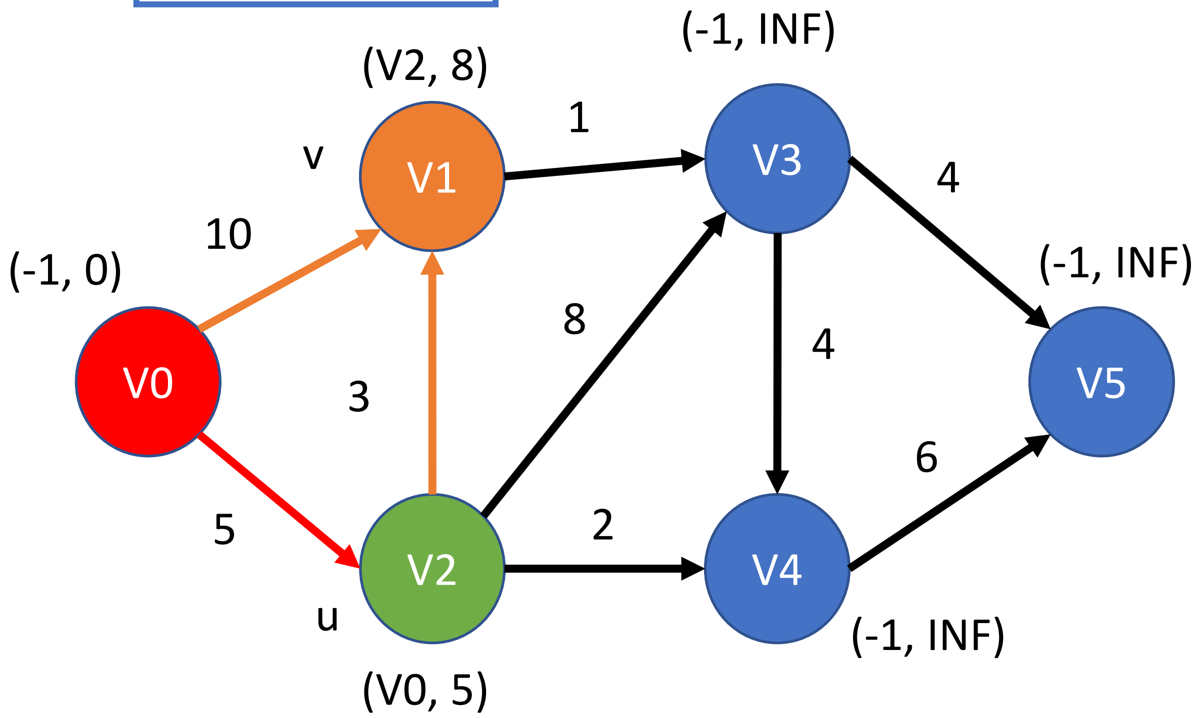

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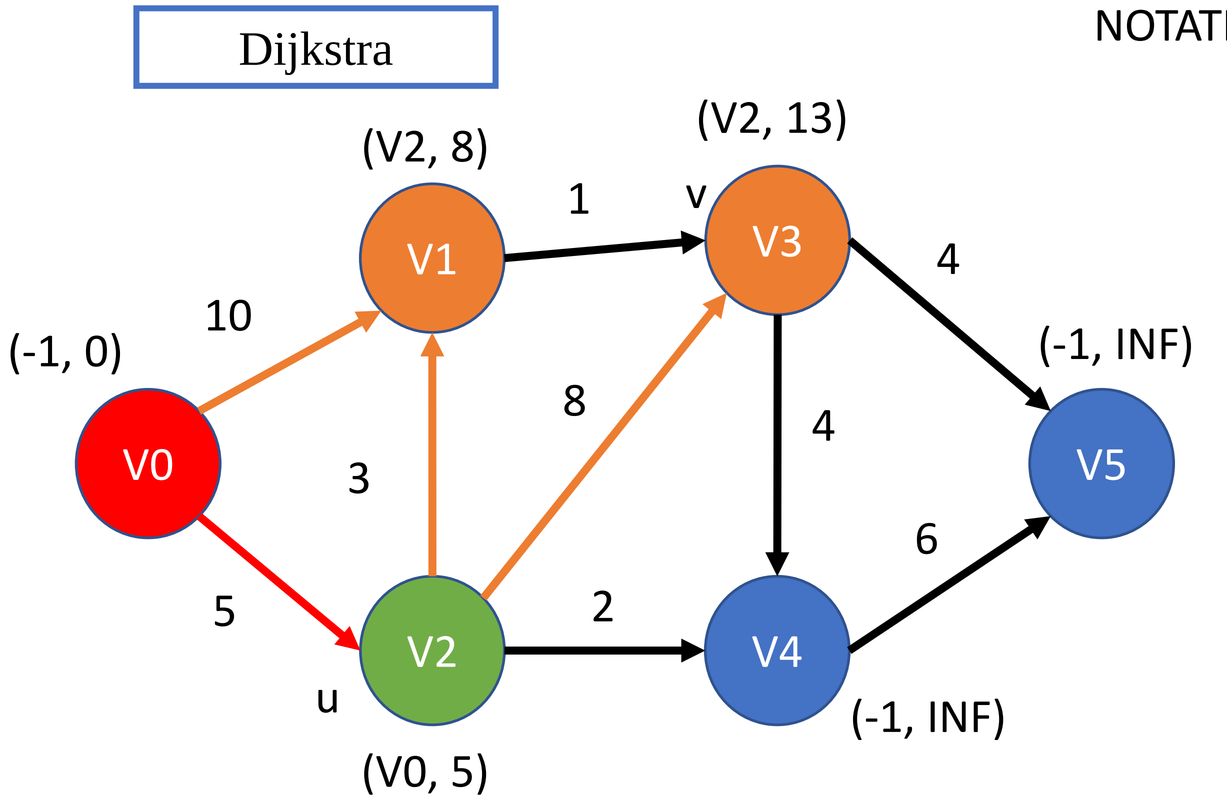

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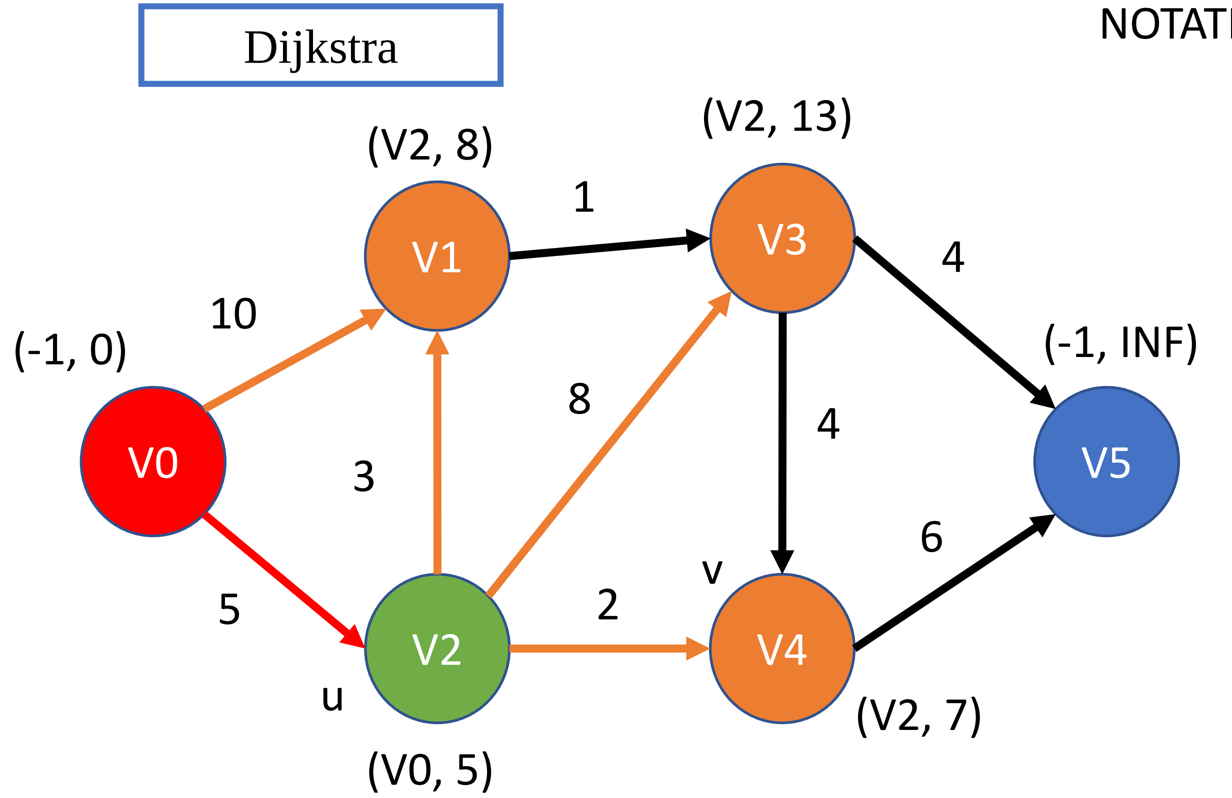

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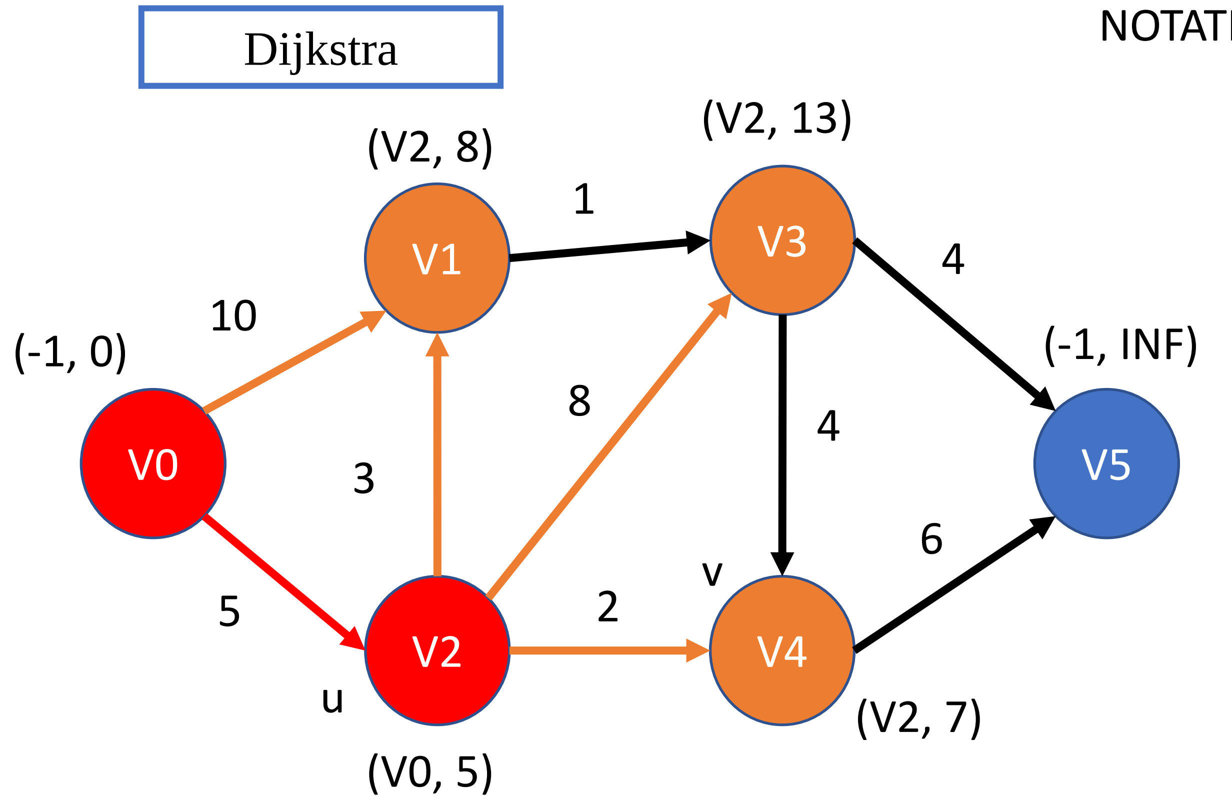

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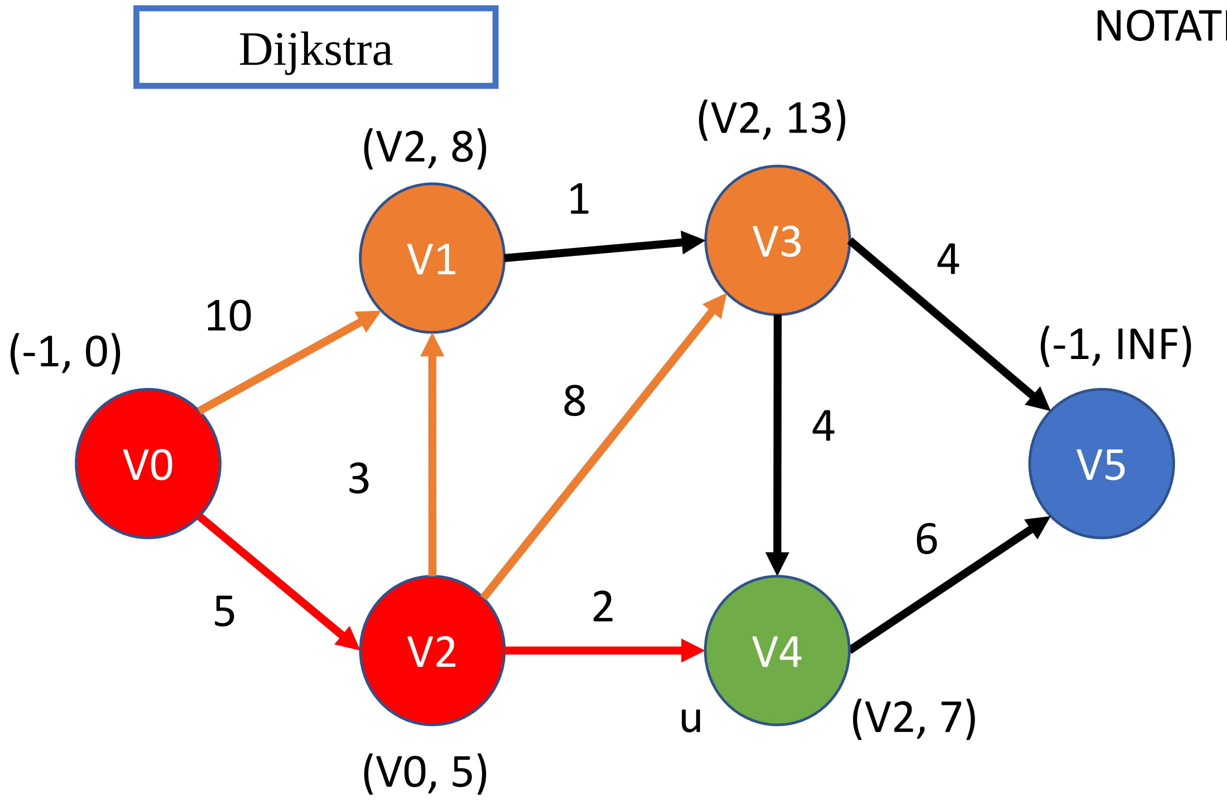

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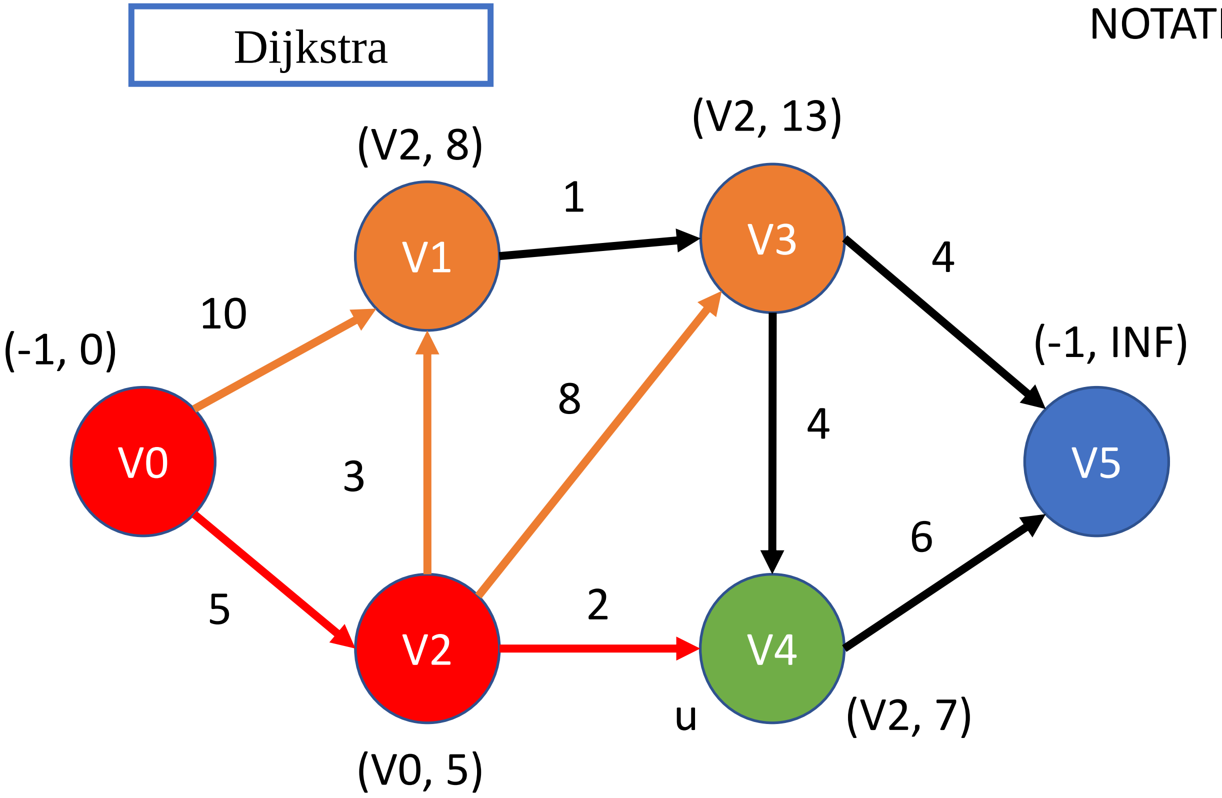

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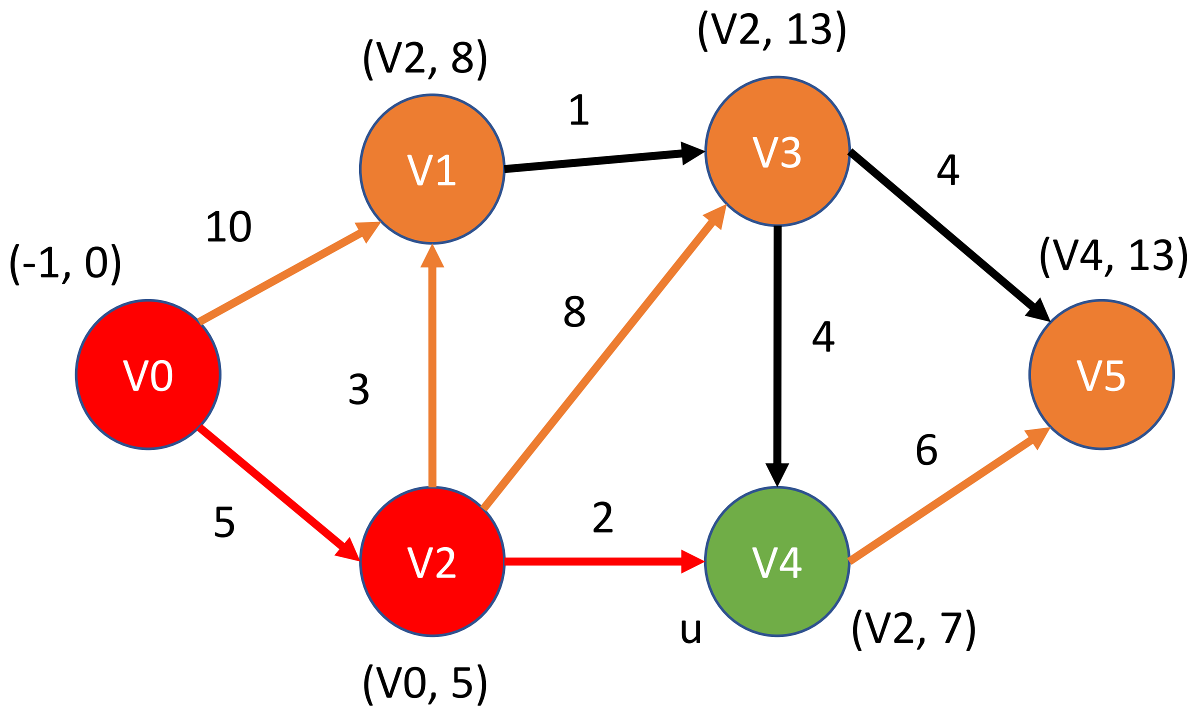

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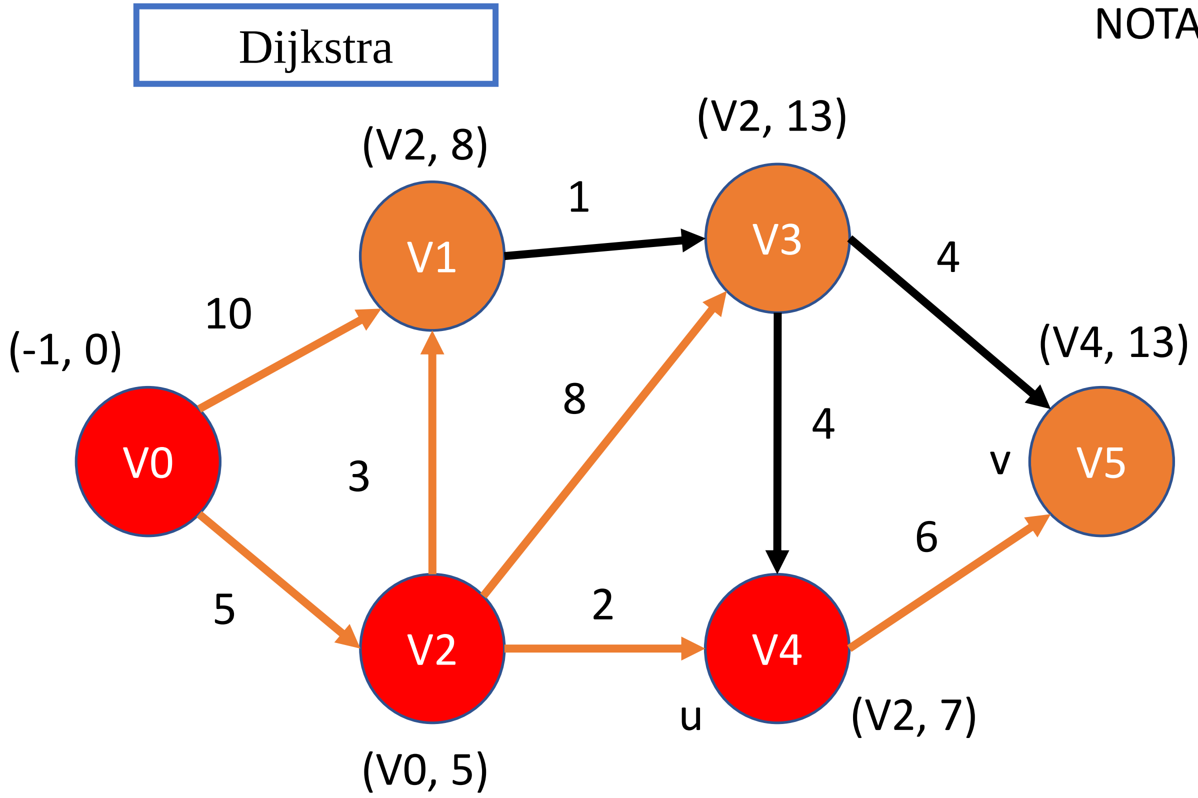

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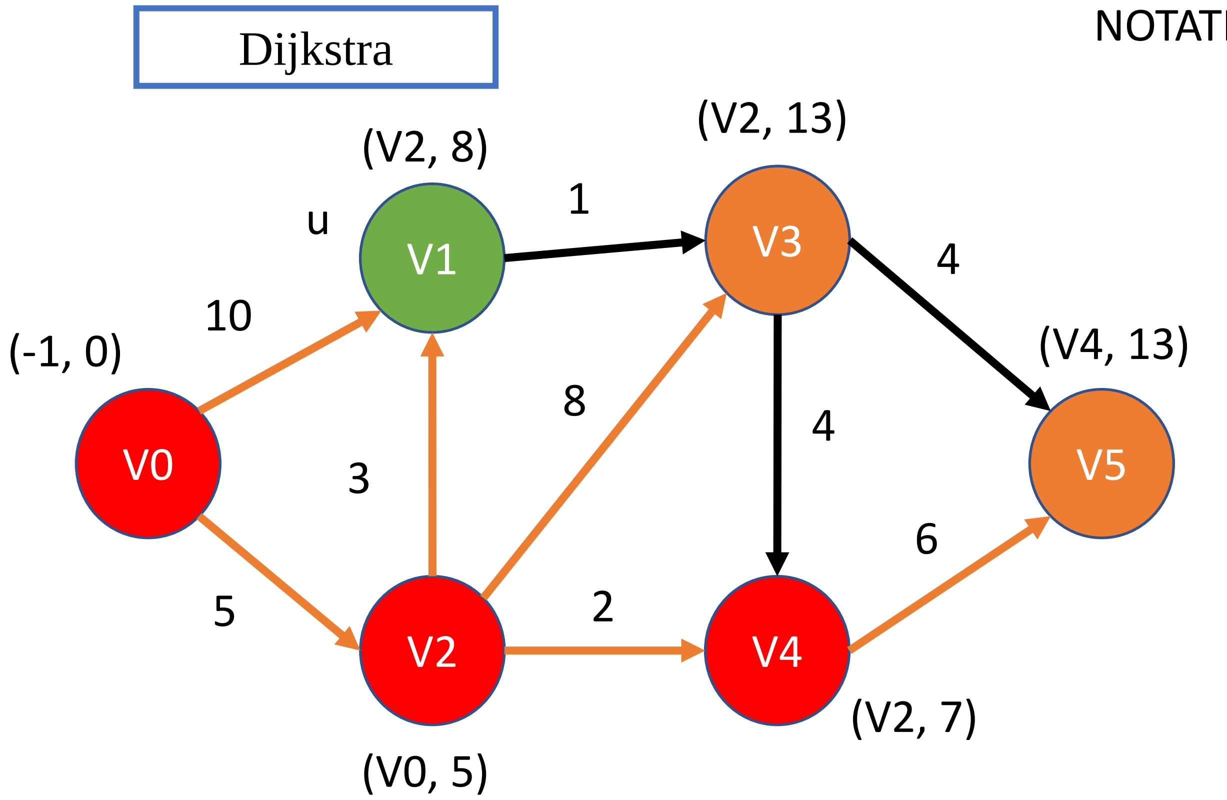

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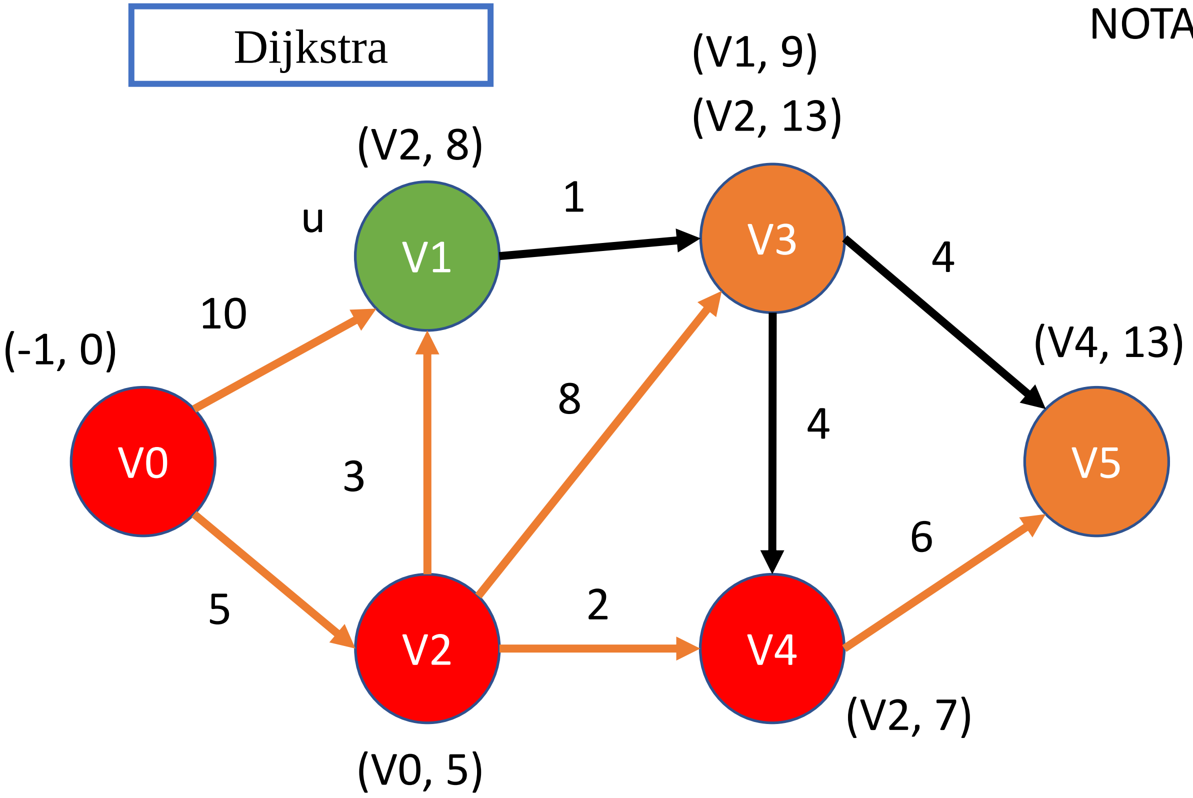

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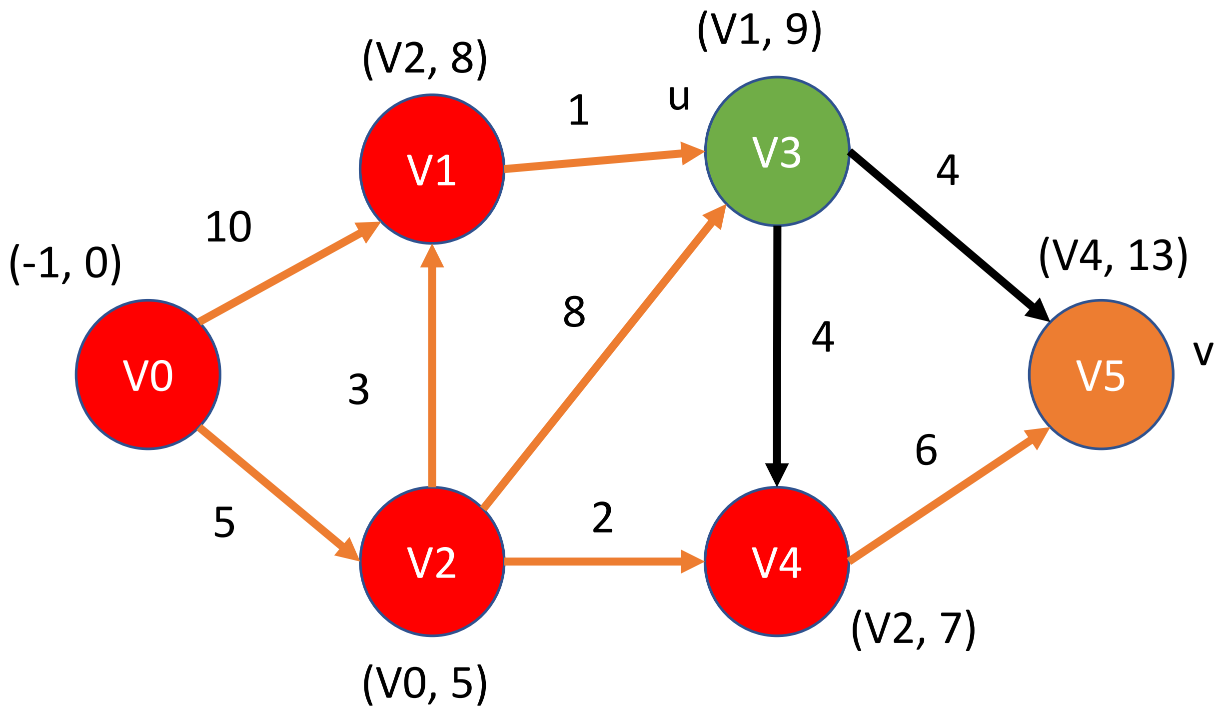

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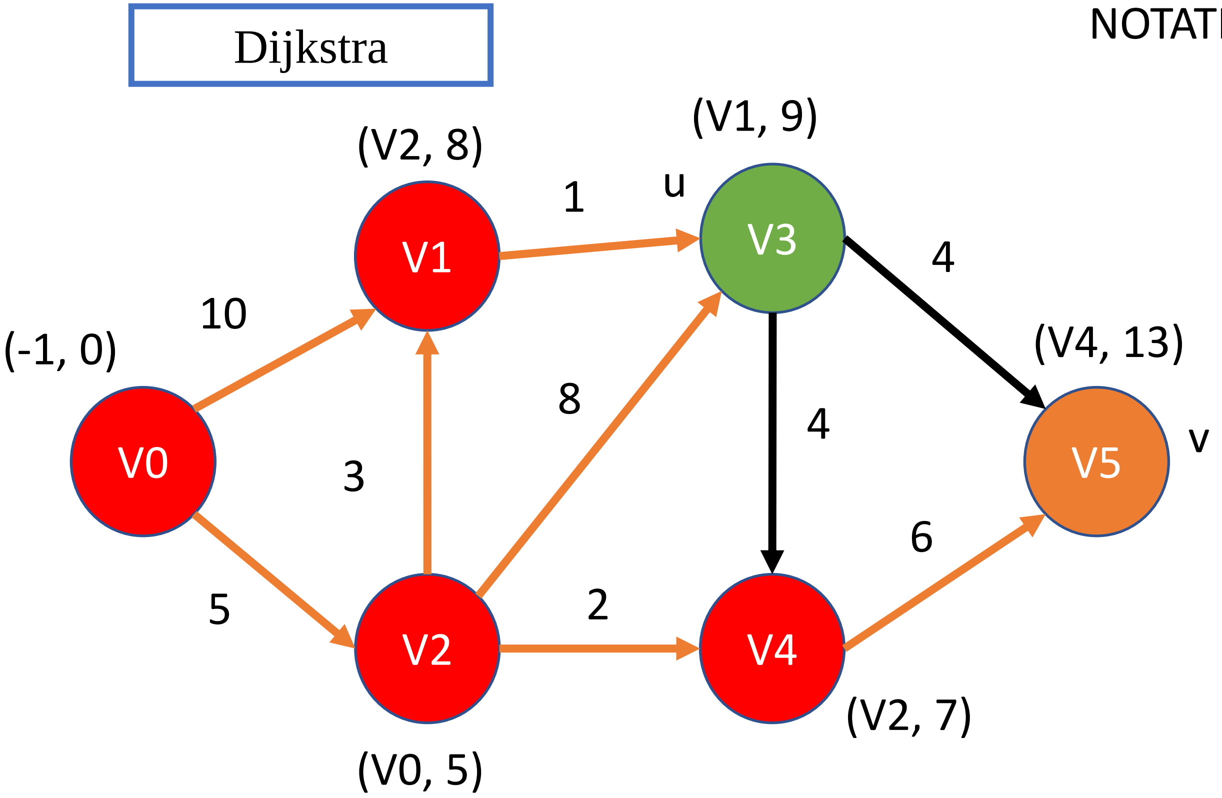

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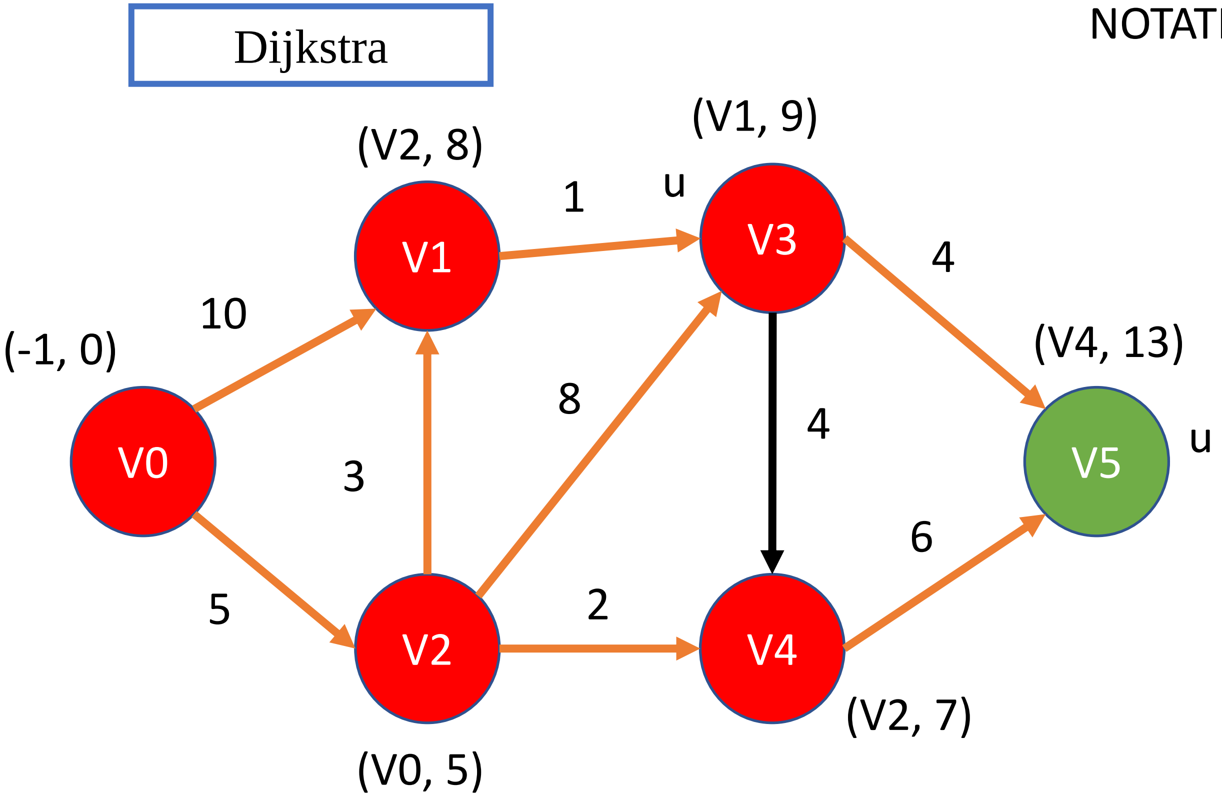

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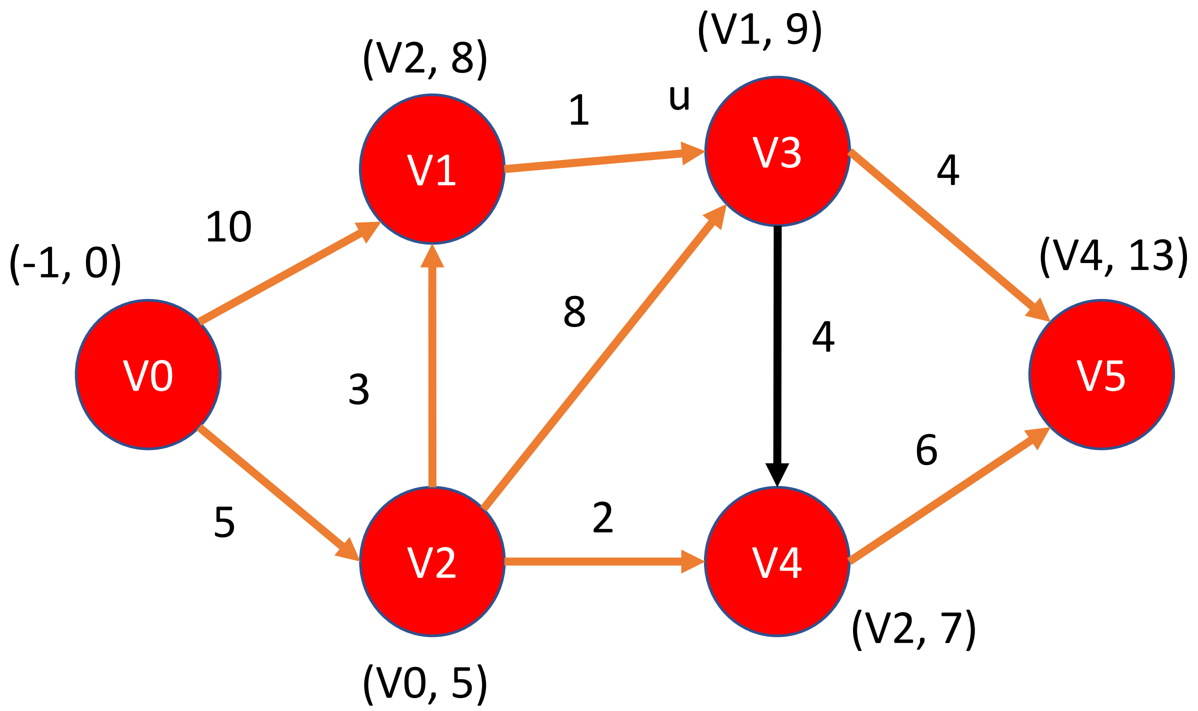
1. Initialize the graph with d(s) = 0, d(v) = INF, for all <u>v ≠ s</u>, and p(v) = -1 for all <u>*v*</u>
2. Make open(v) = True for every v in the graph
3. As long as there is an open vertex:
   * Choose u whose estimate is the smallest among the open
   * Close <u>*u*</u>
   * For every open node <u>*v*</u> adjacent to <u>*u*</u>: relax edge (<u>*u,v*</u>)

Algorithms

Dijkstra



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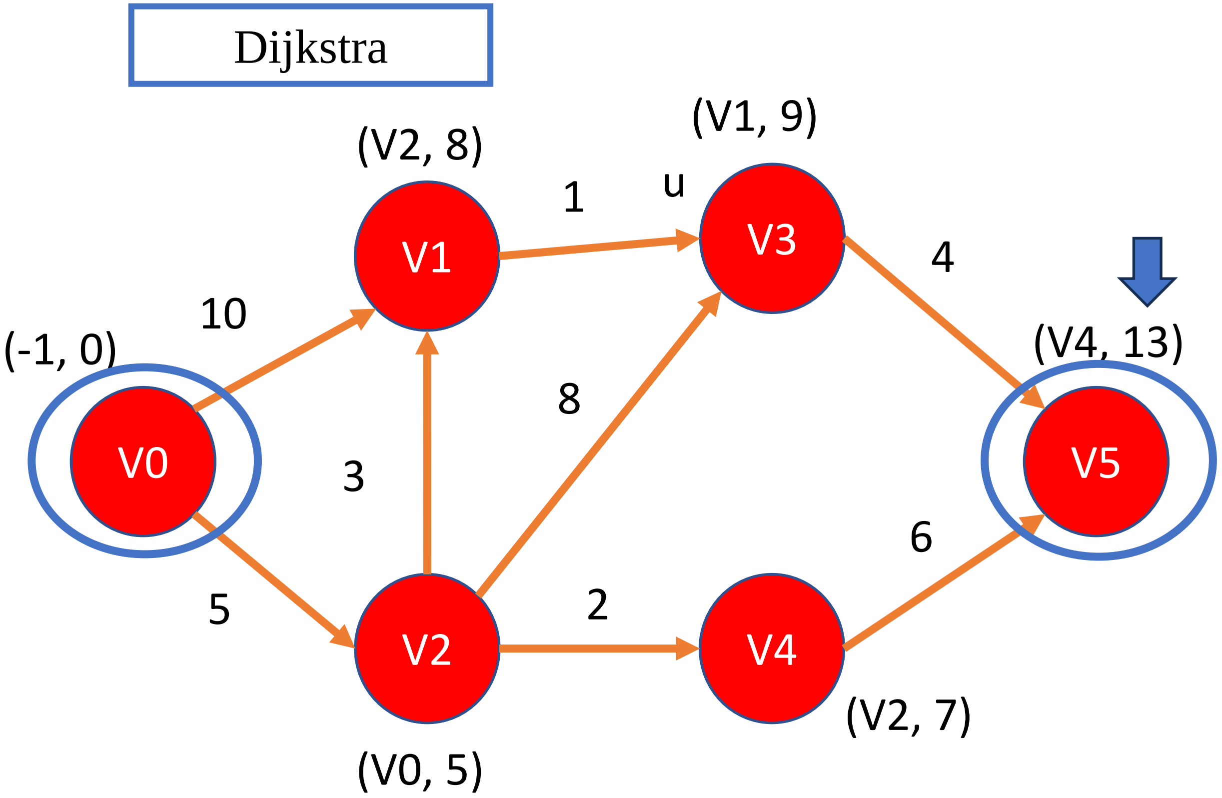
Algorithms



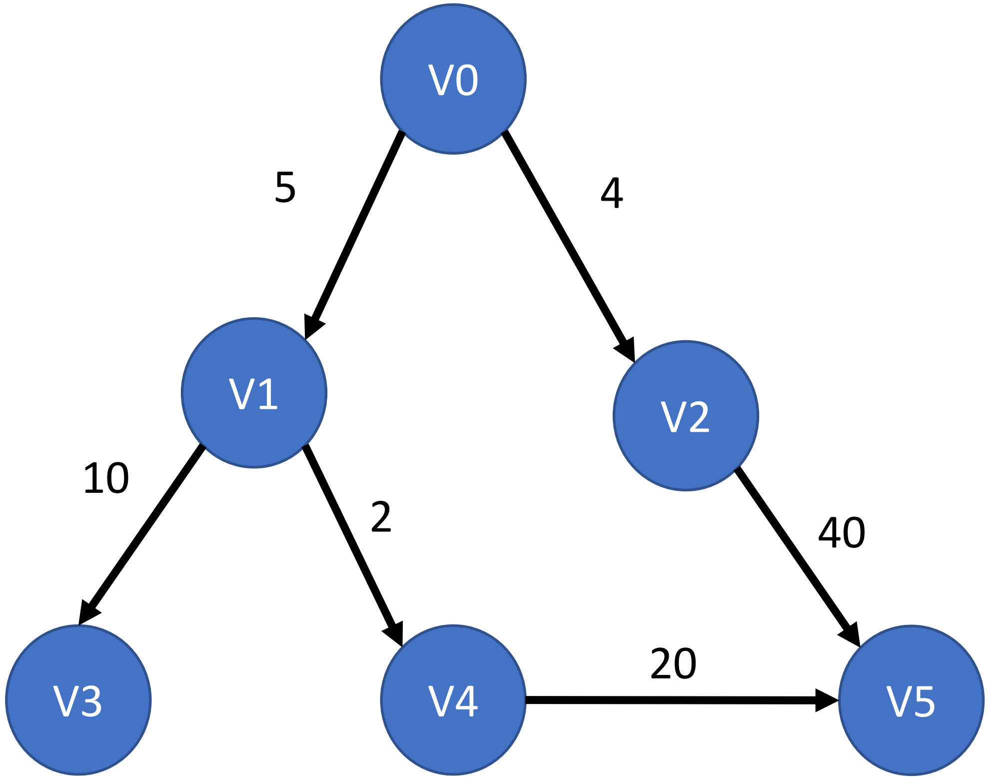
the shortest distance V0 => V5

V5, V4, V2, V0

TOTAL = 13

Algorithms

Dijkstra

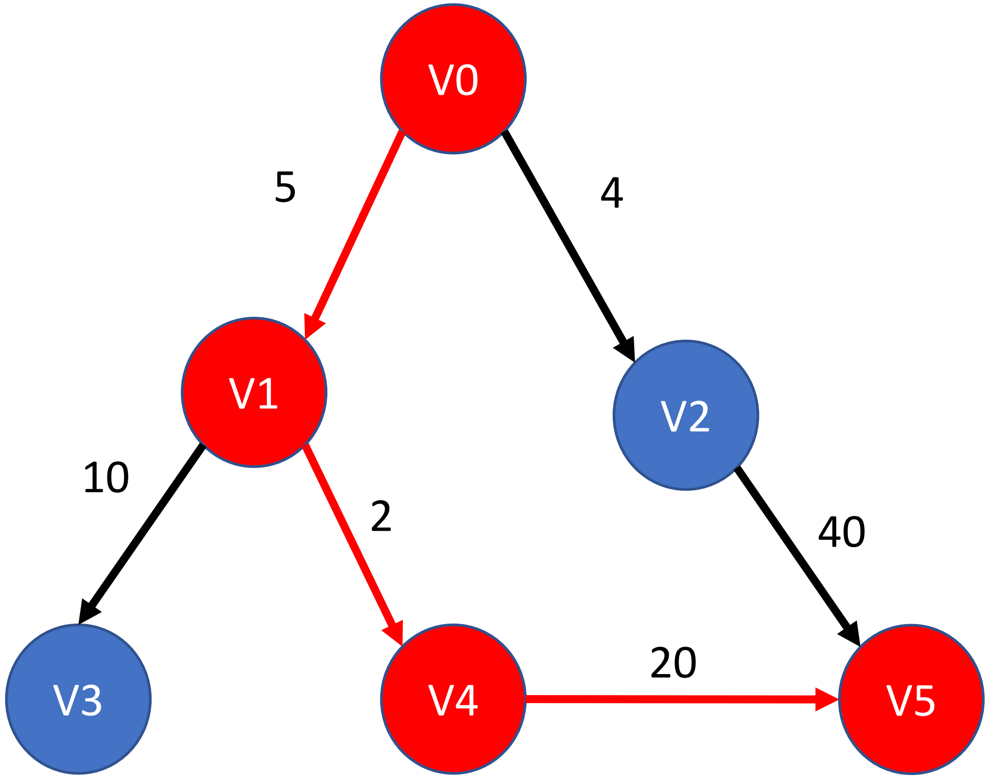



```python
def dijkstra_algorithm(graph, start_node):
    unvisited_nodes = list(graph.get_nodes())
    shortest_path = {}
    previous_nodes = {}
    # We'll use max_value to initialize the "infinity" value of the unvisited nodes
    max_value = sys.maxsize
    for node in unvisited_nodes:
        shortest_path[node] = max_value
    # However, we initialize the starting node's value with 0
    shortest_path[start_node] = 0
    while unvisited_nodes:
        current_min_node = None
        for node in unvisited_nodes: # Iterate over the nodes
            if current_min_node == None:
                current_min_node = node
            elif shortest_path[node] < shortest_path[current_min_node]:
                current_min_node = node
        # The code block below retrieves the current node's neighbors and updates their distances
        neighbors = graph.get_outgoing_edges(current_min_node)
        for neighbor in neighbors:
            tentative_value = shortest_path[current_min_node] + graph.value(current_min_node, neighbor)
            if tentative_value < shortest_path[neighbor]:
                shortest_path[neighbor] = tentative_value
                # We also update the best path to the current node
                previous_nodes[neighbor] = current_min_node
        unvisited_nodes.remove(current_min_node)

    return previous_nodes, shortest_path
```

Algorithms

Dijkstra




```
We found the following best path with a value of 27.
V0 -> V1 -> V4 -> V5
```

Algorithms

A*

A* is an informed search algorithm. Informed Search signifies that the algorithm has extra information, to begin with. It is a complete as well as an optimal solution for solving path and grid problems.

Algorithms

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Complete – It means that it will find all the available paths from start to end.
Optimal – find the least cost from the starting point to the ending point.

Algorithms

A*

A* is an **informed search** algorithm. **Informed Search** signifies that the algorithm has extra information, to begin with. It is a **complete** as well as an **optimal** solution for solving path and grid problems.

$$f(n) = g(n) + h(n)$$

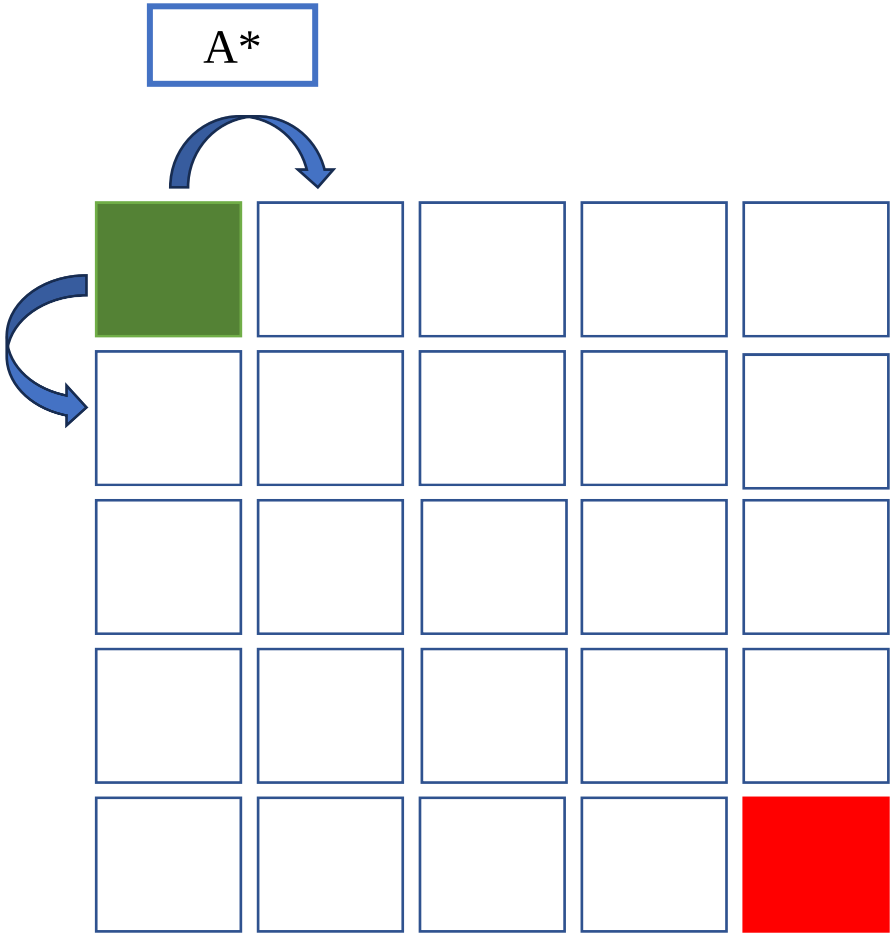
The Algorithm

1. Place the starting node into OPEN and find its f (n) value.
2. Remove the node from OPEN, having the smallest f (n) value.
   * If it is a goal node, then stop and return to success.
   * Else remove the node from OPEN, and find all its successors.
   * Find the f (n) value of all the successors, place them into OPEN, and place the removed node into CLOSE.

Algorithms

manhattan((x1, y1), (x2, y2)) = |x1 — x2| + |y1 — y2|

euclidean((x1, y1), (x2, y2)) = sqrt(x^2 + y^2)

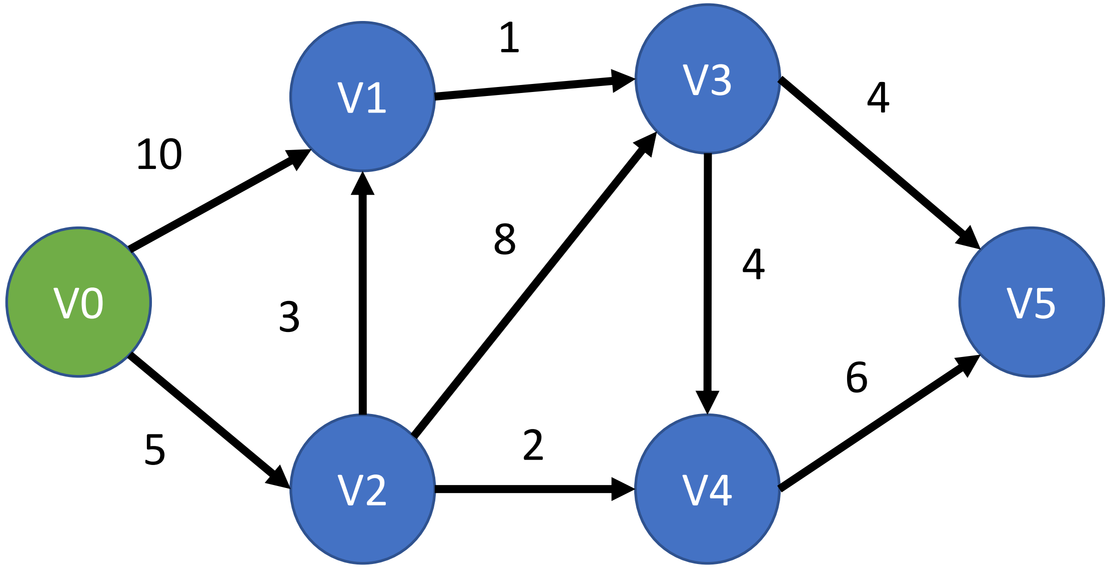


h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*



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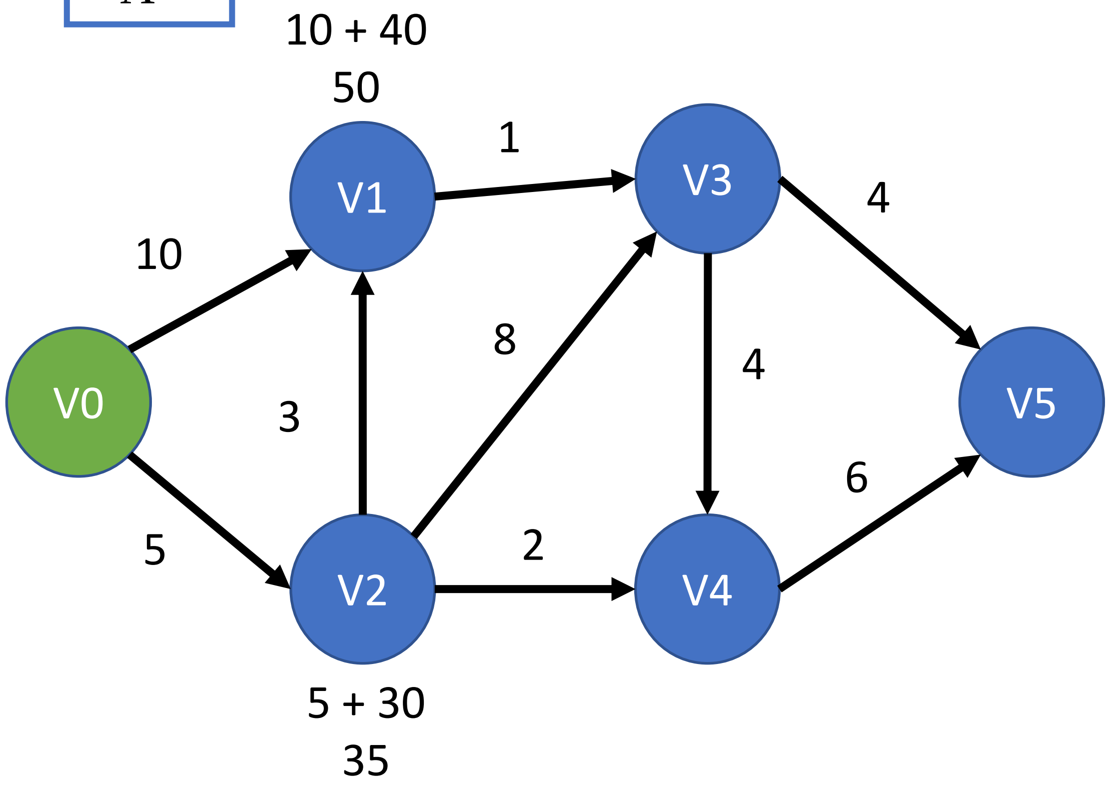
h(n) →

	V5
V0	50
V1	40
V2	30
V3	20
V4	10

Algorithms

A*



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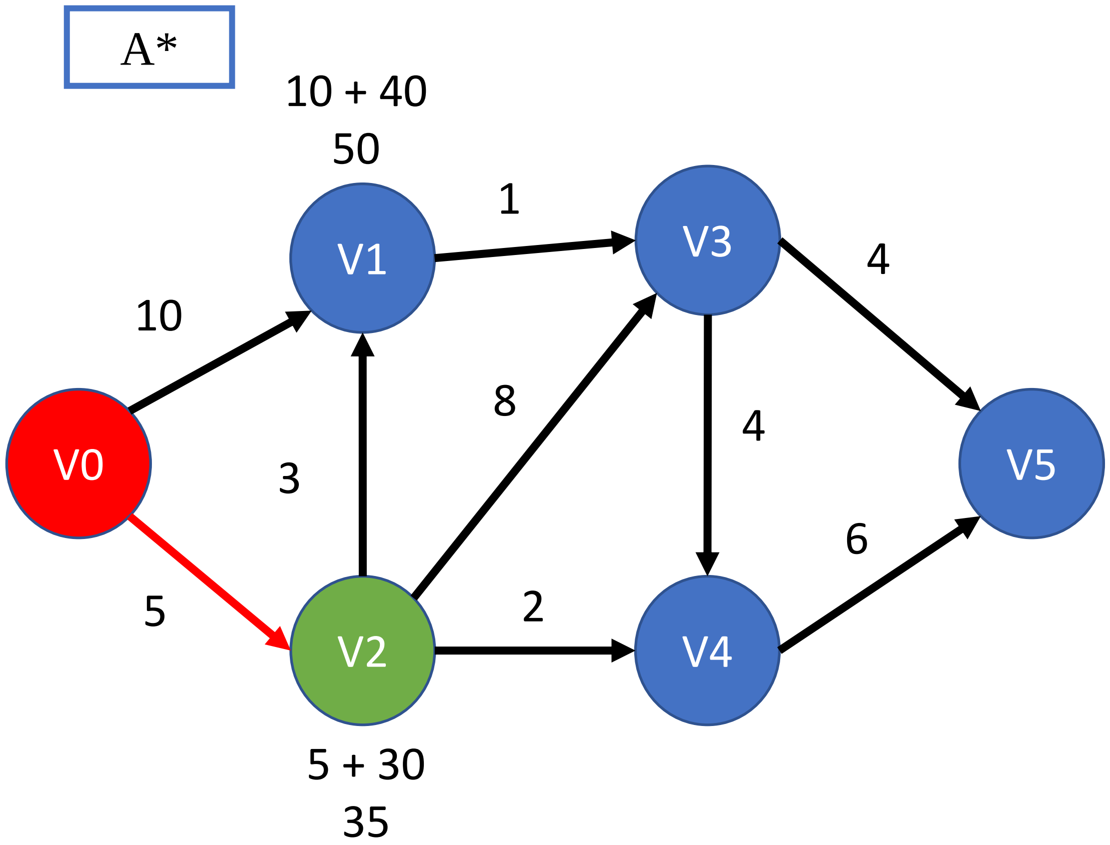

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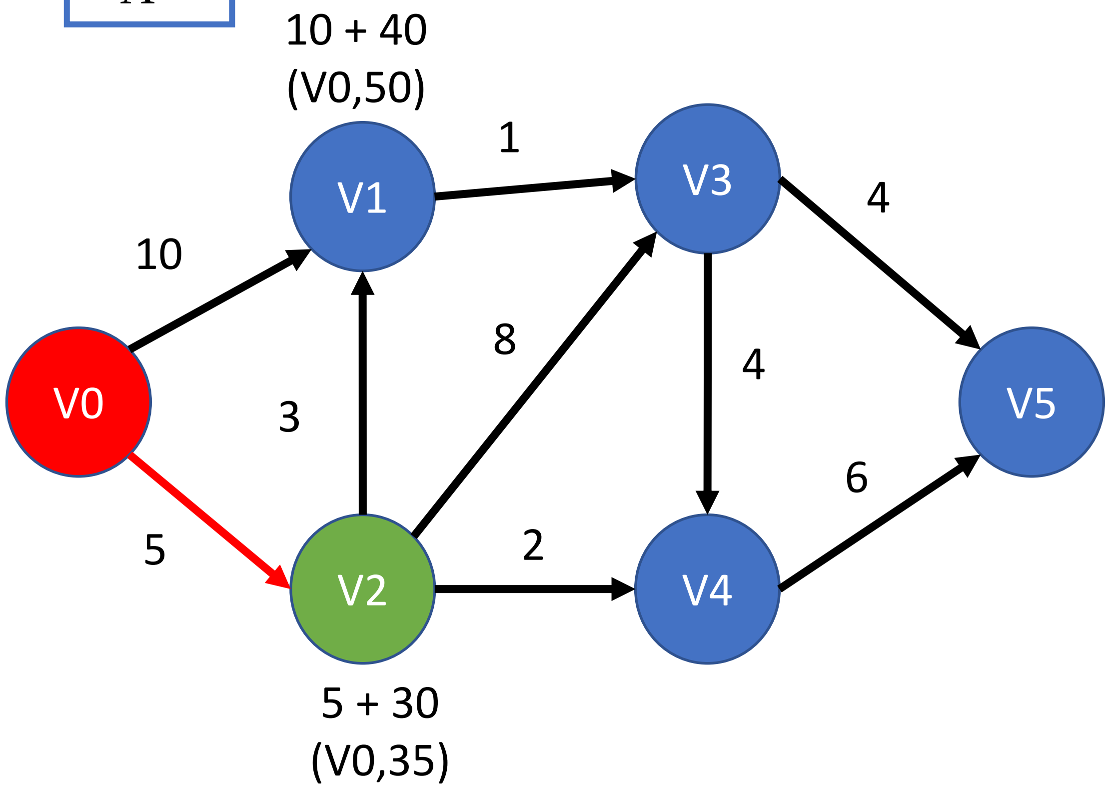

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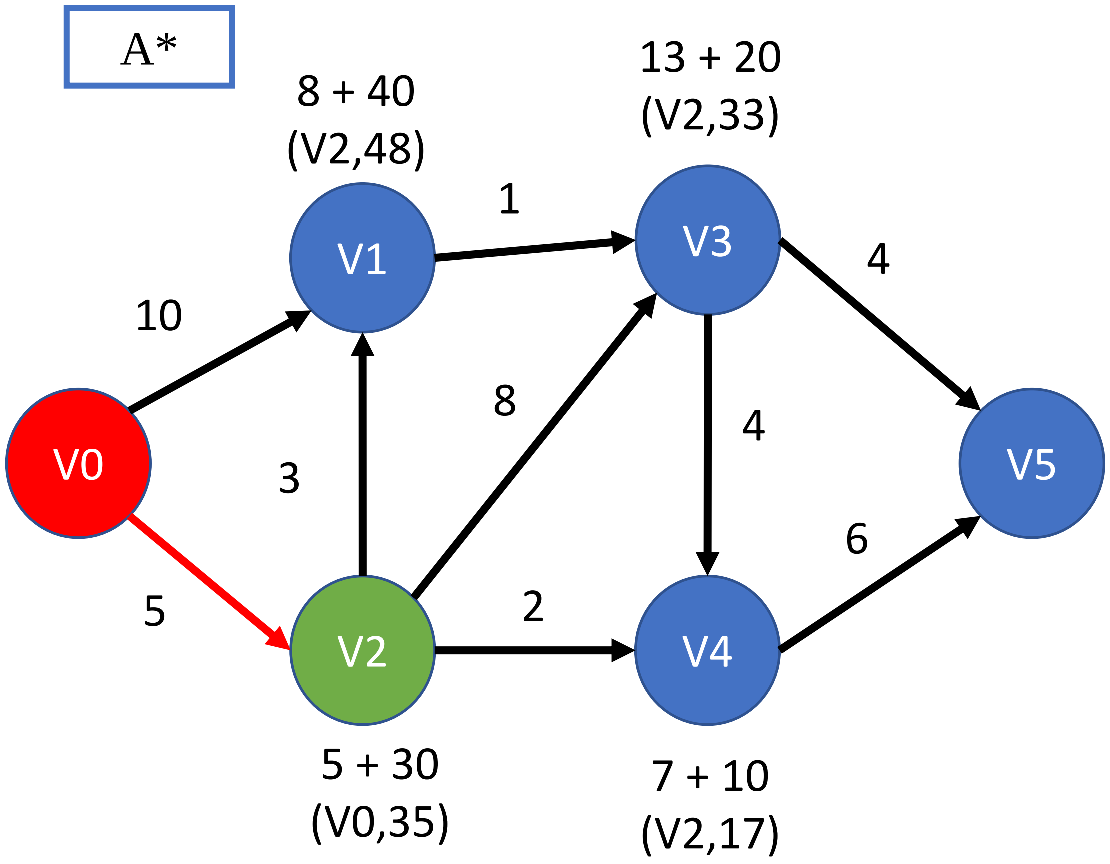

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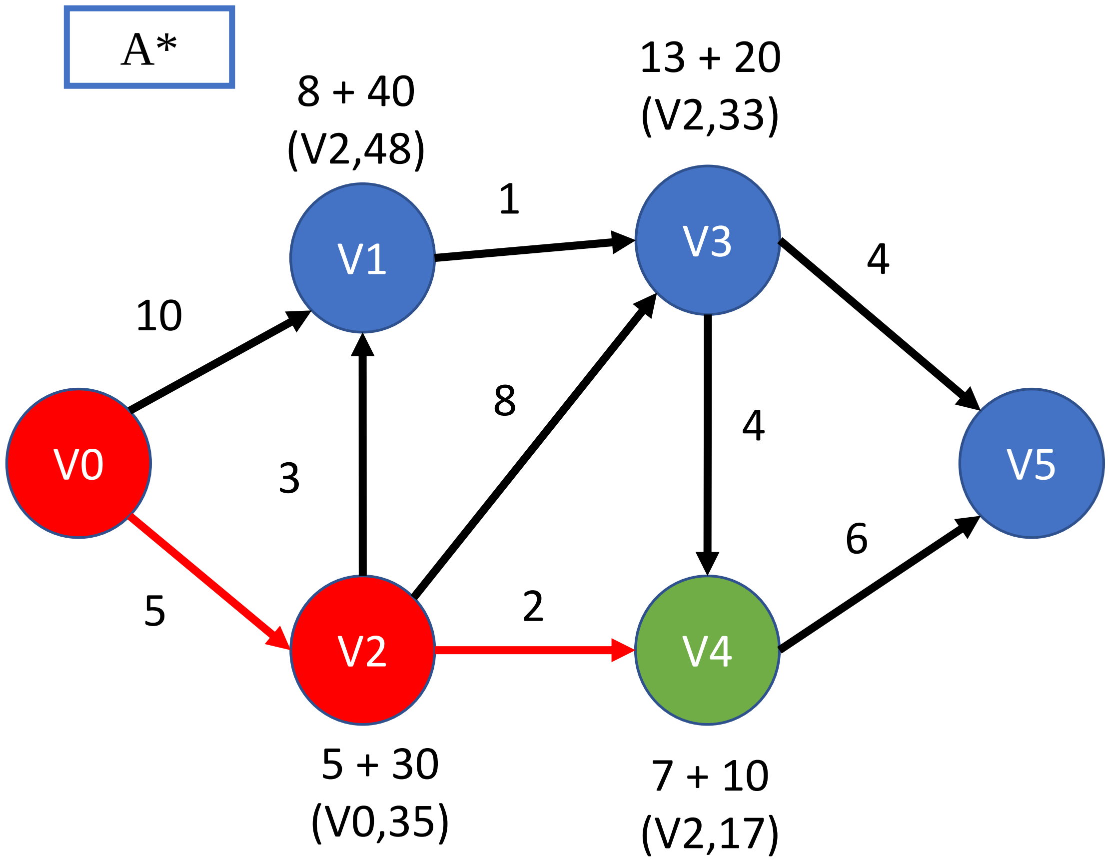

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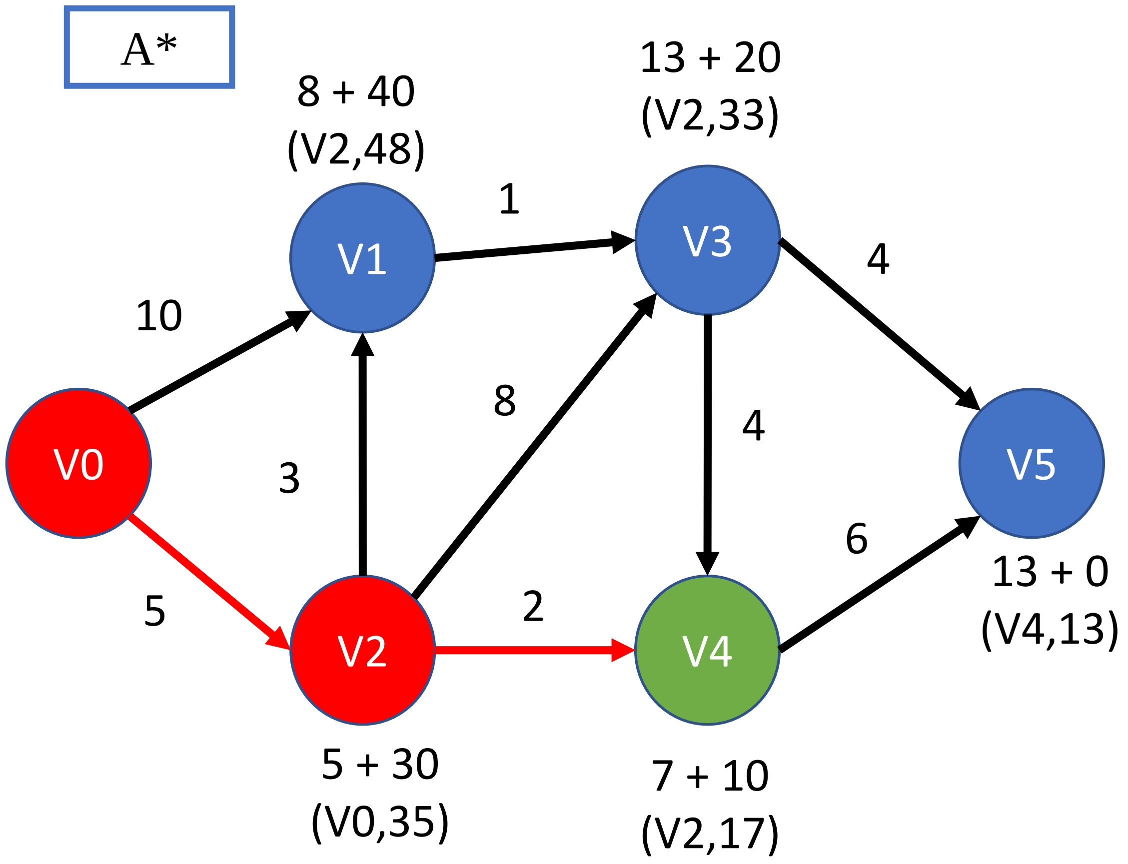

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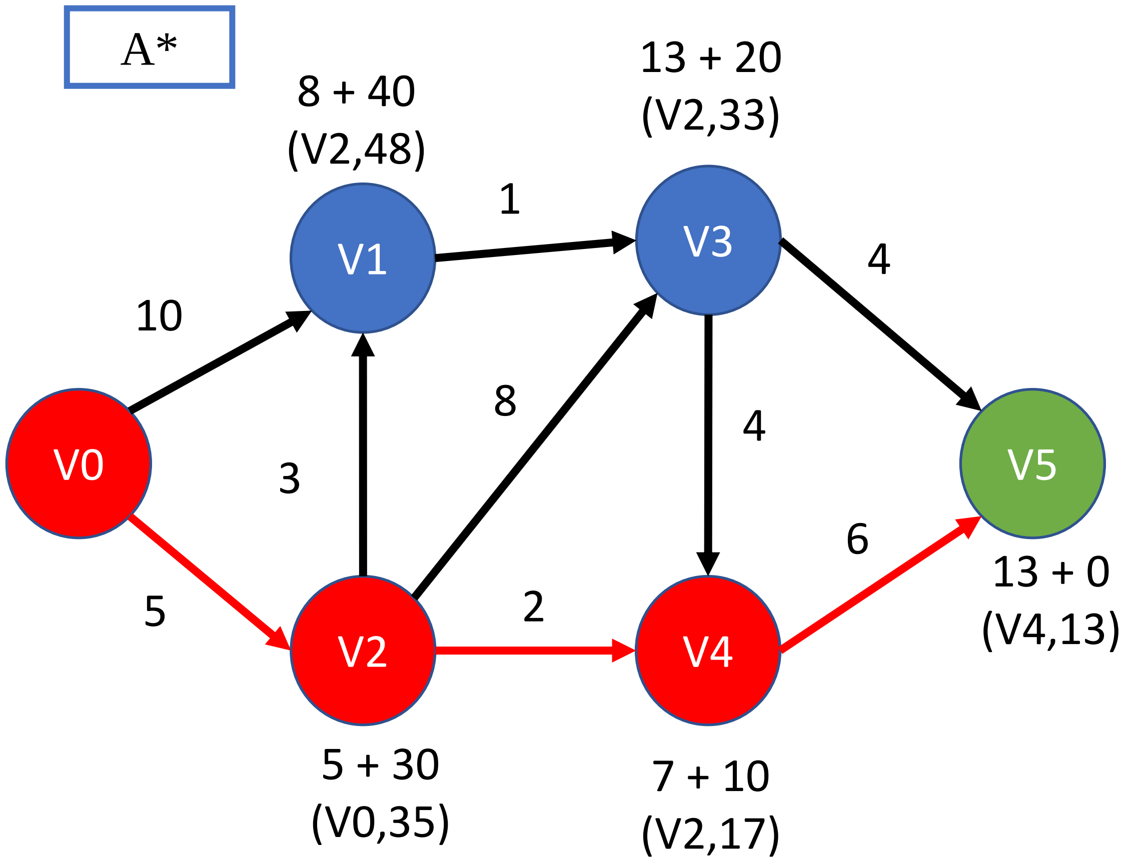

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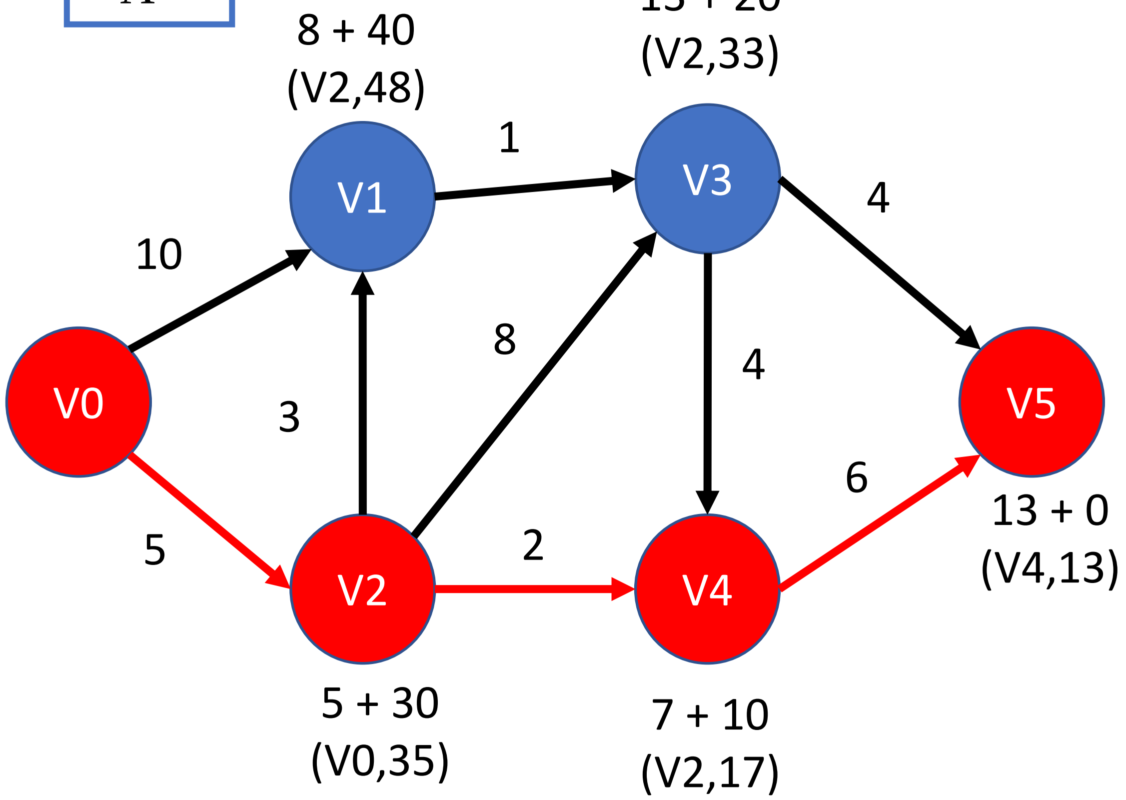

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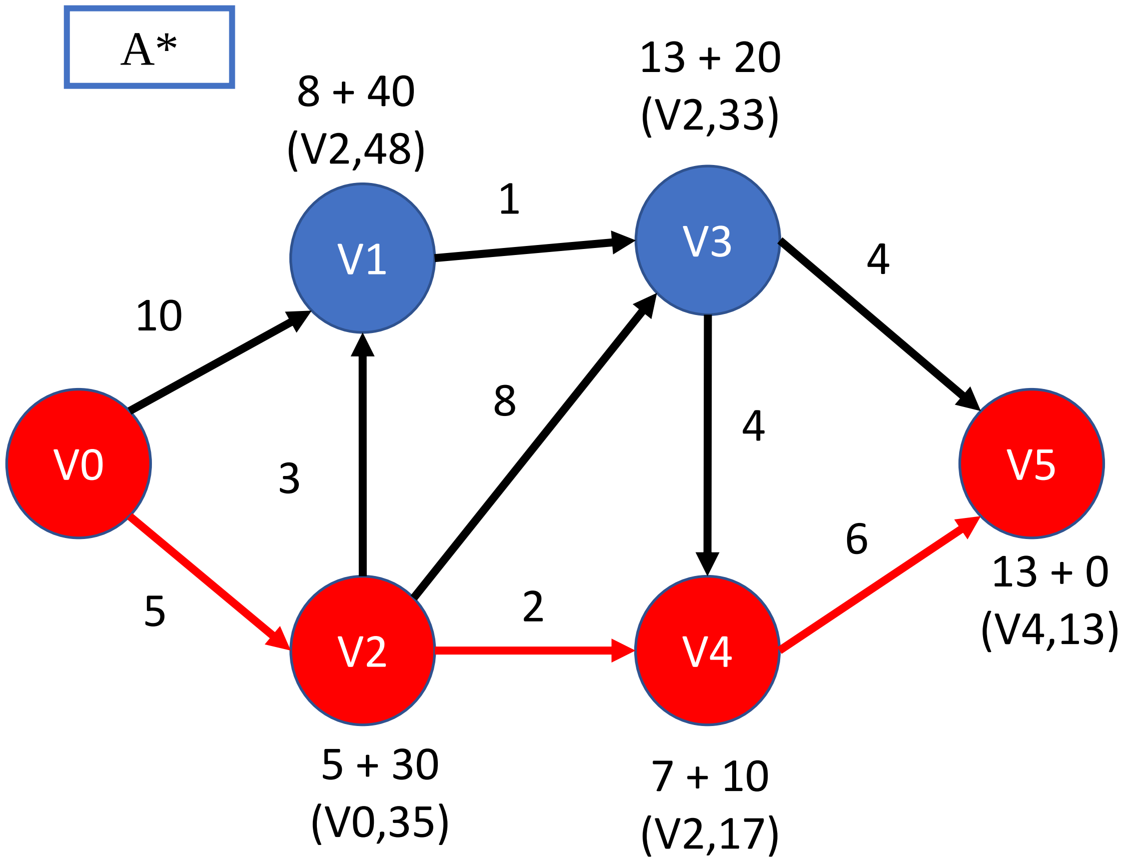

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Algorithms

A*

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1	1	1	1	1
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1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
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1	1	1	1	7+1 8
1	1	1	1	1

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1	1	1	1	8+0 8

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