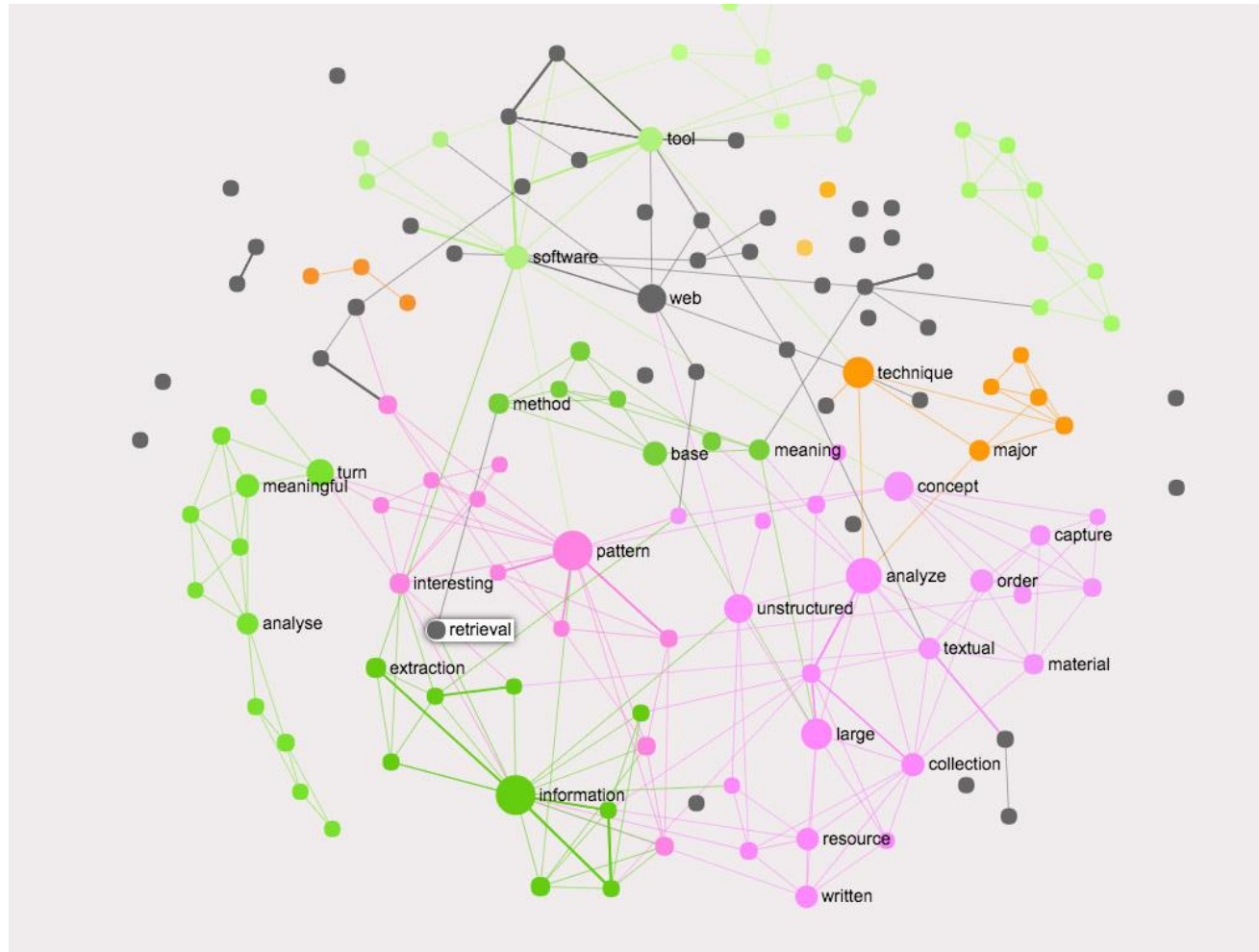# Basic Concepts of Graph Theory
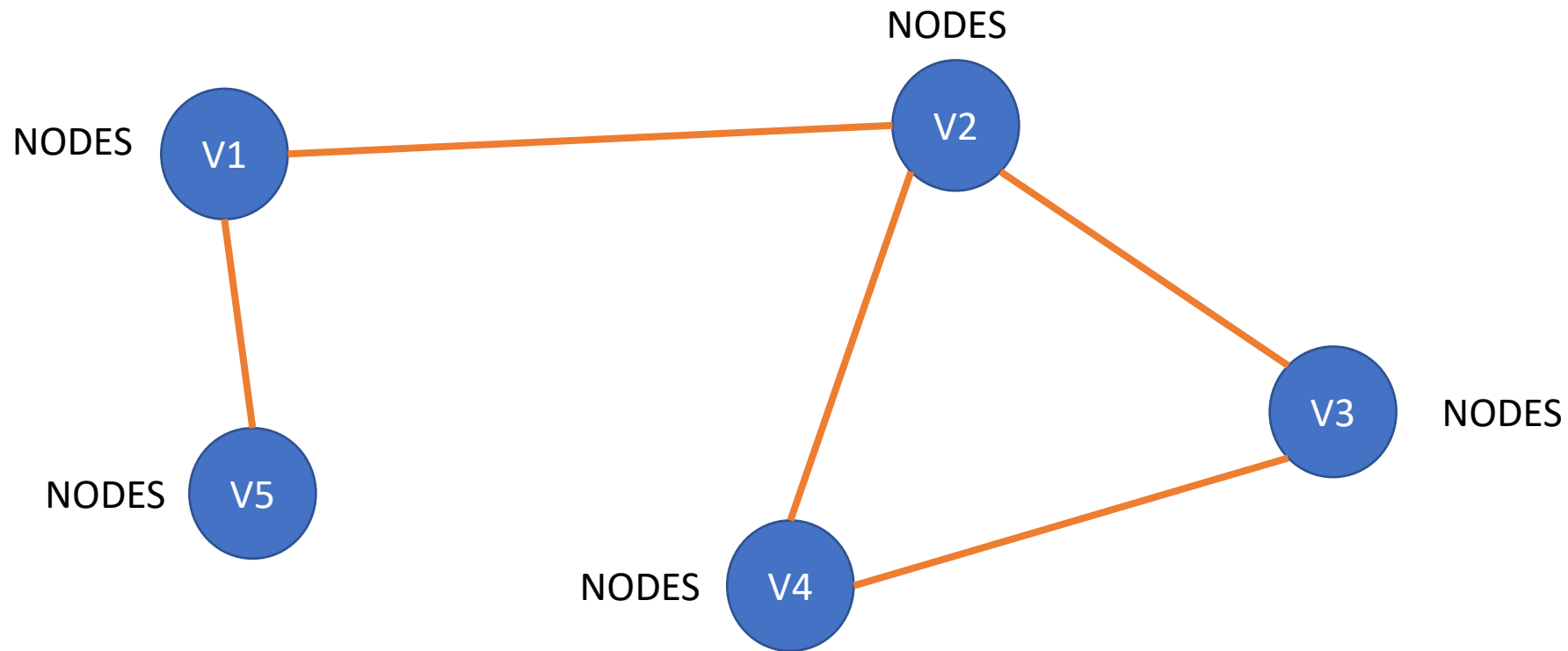
Daniel Nogueira

dnogueira@ipca.pt

Introduction

"Graphs are mathematical structures that allow you to encode relationships between pairs of objects. "

Introduction

"Graphs are mathematical structures that allow you to encode relationships between pairs of **objects.** "
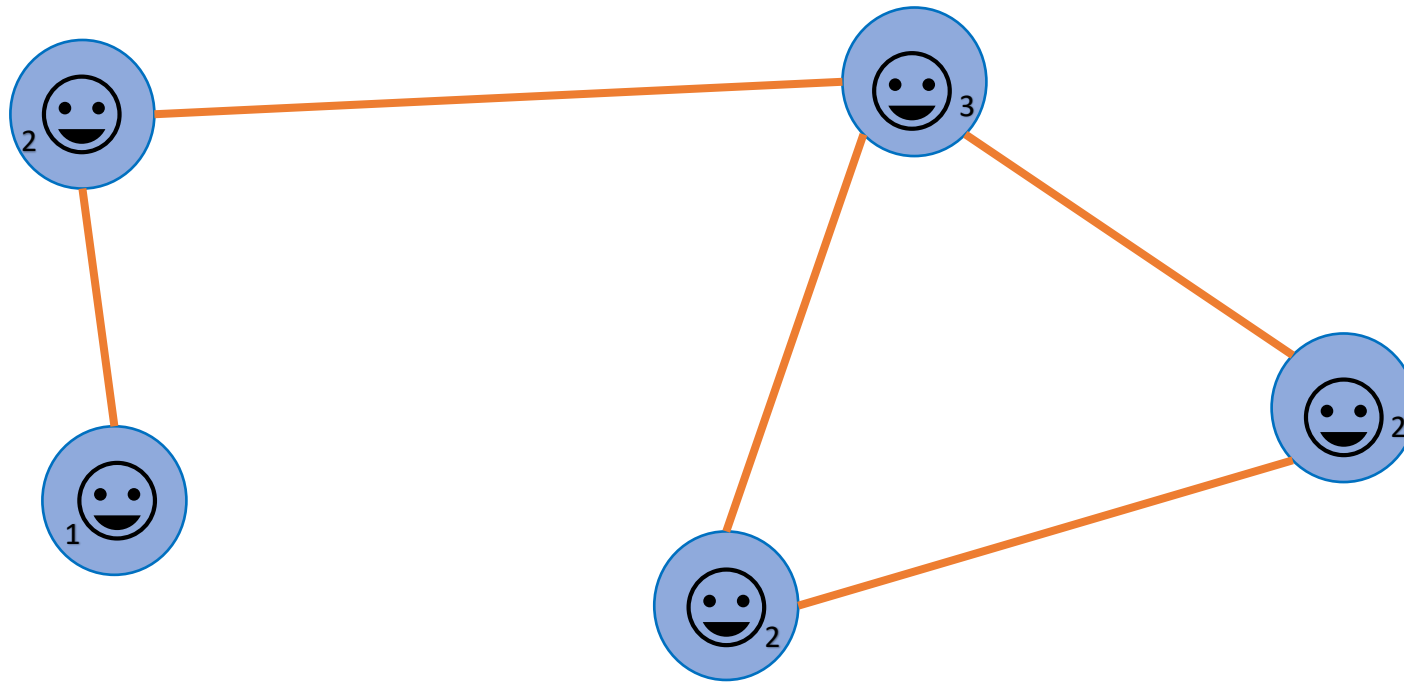
NODES

NODES V1

NODES V2

NODES V3

NODES V5

NODES V4

Introduction

"Graphs are mathematical structures that allow you to encode **relationships** between pairs of **objects.**"
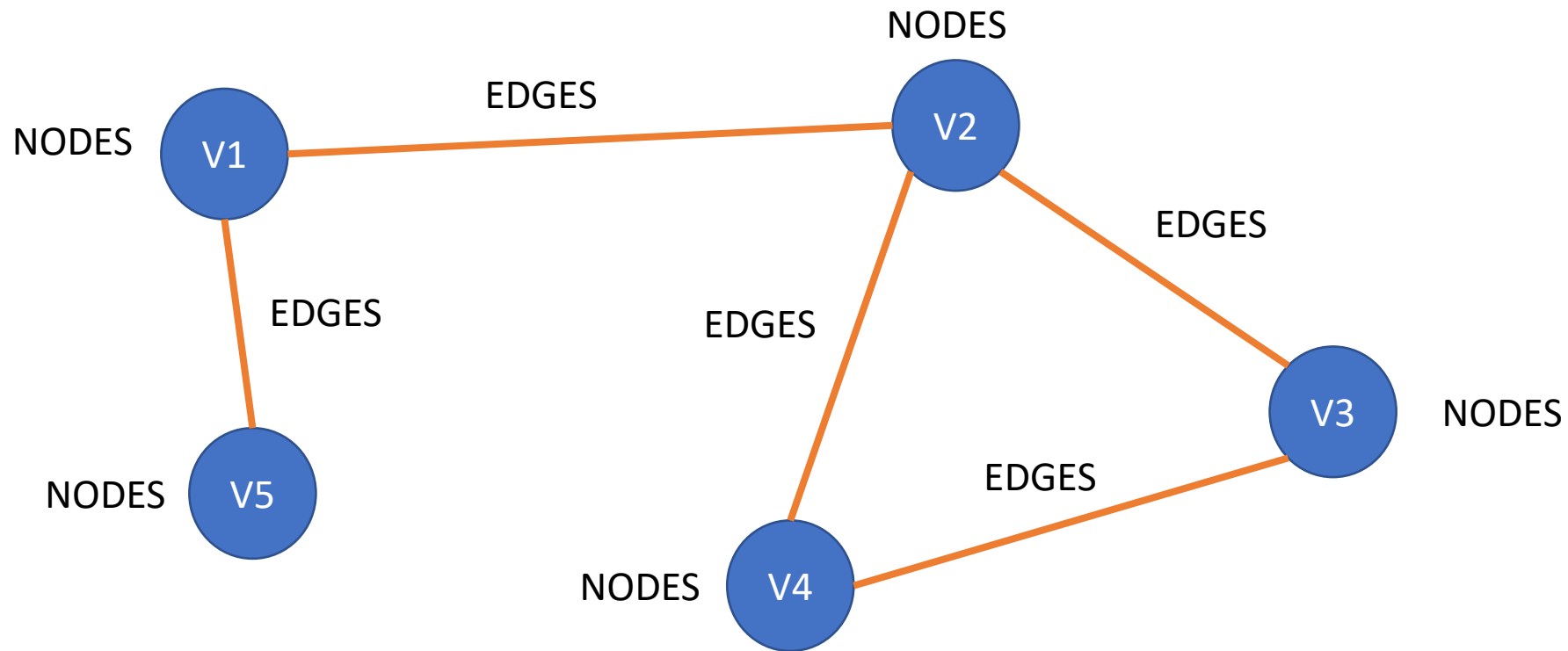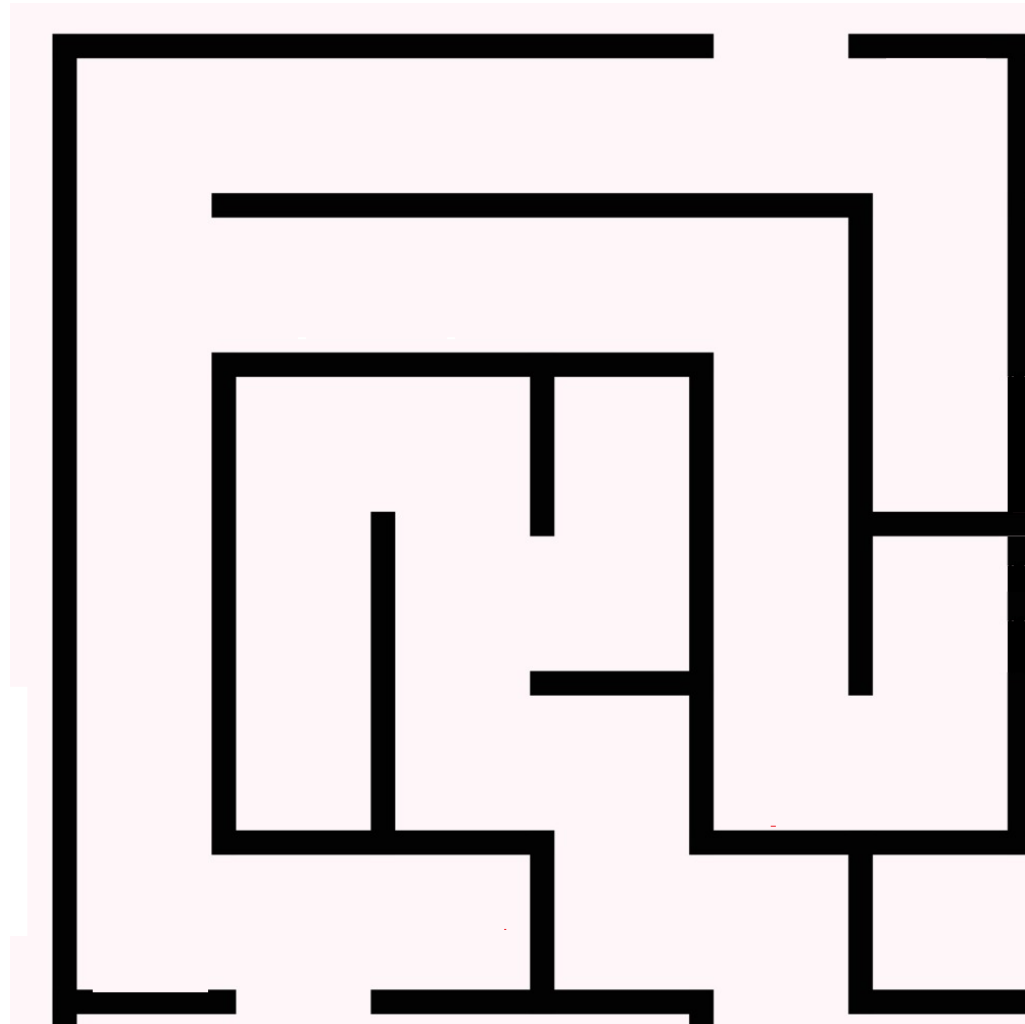
Introduction
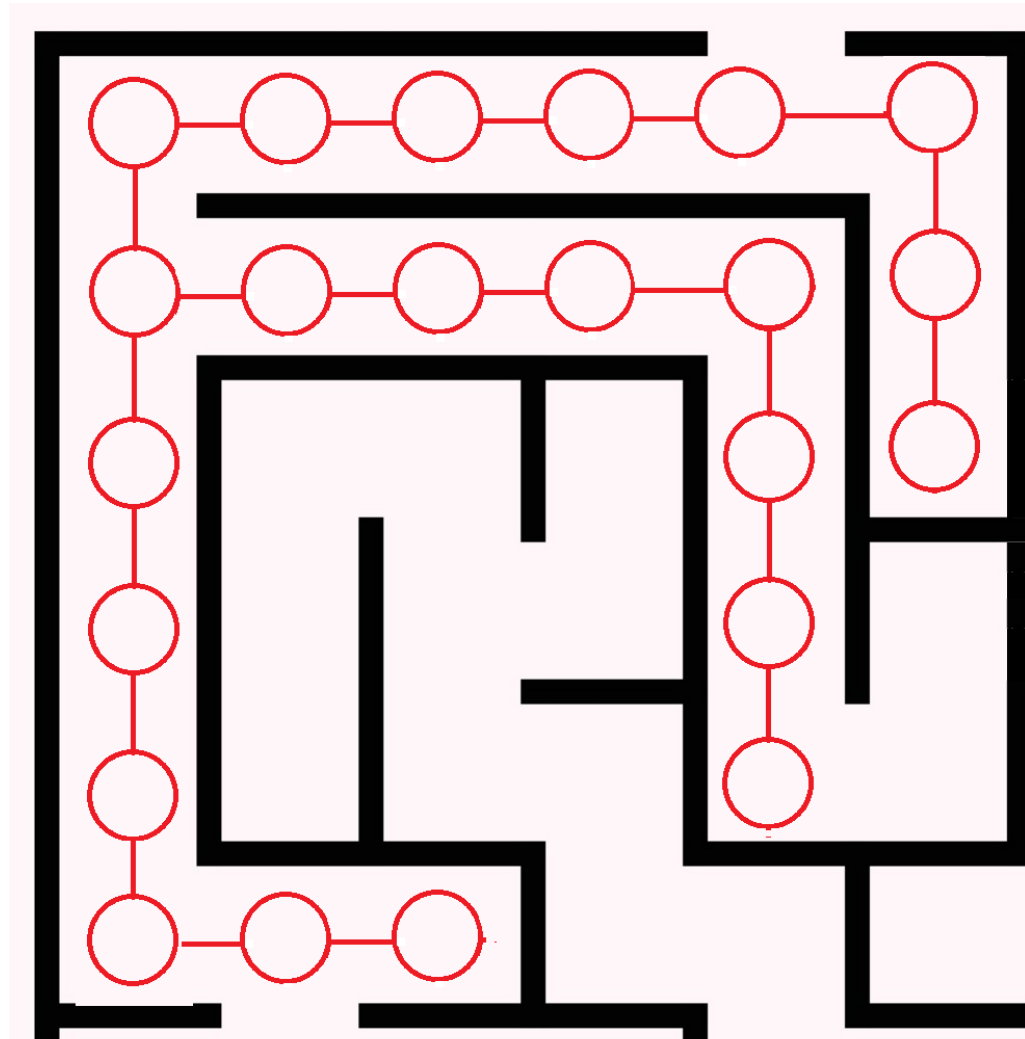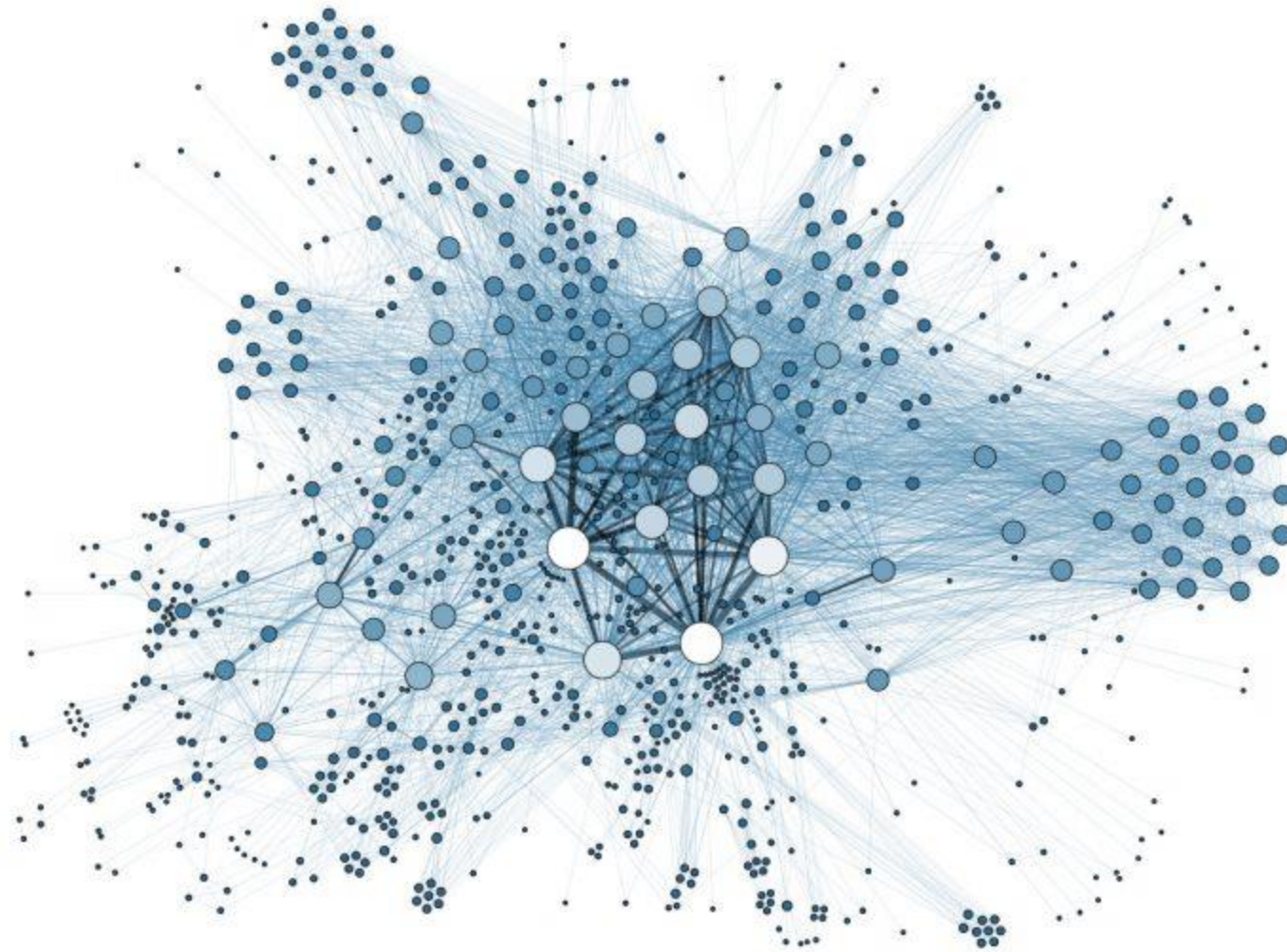
Relationships on social media

Introduction

Maze path finding

Introduction

Maze path finding

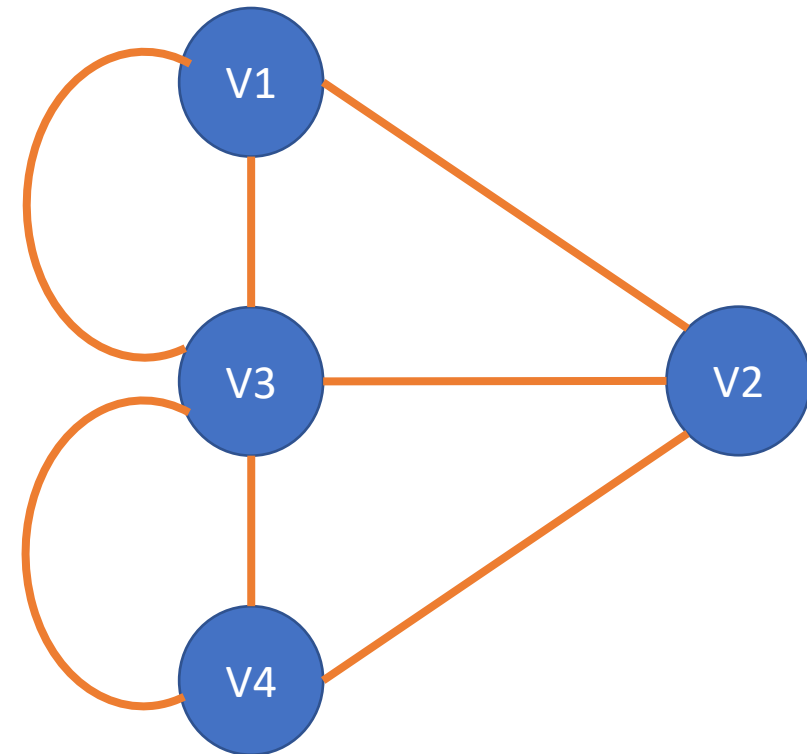# Introduction
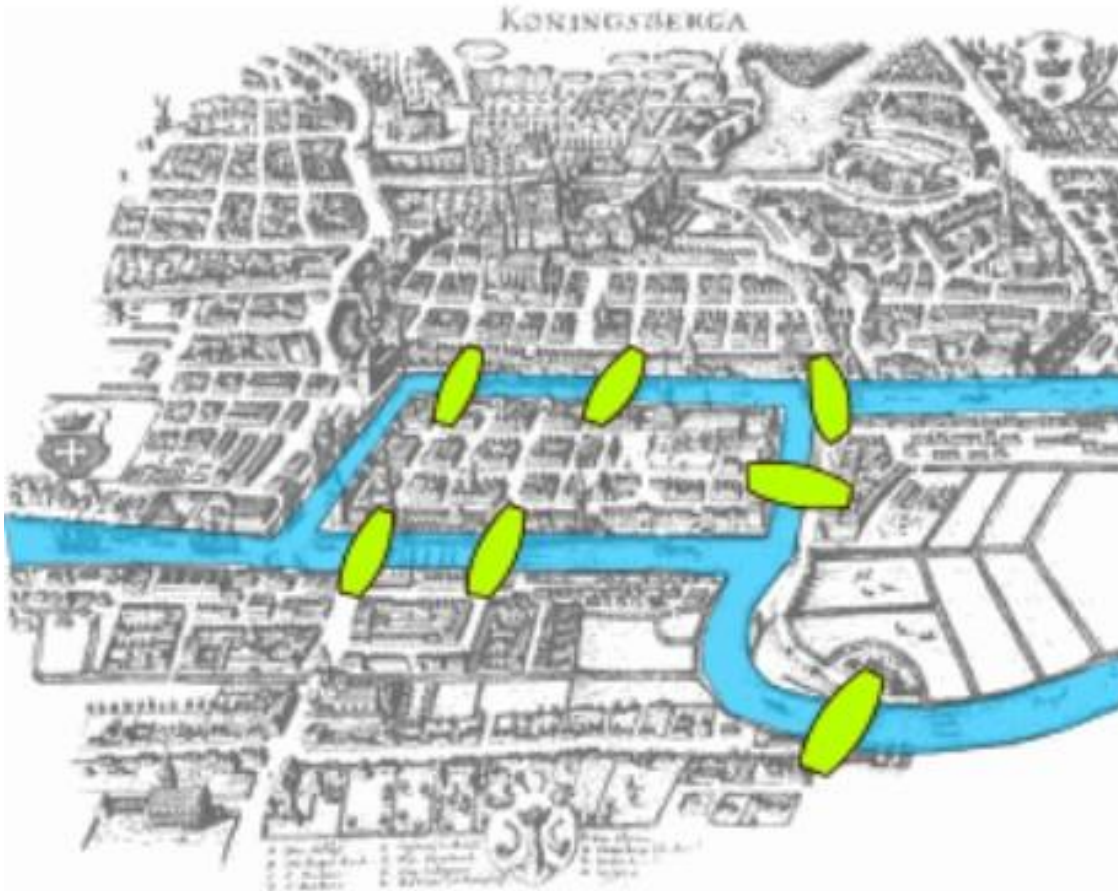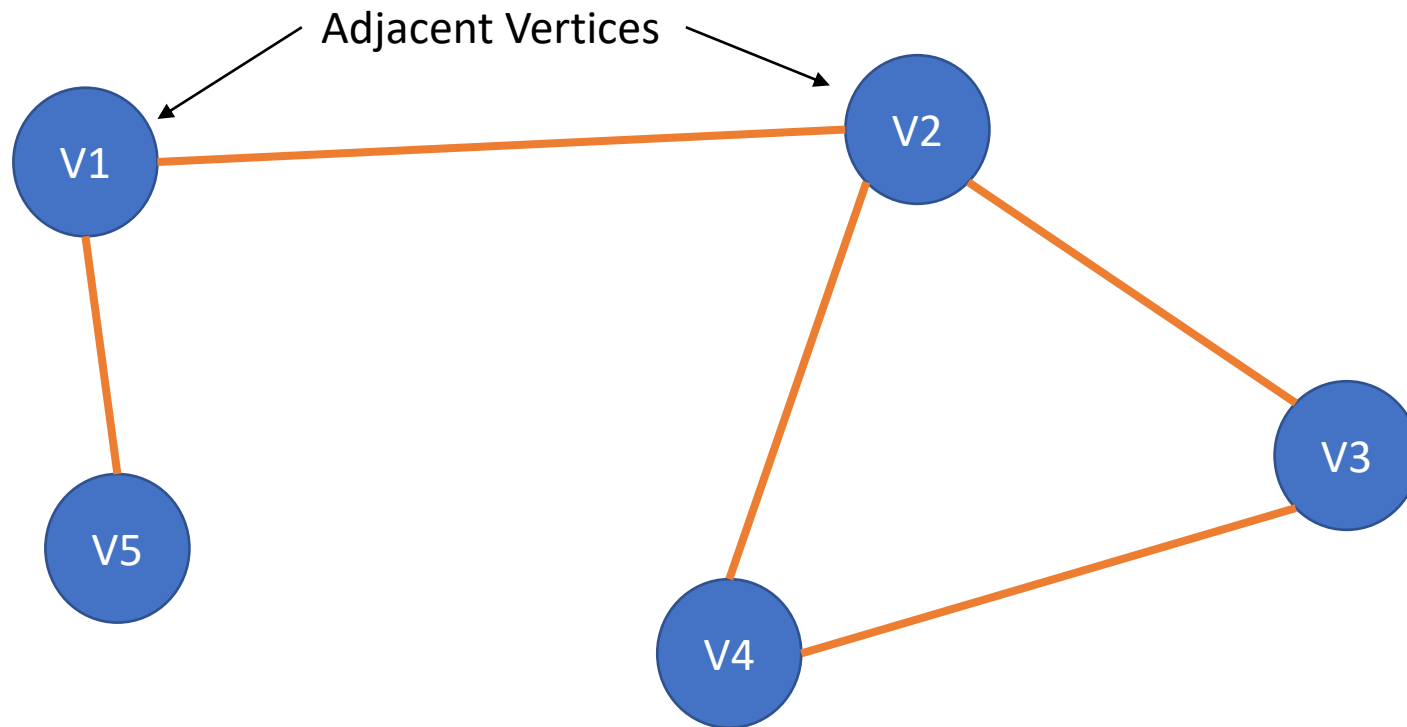

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Introduction
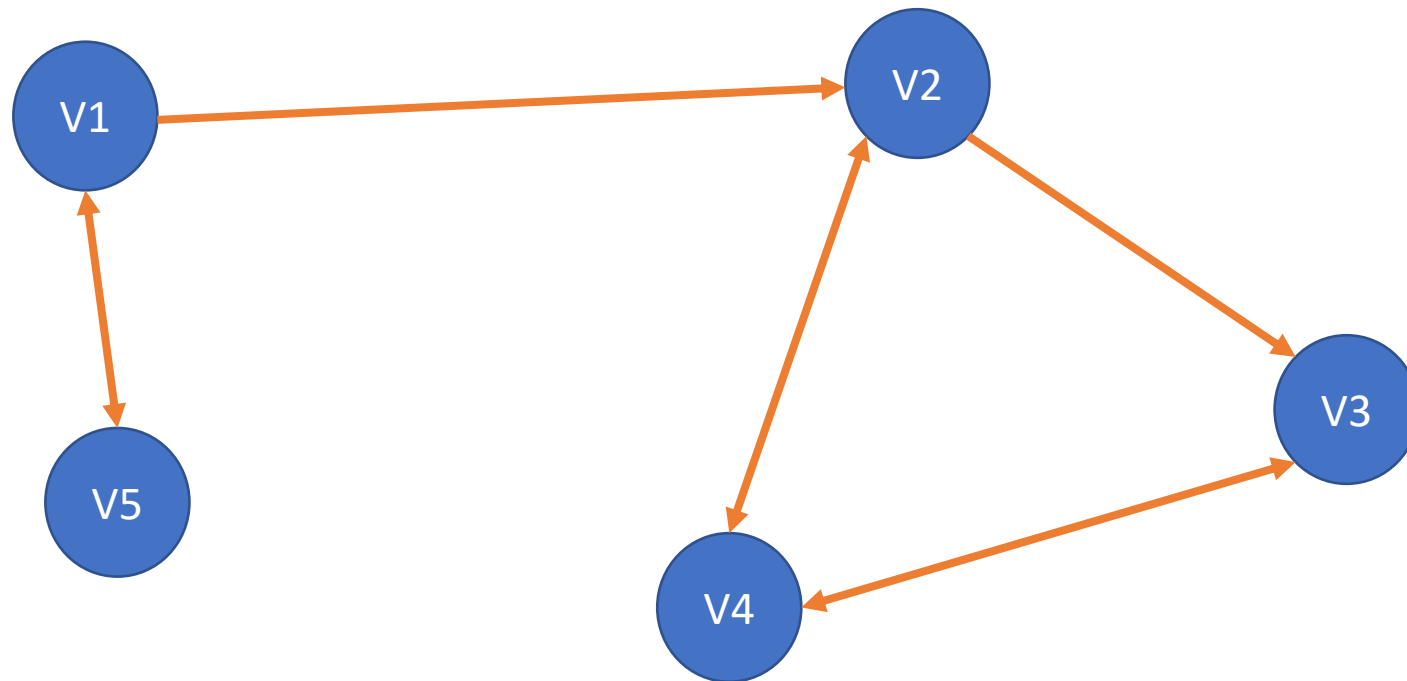
Introduction

Konigsberg Bridges (Leonhard Euler, 1735)

Definitions
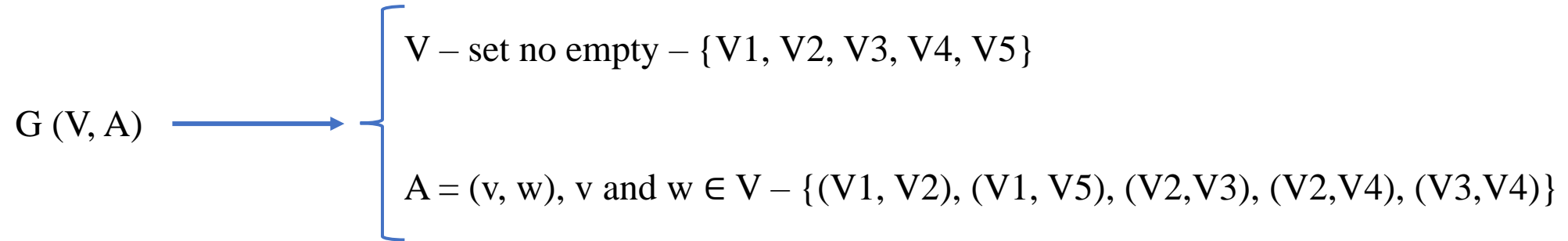
$G (V, A)$

V – set no empty – $\{V1, V2, V3, V4, V5\}$

$A = (v, w)$, v and w $\in V$ – $\{(V1, V2), (V1, V5), (V2,V3), (V2,V4), (V3,V4)\}$

Adjacent Vertices

Definitions

Artificial Intelligence Applied to Games

Definitions

Definitions

(V1, V2) ⟶ ~~(V2, V1)~~

(V1, V5) ⟶ (V5, V1)

Adjacent Vertices

Adjacent Vertices

V1

V2

V3

V4

V5

Definitions

Degree ── Output Degree or Outdegree

Definitions

Output Degree or Outdegree

Degree

Input Degree or Indegree

1
V1

2
V2

2
V3

1
V5

2
V4

Definitions

Degree

Output Degree or Outdegree

Input Degree or Indegree
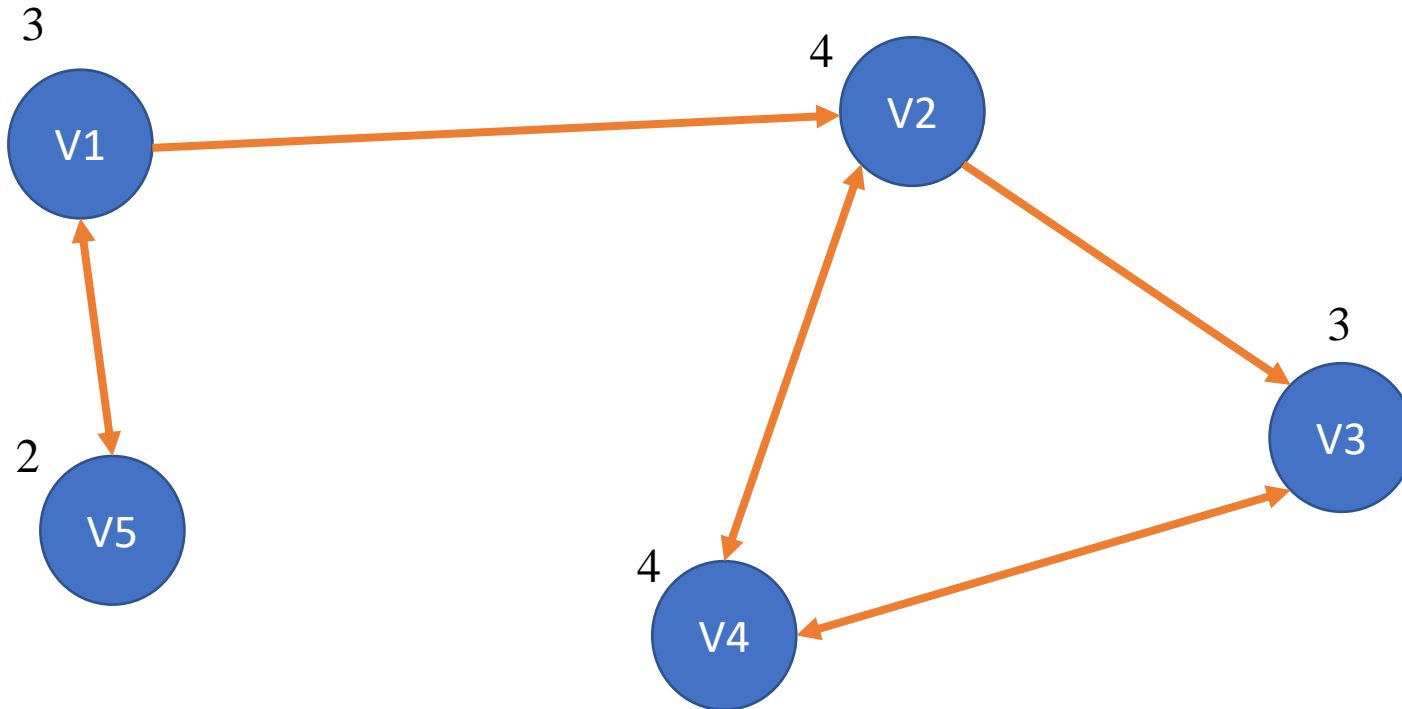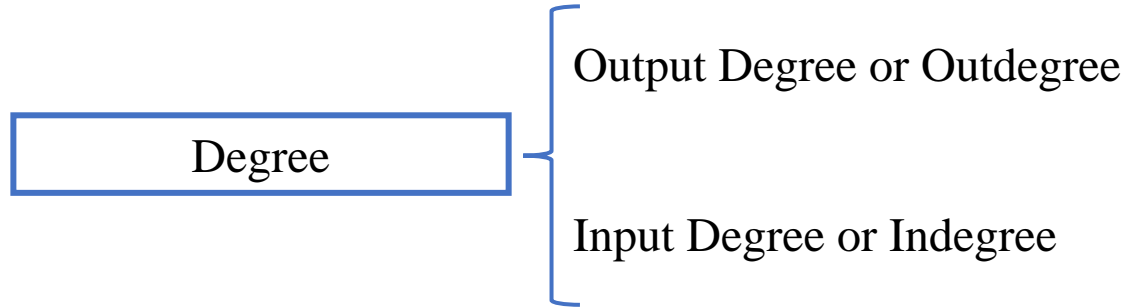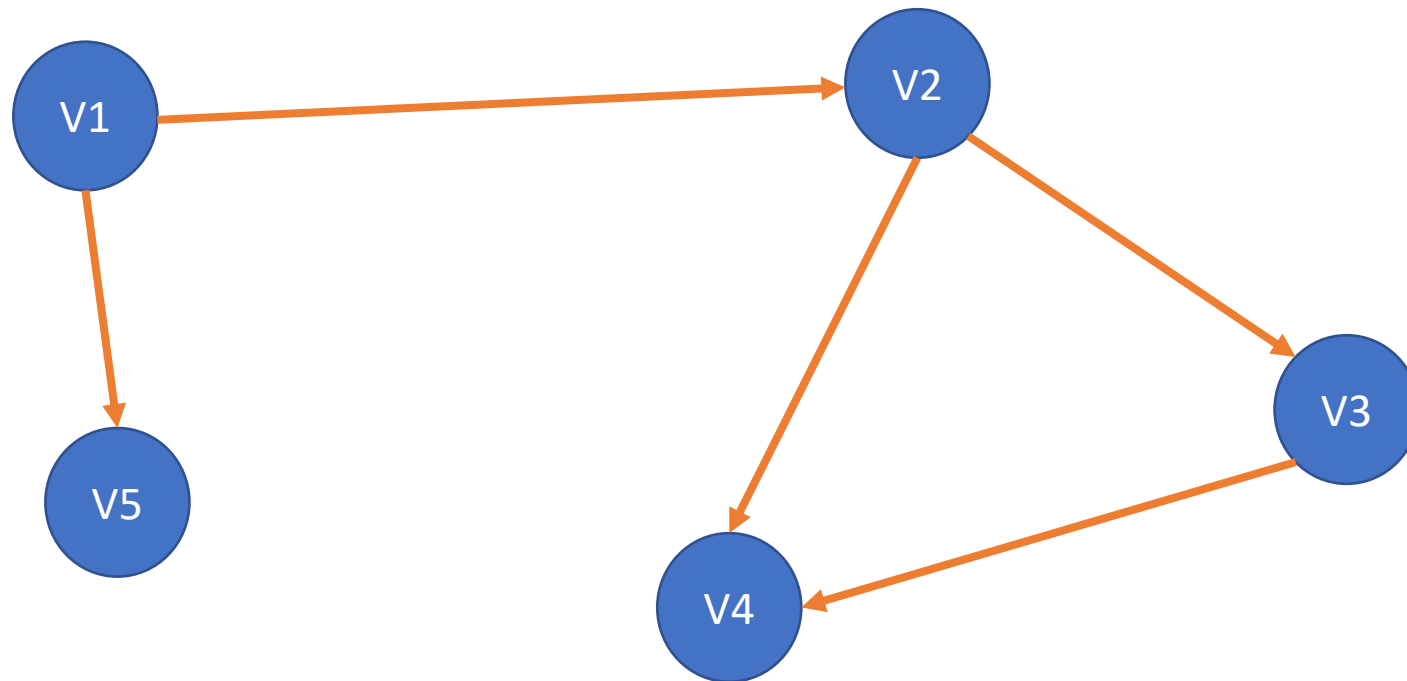
Definitions

(V1, V5)
(V1, V2, V3, V4)
(V1, V2, V4)
(V1, V2)
(V2, V3, V4)
(V2, V3)
(V2, V4)
(V3, V4)
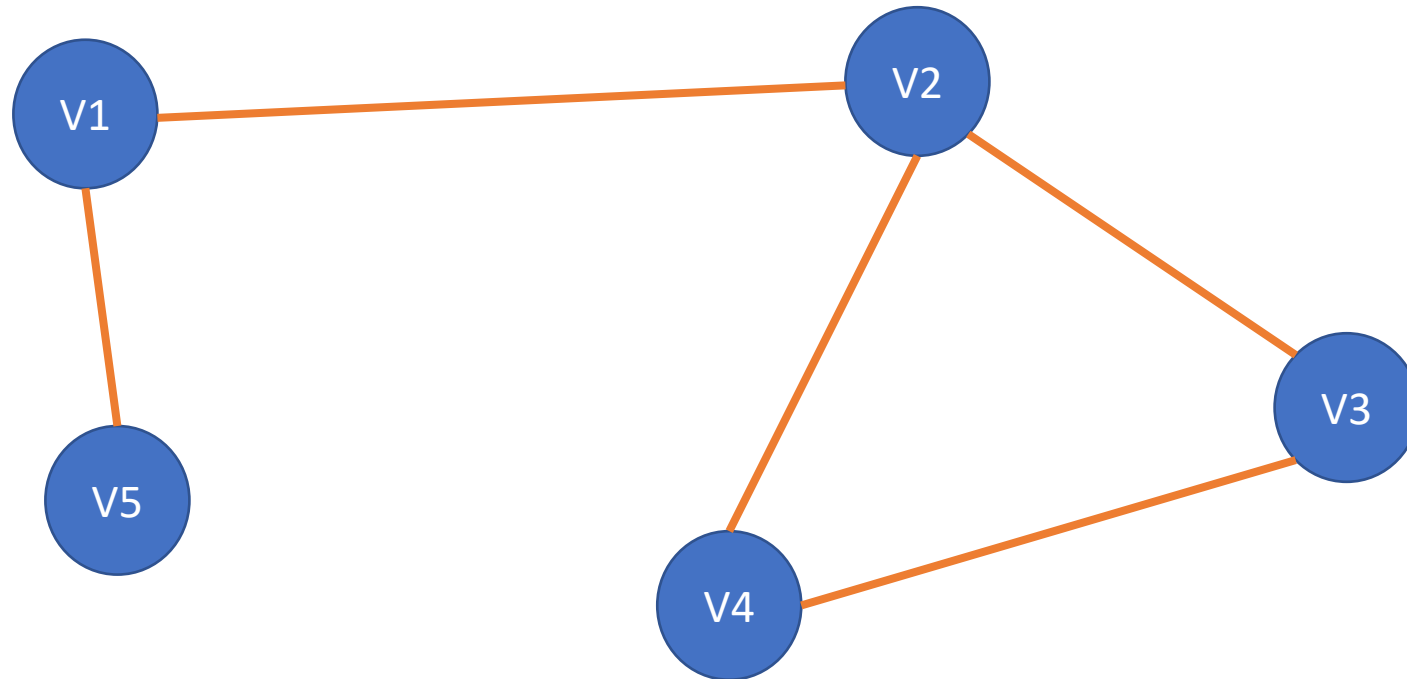
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Definitions

connected graph X disconnected graph

connected graph

Definitions

connected graph   X disconnected graph

disconnected graph

Definitions

connected graph   X   disconnected graph

disconnected graph

Definitions

connected graph   X   disconnected graph

strongly connected graph
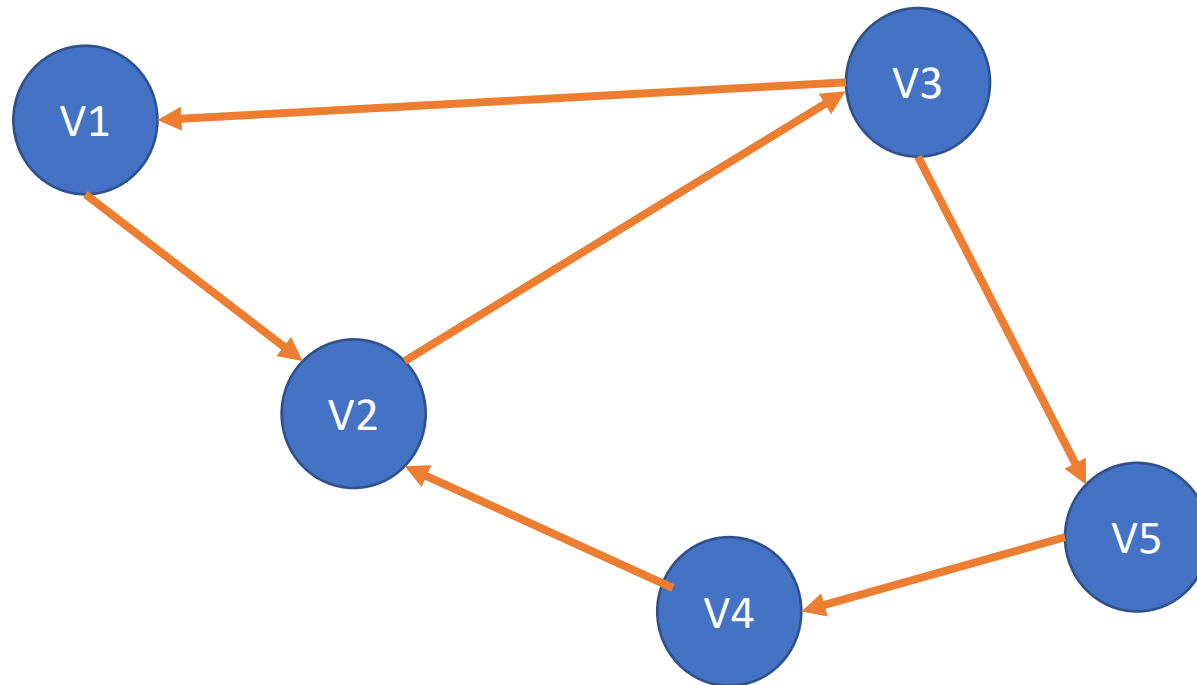
Definitions

connected graph   X   disconnected graph
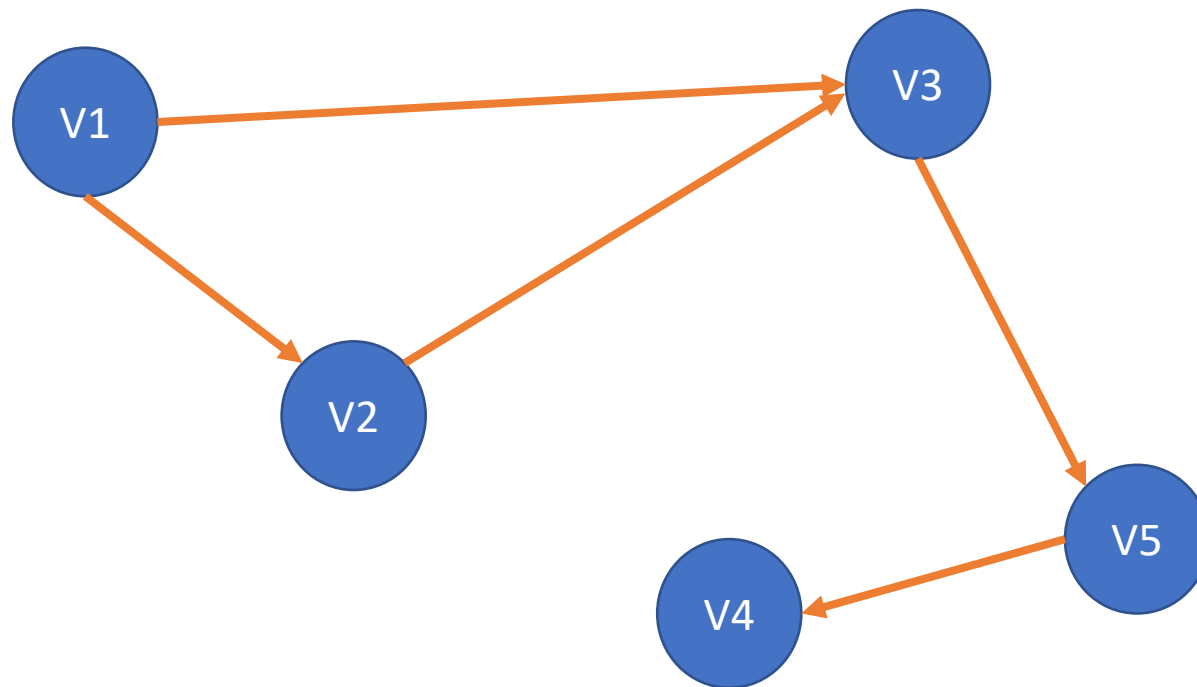
connected graph

Definitions

connected graph   X disconnected graph

weakly connected graph
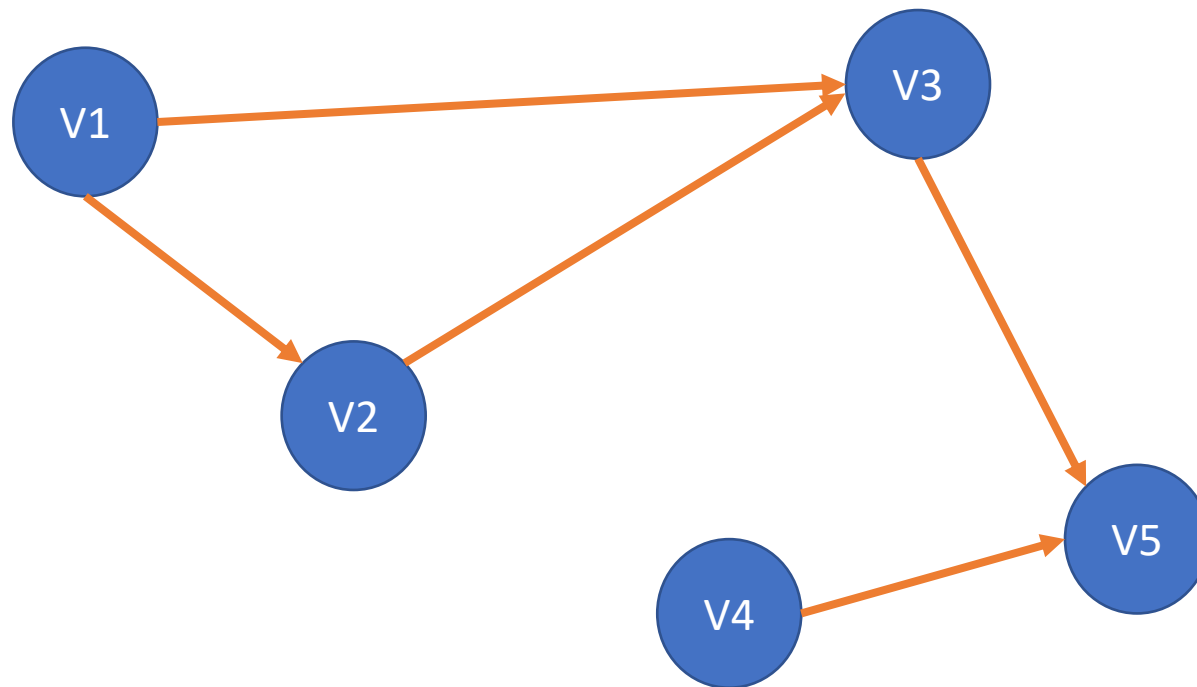
Definitions
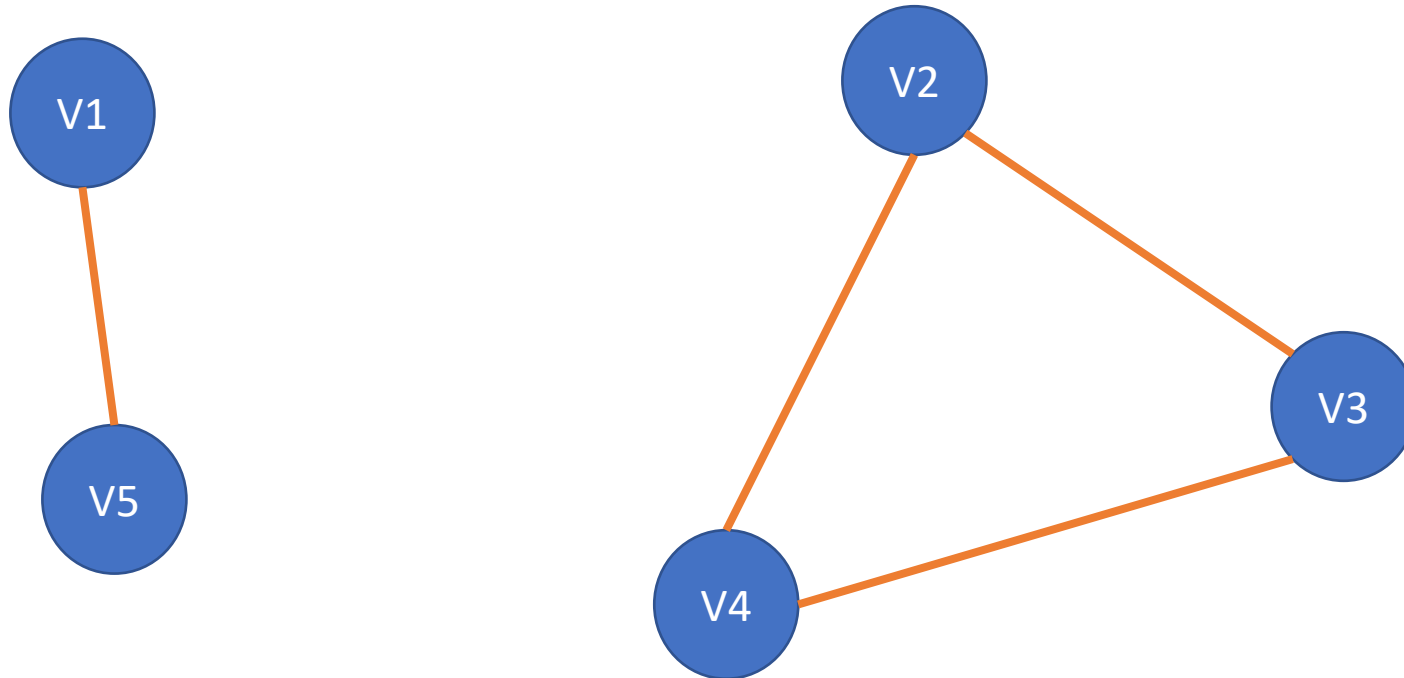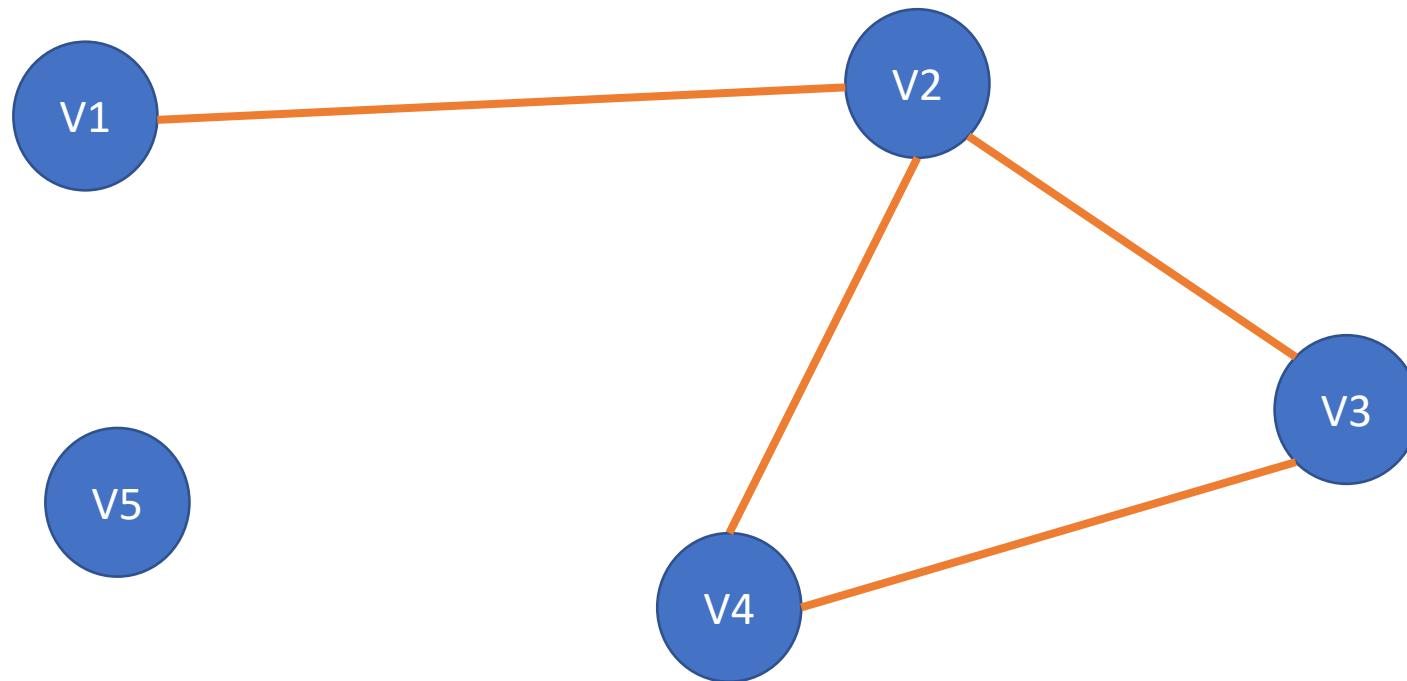
Weights
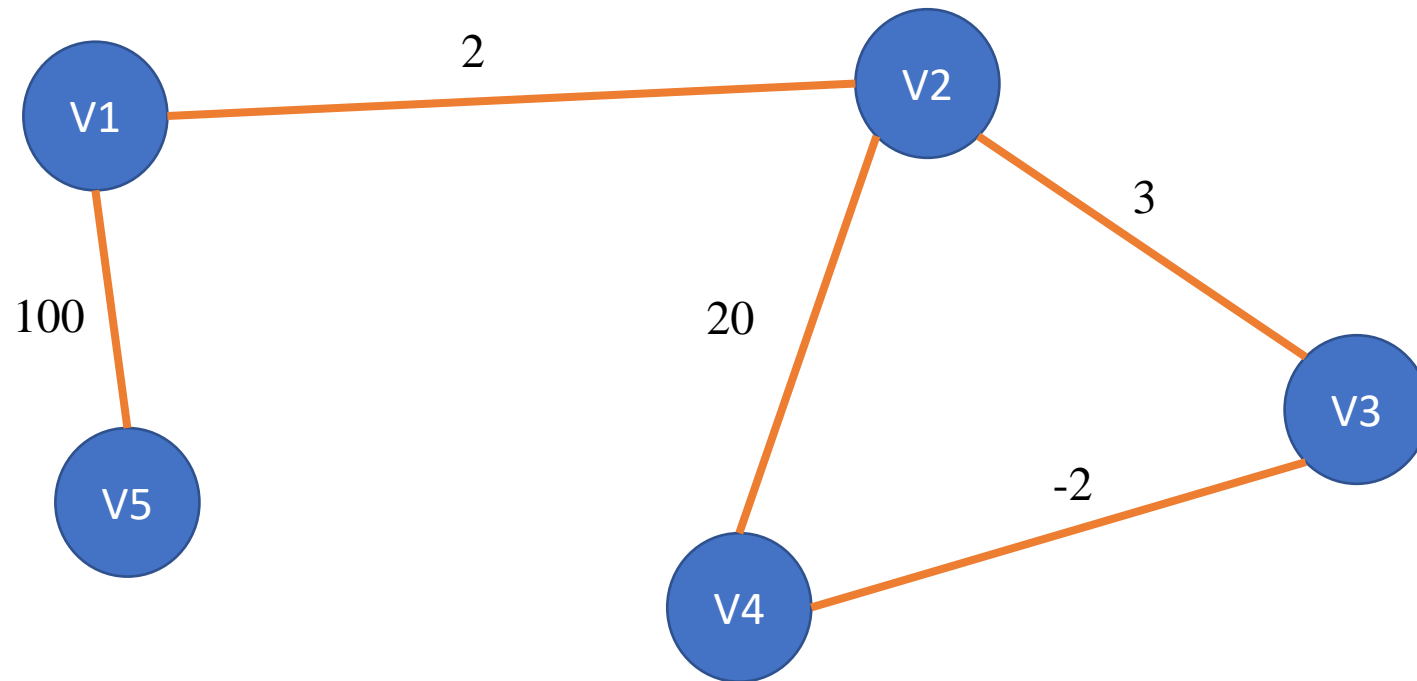
Types of Graphs

Finite Graph

A graph is said to be finite if it has a finite number of vertices and a finite number of edges.
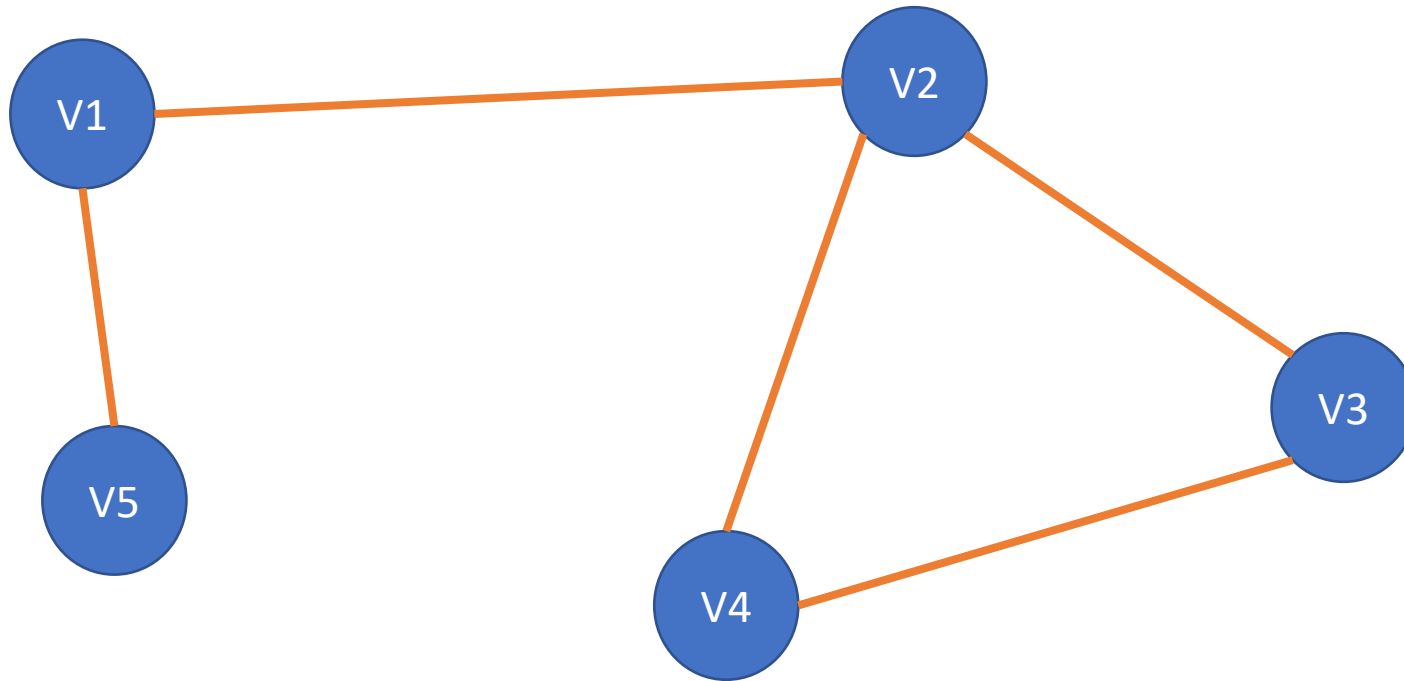
Types of Graphs

Infinite Graph

A graph is said to be infinite if it has an infinite number of vertices as well as an infinite number of edges.
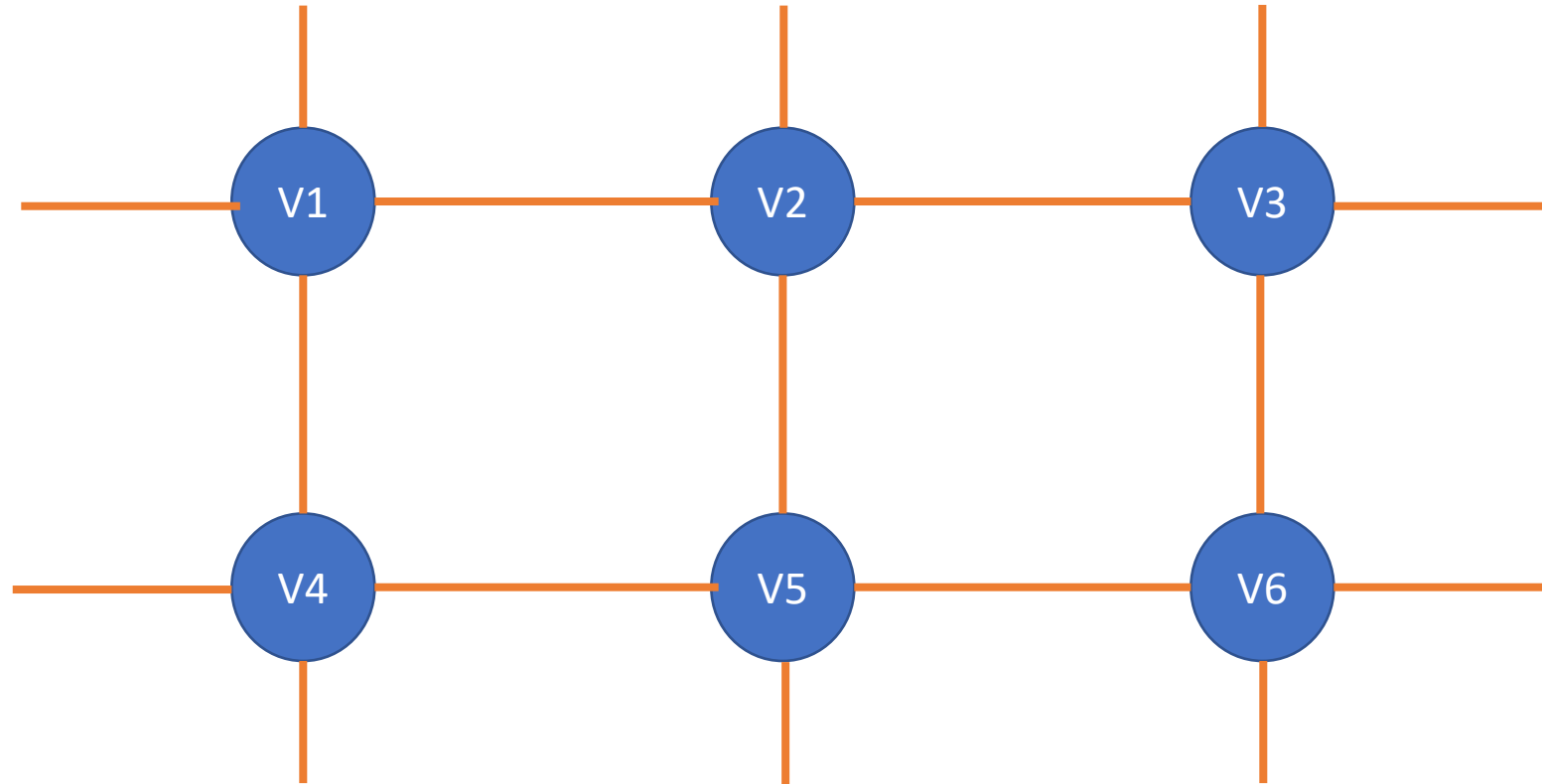
Types of Graphs

Simple Graph

A simple graph is a graph that does not contain more than one edge between the pair of vertices.
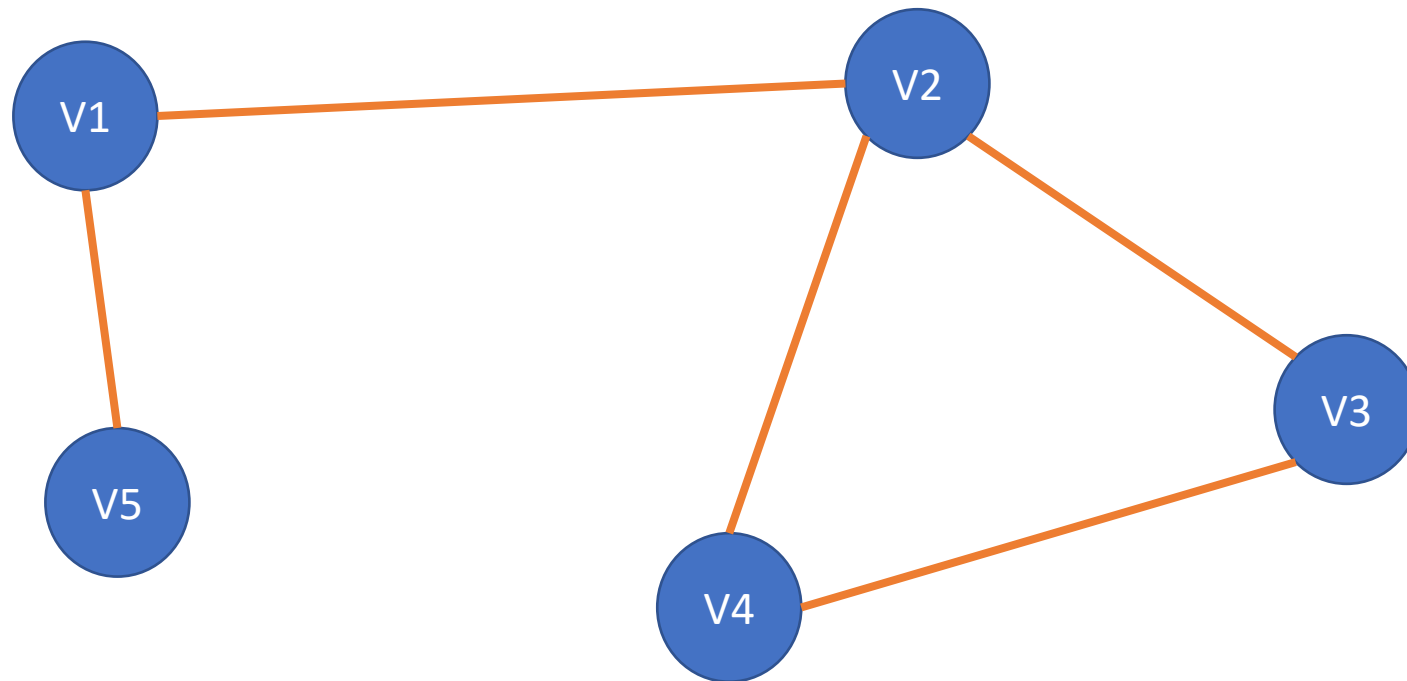
Types of Graphs

Any graph which contains some parallel edges but doesn't contain any self-loop is called a multigraph
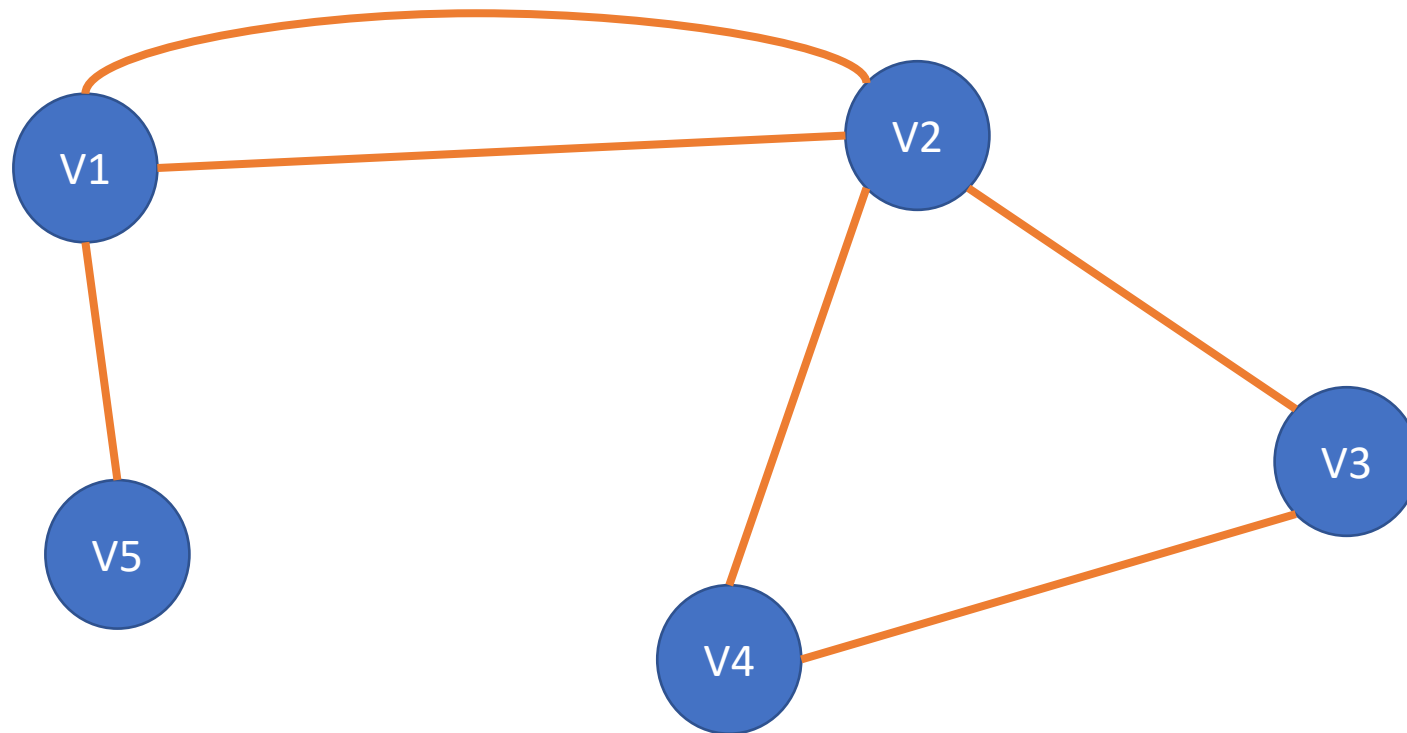
Representation

Adjacent List

Node	
V1	
V2	
V3	
V4	
V5	

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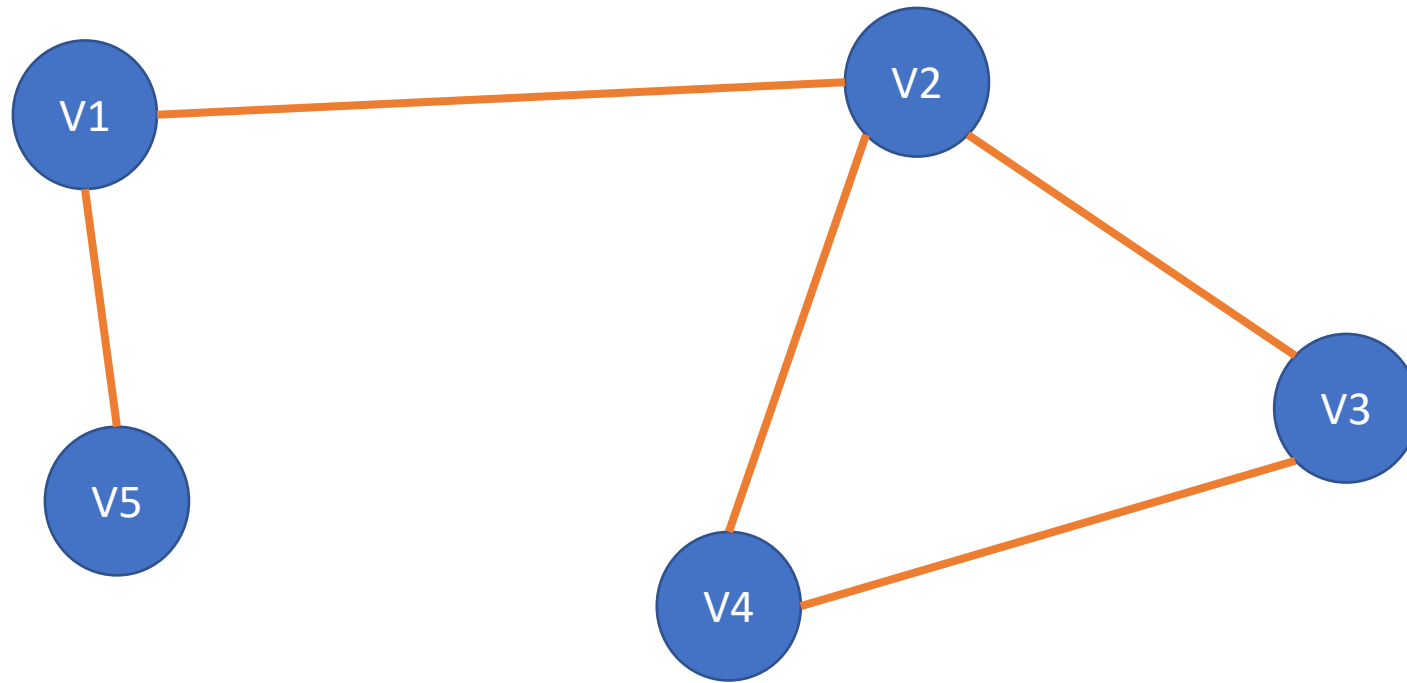
Representation

Node	Connections
V1	V2, V5
V2	V1, V3, V4
V3	V2, V4
V4	V2, V3
V5	V1

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Representation

Representation
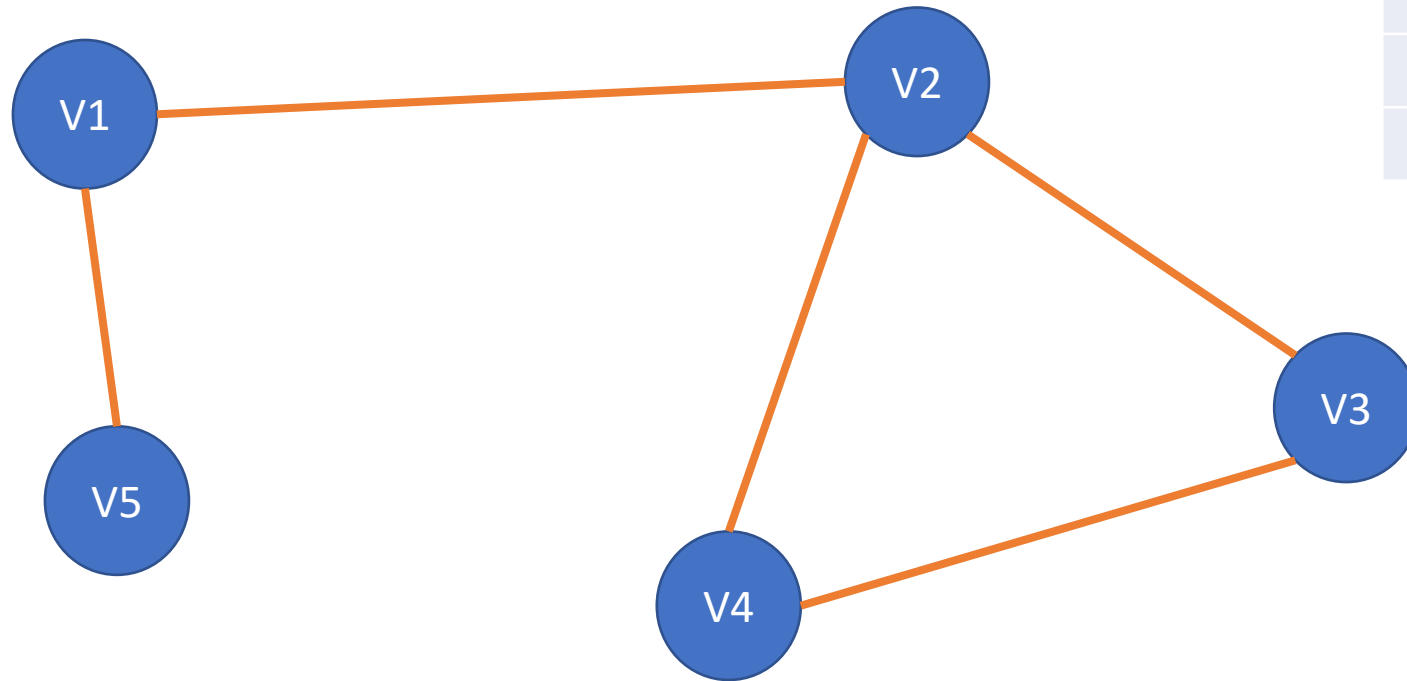
Adjacent Matrix

Node	V1	V2	V3	V4	V5
V1	0	1	0	0	1
V2	1	0	1	1	0
V3	0	1	0	1	0
V4	0	1	1	0	0
V5	1	0	0	0	0

No weighted
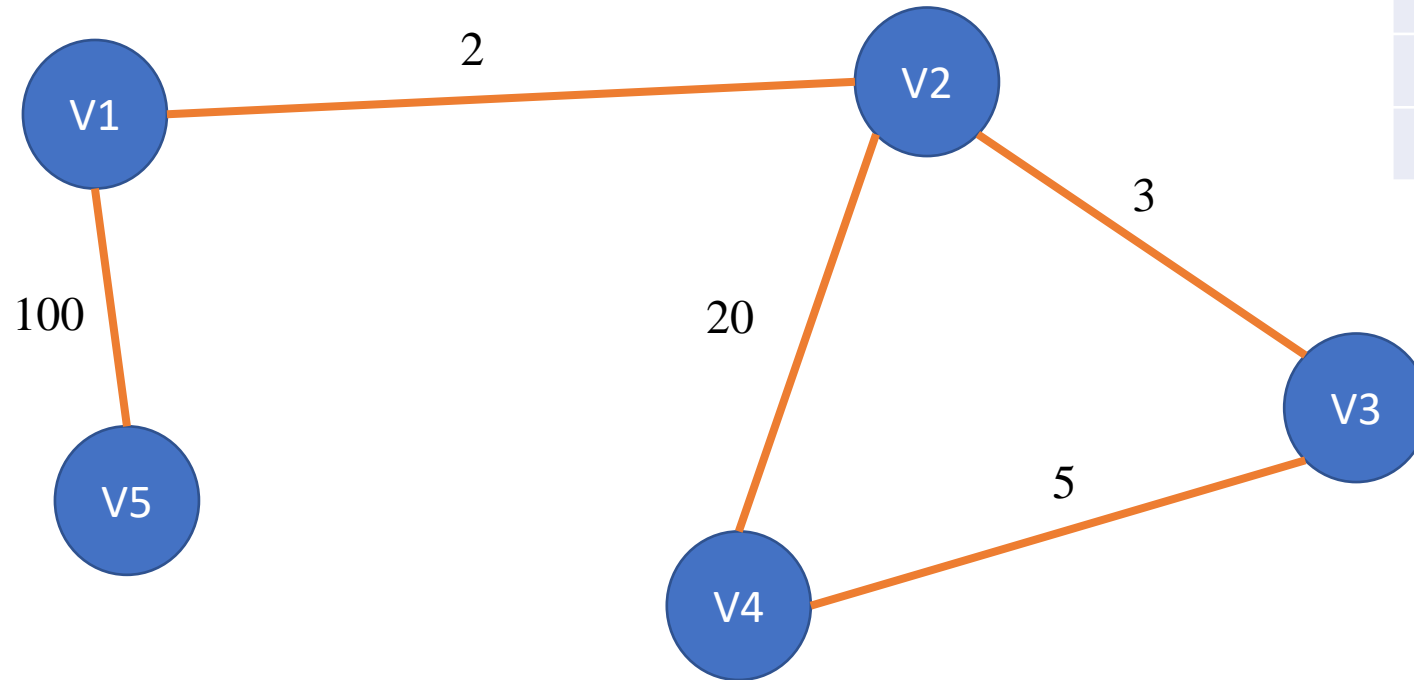
V1

V2

V3

V4

V5

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Representation

Adjacent Matrix

Node	V1	V2	V3	V4	V5
V1	0	2	0	0	100
V2	2	0	3	20	0
V3	0	3	0	5	0
V4	0	20	5	0	0
V5	100	0	0	0	0

Weighted

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Representation
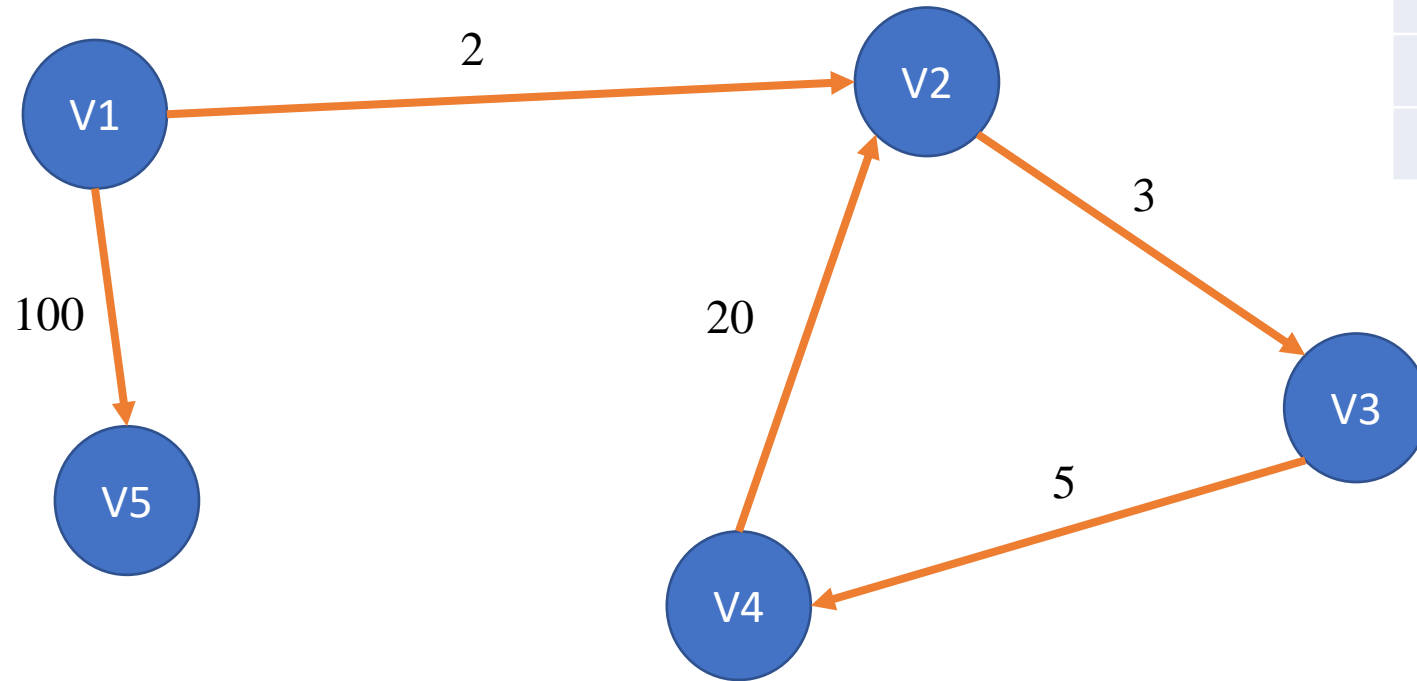
Adjacent Matrix

Node	V1	V2	V3	V4	V5
V1	0	2	0	0	100
V2	0	0	3	0	0
V3	0	0	0	5	0
V4	0	20	0	0	0
V5	0	0	0	0	0

Weighted
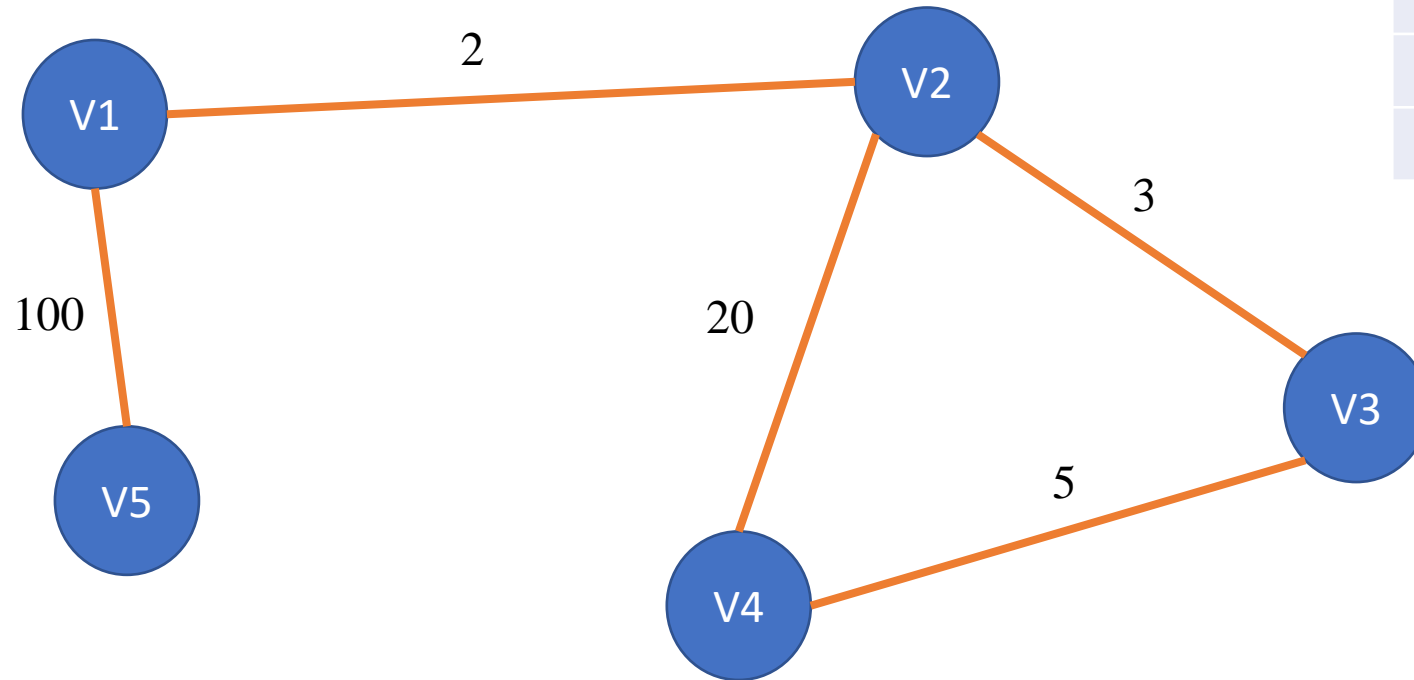
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Representation

Adjacent Matrix

Node	V1	V2	V3	V4	V5
V1	0	2	0	0	100
V2	2	0	3	20	0
V3	0	3	0	5	0
V4	0	20	5	0	0
V5	100	0	0	0	0

Weighted

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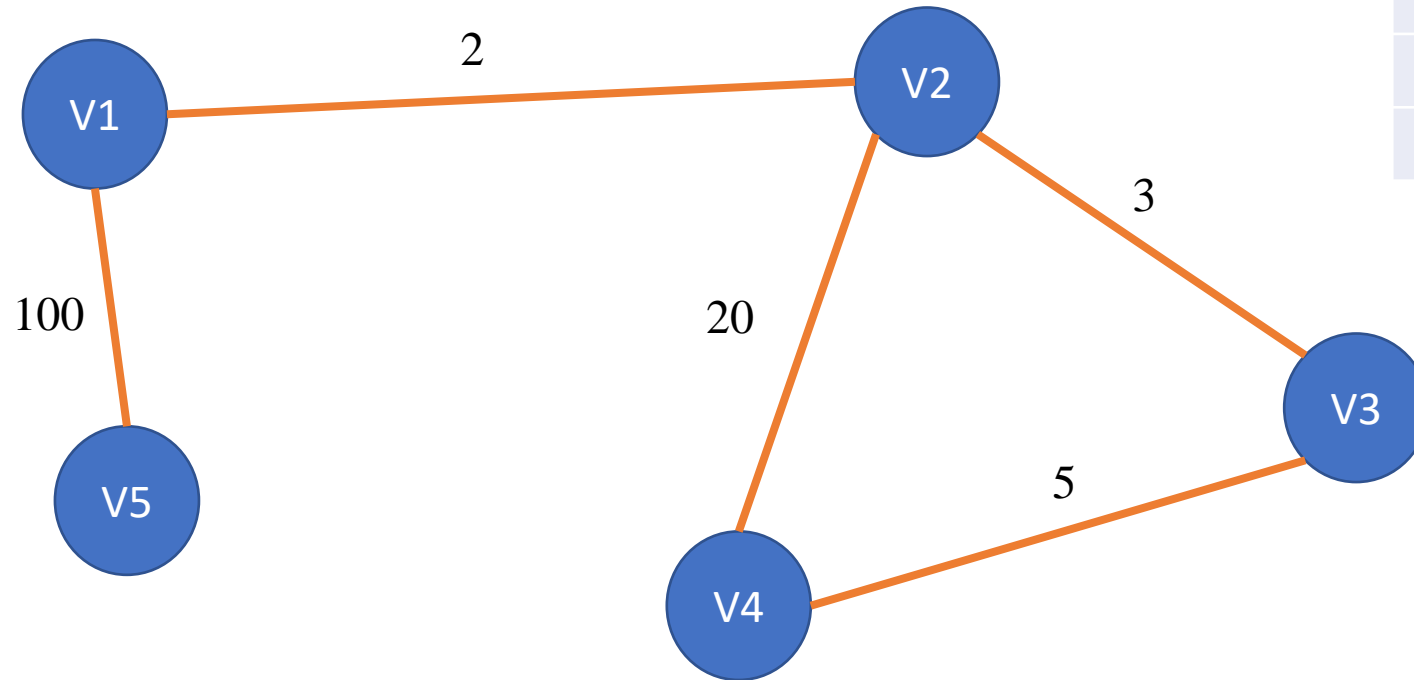
Representation

Node	V1	V2	V3	V4	V5
V1	∞	2	∞	∞	100
V2	2	∞	3	20	∞
V3	∞	3	∞	5	∞
V4	∞	20	5	∞	∞
V5	100	∞	∞	∞	∞

Weighted

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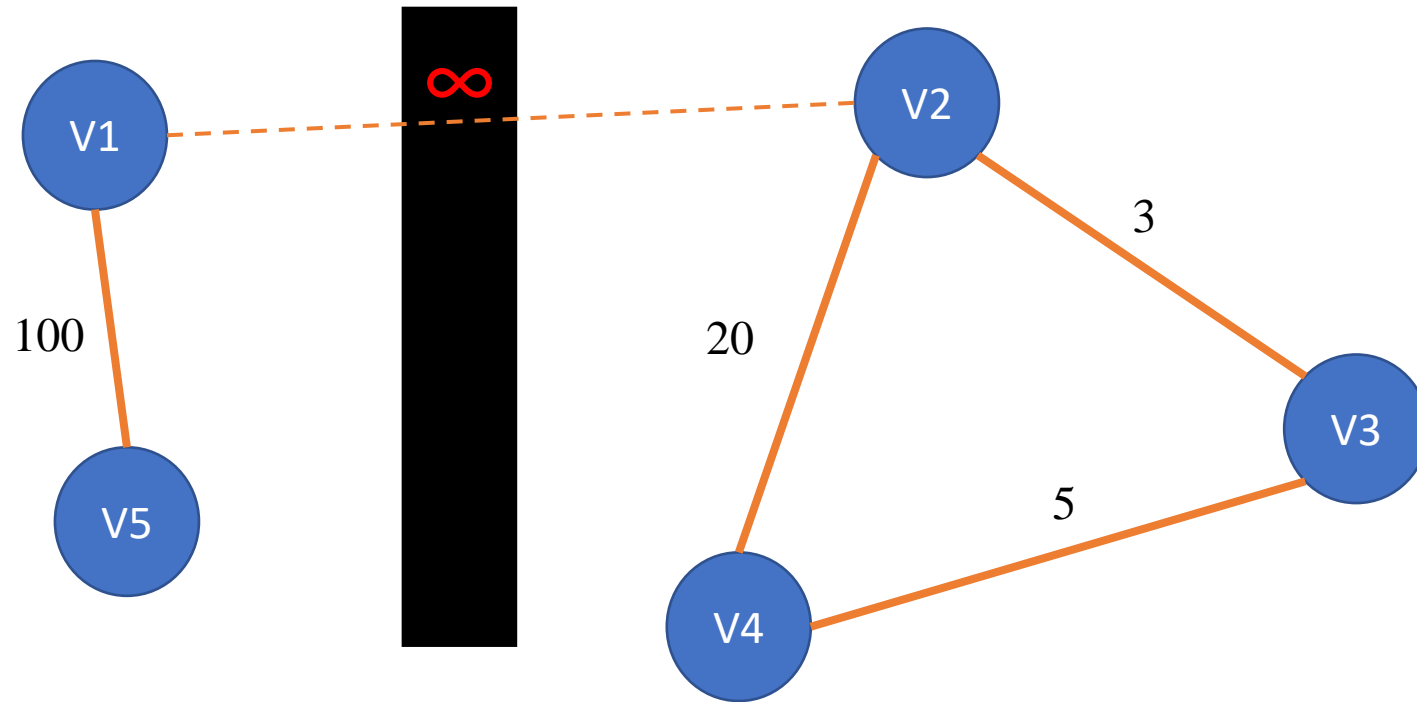
Representation

Representation

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