Path-finding

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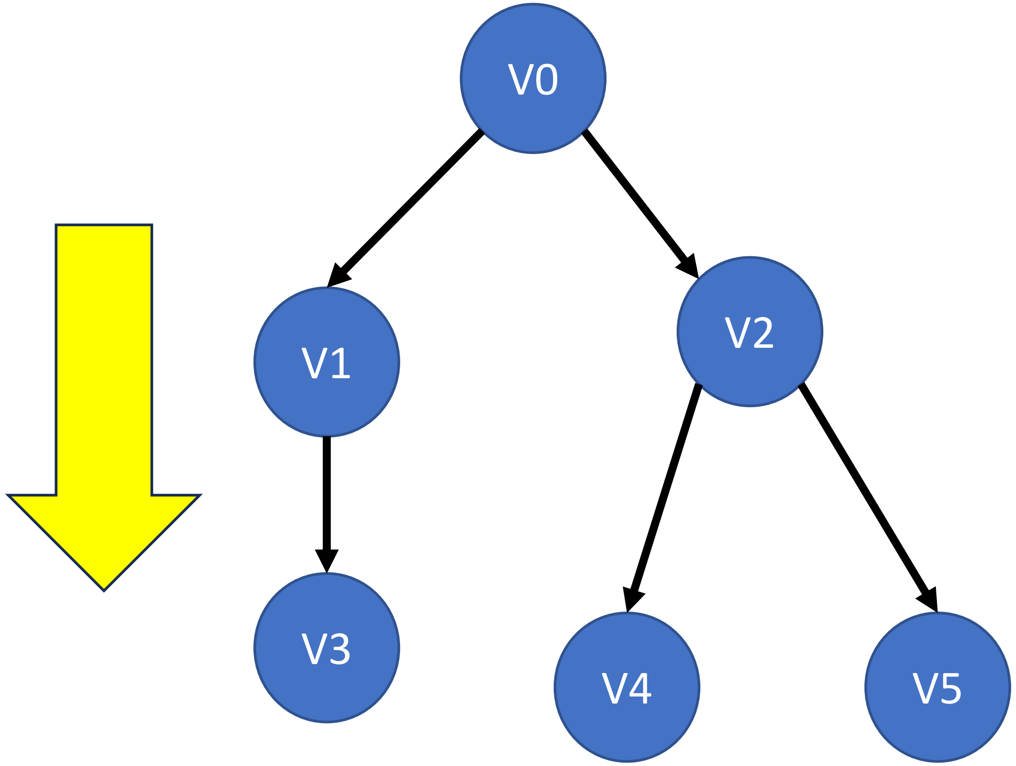
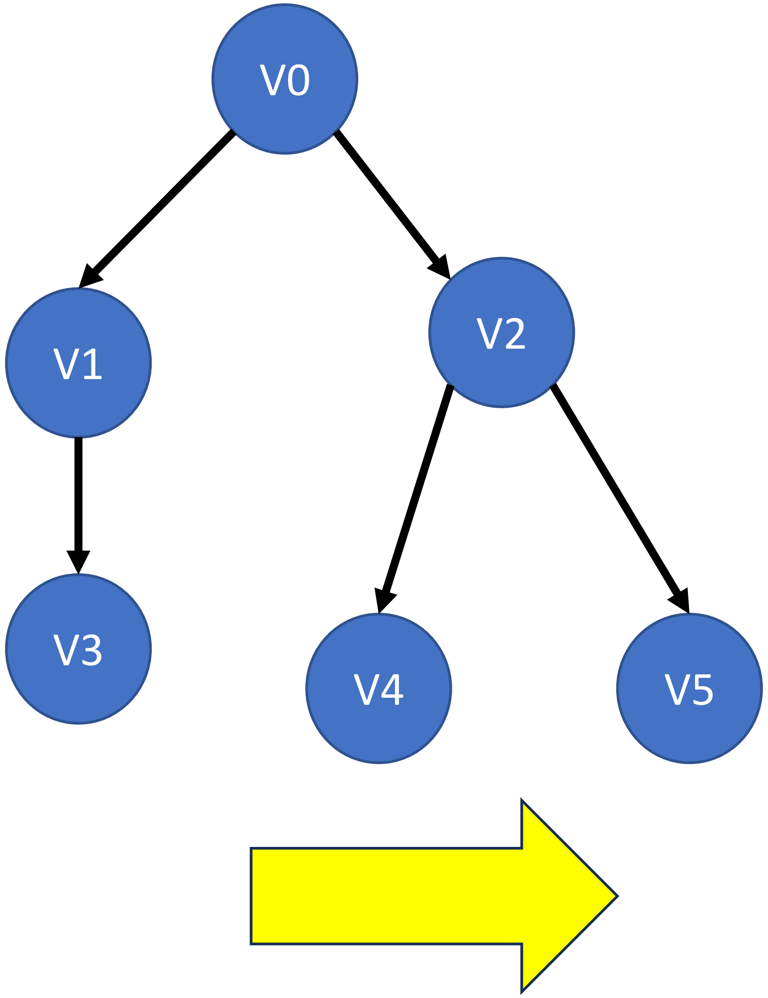
Algorithms

Depth-First Search (DFS)

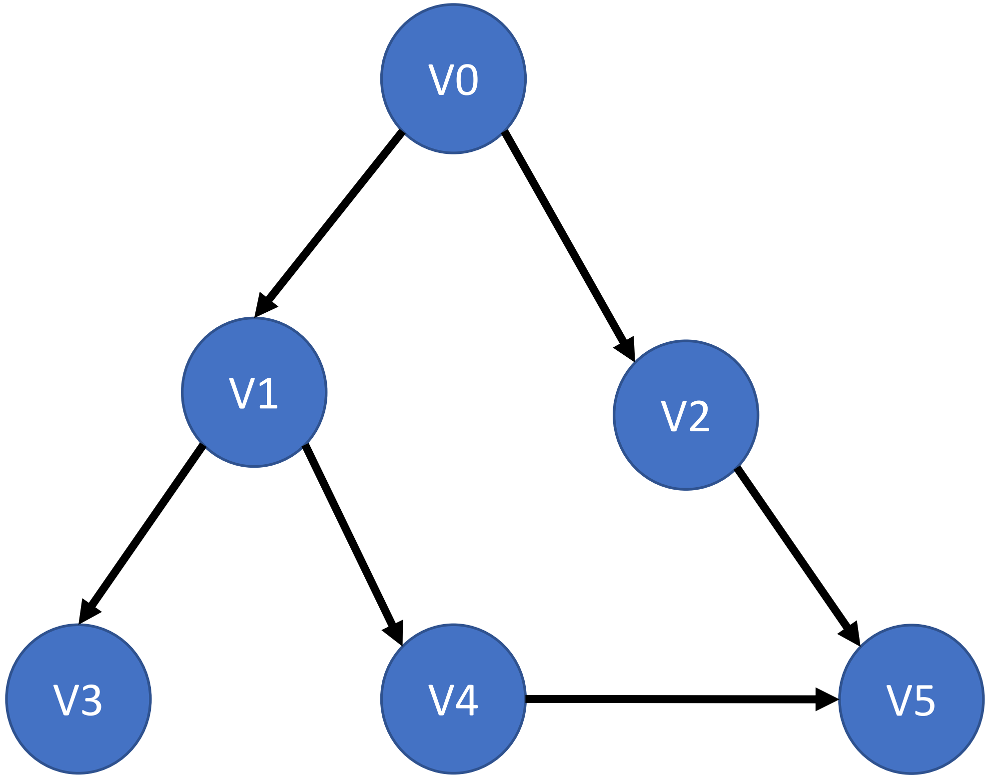


Breadth-First Search (BFS)



Algorithms

Depth-First Search (DFS)



1. Set a start node
2. While this is not an objective or final node (node whose adjacency has already been visited):
   - Choose an adjacent node not yet visited
   - Visit it
3. If it is a non-objective end node:
   - Return to this father
   - If there is a father, repeat. If there is no parent, choose another start node

Algorithms

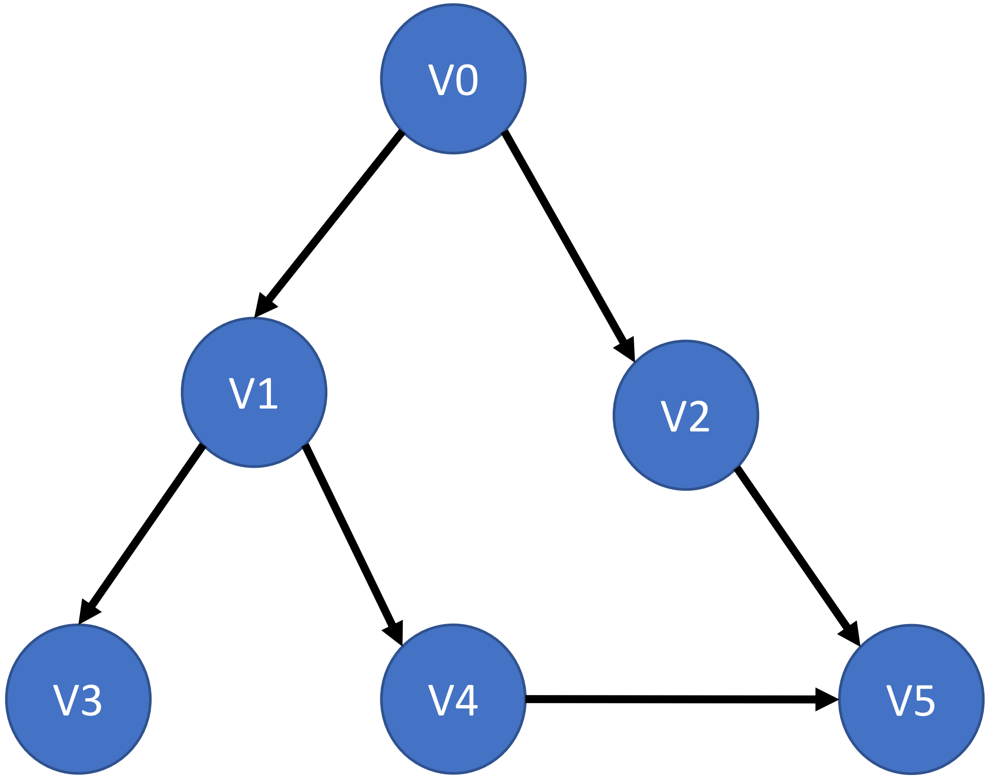
Depth-First Search (DFS)

```
using System;
using System.Collections.Generic;

class Program
{
    static Dictionary<string, List<string>> graph = new Dictionary<string, List<string>>
    {
        { "V0", new List<string> { "V1", "V2" } },
        { "V1", new List<string> { "V3", "V4" } },
        { "V2", new List<string> { "V5" } },
        { "V3", new List<string>() },
        { "V4", new List<string> { "V5" } },
        { "V5", new List<string>() }
    };

    static HashSet<string> visited = new HashSet<string>();

    static void Main(string[] args)
    {
        string startNode = "V0";

        Console.WriteLine("Following is the Depth-First Search:");
        DFS(startNode);

        Console.ReadLine();
    }

    static void DFS(string node)
    {
        if (!visited.Contains(node))
        {
            Console.WriteLine(node);
            visited.Add(node);

            if (graph.ContainsKey(node))
            {
                foreach (string neighbor in graph[node])
                {
                    DFS(neighbor);          }         }      }   }   }
```

```
private void DFSRecursive(string vertex, HashSet<string> visited)
    {
        visited.Add(vertex);
        Console.WriteLine("Visiting vertex: " + vertex);

        foreach (string neighbor in adjacencyList[vertex])
        {
            if (!visited.Contains(neighbor))
            {
                DFSRecursive(neighbor, visited);
            }       }    }  }
class Program
{
    static void Main(string[] args)
    {
        Graph graph = new Graph();

        // Adicionar vértices
        graph.AddVertex("V0");
        graph.AddVertex("V1");
        graph.AddVertex("V2");
        graph.AddVertex("V3");
        graph.AddVertex("V4");
        graph.AddVertex("V5");

        // Adicionar arestas
        graph.AddEdge("V0", "V1");
        graph.AddEdge("V0", "V2");
        graph.AddEdge("V1", "V3");
        graph.AddEdge("V1", "V4");
        graph.AddEdge("V2", "V5");
        graph.AddEdge("V4", "V5");

        Console.WriteLine("DFS starting from vertex V0:");
        graph.DFS("V0");    } }
```

Algorithms

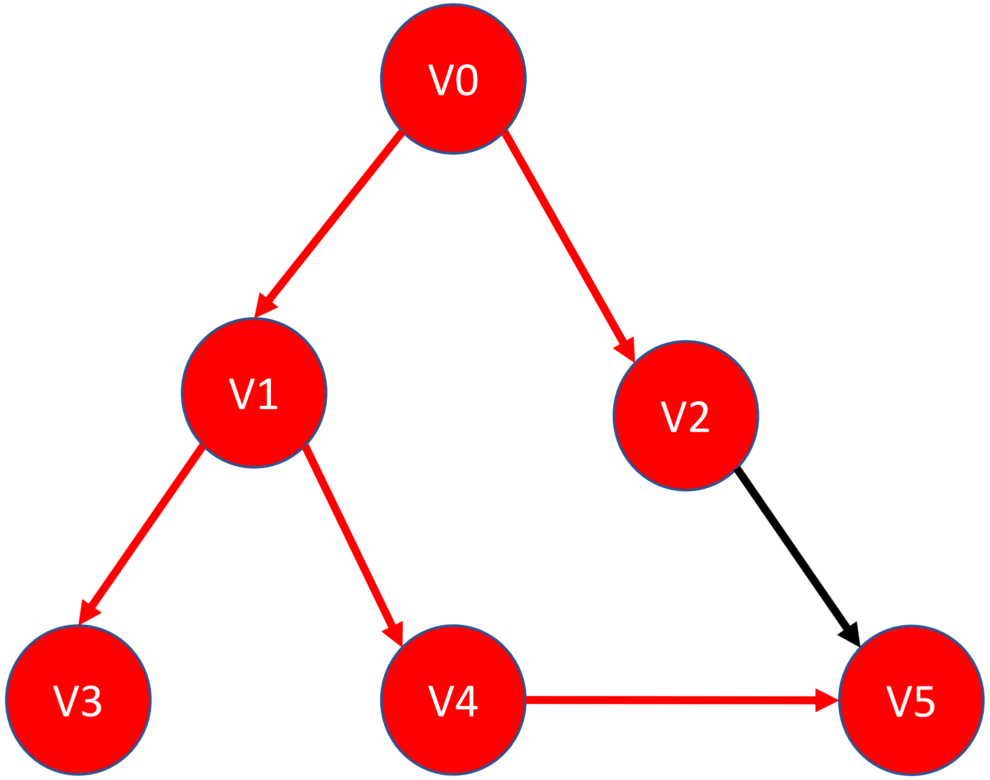
Depth-First Search (DFS)




```
def dfs(graph, vertex, visited):
    # Marcar o vértice como visitado
    visited[vertex] = True
    print("Visitando vértice:", vertex)

    # Recursivamente visitar os vértices adjacentes não visitados
    for neighbor in graph[vertex]:
        if not visited[neighbor]:
            dfs(graph, neighbor, visited)

# Grafo representado como um dicionário de adjacências
graph = {
    'V0': ['V1', 'V2'],
    'V1': ['V3', 'V4'],
    'V2': ['V5'],
    'V3': [],
    'V4': ['V5'],
    'V5': []
}

# Inicializar um vetor de visitados
visited = {vertex: False for vertex in graph}

# Chamar o DFS a partir de todos os vértices não visitados
for vertex in graph:
    if not visited[vertex]:
        dfs(graph, vertex, visited)
```

Algorithms

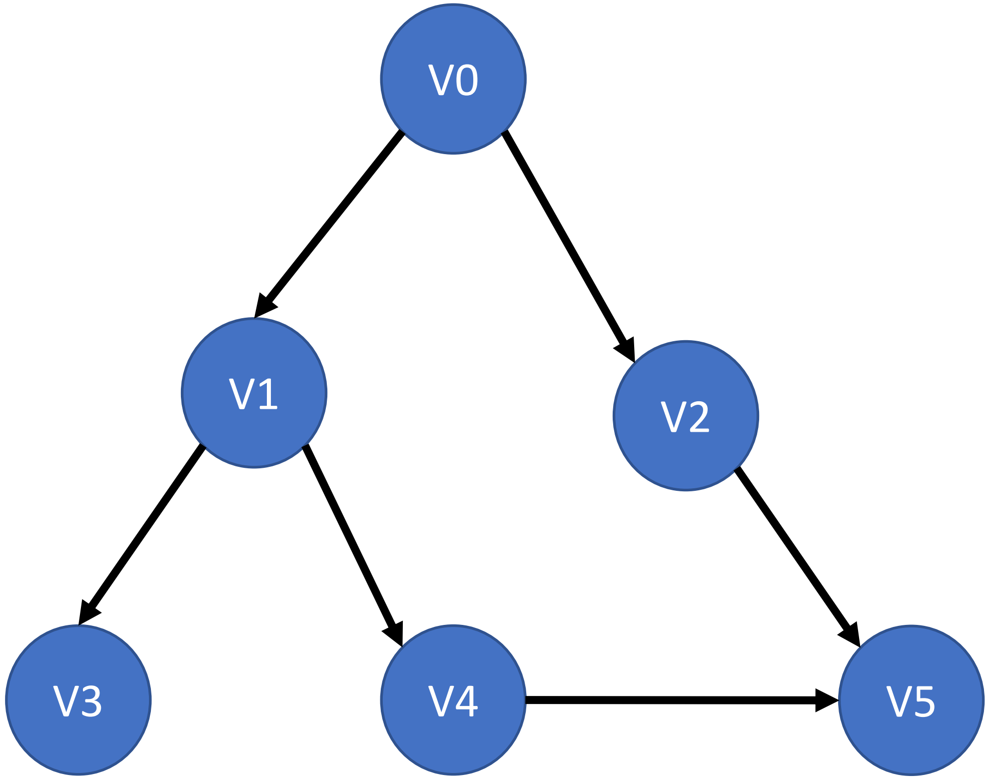
Depth-First Search (DFS)




```
Following is the Depth-First Search
V0
V1
V3
V4
V5
V2
```

Algorithms

Breadth-First Search (BFS)



1. Define an initial node, marking it as explored
2. Put it on the list
3. As long as the queue is not empty:
   - Remove the 1st node from the list, u
   - For each neighbour v of u:
     * If v is not explored:
       ** Mark v as explored
       ** Put v at the end of the list
4. Repeat from another starting node, if there is one

Algorithms

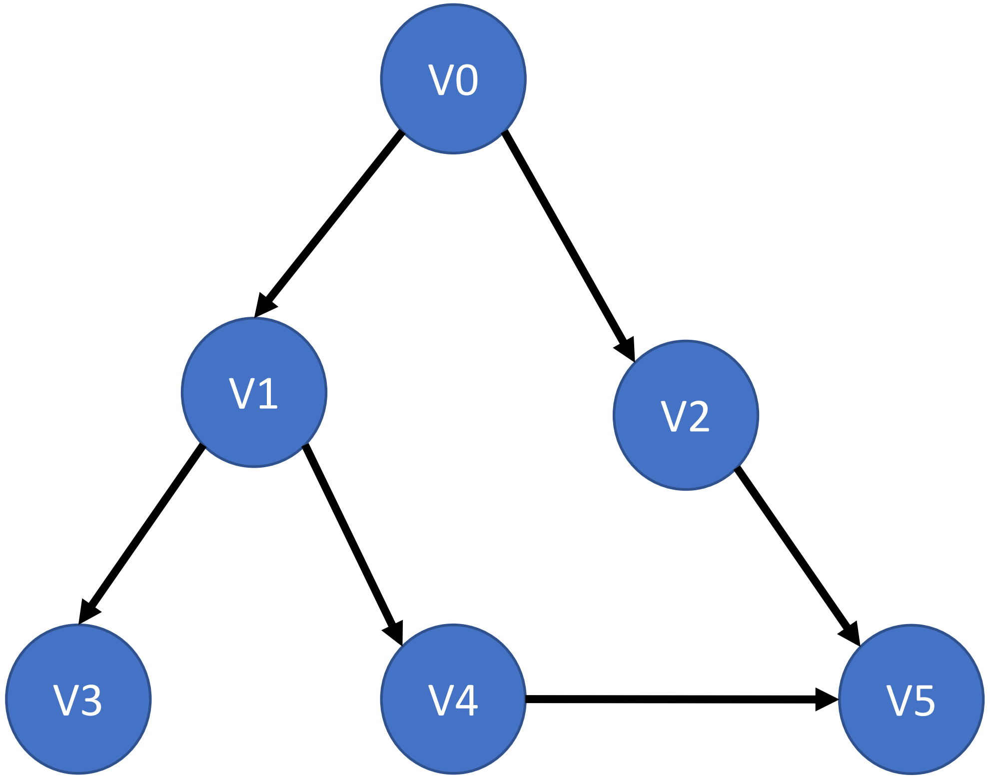
Breadth-First Search (BFS)

```
using System;
using System.Collections.Generic;

class Program
{
   static Dictionary<string, List<string>> graph = new Dictionary<string, List<string>>
   {
      { "V0", new List<string> { "V1", "V2" } },
      { "V1", new List<string> { "V3", "V4" } },
      { "V2", new List<string> { "V5" } },
      { "V3", new List<string>() },
      { "V4", new List<string> { "V5" } },
      { "V5", new List<string>() }
   };

   static List<string> visited = new List<string>();
   static Queue<string> queue = new Queue<string>();

   static void Main(string[] args)
   {
      string startNode = "V0";

      Console.WriteLine("Following is the Breadth-First Search:");
      BFS(startNode);

      Console.ReadLine();
   }
```

```
static void BFS(string node)
   {
      visited.Add(node);
      queue.Enqueue(node);

      while (queue.Count > 0)
      {
         string s = queue.Dequeue();
         Console.Write(s + " ");

         if (graph.ContainsKey(s))
         {
            foreach (string neighbor in graph[s])
            {
               if (!visited.Contains(neighbor))
               {
                  visited.Add(neighbor);
                  queue.Enqueue(neighbor);
               }
            }
         }
      }
   }
}
```

Algorithms

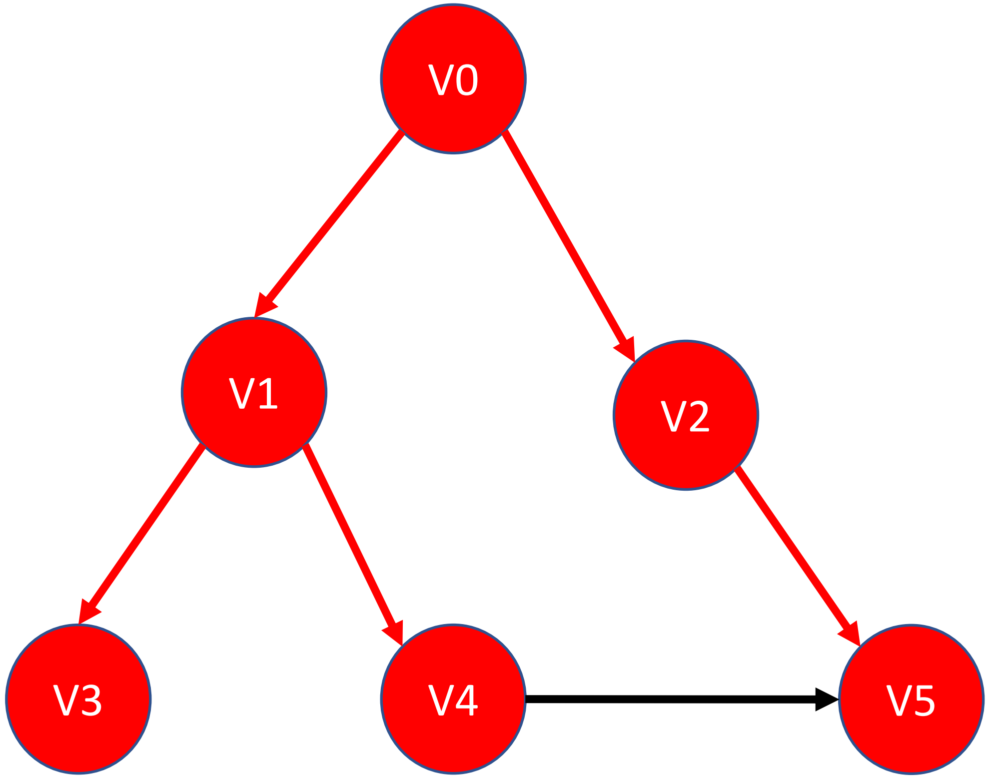
Breadth-First Search (BFS)

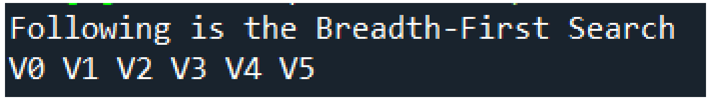



```python
graph = {
    'V0' : ['V1','V2'],
    'V1' : ['V3', 'V4'],
    'V2' : ['V5'],
    'V3' : [],
    'V4' : ['V5'],
    'V5' : []
}

visited = [] # List to keep track of visited nodes.
queue = []     #Initialize a queue

def bfs(visited, graph, node):
  visited.append(node)
  queue.append(node)

  while queue:
    s = queue.pop(0)
    print (s, end = " ")

    for neighbour in graph[s]:
      if neighbour not in visited:
        visited.append(neighbour)
        queue.append(neighbour)

# Driver Code
print("Following is the Breadth-First Search")
bfs(visited, graph, 'V0')
```

Algorithms

Breadth-First Search (BFS)





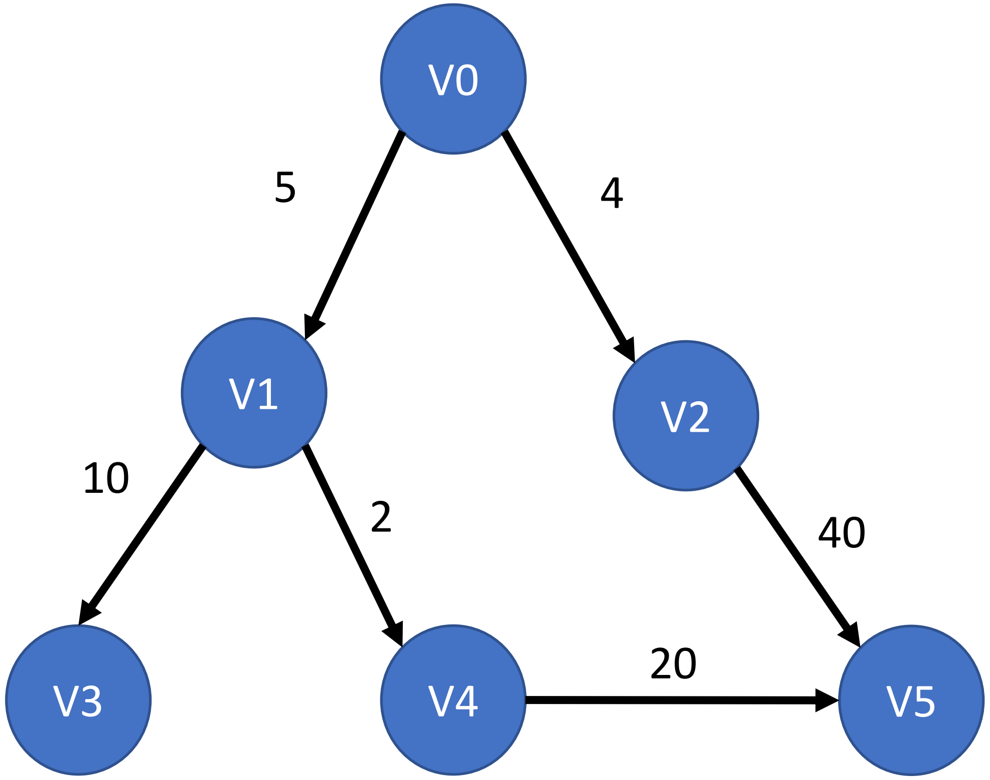
```
Following is the Breadth-First Search
V0 V1 V2 V3 V4 V5
```

DFS V0 V1 V3 V4 V5 V2

BFS V0 V1 V2 V3 V4 V5

Algorithms

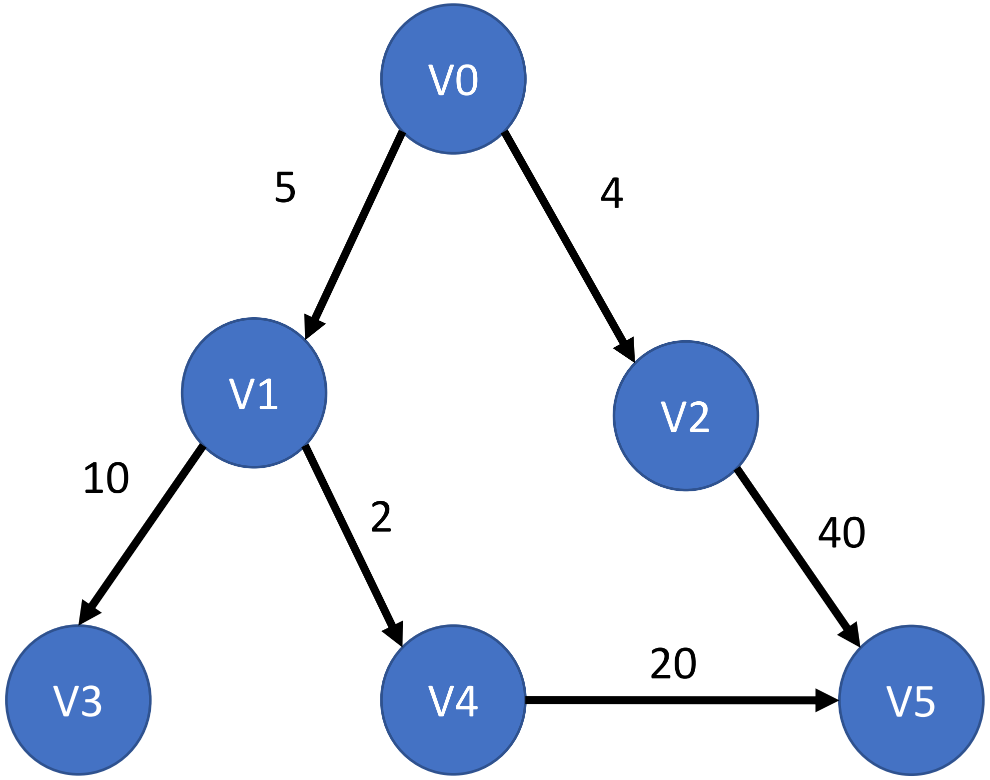
Dijkstra



1. Initialize the graph with d(s) = 0, d(v) = INF, for all v ≠ s, and p(v) = -1 for all v
2. Make open(v) = True for every v in the graph
3. As long as there is an open vertex:
   * Choose u whose estimate is the smallest among the open
   * Close u
   * For every open node v adjacent to u: relax edge (u,v)

Algorithms

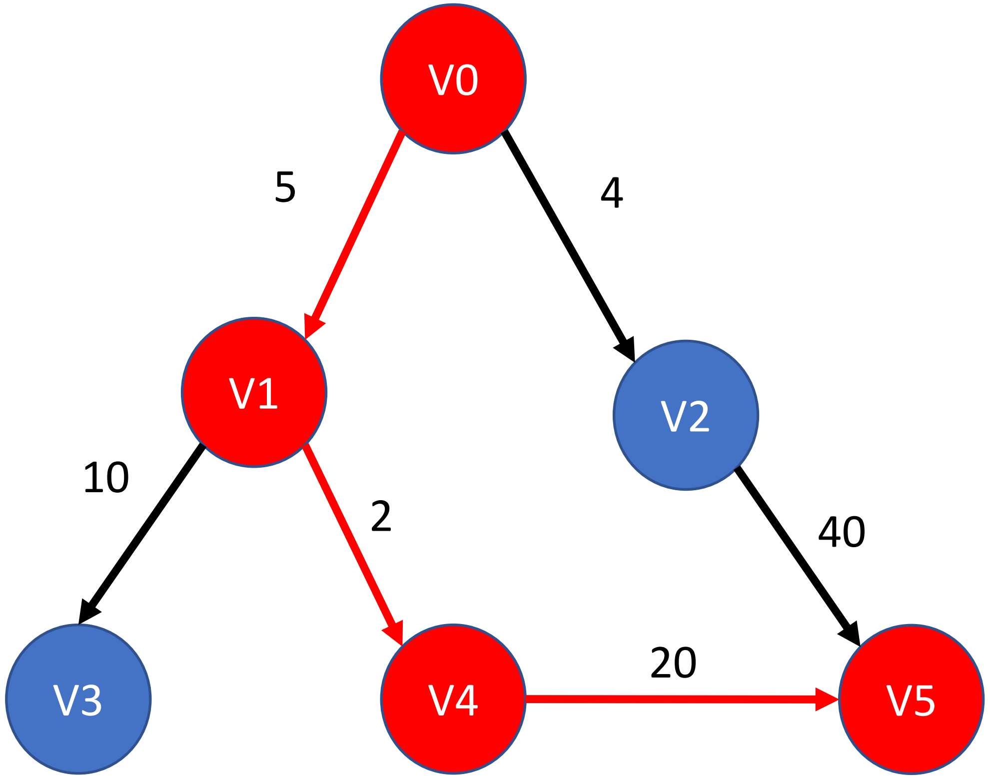
Dijkstra




```python
def dijkstra_algorithm(graph, start_node):
    unvisited_nodes = list(graph.get_nodes())
    shortest_path = {}
    previous_nodes = {}
    # We'll use max_value to initialize the "infinity" value of the unvisited nodes
    max_value = sys.maxsize
    for node in unvisited_nodes:
        shortest_path[node] = max_value
    # However, we initialize the starting node's value with 0
    shortest_path[start_node] = 0
    while unvisited_nodes:
        current_min_node = None
        for node in unvisited_nodes: # Iterate over the nodes
            if current_min_node == None:
                current_min_node = node
            elif shortest_path[node] < shortest_path[current_min_node]:
                current_min_node = node
        # The code block below retrieves the current node's neighbors and updates their distances
        neighbors = graph.get_outgoing_edges(current_min_node)
        for neighbor in neighbors:
            tentative_value = shortest_path[current_min_node] + graph.value(current_min_node, neighbor)
            if tentative_value < shortest_path[neighbor]:
                shortest_path[neighbor] = tentative_value
                # We also update the best path to the current node
                previous_nodes[neighbor] = current_min_node
        unvisited_nodes.remove(current_min_node)

    return previous_nodes, shortest_path
```

Algorithms

Dijkstra




```
We found the following best path with a value of 27.
V0 -> V1 -> V4 -> V5
```

Algorithms

Dijkstra

```
using System;
using System.Collections.Generic;

class Program
{
    static void Main(string[] args)
    {
        Dictionary<string, Dictionary<string, int>> initGraph = new Dictionary<string, Dictionary<string, int>>()
        {
            { "V0", new Dictionary<string, int> { { "V1", 5 }, { "V2", 4 } } },
            { "V1", new Dictionary<string, int> { { "V3", 10 }, { "V4", 2 } } },
            { "V2", new Dictionary<string, int> { { "V5", 40 } } },
            { "V3", new Dictionary<string, int>() },
            { "V4", new Dictionary<string, int> { { "V5", 20 } } },
            { "V5", new Dictionary<string, int>() }
        };

        Graph graph = new Graph(initGraph);
        string startNode = "V0";

        Dictionary<string, int> shortestPath = DijkstraAlgorithm(graph, startNode);
        PrintResult(shortestPath, startNode, "V5");

        Console.ReadLine();
    }
```

```
class Graph
{
    private Dictionary<string, Dictionary<string, int>> graph;

    public Graph(Dictionary<string, Dictionary<string, int>> initGraph)
    {
        graph = new Dictionary<string, Dictionary<string, int>>(initGraph);
    }

    public bool ContainsNode(string node)
    {
        return graph.ContainsKey(node);
    }

    public Dictionary<string, int> GetEdges(string node)
    {
        return graph[node];
    }
}
```

Algorithms

Dijkstra

```
static Dictionary<string, int> DijkstraAlgorithm(Graph graph, string startNode)
 {
    Dictionary<string, int> shortestPath = new Dictionary<string, int>();
    HashSet<string> visited = new HashSet<string>();

    foreach (var node in graph.GetEdges(startNode).Keys)
    {
       shortestPath[node] = int.MaxValue;
    }
    shortestPath[startNode] = 0;
    while (true)
    {
       string currentNode = null;
       int minDistance = int.MaxValue;

       foreach (var node in graph.GetEdges(startNode).Keys)
       {
          if (!visited.Contains(node) && shortestPath[node] < minDistance)
          {
             currentNode = node;
             minDistance = shortestPath[node];
          }
       }
       if (currentNode == null)
       {
          break;
       }
       visited.Add(currentNode);
       foreach (var kvp in graph.GetEdges(currentNode))
       {
          string neighbor = kvp.Key;
          int weight = kvp.Value;
          if (shortestPath[currentNode] + weight < shortestPath[neighbor])
          {
             shortestPath[neighbor] = shortestPath[currentNode] + weight;
          }
       }
    }
    return shortestPath;
 }
 static void PrintResult(Dictionary<string, int> shortestPath, string startNode, string endNode)
 {
    List<string> path = new List<string>();
    string currentNode = endNode;

    while (currentNode != startNode)
    {
       path.Add(currentNode);
       foreach (var kvp in shortestPath)
       {
          if (kvp.Key == currentNode)
          {
             currentNode = kvp.Value.ToString();
             break;
          }
       }
    }

    path.Add(startNode);
    path.Reverse();

    Console.WriteLine($"Shortest Path from {startNode} to {endNode} with a value of {shortestPath[endNode]}:");
    Console.WriteLine(string.Join(" -> ", path));
 }
}
```

Algorithms

A*

A* is an informed search algorithm. Informed Search signifies that the algorithm has extra information, to begin with. It is a complete as well as an optimal solution for solving path and grid problems.

Optimal – find the least cost from the starting point to the ending point. Complete – It means that it will find all the available paths from start to end.

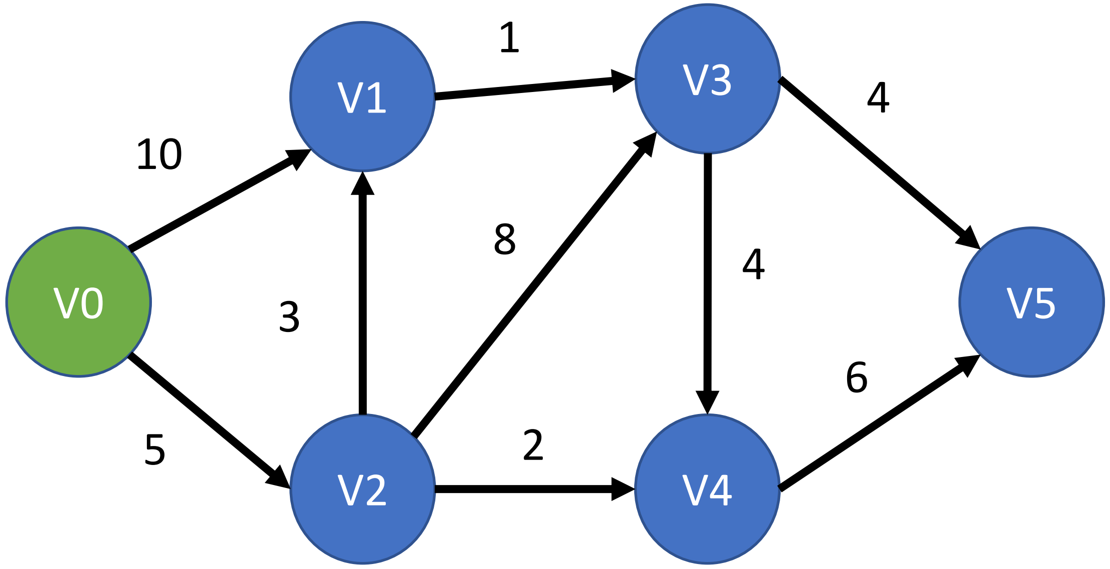
The Algorithm

$$f(n) = g(n) + h(n)$$

1. Place the starting node into OPEN and find its f (n) value.
2. Remove the node from OPEN, having the smallest f (n) value.
   * If it is a goal node, then stop and return to success.
   * Else remove the node from OPEN, and find all its successors.
   * Find the f (n) value of all the successors, place them into OPEN, and place the removed node into CLOSE.

Algorithms

A*



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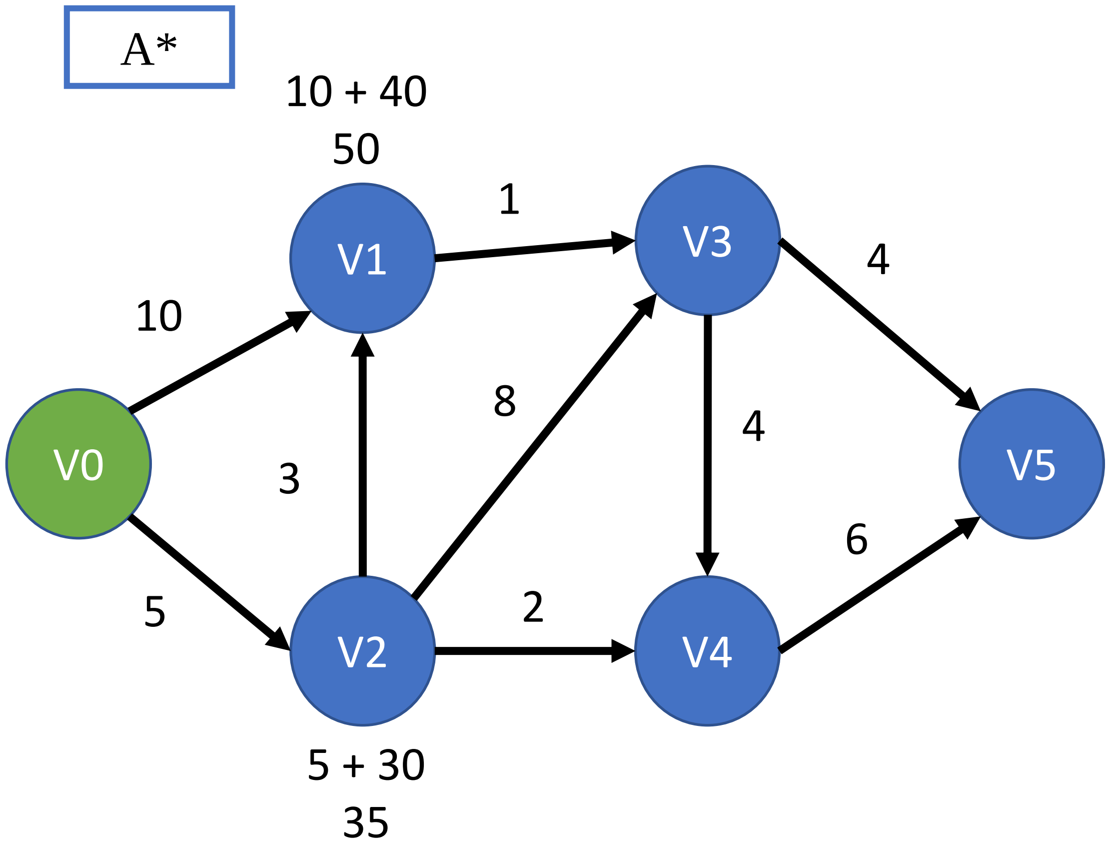
h(n) →

	V5
V0	50
V1	40
V2	30
V3	20
V4	10

Algorithms



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Algorithms

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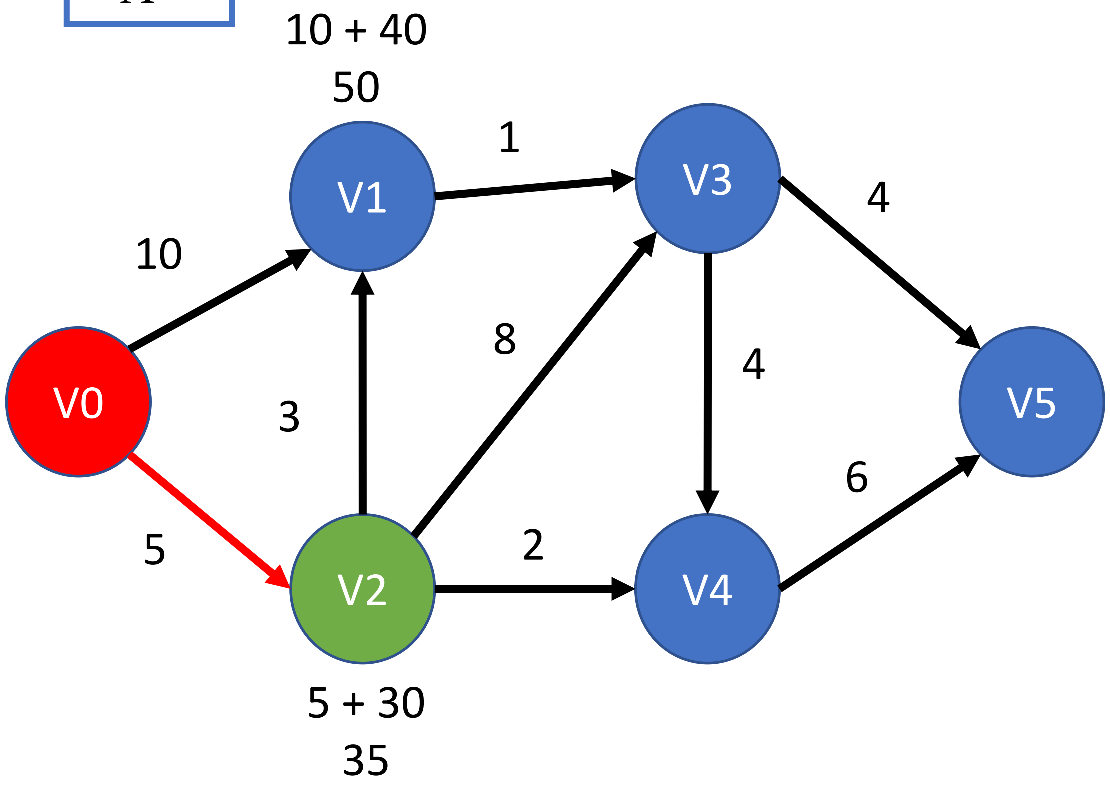

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Algorithms

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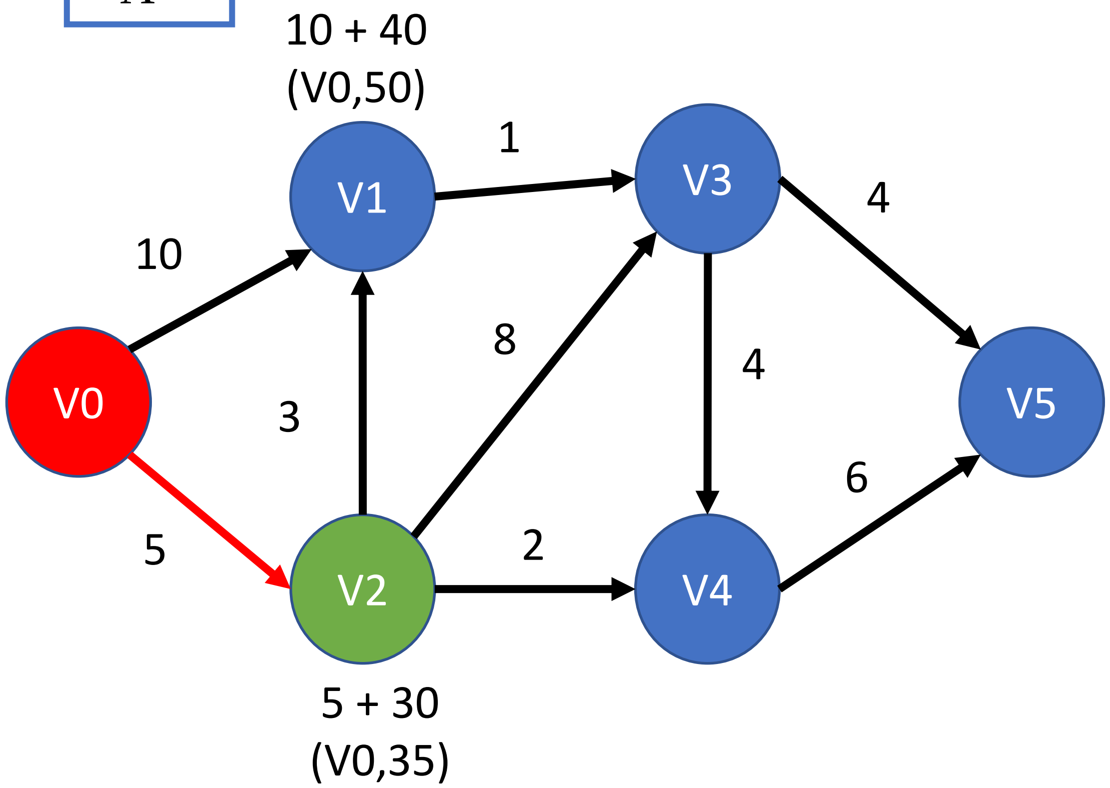

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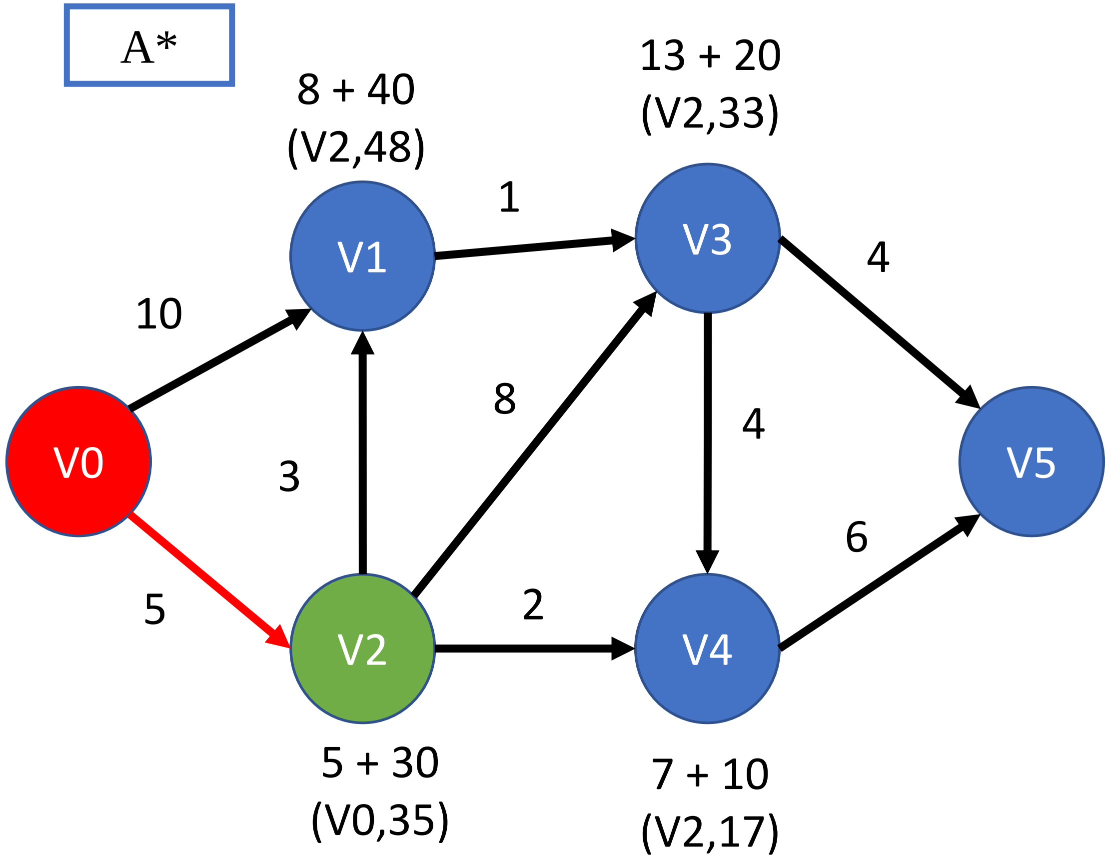
Algorithms



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h(n)

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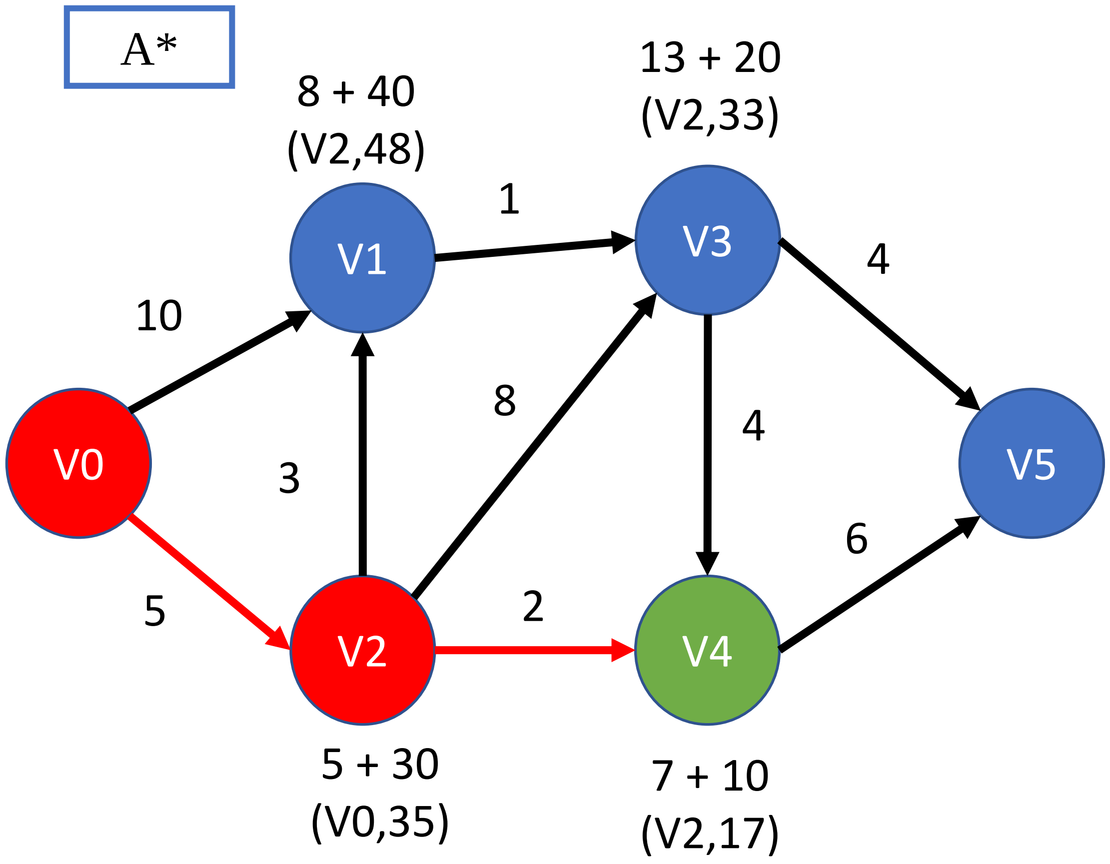
Algorithms



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h(n)

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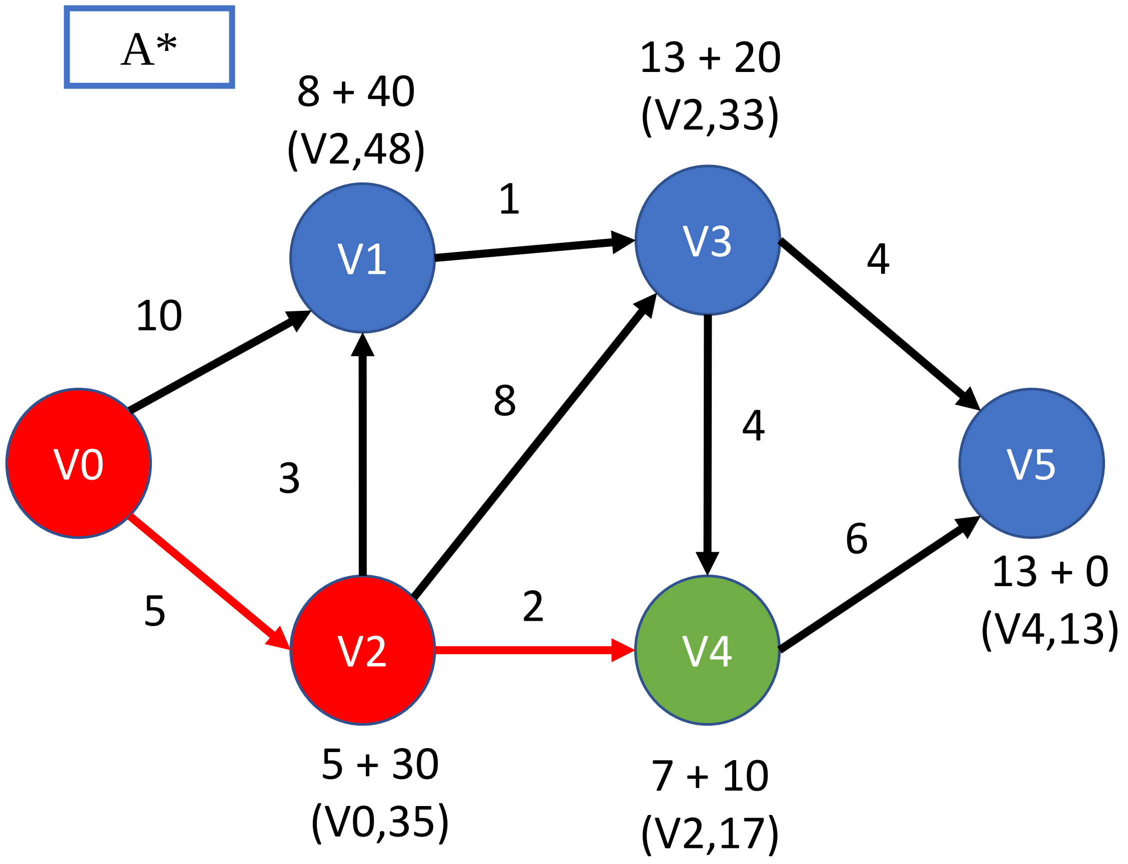
Algorithms



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h(n) →

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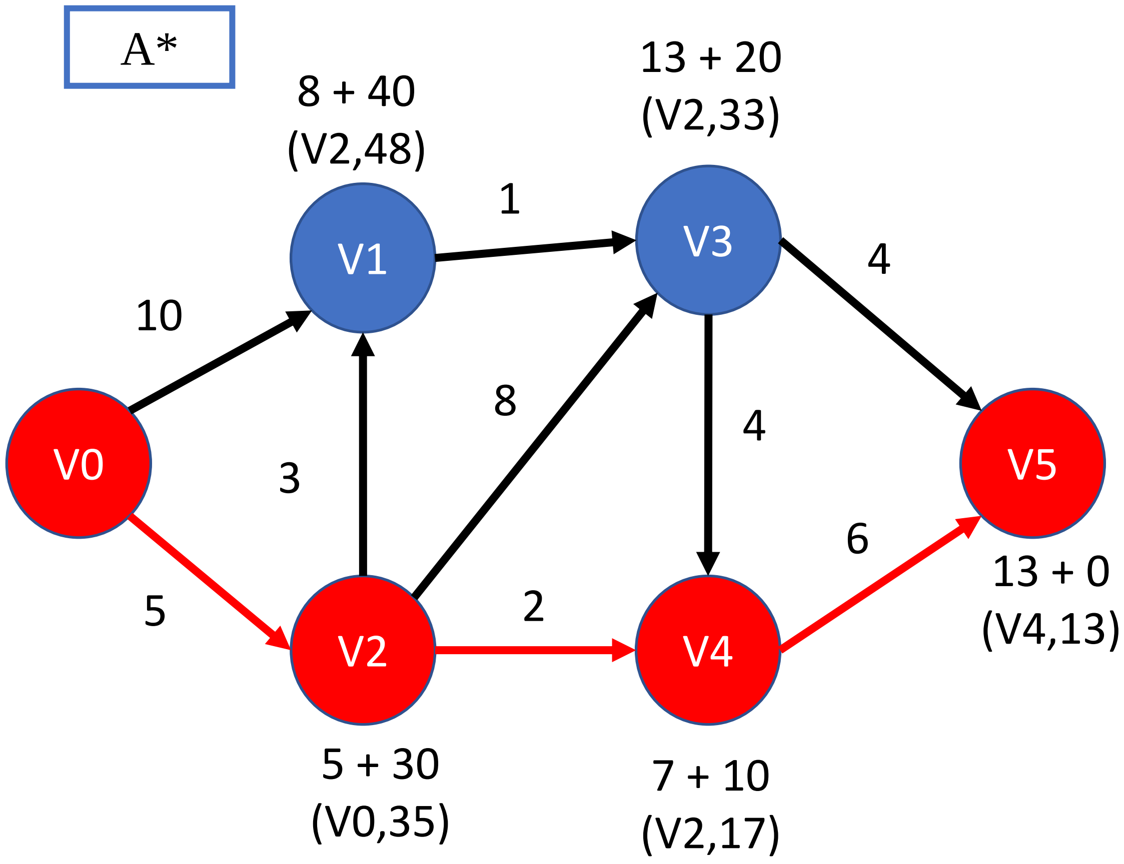
Algorithms



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h(n) →

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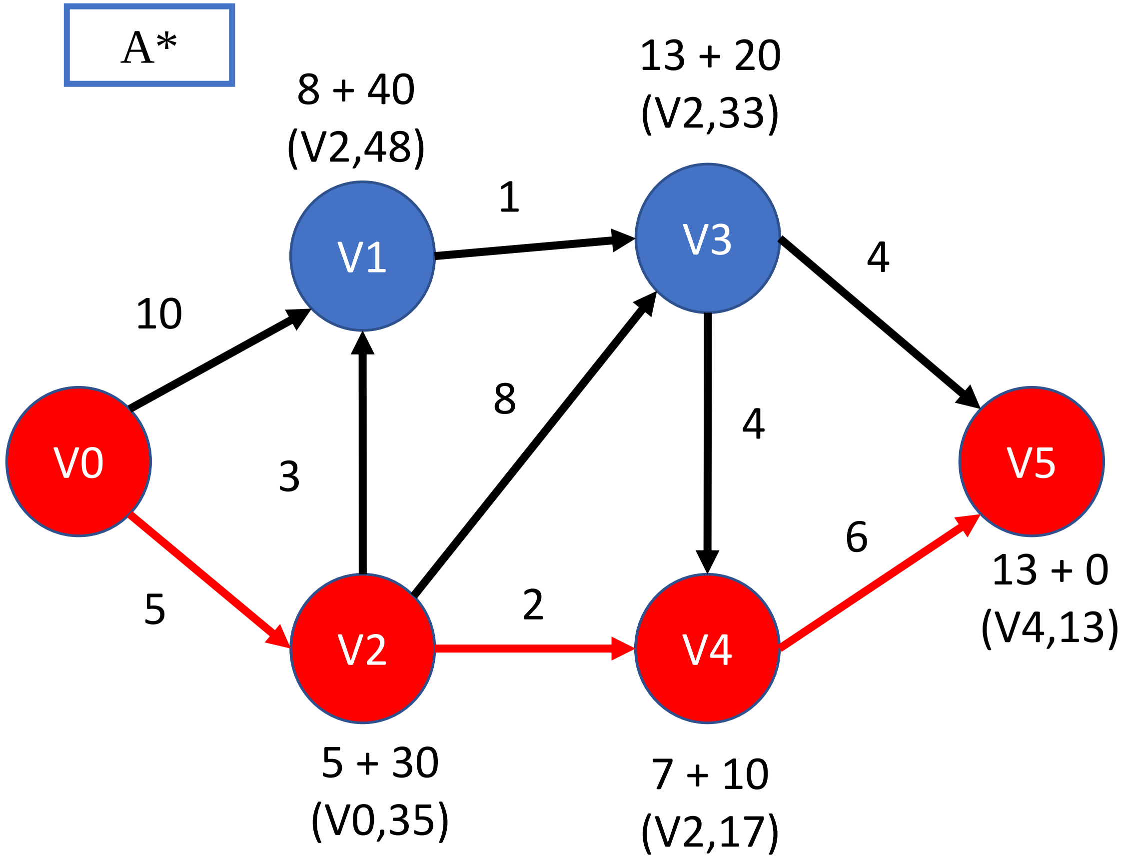
Algorithms



the shortest distance V0 => V5

V5, V4, V2, V0

Algorithms

manhattan((x1, y1), (x2, y2)) = |x1 — x2| + |y1 — y2|

euclidean((x1, y1), (x2, y2)) = sqrt(x^2 + y^2)

A*

0	1	1	1	1
1	1	1	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	1	1	1
1+7 8	1	1	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
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Algorithms

A*

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1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	2+6 8	1	1
1+7 8	2+6 8	1	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
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5	4	3	2	1
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Algorithms

A*

0	1+7 8	2+6 8	1	1
1+7 8	2+6 8	1	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
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6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	2+6 8	3+5 8	1
1+7 8	2+6 8	3+5 8	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	2+6 8	3+5 8	1
1+7 8	2+6 8	3+5 8	1	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

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7	6	5	4	3
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Algorithms

A*

0	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	1
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

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Algorithms

A*

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1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				1
1	1	1	1	1
1	1	1	1	1

h(n)

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1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				1
1	1	1	1	1
1	1	1	1	1

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Algorithms

A*

0	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				6+2 8
1	1	1	1	1
1	1	1	1	1

h(n)

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Algorithms

A*

1	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				6+2 8
1	1	1	1	7+1 8
1	1	1	1	1

h(n)

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7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
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Algorithms

A*

1	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				6+2 8
1	1	1	8+2 10	7+1 8
1	1	1	1	8+0 8

h(n)

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7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

Algorithms

A*

0	1+7 8	2+6 8	3+5 8	4+4 8
1+7 8	2+6 8	3+5 8	4+4 8	5+3 8
1				6+2 8
1	1	1	8+2 10	7+1 8
1	1	1	1	8+0 8

h(n)

8	7	6	5	4
7	6	5	4	3
6	5	4	3	2
5	4	3	2	1
4	3	2	1	0

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