

Summary

Definition

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A , the electric flux through the surface is

$$\Phi_E = EA \cos \theta \quad (24.2)$$

In general, the electric flux through a surface is

$$\Phi_E \equiv \int_{\text{surface}} \vec{E} \cdot d\vec{A} \quad (24.3)$$

Concepts and Principles

Gauss's law says that the net electric flux Φ_E through any closed gaussian surface is equal to the *net* charge q_{in} inside the surface divided by ϵ_0 :

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0} \quad (24.6)$$

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions.

A conductor in **electrostatic equilibrium** has the following properties:

1. The electric field is zero everywhere inside the conductor, whether the conductor is solid or hollow.
2. If the conductor is isolated and carries a charge, the charge resides on its surface.
3. The electric field at a point just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude σ/ϵ_0 , where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. A cubical gaussian surface surrounds a long, straight, charged filament that passes perpendicularly through two opposite faces. No other charges are nearby. (i) Over how many of the cube's faces is the electric field zero? (a) 0 (b) 2 (c) 4 (d) 6 (ii) Through how many of the cube's faces is the electric flux zero? Choose from the same possibilities as in part (i).
2. A coaxial cable consists of a long, straight filament surrounded by a long, coaxial, cylindrical conducting shell. Assume charge Q is on the filament, zero net charge is on the shell, and the electric field is $E_1 \hat{i}$ at a particular point P midway between the filament and the inner surface of the shell. Next, you place the cable into a uniform external field $-E_1 \hat{i}$. What is the x component of the electric field at P then? (a) 0 (b) between 0 and E_1 (c) E_1 (d) between 0 and $-E_1$ (e) $-E_1$
3. In which of the following contexts can Gauss's law *not* be readily applied to find the electric field? (a) near a long, uniformly charged wire (b) above a large, uniformly charged plane (c) inside a uniformly charged ball (d) outside a uniformly charged sphere (e) Gauss's law can be readily applied to find the electric field in all these contexts.
4. A particle with charge q is located inside a cubical gaussian surface. No other charges are nearby. (i) If the particle is at the center of the cube, what is the flux through each one of the faces of the cube? (a) 0 (b) $q/2\epsilon_0$ (c) $q/6\epsilon_0$ (d) $q/8\epsilon_0$ (e) depends on the size of the cube (ii) If the particle can be moved to any point within the cube, what maximum value can the flux through one face approach? Choose from the same possibilities as in part (i).
5. Charges of 3.00 nC, -2.00 nC, -7.00 nC, and 1.00 nC are contained inside a rectangular box with length 1.00 m, width 2.00 m, and height 2.50 m. Outside the box are charges of 1.00 nC and 4.00 nC. What is the electric flux through the surface of the box? (a) 0 (b) $-5.64 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C}$ (c) $-1.47 \times 10^3 \text{ N} \cdot \text{m}^2/\text{C}$ (d) $1.47 \times 10^3 \text{ N} \cdot \text{m}^2/\text{C}$ (e) $5.64 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C}$
6. A large, metallic, spherical shell has no net charge. It is supported on an insulating stand and has a small hole at the top. A small tack with charge Q is lowered on a silk thread through the hole into the interior of the shell. (i) What is the charge on the inner surface of the shell, (a) Q (b) $Q/2$ (c) 0 (d) $-Q/2$ or (e) $-Q$? Choose your answers to the following questions from

the same possibilities. (ii) What is the charge on the outer surface of the shell? (iii) The tack is now allowed to touch the interior surface of the shell. After this contact, what is the charge on the tack? (iv) What is the charge on the inner surface of the shell now? (v) What is the charge on the outer surface of the shell now?

7. Two solid spheres, both of radius 5 cm, carry identical total charges of $2\ \mu\text{C}$. Sphere A is a good conductor. Sphere B is an insulator, and its charge is distributed uniformly throughout its volume. (i) How do the magnitudes of the electric fields they separately create at a radial distance of 6 cm compare? (a) $E_A > E_B = 0$ (b) $E_A > E_B > 0$ (c) $E_A = E_B > 0$ (d) $0 < E_A < E_B$ (e) $0 = E_A < E_B$ (ii) How do the magnitudes of the electric fields they separately create at radius 4 cm compare? Choose from the same possibilities as in part (i).
8. A uniform electric field of $1.00\ \text{N/C}$ is set up by a uniform distribution of charge in the xy plane. What is the electric field inside a metal ball placed $0.500\ \text{m}$ above the xy plane? (a) $1.00\ \text{N/C}$ (b) $-1.00\ \text{N/C}$ (c) 0 (d) $0.250\ \text{N/C}$ (e) varies depending on the position inside the ball
9. A solid insulating sphere of radius 5 cm carries electric charge uniformly distributed throughout its volume. Concentric with the sphere is a conducting spherical shell with no net charge as shown in Figure OQ24.9. The inner radius of the shell is 10 cm, and the outer radius is 15 cm. No other charges are nearby. (a) Rank

the magnitude of the electric field at points A (at radius 4 cm), B (radius 8 cm), C (radius 12 cm), and D (radius 16 cm) from largest to smallest. Display any cases of equality in your ranking. (b) Similarly rank the electric flux through concentric spherical surfaces through points A, B, C, and D.

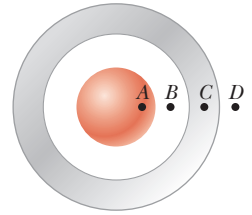


Figure OQ24.9

10. A cubical gaussian surface is bisected by a large sheet of charge, parallel to its top and bottom faces. No other charges are nearby. (i) Over how many of the cube's faces is the electric field zero? (a) 0 (b) 2 (c) 4 (d) 6 (ii) Through how many of the cube's faces is the electric flux zero? Choose from the same possibilities as in part (i).
11. Rank the electric fluxes through each gaussian surface shown in Figure OQ24.11 from largest to smallest. Display any cases of equality in your ranking.

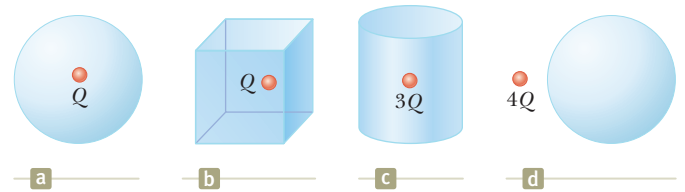


Figure OQ24.11

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Consider an electric field that is uniform in direction throughout a certain volume. Can it be uniform in magnitude? Must it be uniform in magnitude? Answer these questions (a) assuming the volume is filled with an insulating material carrying charge described by a volume charge density and (b) assuming the volume is empty space. State reasoning to prove your answers.
2. A cubical surface surrounds a point charge q . Describe what happens to the total flux through the surface if (a) the charge is doubled, (b) the volume of the cube is doubled, (c) the surface is changed to a sphere, (d) the charge is moved to another location inside the surface, and (e) the charge is moved outside the surface.
3. A uniform electric field exists in a region of space containing no charges. What can you conclude about the net electric flux through a gaussian surface placed in this region of space?
4. If the total charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss's law to find the electric field? Explain.
5. Explain why the electric flux through a closed surface with a given enclosed charge is independent of the size or shape of the surface.
6. If more electric field lines leave a gaussian surface than enter it, what can you conclude about the net charge enclosed by that surface?
7. A person is placed in a large, hollow, metallic sphere that is insulated from ground. (a) If a large charge is placed on the sphere, will the person be harmed upon touching the inside of the sphere? (b) Explain what will happen if the person also has an initial charge whose sign is opposite that of the charge on the sphere.
8. Consider two identical conducting spheres whose surfaces are separated by a small distance. One sphere is given a large net positive charge, and the other is given a small net positive charge. It is found that the force between the spheres is attractive even though they both have net charges of the same sign. Explain how this attraction is possible.
9. A common demonstration involves charging a rubber balloon, which is an insulator, by rubbing it on your hair and then touching the balloon to a ceiling or wall, which is also an insulator. Because of the electrical attraction between the charged balloon and the neutral wall, the balloon sticks to the wall. Imagine now that we have two infinitely large, flat sheets of insulating

material. One is charged, and the other is neutral. If these sheets are brought into contact, does an attractive force exist between them as there was for the balloon and the wall?

10. On the basis of the repulsive nature of the force between like charges and the freedom of motion of

charge within a conductor, explain why excess charge on an isolated conductor must reside on its surface.

11. The Sun is lower in the sky during the winter than it is during the summer. (a) How does this change affect the flux of sunlight hitting a given area on the surface of the Earth? (b) How does this change affect the weather?

Problems

WebAssign

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; 2. intermediate; 3. challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT

Analysis Model tutorial available in Enhanced WebAssign

GP

Guided Problem

M

Master It tutorial available in Enhanced WebAssign

W

Watch It video solution available in Enhanced WebAssign

Section 24.1 Electric Flux

- A flat surface of area 3.20 m^2 is rotated in a uniform electric field of magnitude $E = 6.20 \times 10^5 \text{ N/C}$. Determine the electric flux through this area (a) when the electric field is perpendicular to the surface and (b) when the electric field is parallel to the surface.
- A vertical electric field of magnitude $2.00 \times 10^4 \text{ N/C}$ exists above the Earth's surface on a day when a thunderstorm is brewing. A car with a rectangular size of 6.00 m by 3.00 m is traveling along a dry gravel roadway sloping downward at 10.0° . Determine the electric flux through the bottom of the car.
- A 40.0-cm -diameter circular loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be $5.20 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$. What is the magnitude of the electric field?
- Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4 \text{ N/C}$ as shown in Figure P24.4. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

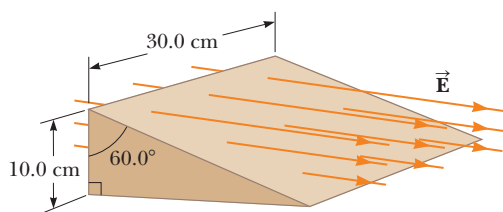


Figure P24.4

- An electric field of magnitude 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long (a) if the plane is parallel to the yz plane, (b) if the plane is parallel to the xy plane, and (c) if the plane contains the y axis and its normal makes an angle of 40.0° with the x axis.

- A nonuniform electric field is given by the expression

$$\vec{E} = ay\hat{i} + bz\hat{j} + cx\hat{k}$$

where a , b , and c are constants. Determine the electric flux through a rectangular surface in the xy plane, extending from $x = 0$ to $x = w$ and from $y = 0$ to $y = h$.

Section 24.2 Gauss's Law

- An uncharged, nonconducting, hollow sphere of radius 10.0 cm surrounds a $10.0\text{-}\mu\text{C}$ charge located at the origin of a Cartesian coordinate system. A drill with a radius of 1.00 mm is aligned along the z axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.
- Find the net electric flux through the spherical closed surface shown in Figure P24.8. The two charges on the right are inside the spherical surface.

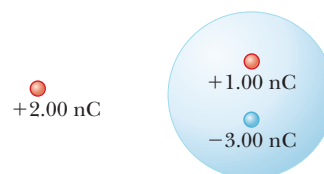


Figure P24.8

- The following charges are located inside a submarine: $5.00 \mu\text{C}$, $-9.00 \mu\text{C}$, $27.0 \mu\text{C}$, and $-84.0 \mu\text{C}$. (a) Calculate the net electric flux through the hull of the submarine. (b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?
- The electric field everywhere on the surface of a thin, spherical shell of radius 0.750 m is of magnitude 890 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface? (b) What is the distribution of the charge inside the spherical shell?

11. Four closed surfaces, S_1 through S_4 , together with the charges $-2Q$, Q , and $-Q$ are sketched in Figure P24.11. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.

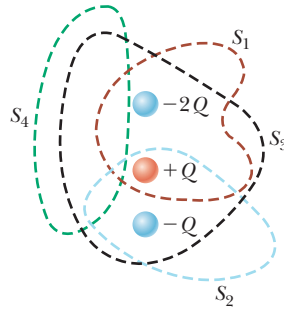


Figure P24.11

12. A charge of $170 \mu\text{C}$ is at the center of a cube of edge 80.0 cm . No other charges are nearby. (a) Find the flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) **What If?** Would your answers to either part (a) or part (b) change if the charge were not at the center? Explain.
13. In the air over a particular region at an altitude of 500 m above the ground, the electric field is 120 N/C directed downward. At 600 m above the ground, the electric field is 100 N/C downward. What is the average volume charge density in the layer of air between these two elevations? Is it positive or negative?
14. A particle with charge of $12.0 \mu\text{C}$ is placed at the center of a spherical shell of radius 22.0 cm . What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do the results depend on the radius? Explain.

15. (a) Find the net electric flux through the cube shown in Figure P24.15. (b) Can you use Gauss's law to find the electric field on the surface of this cube? Explain.

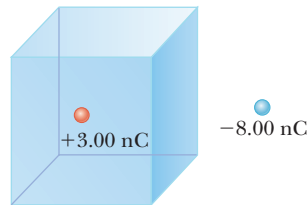


Figure P24.15

16. (a) A particle with charge q is located a distance d from an infinite plane. Determine the electric flux through the plane due to the charged particle. (b) **What If?** A particle with charge q is located a *very small* distance from the center of a *very large* square on the line perpendicular to the square and going through its center. Determine the approximate electric flux through the square due to the charged particle. (c) How do the answers to parts (a) and (b) compare? Explain.

17. An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown in Figure P24.17. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases, where (a) $R < d$ and (b) $R > d$.

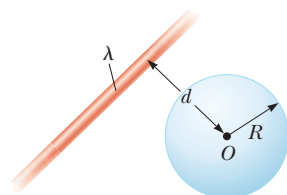


Figure P24.17

18. Find the net electric flux through (a) the closed spherical surface in a uniform electric field shown in Figure P24.18a and (b) the closed cylindrical surface shown in Figure P24.18b. (c) What can you conclude about the charges, if any, inside the cylindrical surface?

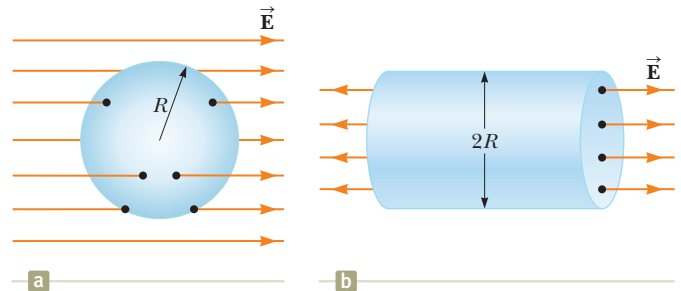


Figure P24.18

19. A particle with charge $Q = 5.00 \mu\text{C}$ is located at the center of a cube of edge $L = 0.100 \text{ m}$. In addition, six other identical charged particles having $q = -1.00 \mu\text{C}$ are positioned symmetrically around Q as shown in Figure P24.19. Determine the electric flux through one face of the cube.

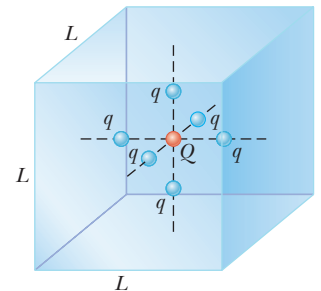


Figure P24.19

Problems 19 and 20.

20. A particle with charge Q is located at the center of a cube of edge L . In addition, six other identical charged particles q are positioned symmetrically around Q as shown in Figure P24.19. For each of these particles, q is a negative number. Determine the electric flux through one face of the cube.

21. A particle with charge Q is located a small distance δ immediately above the center of the flat face of a hemisphere of radius R as shown in Figure P24.21. What is the electric flux (a) through the curved surface and (b) through the flat face as $\delta \rightarrow 0$?

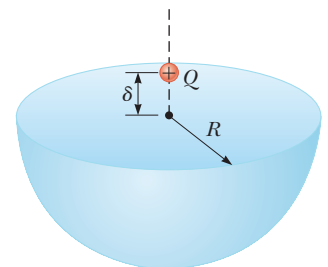


Figure P24.21

22. Figure P24.22 (page 742) represents the top view of a cubic gaussian surface in a uniform electric field \vec{E} oriented parallel to the top and bottom faces of the cube. The field makes an angle θ with side ①, and the area of each face is A . In symbolic form, find the electric flux through (a) face ①, (b) face ②, (c) face ③, (d) face ④, and (e) the top and bottom faces of the cube. (f) What

is the net electric flux through the cube? (g) How much charge is enclosed within the gaussian surface?

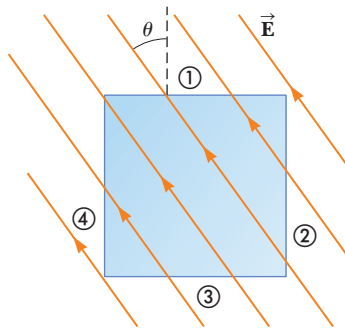


Figure P24.22

Section 24.3 Application of Gauss's Law to Various Charge Distributions

23. In nuclear fission, a nucleus of uranium-238, which contains 92 protons, can divide into two smaller spheres, each having 46 protons and a radius of 5.90×10^{-15} m. What is the magnitude of the repulsive electric force pushing the two spheres apart?

24. The charge per unit length on a long, straight filament is $-90.0 \mu\text{C}/\text{m}$. Find the electric field (a) 10.0 cm, (b) 20.0 cm, and (c) 100 cm from the filament, where distances are measured perpendicular to the length of the filament.

25. A 10.0-g piece of Styrofoam carries a net charge of $-0.700 \mu\text{C}$ and is suspended in equilibrium above the center of a large, horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?

26. Determine the magnitude of the electric field at the surface of a lead-208 nucleus, which contains 82 protons and 126 neutrons. Assume the lead nucleus has a volume 208 times that of one proton and consider a proton to be a sphere of radius 1.20×10^{-15} m.

27. A large, flat, horizontal sheet of charge has a charge per unit area of $9.00 \mu\text{C}/\text{m}^2$. Find the electric field just above the middle of the sheet.

28. Suppose you fill two rubber balloons with air, suspend both of them from the same point, and let them hang down on strings of equal length. You then rub each with wool or on your hair so that the balloons hang apart with a noticeable separation between them. Make order-of-magnitude estimates of (a) the force on each, (b) the charge on each, (c) the field each creates at the center of the other, and (d) the total flux of electric field created by each balloon. In your solution, state the quantities you take as data and the values you measure or estimate for them.

29. Consider a thin, spherical shell of radius 14.0 cm with a total charge of $32.0 \mu\text{C}$ distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

30. A nonconducting wall carries charge with a uniform density of $8.60 \mu\text{C}/\text{cm}^2$. (a) What is the electric field 7.00 cm in front of the wall if 7.00 cm is small compared

with the dimensions of the wall? (b) Does your result change as the distance from the wall varies? Explain.

31. A uniformly charged, straight filament 7.00 m in length has a total positive charge of $2.00 \mu\text{C}$. An uncharged cardboard cylinder 2.00 cm in length and 10.0 cm in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.

32. Assume the magnitude of the electric field on each face of the cube of edge $L = 1.00$ m in Figure P24.32 is uniform and the directions of the fields on each face are as indicated. Find (a) the net electric flux through the cube and (b) the net charge inside the cube. (c) Could the net charge be a single point charge?

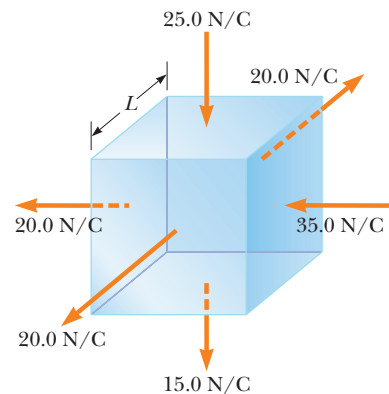


Figure P24.32

33. Consider a long, cylindrical charge distribution of radius R with a uniform charge density ρ . Find the electric field at distance r from the axis, where $r < R$.

34. A cylindrical shell of radius 7.00 cm and length 2.40 m has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is $36.0 \text{ kN}/\text{C}$. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

35. A solid sphere of radius 40.0 cm has a total positive charge of $26.0 \mu\text{C}$ uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere.

36. Review. A particle with a charge of -60.0 nC is placed at the center of a nonconducting spherical shell of inner radius 20.0 cm and outer radius 25.0 cm. The spherical shell carries charge with a uniform density of $-1.33 \mu\text{C}/\text{m}^3$. A proton moves in a circular orbit just outside the spherical shell. Calculate the speed of the proton.

Section 24.4 Conductors in Electrostatic Equilibrium

37. A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of $30.0 \text{ nC}/\text{m}$. Find the electric field (a) 3.00 cm, (b) 10.0 cm, and (c) 100 cm from the

axis of the rod, where distances are measured perpendicular to the rod's axis.

38. Why is the following situation impossible? A solid copper sphere of radius 15.0 cm is in electrostatic equilibrium and carries a charge of 40.0 nC. Figure P24.38 shows the magnitude of the electric field as a function of radial position r measured from the center of the sphere.

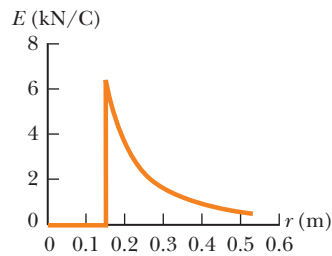


Figure P24.38

39. A solid metallic sphere of radius a carries total charge Q . No other charges are nearby. The electric field just outside its surface is $k_e Q/a^2$ radially outward. At this close point, the uniformly charged surface of the sphere looks exactly like a uniform flat sheet of charge. Is the electric field here given by σ/ϵ_0 or by $\sigma/2\epsilon_0$?
40. A positively charged particle is at a distance $R/2$ from the center of an uncharged thin, conducting, spherical shell of radius R . Sketch the electric field lines set up by this arrangement both inside and outside the shell.
41. A very large, thin, flat plate of aluminum of area A has a total charge Q uniformly distributed over its surfaces. Assuming the same charge is spread uniformly over the upper surface of an otherwise identical glass plate, compare the electric fields just above the center of the upper surface of each plate.
42. In a certain region of space, the electric field is $\vec{E} = 6.00 \times 10^3 x^2 \hat{i}$, where \vec{E} is in newtons per coulomb and x is in meters. Electric charges in this region are at rest and remain at rest. (a) Find the volume density of electric charge at $x = 0.300$ m. *Suggestion:* Apply Gauss's law to a box between $x = 0.300$ m and $x = 0.300$ m + dx . (b) Could this region of space be inside a conductor?
43. Two identical conducting spheres each having a radius of 0.500 cm are connected by a light, 2.00-m-long conducting wire. A charge of 60.0 μC is placed on one of the conductors. Assume the surface distribution of charge on each sphere is uniform. Determine the tension in the wire.
44. A square plate of copper with 50.0-cm sides has no net charge and is placed in a region of uniform electric field of 80.0 kN/C directed perpendicularly to the plate. Find (a) the charge density of each face of the plate and (b) the total charge on each face.
45. A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ , and the cylinder has a net charge per unit length of 2λ . From this information, use Gauss's law to find (a) the charge per unit length on the inner surface of the cylinder, (b) the charge per unit length on the outer surface of the cylinder, and (c) the electric field outside the cylinder a distance r from the axis.
46. A thin, square, conducting plate 50.0 cm on a side lies in the xy plane. A total charge of 4.00×10^{-8} C is placed

on the plate. Find (a) the charge density on each face of the plate, (b) the electric field just above the plate, and (c) the electric field just below the plate. You may assume the charge density is uniform.

47. A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC . A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of $-4.00 \mu\text{C}$. Find the electric field at (a) $r = 1.00$ cm, (b) $r = 3.00$ cm, (c) $r = 4.50$ cm, and (d) $r = 7.00$ cm from the center of this charge configuration.

Additional Problems

48. Consider a plane surface in a uniform electric field as in Figure P24.48, where $d = 15.0$ cm and $\theta = 70.0^\circ$. If the net flux through the surface is $6.00 \text{ N} \cdot \text{m}^2/\text{C}$, find the magnitude of the electric field.

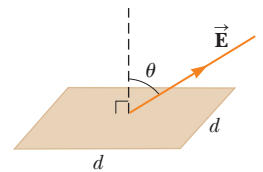


Figure P24.48

Problems 48 and 49.

49. Find the electric flux through the plane surface shown in Figure P24.48 if $\theta = 60.0^\circ$, $E = 350 \text{ N/C}$, and $d = 5.00$ cm. The electric field is uniform over the entire area of the surface.
50. A hollow, metallic, spherical shell has exterior radius 0.750 m, carries no net charge, and is supported on an insulating stand. The electric field everywhere just outside its surface is 890 N/C radially toward the center of the sphere. Explain what you can conclude about (a) the amount of charge on the exterior surface of the sphere and the distribution of this charge, (b) the amount of charge on the interior surface of the sphere and its distribution, and (c) the amount of charge inside the shell and its distribution.

51. A sphere of radius $R = 1.00$ m surrounds a particle with charge $Q = 50.0 \mu\text{C}$ located at its center as shown in Figure P24.51. Find the electric flux through a circular cap of half-angle $\theta = 45.0^\circ$.

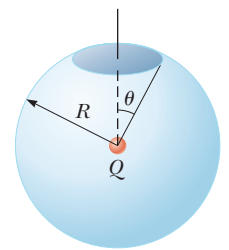


Figure P24.51

Problems 51 and 52.

52. A sphere of radius R surrounds a particle with charge Q located at its center as shown in Figure P24.51. Find the electric flux through a circular cap of half-angle θ .

53. A very large conducting plate lying in the xy plane carries a charge per unit area of σ . A second such plate located above the first plate at $z = z_0$ and oriented parallel to the xy plane carries a charge per unit area of -2σ . Find the electric field for (a) $z < 0$, (b) $0 < z < z_0$, and (c) $z > z_0$.

54. A solid, insulating sphere of radius a has a uniform charge density throughout its volume and a total charge Q . Concentric with this sphere is an uncharged, conducting, hollow sphere whose inner and outer radii are b and c as shown in Figure P24.54 (page 744). We wish to

understand completely the charges and electric fields at all locations. (a) Find the charge contained within a sphere of radius $r < a$. (b) From this value, find the magnitude of the electric field for $r < a$. (c) What charge is contained within a sphere of radius r when $a < r < b$? (d) From this value, find the magnitude of the electric field for r when $a < r < b$. (e) Now consider r when $b < r < c$. What is the magnitude of the electric field for this range of values of r ? (f) From this value, what must be the charge on the inner surface of the hollow sphere? (g) From part (f), what must be the charge on the outer surface of the hollow sphere? (h) Consider the three spherical surfaces of radii a , b , and c . Which of these surfaces has the largest magnitude of surface charge density?

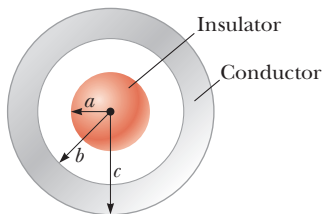


Figure P24.54

Problems 54, 55, and 57.

55. A solid insulating sphere of radius $a = 5.00$ cm carries a net positive charge of $Q = 3.00$ μC uniformly distributed throughout its volume. Concentric with this sphere is a conducting spherical shell with inner radius $b = 10.0$ cm and outer radius $c = 15.0$ cm as shown in Figure P24.54, having net charge $q = -1.00$ μC . Prepare a graph of the magnitude of the electric field due to this configuration versus r for $0 < r < 25.0$ cm.

56. Two infinite, nonconducting sheets of charge are parallel to each other as shown in Figure P24.56. The sheet on the left has a uniform surface charge density σ , and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets. (d) **What If?** Find the electric fields in all three regions if both sheets have positive uniform surface charge densities of value σ .

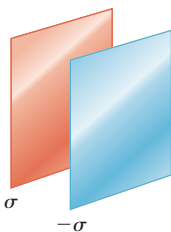


Figure P24.56

57. For the configuration shown in Figure P24.54, suppose $a = 5.00$ cm, $b = 20.0$ cm, and $c = 25.0$ cm. Furthermore, suppose the electric field at a point 10.0 cm from the center is measured to be 3.60×10^3 N/C radially inward and the electric field at a point 50.0 cm from the center is of magnitude 200 N/C and points radially outward. From this information, find (a) the charge on the insulating sphere, (b) the net charge on the hollow conducting sphere, (c) the charge on the inner surface of the hollow conducting sphere, and (d) the charge on the outer surface of the hollow conducting sphere.

58. An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q . A spherical gaussian surface of radius r , which shares a common center with the insulating sphere, is inflated starting from $r = 0$. (a) Find an expression for the electric flux passing through the surface of the gaussian sphere as a function of r for $r < a$. (b) Find an expression for the electric flux for $r > a$. (c) Plot the flux versus r .

59. A uniformly charged spherical shell with positive surface charge density σ contains a circular hole in its surface. The radius r of the hole is small compared with the radius R of the sphere. What is the electric field at the center of the hole? *Suggestion:* This problem can be solved by using the principle of superposition.

60. An infinitely long, cylindrical, insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ . A line of uniform linear charge density λ is placed along the axis of the shell. Determine the electric field for (a) $r < a$, (b) $a < r < b$, and (c) $r > b$.

Challenge Problems

61. A slab of insulating material has a nonuniform positive charge density $\rho = Cx^2$, where x is measured from the center of the slab as shown in Figure P24.61 and C is a constant. The slab is infinite in the y and z directions. Derive expressions for the electric field in (a) the exterior regions ($|x| > d/2$) and (b) the interior region of the slab ($-d/2 < x < d/2$).

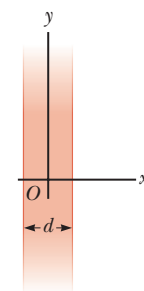


Figure P24.61

Problems 61 and 69.

62. **Review.** An early (incorrect) model of the hydrogen atom, suggested by J. J. Thomson, proposed that a positive cloud of charge $+e$ was uniformly distributed throughout the volume of a sphere of radius R , with the electron (an equal-magnitude negatively charged particle $-e$) at the center. (a) Using Gauss's law, show that the electron would be in equilibrium at the center and, if displaced from the center a distance $r < R$, would experience a restoring force of the form $F = -Kr$, where K is a constant. (b) Show that $K = k_e e^2 / R^3$. (c) Find an expression for the frequency f of simple harmonic oscillations that an electron of mass m_e would undergo if displaced a small distance ($< R$) from the center and released. (d) Calculate a numerical value for R that would result in a frequency of 2.47×10^{15} Hz, the frequency of the light radiated in the most intense line in the hydrogen spectrum.

63. A closed surface with dimensions $a = b = 0.400$ m and $c = 0.600$ m is located as shown in Figure P24.63. The left edge of the closed surface is located at position $x = a$. The electric field throughout the region is non-uniform and is given by $\vec{E} = (3.00 + 2.00x^2)\hat{i}$ N/C, where x is in meters. (a) Calculate the net electric flux

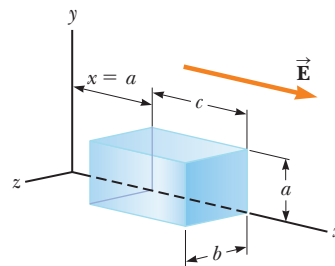


Figure P24.63

leaving the closed surface. (b) What net charge is enclosed by the surface?

- 64.** A sphere of radius $2a$ is made of a nonconducting material that has a uniform volume charge density ρ . Assume the material does not affect the electric field. A spherical cavity of radius a is now removed from the sphere as shown in Figure P24.64. Show that the electric field within the cavity is uniform and is given by $E_x = 0$ and $E_y = \rho a / 3\epsilon_0$.

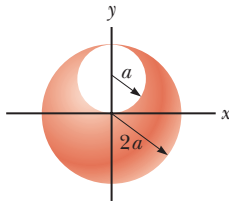


Figure P24.64

- 65.** A spherically symmetric charge distribution has a charge density given by $\rho = a/r$, where a is constant. Find the electric field within the charge distribution as a function of r . *Note:* The volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2 dr$.
- 66.** A solid insulating sphere of radius R has a nonuniform charge density that varies with r according to the expression $\rho = Ar^2$, where A is a constant and $r < R$ is measured from the center of the sphere. (a) Show that the magnitude of the electric field outside ($r > R$) the sphere is $E = AR^5 / 5\epsilon_0 r^2$. (b) Show that the magnitude of the electric field inside ($r < R$) the sphere is $E = Ar^3 / 5\epsilon_0$. *Note:* The volume element dV for a spherical shell of radius r and thickness dr is equal to $4\pi r^2 dr$.

- 67.** An infinitely long insulating cylinder of radius R has a volume charge density that varies with the radius as

$$\rho = \rho_0 \left(a - \frac{r}{b} \right)$$

where ρ_0 , a , and b are positive constants and r is the distance from the axis of the cylinder. Use Gauss's law to determine the magnitude of the electric field at radial distances (a) $r < R$ and (b) $r > R$.

- 68.** A particle with charge Q is located on the axis of a circle of radius R at a distance b from the plane of the circle (Fig. P24.68). Show that if one-fourth of the electric flux from the charge passes through the circle, then $R = \sqrt{3}b$.

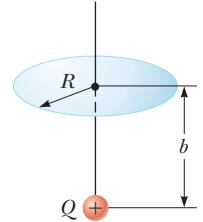


Figure P24.68

- 69. Review.** A slab of insulating material (infinite in the y and z directions) has a thickness d and a uniform positive charge density ρ . An edge view of the slab is shown in Figure P24.61. (a) Show that the magnitude of the electric field a distance x from its center and inside the slab is $E = \rho x / \epsilon_0$. (b) **What If?** Suppose an electron of charge $-e$ and mass m_e can move freely within the slab. It is released from rest at a distance x from the center. Show that the electron exhibits simple harmonic motion with a frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{\rho e}{m_e \epsilon_0}}$$