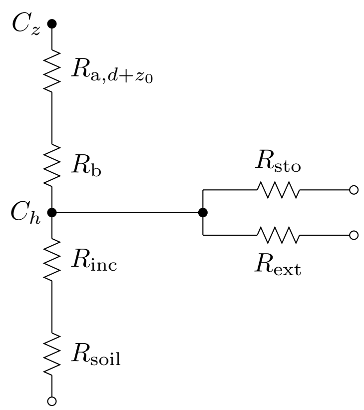
DO$_3$SE Multi-Layer Resistance Model

1 Introduction

This document briefly describes the new DO$_3$SE multi-layer resistance model, how it applies to ozone concentration throughout the canopy, and how it relates to the method DO$_3$SE currently uses.

2 Existing model

The existing single-layer, "big leaf" model can be described with the following resistance diagram:



Using this resistance model to bring a concentration, C_z, from an arbitrary height (above the canopy), z, down to the canopy height, h, follows this formulation:

$$\mathrm{Ra}\left(z_1, z_2\right) = \left(\frac{1}{ku^*}\right) \ln\left(\frac{z_2 - d}{z_1 - d}\right) \quad (1)$$

$$R_{a,h} = \mathrm{Ra}\left(h, z\right) \quad (2)$$

$$R_{a,d+z_0} = \mathrm{Ra}\left(d + z_0, z\right) \quad (3)$$

$$\frac{1}{R_{sur}} = \frac{1}{R_{sto}} + \frac{1}{R_{ext}} + \frac{1}{R_{inc} + R_{soil}} \quad (4)$$

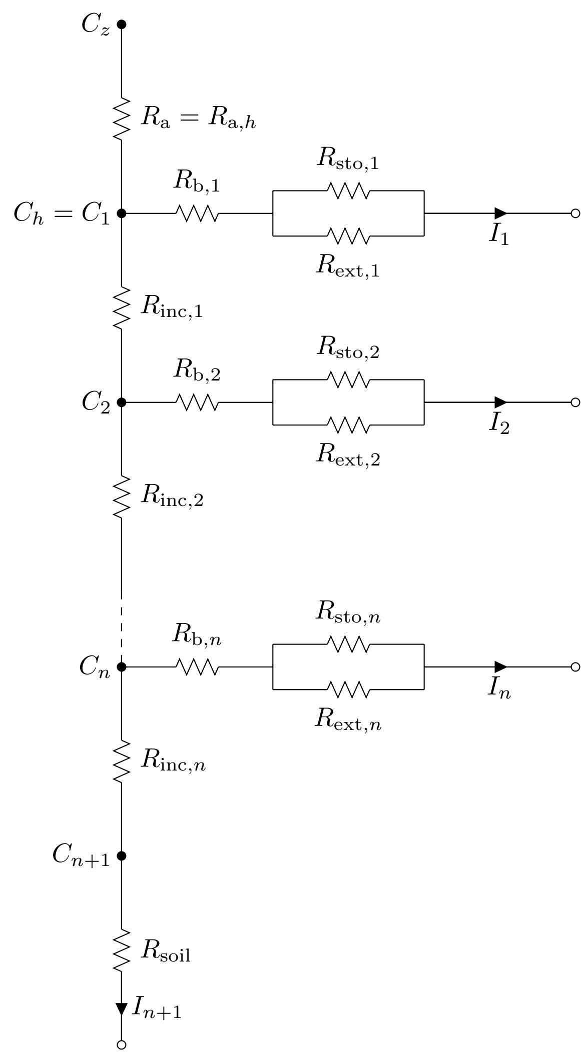
$$V_d = \frac{1}{R_{a,d+z_0} + R_b + R_{sur}} \quad (5)$$

$$C_h = C_z\left(1 - \frac{R_{a,h}}{V_d}\right) \quad (6)$$

The $R_{a,d+z_0} + R_b$ pathway represents the aerodynamic resistance and boundary layer resistance to ozone penetrating into the canopy from a height of z. The $R_{inc} + R_{soil}$ pathway is the resistance to ozone passing through the canopy and into the soil. The R_{sto} and R_{ext} pathways are the resistances to deposition to the vegetation. The aerodynamic resistance to the top of the canopy is different to inside the canopy, hence $R_{a,h}$ in the C_h calculation.

3 New resistance model

The new multi-layer model acts like several "big leaves" stacked on top of each other, with resistances to passing through each layer. The following diagram describes the model for n vegetation layers:



3.1 Single-layer analogue

The multi-layer resistance model can be simplified to a single-layer resistance model by combining resistances. The "surface resistance", R_{sur}, of a layer is defined as the total resistance to deposition to that layer. The "total resistance", R_{total}, of a layer is the total resistance to deposition, both to the vegetation and the ground surface, for that layer and everything below it; $R_{total,1}$ therefore corresponds to R_{sur} in the old model.

$$R_{\mathrm{sur},i} = R_{\mathrm{b},i} + 1/\left(\frac{1}{R_{\mathrm{sto},i}} + \frac{1}{R_{\mathrm{ext},i}}\right) \quad (7)$$

$$R_{\mathrm{total},i} = 1/\left(\frac{1}{R_{\mathrm{sur},i}} + \frac{1}{R_{\mathrm{inc},i} + R_{\mathrm{total},i+1}}\right) \quad (8)$$

$$R_{\mathrm{total},n+1} = R_{\mathrm{soil}} \quad (9)$$

This allows us to use the old formulation for ozone concentration at the canopy height. It needs to be modified slightly, since the position of R_{b} has changed and is incorporated into $R_{\mathrm{total},1}$:

$$V_d = \frac{1}{R_{\mathrm{a},d+z_0} + R_{\mathrm{total},1}} \quad (10)$$

$$C_1 = C_h = C_z\left(1 - \frac{R_{\mathrm{a},h}}{V_d}\right) \quad (11)$$

3.2 Multi-layer concentration

The intent is to use the multi-layer model to obtain ozone concentrations for each layer. This can be done with an analytical solution to the described resistance system, given all resistances and the value of C_z.

Firstly, resistance values are arranged in vectors corresponding to the layers; there is one more layer than the number of vegetation layers to account for deposition to the soil:

$$R = \begin{pmatrix} R_{\mathrm{a}} \\ R_{\mathrm{inc},1} \\ \vdots \\ R_{\mathrm{inc},n} \end{pmatrix} \quad (12)$$

$$r = \begin{pmatrix} R_{\mathrm{sur},1} \\ \vdots \\ R_{\mathrm{sur},n} \\ R_{\mathrm{soil}} \end{pmatrix} \quad (13)$$

Then they are built into a matrix that describes the simultaneous equations for ozone concentration at each layer:

$$A = \begin{pmatrix} R_1 & R_1 & \dots & R_1 \\ 0 & R_2 & \dots & R_2 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & R_{n+1} \end{pmatrix} \quad (14)$$

$$B = \begin{pmatrix} r_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & r_n & 0 \\ 0 & 0 & 0 & r_{n+1} \end{pmatrix} \quad (15)$$

$$D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ r_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & r_n & 0 \end{pmatrix} \quad (16)$$

$$X = A + B - D \quad (17)$$

For example, with 2 vegetation layers:

$$X = \begin{pmatrix} R_{\mathrm{a}} + R_{\mathrm{sur},1} & R_{\mathrm{a}} & R_{\mathrm{a}} \\ -R_{\mathrm{sur},1} & R_{\mathrm{inc},1} + R_{\mathrm{sur},2} & R_{\mathrm{inc},1} \\ 0 & -R_{\mathrm{sur},2} & R_{\mathrm{inc},2} + R_{\mathrm{soil}} \end{pmatrix} \quad (18)$$

Finally, we solve the following system of linear equations for $C_1, \dots, C_{n+1}$:

$$X \begin{pmatrix} C_1/r_1 \\ \vdots \\ C_n/r_n \\ C_{n+1}/r_{n+1} \end{pmatrix} = \begin{pmatrix} C_z \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (19)$$

3.3 Discrepancies

If used as described above, with C_z being the ozone concentration at some height above the canopy, z, and R_{a} corresponding to $R_{\mathrm{a},h}$ from the single-layer model, then the output C_1 should be the ozone concentration at the canopy height. However, this does not make any use of $R_{\mathrm{a},d+z_0}$ and therefore differs from the single-layer result; it instead acts as if $V_d = 1/(R_{\mathrm{a},h} + R_{\mathrm{total},1})$, i.e. simply:

$$C_h = C_z \frac{R_{\mathrm{total},1}}{R_{\mathrm{a},h} + R_{\mathrm{total},1}} \quad (20)$$